MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ A2

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POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2-2}$. 5+15+5
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln(4-x^2)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-2x-(2+1)}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x+2y-z+u &= 2 \\ 2x+5y-z+2u &= 3 \\ 3x-y-2z+u &= 2 \\ x-y+3z-5u &= 2 \end{aligned}$$

MOBITEL

6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 2

1. $z^4 - (4-i)^2 = 0$

$w = 4-i$

$z = \sqrt[4]{(4-i)^2}$

$= \sqrt{4-i}$

$|w| = \sqrt{16+1}$

$= \sqrt{17}$

$z_k = \sqrt{17}\left(\cos\dfrac{\frac{\pi}{12}+2k\pi}{2} + i\sin\dfrac{\frac{\pi}{12}+2k\pi}{2}\right)$

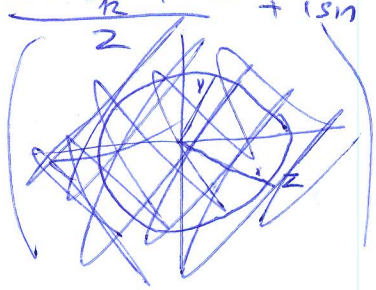
$z_0 = \sqrt{17}\left(\cos\dfrac{\pi}{24} + i\sin\dfrac{\pi}{24}\right)$

$z_1 = \sqrt{17}\left(\cos\dfrac{-\frac{\pi}{12}+2\pi}{2} + i\sin\dfrac{-\frac{\pi}{12}+2\pi}{2}\right)$

$\cos\phi = \dfrac{x}{|w|} = \dfrac{4}{\sqrt{17}} = 14^\circ 2' 11''$

$\sin\phi = \dfrac{y}{|w|} = \dfrac{-1}{\sqrt{17}} = 104^\circ 2' 11''$

$z = \sqrt{17}\left(\dfrac{4}{\sqrt{17}} - i\dfrac{1}{\sqrt{17}}\right)$



NASTAVNIK PRONAŠAO SAKRIVEN UPALJEN MOBITEL NA SJEDALICI

STUDENT IZBAČEN

2. $f(x) = x - \sqrt{x^2-2}$

DOMENA: $x^2-2 \geq 0$

$x^2-2=0 \ldots x^2=2 \ldots x=\pm\sqrt{2}$

$f'(x) = 1 - \dfrac{x}{\sqrt{x^2-2}}$

$$f''(x) = \frac{-\sqrt{x^2-2} + x \cdot \frac{x}{\sqrt{x^2-2}}}{x^2-2}$$

$$= \frac{\frac{-x^2-2+x^2}{\sqrt{x^2-2}}}{x^2-2} = \frac{-2}{(x^2-2)\sqrt{x^2-2}}$$

ASIMPTOTE:

VERTIKALNE

$$\lim_{x \to \sqrt{2}} = (x - \sqrt{x^2-2}) = (\sqrt{2} - \sqrt{2-2}) = \sqrt{2}$$ NEMA

$$\lim_{x \to -\sqrt{2}} (-x - \sqrt{x^2-2}) = (-\sqrt{2} - \sqrt{2-2}) = -\sqrt{2}$$ NEMA

HORIZONTALNE

$$\lim_{x \to \pm\infty} \left| x - \sqrt{x^2-2} \cdot \frac{x + \sqrt{x^2-2}}{x + \sqrt{x^2-2}} \right| = \lim_{x \to \pm\infty} \frac{x^2 - x^2 - 2}{x + \sqrt{x^2-2}}$$

Postoji horizontalna asimptota $y=0$ $= \frac{-2}{\infty} = 0$

KOSE:

$$\lim_{x \to \pm\infty} = \frac{x - \sqrt{x^2-2}}{x} = 1 - \lim_{x \to \pm\infty} \frac{\sqrt{x^2-2}}{x} = 1$$

$$\lim_{x \to \pm\infty} = (x - \sqrt{x^2-2} - x) = -\infty$$ NEMA KOSE ASIMPTOTE

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3. $f(x) = \ln(4-x^2)$ $D_f = \langle -\infty, -2 \rangle \cup \langle 2, \infty \rangle$

DOMENA: $4 - x^2 \geq 0$

$x^2 = 4 \quad \dots \quad x = \pm 2$

	$-\infty$		-2		2		$+\infty$
$(4-x^2)$		$+$	0	$-$	0	$+$	

$$f'(x) = \frac{-2x}{4-x^2}$$

$$f''(x) = \frac{-8 + 2x - \overbrace{-2x \cdot 2x}^{-2x^2}}{(4-x^2)^2} = \frac{-8 + 2x^2 - 4x^2}{(4-x^2)^2}$$

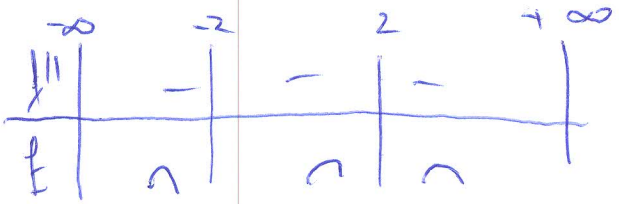
$\Rightarrow$ 2. LIST

IME I PREZIME: BROJ INDEKSA:

3. NASTAVAK...

$$= -2 \cdot \frac{x^2+4}{(4-x^2)^2} = 0$$

$x^2+4=0 \qquad x \neq \pm 2$



5. $x+2y-z+u=2$

$2x+5y-z+2u=3$

$3x-y-2z+u=2$

$x-y+3z-5u=2$

$$\begin{array}{cccc|c|l} 1 & 2 & -1 & 1 & 2 & \cdot(-2) \\ 2 & 5 & -1 & 2 & 3 & \\ 3 & -1 & -2 & 1 & 2 & \cdot(-1) \\ 1 & -1 & 3 & -5 & 3 & \cdot 5 \end{array} \sim \begin{array}{cccc|c|l} 1 & 2 & -1 & 1 & 2 & \cdot 1 \\ 0 & 1 & 1 & 0 & -1 & \cdot 1 \\ 2 & -3 & -1 & 0 & 0 & \\ 6 & 9 & -2 & 0 & 12 & \cdot 2 \end{array}$$

$$\begin{array}{cccc|c|l} 1 & 3 & 0 & 1 & 1 & \cdot 3 \\ 0 & 1 & 1 & 0 & -1 & \cdot 1 \\ 2 & -2 & 0 & 0 & -1 & \cdot 1 \\ 6 & 11 & 0 & 0 & 10 & \cdot 11 \end{array} \sim \begin{array}{cccc|c} 1 & 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & -1 \\ 1 & -1 & 0 & 0 & -\frac{1}{2} \\ 6 & 11 & 0 & 0 & 10 \end{array}$$

$$\begin{array}{cccc|c} 4 & 6 & 0 & 1 & -\frac{1}{2} \\ 1 & 0 & 1 & 0 & -\frac{3}{2} \\ 1 & -1 & 0 & 0 & -\frac{1}{2} \\ 17 & 0 & 0 & 0 & -\frac{9}{2} \end{array}$$

$4x+u=-\frac{1}{2}$

$x+z=-\frac{3}{4}$

$x-y=-\frac{1}{2}$

$17x=\frac{9}{2}$

$x=\frac{9}{24}$

$$U = -\frac{1}{2} - 4 \cdot \frac{9}{24} = -2$$

$$z = -\frac{3}{2} - \frac{9}{24} = -\frac{15}{8}$$

$$y = \frac{1}{2} + \frac{9}{24} = \frac{7}{8}$$

Rješenje

$$\begin{bmatrix} \frac{9}{24} \\ \frac{7}{8} \\ -\frac{15}{8} \\ -2 \end{bmatrix}$$

Provjera:

$$\frac{9}{24} + 2 \cdot \frac{7}{8} + \frac{15}{8} - 2 = 2 = 2 \text{ V}$$

$$2 \cdot \frac{9}{24} + 5\frac{7}{8} + \frac{15}{8} - 4 = 3 = 3 \text{ V}$$

$$3 \cdot \frac{9}{24} - \frac{7}{8} + 2\frac{15}{8} - 2 = 2 = 2 \text{ V}$$

$$\frac{2}{24} - \frac{7}{8} - 3 \cdot \frac{15}{8} - 5 \cdot (-2) = 2 \text{ V}$$

KOMPLEKSNA RAVNINA

1. $z^4 - (4-i)^2 = 0$

2. $f(x) = x - \sqrt{x^2 - 2}$ (DOMENA) "Derivacija"

3. Domena (PARNOST) ZAKLIVSEN" (GRAFA (1)) $g(x) = \ln|4 - x^2|$

$x \neq 0$

$x + 2y - z + v = 2 \mid \cdot(-2)$

$-2x - 4y + 2z - 2v$

$2x + 5y - z + 2v = 3$

$y + z = 3$

$-4y + 2z = -4$

$+5y + z = 3 \mid \cdot 2$

$+10y - 2z = -6$

$-2x - 4y + 2z - 2v$

$x + 2y - z + 2v = 2 \mid \cdot -2$

$2x + 5y - z + 2v = 3$

$3x - y - 2z + v = 3$

$x - y + 3z - 5v = 2$

$1 + 2 - 1 + y = 2$

$2 + 5 - 1 + 2 = 3$

$3 - y - 2 + v = 2$

$1 - y + 3 - 5 = 2$

$\lim_{x \to \infty} \frac{e^x}{x^4} = \lim_{x \to \infty} \frac{-}{+} = -$

1.

$2^4 - (4-i)^2 = 0 \Rightarrow 2 = \sqrt[4]{(4-i)^2} = \sqrt{4-i}$

$w = 4 - i$

$2 = \sqrt{17}\left(\frac{4}{\sqrt{17}} - i\frac{1}{\sqrt{17}}\right)$ $\cos\phi =$