MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: RJEŠENJE **BROJ INDEKSA:**

Kod kojeg nastavnika želite ustmeni?

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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3. Neka je kvadrat $K = \{(x, y, z) : x = 3,\ y \in [-2, 2],\ z \in [-2, 2]\}$. Kako preko potencijalnog polja izračunati $\iint_{\partial K} ydy$? 20

4. Izračunati volumen područja unutar cilindra $x^2 + y^2 = 3$, koji je između plašta stošca $z = \sqrt{x^2 + y^2}$ i ravnine $x + y + z = 8$. 20

5. Izračunati volumen tijela omeđenog ravninama $x = 0$, $x = 1$, $y = 0$, $z = 0$, $z = 1 - y$.? 20

Ukupno:

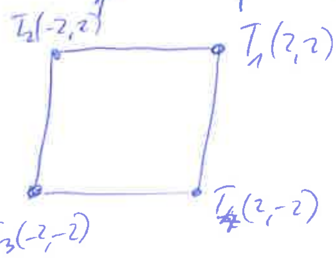
1) — SAMI —

2) VIDI VUKUŠIĆ

3) POGREŠKA U OZNACI: TREBA BITI $\int_{\partial K} y\,dy$. OBZIROM DA SE U ZADATKU SPOMINJE POTENCIJALNO POLJE, I ∂K TREBALO JE BITI JASNO DA DVOSTRUKI INTEGRAL NE ŠTIMA, VEĆ TREBA STAVITI JEDNOSTRUKI

POTENCIJALNO POLJE ~~...~~ $f = -\frac{y^2}{2}$, $-\nabla f = -\begin{pmatrix} 0 \\ -y \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ y \end{pmatrix} = w$

RUB KVADRATA



JE ZATVORENA KRIVULJA KOJOJ MOŽEMO PROGLASITI POČETAK I KRAJ U BILO KOJOJ TOČKI.

STOGA

$$\int_{\partial K} w \cdot ds = \int_{T_1}^{T_1} w \cdot ds = f(T_1) - f(T_1) = 0$$

4) VIDI MATIJAŠEVIĆ

5) VIDI VUKAŠINA

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IME I PREZIME: Ines Kurfirst Vukušić

BROJ INDEKSA: 17-2-0223-2012

Kod kojeg nastavnika želite ustmeni? Uglešić

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 40

1) $x'''(t) - \cancel{(x)}\, x(t) = t$, $x(0) = x'(0) = 1$, $x''(0) = 2$

$x'''(t) - x(t) = t \,/ L$

$s^3 L(x)(s) - s^2 x(0) - s x'(0) - x''(0) - L(x)(s) = \frac{1}{s^2}$

$s^3 Y(s) - s^2 - \ ) - Y(s) - \frac{1}{s^2}$

$Y(s)(s^3 - 1) = \frac{1}{s^2} + s^2 + s + 2$

$Y(s)(s^3 - 1) = \frac{1 + s^4 + s^3 + 2s^2}{s^2} \Big| \frac{1}{s^3 - 1}$

$Y(s) = \frac{s^4 + s^3 + 2s^2 + 1}{s^2(s^3 - 1)} = \frac{s^4 + s^3 + 2s^2 + 1}{s^2(s-1)(s^2+s+1)} =$

$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} + \frac{Ds + E}{s^2 + s + 1} \Big| \cdot s^2(s-1)(s^2+s+1)$

$s^4 + s^3 + 2s^2 + 1 = As(s-1)(s^2+s+1) + B(s-1)(s^2+s+1) +$
$+ Cs^2(s^2+s+1) + (Ds+E)s^2(s-1)$

$s^4 + s^3 + 2c^2 + 1 = (As^2 - As)(s^2+s+1) + B(s^3 + \cancel{s^2} + \cancel{s} - \cancel{s^2} - \cancel{s} - 1) +$
$+ Cs^4 + Cs^3 + Cs^2 + (Ds+E)(s^3 - s^2)$

$s^4 + s^3 + 2s^2 + 1 = As^4 + \cancel{As^3} + \cancel{As^2} - \cancel{As^3} - \cancel{As^2} - As + Bs^3 - B +$
$+ Cs^4 + Cs^3 + Cs^2 + Ds^4 - Ds^3 + Es^3 - Es^2$

$s^4 + s^3 + 2s^2 + 1 = s^4(A + C + D) + s^3(B + C - D + E) +$
$+ s^2(C - E) + s(-A) + (-B)$

$-B=1 \Rightarrow \boxed{B=-1}$

$-A=0 \Rightarrow \boxed{A=0}$

$C-E=2 \Rightarrow C=2+E$

$-1+C-D+E=1$

$C+D=1$

$D=1-C=$

$=1-2-E$

$\cancel{-1}+\cancel{2}+E-1+2+E+E=\cancel{1}$

$3E=-1 \quad \boxed{E=-\frac{1}{3}}$

$D=-\frac{2}{3} \qquad C=\frac{5}{3}$

$Y(s)=\frac{-1}{s^2}+\frac{5}{3}\frac{1}{s-1}-\frac{1}{3}\frac{2s+1}{s^2+s+1}$

$\Rightarrow X(t)=-t+\frac{5}{3}e^x-\frac{1}{3}$

PROVJERA

$x(0)=\frac{4}{3}$ ×

2. $r=3$

$\iint_S 3\,dx\,dy$

gornja polusfera oko $T(0,0)$ sa $r=3$

$w=\begin{pmatrix}0\\0\\3\end{pmatrix}$ ✓

parametrizacija (sa D na S)

$r(u,v)=\left(u,v,\sqrt{3-u^2-v^2}\right)$ ✓

$\frac{dr}{du}=\begin{pmatrix}1\\0\\\frac{-u}{\sqrt{3-u^2-v^2}}\end{pmatrix}, \quad \frac{dr}{dv}=\begin{pmatrix}0\\1\\\frac{-v}{\sqrt{3-u^2-v^2}}\end{pmatrix}$

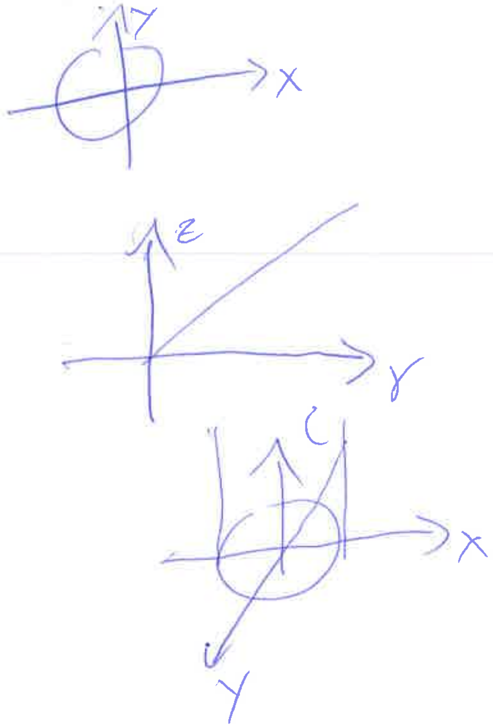
$\vec{n}=\begin{pmatrix}\frac{u}{\sqrt{3-u^2-v^2}}\\\frac{v}{\sqrt{3-u^2-v^2}}\\1\end{pmatrix}$

$* = \iint_D w\vec{n}\,du\,dv = \iint_D \begin{pmatrix}0\\0\\3\end{pmatrix}\vec{n}\,du\,dv = 3\cdot P(D) =$

$=3\cdot$[illegible]$\cdot\pi=27\pi$ ✓

4. $x^2+y^2=3$ Ineskuti... [illegible]

$d=\sqrt{x^2+y^2}$



$z=\sqrt{3}$ $r=\sqrt{3}$

$z=8-x-y$

$r\in[0,\sqrt{3}]$

$\varphi\in[0,2\pi]$

$z\in[r, 8-r\cos\varphi-r\sin\varphi]$

cilindrične koordinate $\left.\begin{array}{l} x=r\cos\varphi \\ y=r\sin\varphi \\ z=z \end{array}\right\}$

2π $\sqrt{3}$ $8-r\cos\varphi-r\sin\varphi$

$$V=\int_0^{2\pi}\int_0^{\sqrt{3}}\int_r^{1}\boxed{r}\,dz\,dr\,d\varphi=\int_0^{2\pi}\int_0^{\sqrt{3}} z\Big|_r^{8r\cos\varphi-r\sin\varphi} r\,dr\,d\varphi=$$

FAKTOR

$$=\int_0^{2\pi}\int_0^{\sqrt{3}}(8-r\cos\varphi-r\sin\varphi\, r)\,dr\,d\varphi=$$

$$=\int_0^{2\pi}\left(8r-\frac{r^2}{2}\cos\varphi-\frac{r^2}{2}\sin\varphi\,\frac{r^2}{2}\right)\Big|_0^{\sqrt{3}}d\varphi=$$

$$=\int_0^{2\pi}\left(8\sqrt{3}-\frac{3}{2}\cos\varphi-\frac{3}{2}\sin\varphi-\frac{3}{2}\right)d\varphi=\left(8\sqrt{3}\varphi-\frac{3}{2}\sin\varphi+\frac{3}{2}\cos\varphi-\frac{3}{2}\varphi\right)\Big|^{2\pi}$$

$$=16\sqrt{3}\pi+\cancel{\frac{3}{2}}-3\pi-\frac{3}{2}\left(16\sqrt{3}-3\right)\pi$$

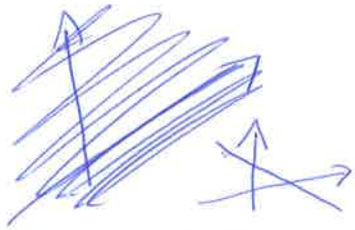
⑤ $x=0, \quad x=1$ Ineskurfrat Valuušić

$y=0$

$z=0$

$z=1-y$ $\quad 1-y=0$

$y=1$



$y \mid 0 0 0 0 0 0 0 0 0 0$

$z \mid 0 0 0 0 0 0$

$$V=\int_0^1\int_0^1\int_0^{1-y} 1 \, dz\, dy\, dx = \int_0^1\int_0^1 z\Big|_0^{1-y} dy\, dx =$$

$$=\int_0^1\int_0^1 (1-y)\, dy\, dx = \int_0^1 \left(y-\frac{y^2}{2}\right)\Big|_0^1 dx =$$

$$=\int_0^1 \frac{1}{2}\, dx = \frac{1}{2}$$

③ $$\int_{(3,-2,-2)}^{(3,2,2)} y\, dy =$$

$= f(3,2,2) - f(3,-2,-2) =$

$=0$

$$-\nabla f = \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}$$

$$-\frac{\partial f}{\partial y} = y \,/\int$$

$$f(x,y,z) = \frac{y^2}{2} + \varphi(x,z) \,/\partial x$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x} = 0 \,/\int$$

$\varphi(x,z)=\varphi(z)$

$$f(x,y,z) = \frac{y^2}{2} + \varphi(z) \,/\partial z$$

$$\frac{\partial f}{\partial z} = \varphi'(z)=0$$

$\varphi(z) = C$

$$f(x,y,z) = \frac{y^2}{2}$$

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IME I PREZIME: Roko Matijašević BROJ INDEKSA: 17-1-0073-2011	Broj ↓ bodova
Kod kojeg nastavnika želite ustmeni? Nikica Uglešić	

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Ukupno: 40

4. $x^2+y^2=3$ - cilindar (valjak)

$z=\sqrt{x^2+y^2}$ - stožac $\Rightarrow x^2+y^2=z^2$

$x+y+z=8$

$x=r\cos\varphi$

$y=r\sin\varphi$

$(r\cos\varphi)^2+(r\sin\varphi)^2=3$

$r^2\cos^2\varphi+r^2\sin^2\varphi=3$

$r^2\cdot\underbrace{(\cos^2\varphi+\sin^2\varphi)}_{1}=3$

$r^2=3 \, / \sqrt{\ }$

$r=\sqrt{3}$

$(r\cos\varphi)^2+(r\sin\varphi)^2=z^2$

$r^2\cos^2\varphi+r^2\sin^2\varphi=z^2$

$r^2\underbrace{(\cos^2\varphi+\sin^2\varphi)}_{1}=z^2$

$r^2=z^2 \, / \sqrt{\ }$

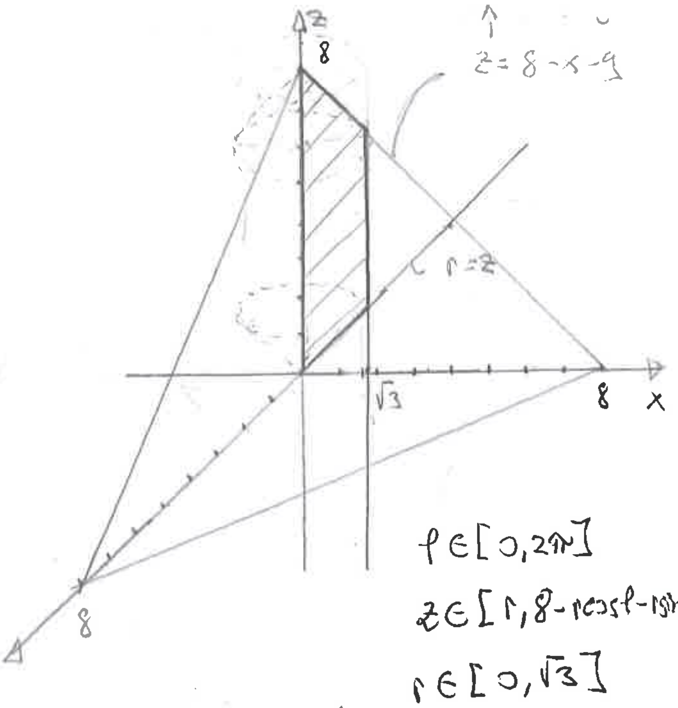
$r=\sqrt{z^2}$

$r=z$

$\frac{1}{2}\cdot\frac{3}{1}=\frac{3}{2}$

$z=8-r\cos\varphi-r\sin\varphi$

$z=8-x-y$



$x:0 \quad x=-y-z+8$

$y:0 \quad x=-0-0+8=8$

$z:0 \quad y=-x-z+8$

$y=-0-0+8=8$

$z=-x-y+8$

$z=-0-0+8=8$

$\varphi\in[0,2\pi]$

$z\in[r, 8-r\cos\varphi-r\sin\varphi]$

$r\in[0,\sqrt{3}]$

$$V=\int_0^{2\pi}\int_0^{\sqrt{3}}\int_r^{8-r\cos\varphi-r\sin\varphi} r\,dz\,dr\,d\varphi=\int_0^{2\pi}\int_0^{\sqrt{3}} r\,[z]_r^{8-r\cos\varphi-r\sin\varphi}\,dr\,d\varphi$$

$$V=\int_0^{2\pi}\int_0^{\sqrt{3}} r\,[8-r\cos\varphi-r\sin\varphi-(r)]\,dr\,d\varphi=V=\int_0^{2\pi}\int_0^{\sqrt{3}}[8r-r^2\cos\varphi-r^2\sin\varphi-r^2]\,dr\,d\varphi$$

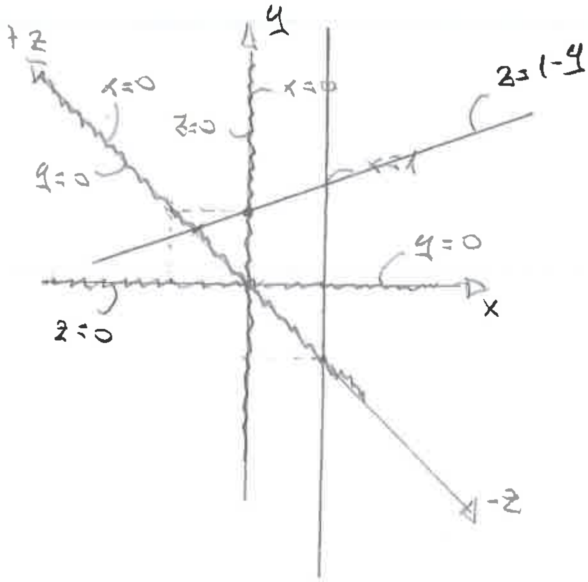
$$V=\int_0^{2\pi}\left[8\frac{r^2}{2}-\frac{r^3}{3}\cos\varphi-\frac{r^3}{3}\sin\varphi-\frac{r^3}{3}\right]_0^{\sqrt{3}}d\varphi=\int_0^{2\pi}\left[\left(8\frac{(\sqrt{3})^2}{2}-\frac{(\sqrt{3})^3}{3}\cos\varphi-\frac{(\sqrt{3})^3}{3}\sin\varphi-\frac{(\sqrt{3})^3}{3}\right)-(0)\right]d\varphi$$

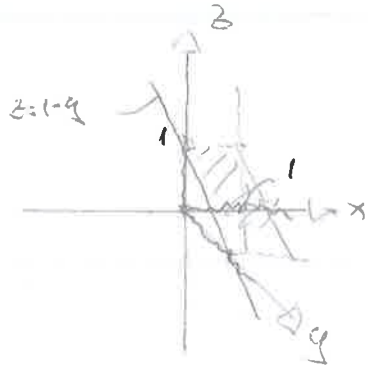
$$=\int_0^{2\pi}\left[(12-1{,}73\cos\varphi-1{,}73\sin\varphi-1{,}73)\right]d\varphi$$

$$\int_0^{2\pi} [12 - 1{,}73\cos\varphi - 1{,}73\sin\varphi - 1{,}73]\, d\varphi = [12\varphi + 1{,}73\sin\varphi - 1{,}73\cos\varphi - 1{,}73\varphi]_0^{2\pi}$$

$$= 62{,}798 + 1{,}73 = 64{,}528$$

5. $x=0$ ✓
$x=1$ ✓
$y=0$ ✓
$z=0$ ✓
$z=1-y \Rightarrow y=-z+1$ ✓





$z=1-y$	1	0
y	0	1

$z=1-0$
$z=1$
$0=1-y$
$y=1$

$$V = \int_0^1\int_0^1\int_0^{-z+1} 1 \, dy\, dz\, dx$$

$$= \int_0^1\int_0^1 [y]_0^{-z+1} dz\,dx = \int_0^1\int_0^1 [-z+1-0]\,dz\,dx = \int_0^1\int_0^1 [-z+1]\,dz\,dx = \int_0^1 \left[-\frac{z^2}{2}+z\right]_0^1 dx$$

$$= \int_0^1 \left[-\frac{1^2}{2}+1\right] - \left[-\frac{0^2}{2}+0\right] dx = \int_0^1 \frac{1}{2}\,dx = \left[\frac{1}{2}x\right]_0^1 = \left[\frac{1}{2}\cdot 1\right] - \left[\frac{1}{2}\cdot 0\right] = \frac{1}{2}$$ ✓

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Ukupno: 20

(1.)

$x'''(t) - x(t) = t \quad x(0) = x'(0) = 1$

$x''(0) = 2$

$s^3F(s) - s^2f(0) - sf'(0) - f''(0) - F(s) = \frac{1}{s^2}$

$s^3F(s) - s^2 - s - 2 - F(s) = \frac{1}{s^2}$

e^t

$\frac{1}{(s-1)}$

$F(s)(s^3 - 1) = \frac{1}{s^2} + s^2 + s + 2 \quad / : s^3 - 1$

$F(s) = \dfrac{\frac{1}{s^2} + s^2 + s + 2}{s^3 - 1} = \dfrac{\frac{1}{s^2} + s^2 + s + 2}{(s-1)(s^2+s+1)} = \dfrac{\frac{1}{s^2} + 1}{(s-1)(s^2+s+1)} + \dfrac{\cancel{s^2+s+1}}{(s-1)\cancel{(s^2+s+1)}}$

$s^3 - s^2$

$\dfrac{1 + s^2}{s^2(s-1)(s^2+s+1)} = \dfrac{\cancel{A}}{\cancel{s}} + \dfrac{B}{s^2} + \dfrac{C}{s-1} + \dfrac{D + Es}{s^2+s+1} \quad / \cdot s^2(s-1)(s^2+s+1)$

$s(s^3-1) \qquad s^4 + s^3 + 1$

$1 + s^2 = A(s(s-1)(s^2+s+1)) + B(s-1)(s^2+s+1) + C(s^2(s^2+s+1)) + Ds^3 - Ds^2 + Es^4 - Es^3$

$1 + s^2 = As^4 - As + Bs^3 - B + Cs^4 + Cs^2 + C + Ds^3 - Ds^2 + Es^4 - Es^2$

$A + C + E = 0 \qquad A = 0$

$B + D = 0 \qquad B = -1$

$C + D - E = 1 \qquad C = 0$

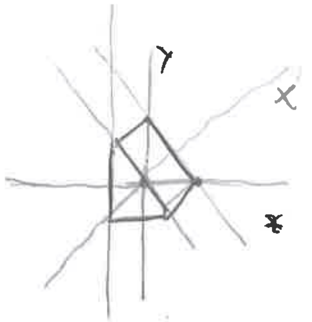
$A = 0 \qquad D = 1$

$-B + C = 1 \qquad E = 0$

IME I PREZIME: Ivan Vukašina **BROJ INDEKSA:** 1720182-12

(5.)

$x=0$
$x=1$
$y=0$
$z=0$
$z=1-y$
$y=1$



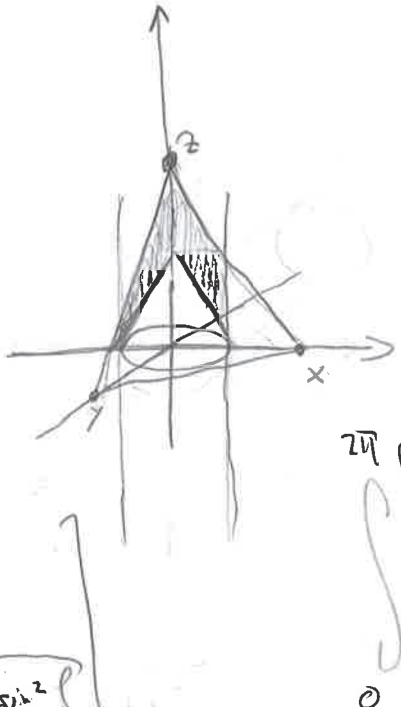
$$\int_0^1\int_0^1\int_0^{1-y} 1 \, dz\,dy\,dx = \int_0^1\int_0^1 z\Big|_0^{1-y} dy\,dx = \int_0^1\int_0^1 1-y \; dy\,dx$$

$$\int_0^1 y-\frac{y^2}{2}\Big|_0^1 dx = \int_0^1 1-\frac{1}{2}-\left(0-\frac{0}{2}\right) dx = \int_0^1 \frac{1}{2}\,dx = \frac{1}{2}x\Big|_0^1$$

$$=\frac{1}{2}$$

(4)

$x^2+y^2=3$
$z=\sqrt{x^2+y^2}$
$x+y+z=8$



$\iiint 1\,dv = ?$

$x=8-y-z$
$y=8-x-y$
$z=8-x-y$

$$\left[\begin{array}{l} x=r\cos\varphi \\ y=r\sin\varphi \\ z=\sqrt{r^2\cos^2\varphi+r^2\sin^2\varphi} \end{array}\right]$$

$z=r$

$r^2=3 \quad r=\sqrt{3}$

$$\int_0^{2\pi}\int_0^{\sqrt{3}}\int_{\sqrt{x^2+y^2}}^{8-y-x} r\,dz\,dr\,d\varphi$$

→

$$\int_0^{2\pi}\int_0^{\sqrt{3}}\int_r^{8-r\cos\varphi-r\sin\varphi} 1 \, dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^{\sqrt{3}} z\Big|_r^{8-r\cos\varphi-r\sin\varphi} dr\,d\varphi$$

r

$$= \int_0^{2\pi}\int_0^{\sqrt{3}} 8-r\cos\varphi-r\sin\varphi-r \, dr\,d\varphi$$

$$= \int_0^{2\pi} 8r-\frac{r^2}{2}\cos\varphi-\frac{r^2}{2}\sin\varphi-\frac{r^2}{2}\Big|_0^{\sqrt{3}} d\varphi$$

$$= \int_0^{2\pi} 8\sqrt{3}-\frac{3}{2}\cos\varphi-\frac{3}{2}\sin\varphi-\frac{3}{2}\, d\varphi = 8\sqrt{3}\varphi-\frac{3}{2}\sin\varphi+\frac{3}{2}\cos\varphi-\frac{3}{2}\varphi\Big|_0^{2\pi}$$

$$= \left(8\sqrt{3}\cdot 2\pi - \cancel{\frac{3}{2}0} + \frac{3}{2} - \frac{3}{\cancel{2}}\cdot\cancel{2}\pi\right) - \left(\cancel{0} - \cancel{0} + \frac{3}{2} - \cancel{0}\right)$$

$$= \left(16\sqrt{3}\pi + \frac{3}{2} - 3\pi\right) - \frac{3}{2}\Big) = 77.63$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano. IME I PREZIME: TOMISLAV GLAVAN BROJ INDEKSA: 17-0115-2011 Kod kojeg nastavnika želite ustmeni? PROFESOR N. UGLEŠIĆ	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) - x(t) = t, \quad x(0) = x'(0) = 1,\ x''(0) = 2.$$

2. Neka je S gornja polusfera radijusa $r = 3$ sa centrom u ishodištu ($z \geq 0$) orijentirana prema van. Izračunati $\iint\limits_S 3\mathrm{d}x\mathrm{d}y$? Eventualno može pomoći rot $(3x\mathbf{j}) = 3\mathbf{k}$. 20

3. Neka je kvadrat $K = \{(x, y, z) : x = 3,\ y \in [-2, 2],\ z \in [-2, 2]\}$. Kako preko potencijalnog polja izračunati $\iint\limits_{\partial K} y\mathrm{d}y$? 20

~~4.~~ Izračunati volumen područja unutar cilindra $x^2 + y^2 = 3$, koji je između plašta stošca $z = \sqrt{x^2 + y^2}$ i ravnine $x + y + z = 8$. ~~20~~

~~5.~~ Izračunati volumen tijela omeđenog ravninama $x = 0$, $x = 1$, $y = 0$, $z = 0$, $z = 1 - y$.? 20

Ukupno: 20

Volumen tijela ?

(5) $X=0,\ x=1,\ y=0,\ z=1-y$

$\hookrightarrow z=0,\ y=0$

TOMISLAV GLAVAN

BROJ INDEKSA 17-0115-2011

$$\int_0^1\int_0^1\int_0^{1-y} dz\,dy\,dx = \int_0^1\int_0^1 z\Big|_0^{1-y} dy\,dx =$$

$$\int_0^1\int_0^1 (1-y)\,dy\,dx = \int_0^1 y-\frac{y^2}{2}\Big|_0^1 dx$$

$$= \int_0^1 \left(1-\frac{1}{2}\right)dx = \int_0^1 dx = \frac{1}{2}x\Big|_0^1 = \frac{1}{2}$$

Volumen područja unutar cilindra $x^2+y^2=3$ ...

(4) $x^2+y^2=3$ IZMEĐU $z=\sqrt{x^2+y^2}$ $x+y+z=8$

$x=r\cos\varphi$

$y=r\sin\varphi$

$z=z$

$z=\sqrt{r}$

$z=8-x-y$

$z=8-r(\cos\varphi+\sin\varphi)$

$$dV=r-\sqrt{r}$$

$$\int_0^{2\pi}\int_0^{\sqrt{3}}\int_{\sqrt{r}}^{8-r(\cos\varphi+\sin\varphi)} \sqrt{3}\,dz\,dr\,d\varphi = \sqrt{3}\int_0^{2\pi}\int_0^{\sqrt{3}} 8-r(\cos\varphi+\sin\varphi)\cdot r^{\frac{1}{2}}\,dr\,d\varphi$$

$$= \sqrt{3}\int_0^{2\pi}\left(8r-\frac{r^2}{2}(\cos\varphi+\sin\varphi)-\frac{r^{\frac{3}{2}}}{\frac{3}{2}}\right)\Big|_0^{\sqrt{3}} d\varphi$$

$$= \sqrt{3}\int_0^{2\pi} 8\sqrt{3}-\frac{3}{2}(\cos\varphi+\sin\varphi)-\frac{3}{2}(\sqrt{3})\,d\varphi =$$

$$= \sqrt{3}\left(8\sqrt{3}\varphi-\underbrace{\frac{3}{2}(-\sin\varphi+\cos\varphi)}_{0}-\frac{3}{2}\sqrt{3}^{\frac{3}{2}}\varphi\right)\Big|_0^{2\pi}$$

$$= \sqrt{3} \cdot 8\sqrt{3} \cdot 2\pi - \frac{2}{3} \cdot \sqrt{3} \cdot (\sqrt{3})^{\frac{3}{2}} \cdot 2\pi$$

$$= 2\pi\left(8 \cdot 3 + \frac{2}{3} \cdot 3\right) = 52\pi$$

2) $r=3$

$$\iint_S 3\,dx\,dy$$

$$z(x,y) = x^2 + y^2$$

$$ds = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} = \sqrt{1 + (2x) + (2y)^2}$$

$$= \sqrt{}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: VEDRAN ČIZMIN **BROJ INDEKSA:**

Kod kojeg nastavnika želite ustmeni?

POPUNJAVA NASTAVNIK Broj ↓ bodova

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4. Izračunati volumen područja unutar cilindra $x^2 + y^2 = 3$, koji je između plašta stošca $z = \sqrt{x^2 + y^2}$ i ravnine $x + y + z = 8$. 20

5. Izračunati volumen tijela omeđenog ravninama $x = 0$, $x = 1$, $y = 0$, $z = 0$, $z = 1 - y$.? 20

Ukupno:

20

VEDRAN ČIZMIN

(5.) $x=0 \quad x=1 \quad y=0 \quad z=0 \quad z=1-y$

$$\Downarrow$$

$$0=1-y$$

$$y=1$$

$$\int_0^1\int_0^1\int_0^{1-y} 1 \, dz\,dy\,dx = \int_0^1\int_0^1 z\Big|_0^{1-y} dy\,dx = \int_0^1\int_0^1 1-y \, dy\,dx = \int_0^1 y-\frac{y^2}{2}\Big|_0^1 dx$$

$$= \int_0^1 1-\frac{1}{2}-0 \, dx = \int_0^1 \frac{1}{2} dx = \frac{1}{2}x\Big|_0^1 = \frac{1}{2}$$ //

(4.) $x^2+y^2=3 \qquad z=\sqrt{x^2+y^2} \qquad x+y+z=8$

$$r^2=3$$

$$\boxed{r=\pm\sqrt{3}}$$

$$z=\sqrt{r^2}$$

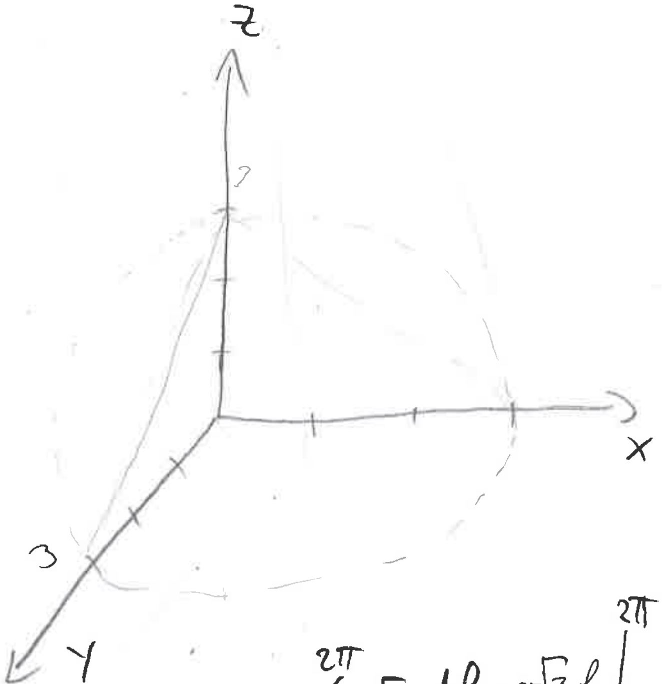
$$z=r$$

$\varphi \in [0, 2\pi]$

$r \in [-\sqrt{3}, \sqrt{3}]$

$z \in [0, r]$

$$r\,dr\,dz\,d\varphi = dx\,dy\,dz$$



$$\int_0^{2\pi}\int_{-\sqrt{3}}^{\sqrt{3}}\int_0^{r} 1 \; r\,dz\,dr\,d\varphi = \int_0^{2\pi}\int_{-\sqrt{3}}^{\sqrt{3}} r z\Big|_0^{r} dr\,d\varphi$$

$$= \int_0^{2\pi}\int_{-\sqrt{3}}^{\sqrt{3}} r^2\,dr\,d\varphi = \int_0^{2\pi} \frac{r^3}{3}\Big|_{-\sqrt{3}}^{\sqrt{3}} d\varphi$$

$$= \int_0^{2\pi} \frac{(\sqrt{3})^3}{3} - \frac{(-\sqrt{3})^3}{3} \, d\varphi = \int_0^{2\pi} \frac{\cancel{3}\sqrt{3}}{\cancel{3}} + \frac{\cancel{3}\sqrt{3}}{\cancel{3}} d\varphi$$

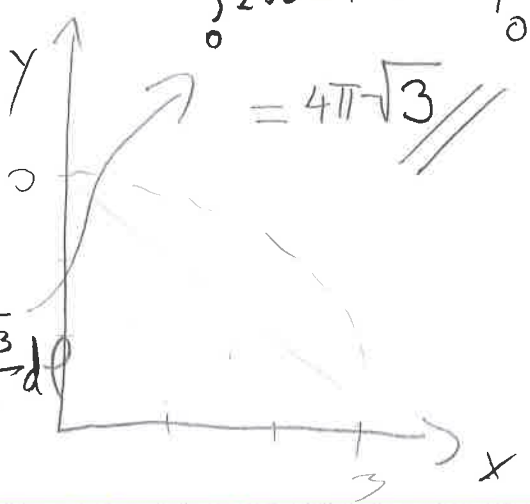
$$\int_0^{2\pi} 2\sqrt{3}\, d\varphi = 2\sqrt{3}\varphi\Big|_0^{2\pi}$$

$$= 4\pi\sqrt{3}$$ //

$x+y=8-\sqrt{3}$

$y=8-\sqrt{3}-x$

x	0	1	1
y	6,26	6,88	6



MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano. IME I PREZIME: AUGUSTIN PTIČAR BROJ INDEKSA: 17-1-0055-2011 Kod kojeg nastavnika želite ustmeni?	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

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5. Izračunati volumen tijela omeđenog ravninama $x = 0$, $x = 1$, $y = 0$, $z = 0$, $z = 1 - y$.? ~~20~~

Ukupno: 0

1.

$$X'''(t) - x(t) = t \ ; \quad X(0) = x'(0) = 1 \ , \ x''(0) = 2$$

$$s^3 X(s) - s^2 x(0) - s x'(0) - x''(0) - s X(s) - x(0) = \frac{1}{s^2}$$

$$s^3 X(s) - s X(s) = \frac{1}{s^2} + s^2 \underset{=1}{x(0)} + s \underset{=1}{x'(0)} + \underset{=2}{x''(0)} + \underset{=1}{x(0)}$$

$$s^3 X(s) - s X(s) = \frac{1}{s^2} + s^2 + s + 2 + 1$$

$$s^3 X(s) - s X(s) = \frac{1}{s^2} + s^2 + s + 3$$

$$X(s)\left(s^3 - s\right) = \frac{1}{s^2} + s^2 + s + 3$$

$$X(s)\left(s^3 - s\right) = \frac{1 + s^4 + s^3 + 3s^2}{s^2} \Big/ \cdot \left(\frac{1}{s^3 - s}\right)$$

→

$$X(s) = \frac{1+s^4+s^3+3s^2}{s^2} \cdot \frac{1}{(s^3-s)}$$

$$X(s) = \frac{1+s^4+s^3+3s^2}{s^2(s^3-s)} = \frac{1+s^4+s^3+3s^2}{s^5-s^3} = \frac{1+s^4+s^3+3s^2}{s^3(s^2-1)}$$

$\Downarrow$

$s^3 = 0$ $\quad$ $s^2 = 1$

$s_1 = 0$ $\quad$ $s = \pm\sqrt{1}$

$s_2 = 0$ $\quad$ $s_4 = +1$

$s_3 = 0$ $\quad$ $s_5 = -1$

$\Downarrow$

$(s-1)(s+1)$

$$\frac{1+s^4+s^3+3s^2}{s^3(s-1)(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{(s-1)} + \frac{E}{(s+1)} \Bigg/ \cdot \left(s^3(s-1)(s+1)\right)$$

$$1+s^4+s^3+3s^2 = A(s^2(s-1)(s+1)) + B(s(s-1)(s+1)) + C(s-1)(s+1) + D(s^3(s+1)) + E(s^3(s-1))$$

$$1+s^4+s^3+3s^2 = A(s^2(s^2-1)) + B(s(s^2-1)) + C(s^2-1) + D(s^3(s+1)) + E(s^3(s-1))$$

$$= A(s^4-s^2) + B(s^3-s) + C(s^2-1) + D(s^4+s^3) + E(s^4-s^3)$$

$$1+s^4+s^3+3s^2 = As^4 - As^2 + Bs^3 - Bs + Cs^2 - C + Ds^4 + Ds^3 + Es^4 - Es^3$$

$s=0 \Rightarrow 1 = -C$

$C = 1$

uz $s^4 \Rightarrow 1 = A+B+D+E$

$s^3 \Rightarrow 1 = D - E$

$s^2 \Rightarrow 3 = -A + C$

$s \Rightarrow 0 = -B$

$B = 0$

$B = 0$

$\boxed{C = 1}$

$A = -2$

$B = 0$

$C = 1$

$D = 2$

$E = 1$

$3 = -A + C$

$A = C - 3$

$A = 1 - 3$

$\boxed{A = -2}$

$3 - D = E$

$\boxed{E = 1}$

$1 = -2 + 0 + D + E$

$1 = -2 + D + E$

$\boxed{3 - D = E}$

$1 = D - (3 - D)$

$1 = D - 3 + D$

$1 = 2D - 3$

$2D = 4$

$D = 2$

AUGUSTIN PTIČAR

$$\frac{1+s^4+s^3+3s}{s^3(s-1)(s+1)} = -\frac{2}{s} + 0 + \frac{1}{s^2} + \frac{2}{(s-1)} + \frac{1}{s+1}$$

PROVJERA:

$x(0) = -2+1+1=0$ ✗

$$\frac{1+s^4+s^3+3s}{s^3(s-1)(s+1)} = -\frac{2}{s} + \frac{1}{s^2} + \frac{2}{s-1} + \frac{1}{s+1}$$

$$X(s) = -\frac{2}{s} + \frac{1}{s^2} + \frac{2^*}{s-1} + \frac{1}{s+1}$$

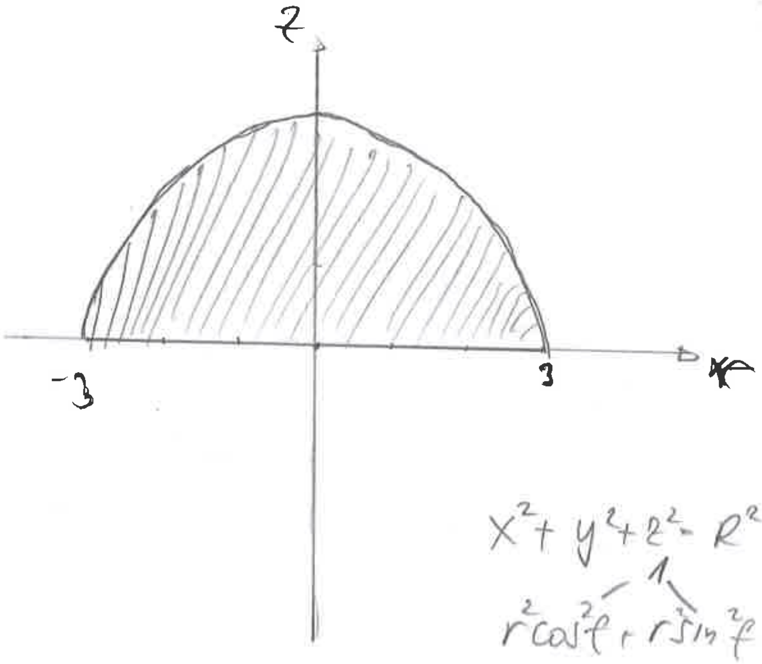
$$X(t) = -2 + t + e^{-t} + 2e^{t}$$

$*$ $\frac{2}{s-1} = 2\left(\frac{1}{(s-1)}\right)$

2. $r=3$

$z \geq 0$

$\iint_S 3\,dx\,dy$



$x^2+y^2+z^2=R^2$

$r^2\cos^2\varphi + r^2\sin^2\varphi + z^2 = R^2$

$r^2+z^2=9$

$z^2 = 9-r^2$

$z = \sqrt{9-r^2}$

$r = [0, 3]$

$\varphi = [0, \pi]$

$z = [0, \sqrt{9-r^2}]$

$$\int_0^{\pi}\int_0^{3}\int_0^{\sqrt{9-r^2}} 3\,\cancel{dx}\,dy\,\cdot = \int_0^{\pi}\int_0^{3}\int_0^{\sqrt{9-r^2}} 3\,r\,dz\,dr\,d\rho =$$

$$= \int_0^{\pi} d\rho \int_0^{3} 3r\,dr \int_0^{\sqrt{9-r^2}} dz = \int_0^{\pi} d\rho \int_0^{3} 3r\,dr \;\; z\Big|_0^{\sqrt{9-r^2}} =$$

$$= \int_0^{\pi} d\rho \int_0^{3} 3r\,dr \cdot \sqrt{9-r^2} =$$

$$= \int_0^{\pi} d\rho \int_0^{3} 3r\sqrt{9-r^2}\,dr \qquad * \left[\begin{array}{c} 9-r^2 = t \,/\, d \\ -2r = dt \\ r = -\frac{1}{2}dt \end{array}\right]$$

$$= \int_0^{\pi} d\rho \int_0^{3} 3\sqrt{t}\cdot\left(-\frac{1}{2}\right)dt = \int_0^{\pi} d\rho - \frac{3}{2}\int_0^{3} \sqrt{t}\,dt$$

$$= \int_0^{\pi} d\rho - \frac{3}{2}\int t^{\frac{1}{2}}\,dt = \int_0^{\pi} d\rho - \frac{3}{2}\,\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}$$

$$= \int_0^{\pi} d\rho - \frac{3}{2}\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}} = \int_0^{\pi} d\rho - \frac{\cancel{3}}{\cancel{2}}\cdot\frac{\cancel{2}\,t^{\frac{3}{2}}}{\cancel{3}}\Bigg|_0^{3}$$

$$= \int_0^{\pi} d\rho - t^{\frac{3}{2}}\Big|_0^{3} = \int_0^{\pi} d\rho - (9-r^2)^{\frac{3}{2}}\Big|_0^{3}$$

$$= \int_0^{\pi} d\rho\,(-0)^{\frac{3}{2}=0} = \int_0^{\pi} d\rho = \rho\Big|_0^{\pi} = \pi - 0 = \pi$$

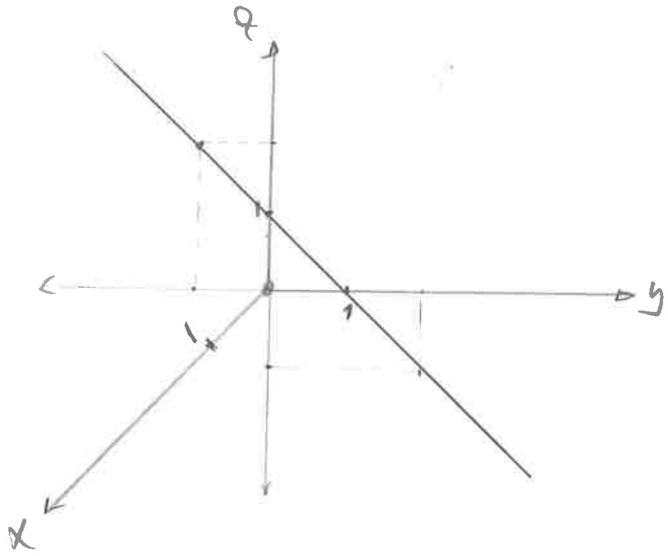
5. $x=0$

$x=1$

$y=0$

$z=0$

$\underbrace{z=1-y}$



y	0	1	-1	-2	2
$z=1-y)$	1	0	2	3	-1

$$\underbrace{\int_0^1 \int_0^1 \int_0^{1-y} 1 \, dx\,dy\,dz} = \underbrace{\int_0^1 \int_0^1 \int_0^{1-y} 1 \, r dr d}$$

POGREŠAN
POREDAK
INTEGRACIJE

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IME I PREZIME: ANTE [illegible] BROJ INDEKSA: 17-2-0247-2012

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4. Izračunati volumen područja unutar cilindra $x^2 + y^2 = 3$, koji je između plašta stošca $z = \sqrt{x^2+y^2}$ i ravnine $x + y + z = 8$. 20

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Ukupno: $\emptyset$

1. $x'''(t) - x(t) = t$

$x(0) = x'(0) = 1$

$x''(0) = 2$

$s^3 \mathcal{L}[x] - s^2 x(0) - s x'(0) - x''(0) - \mathcal{L}[x] = \mathcal{L}[t]$

$s^3 \mathcal{L}[x] - s^2 - s - 2 - \mathcal{L}[x] = \frac{1}{s^2}$

$s^3 \mathcal{L}[x] - \mathcal{L}[x] = \frac{1}{s^2} + s^2 + s + 2$

$\mathcal{L}[x](s^3 - 1) = \frac{1 + s^4 + s^3 + 2s^2}{s^2}$

$\mathcal{L}[x](s^3 - 1) = \frac{s^4 + s^3 + 2s^2 + 1}{s^2}$

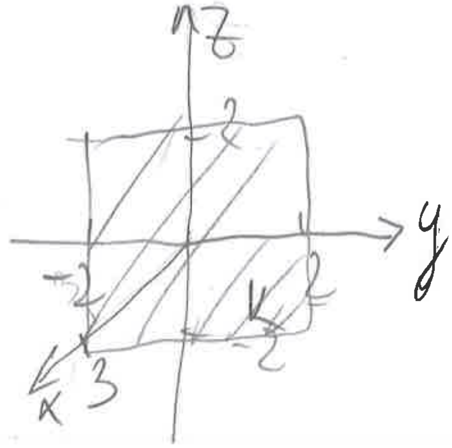
$\mathcal{L}[x] = \frac{s^4 + s^3 + 2s^2 + 1}{s^2(s^3 - 1)}$

3.

$x = 3$

$y \in [-2, 2]$

$z \in [-2, 2]$



$\iint_{\partial K} y\,dy = 2$

$E = \frac{q}{4\pi R^2}$

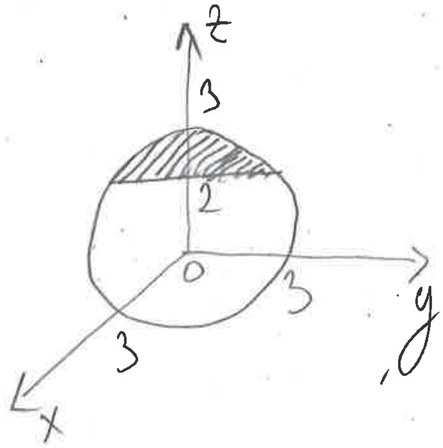
ANTE ŠIŠAK

4. $x^2+y^2=3$ unutar cilindra

$z=\sqrt{x^2+y^2}$

$x+y+z=8$

$V=?$



$x^2+y^2+z^2=9$ $\qquad z\geq 2$

$y=r\cos\varphi$

$z=r\sin\varphi$

$x=x$

$$V=\int_0^{2\pi}d\varphi\cdot\int_0^3 r\,dr\cdot\int_2^3\frac{1}{5}\cdot\frac{4}{3}r^3\pi\,dz=$$

$$V=\varphi\Big|_0^{2\pi}\cdot\frac{r^2}{2}\Big|_0^3\cdot\frac{4}{15}\int_2^3 r^3\pi\,dz+c=$$

$$=\varphi\Big|_0^{2\pi}\cdot\frac{r^2}{2}\Big|_0^3\cdot\frac{4\pi}{15}\cdot\frac{r^3}{\ln r}\cdot\left(z\Big|_2^3\right)=2\pi\cdot4\cdot\frac{4\pi}{15}\cdot\frac{r^3}{\ln r}\cdot\left(z^3-z^2\right)=$$

$$V=\frac{32\pi^2}{15}\cdot\frac{r^3\cdot(z^3-z^2)}{\ln r}$$

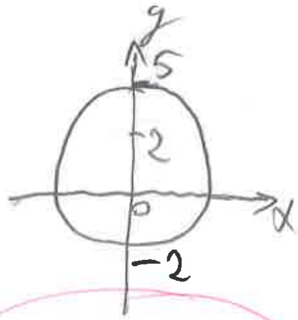
2. NASTAVAK

$$I=\int_0^{2\pi}d\varphi\cdot3\int_0^2 r\,dr\cdot\int_{2-z}^{z^2+y^2=4}\left(\frac{(\sqrt3 r\cos\varphi)^2}{3}+\frac{(\sqrt3 r\sin\varphi)^2}{3}\right)dz=$$

$$=\int_0^{2\pi}d\varphi\cdot3\int_0^2 r\,dr\cdot\int^{z^2+y^2=4}\underbrace{(r^2\cos^2\varphi+r^2\sin^2\varphi)}_{r^2\cdot(\cos^2+\sin^2\varphi)}dz=\int_0^{2\pi}d\varphi\cdot3\int_0^2 r\,dr\cdot\int_{2-z}^{z^2+y^2=4}r^2\,dz=$$

$$=2\pi\cdot\frac{3r^2}{2}\Big|_0^2\cdot\frac{r^2}{\ln r}\Big|_{2-z}^{z^2+y^2=4}=2\pi\cdot\frac{3r^4}{4}\cdot\left[\left(\frac{z^4+y^4}{\ln(z^2+y^2)}\right)-\left(\frac{4-4z+z^2}{\ln(2-z)}\right)\right]$$

5. / S gornja polusfera s $r=3$ s $(z\geq 0)$ prema van



$$\iint_S 3\,dx\,dy$$

$r=4$

$T(0,2)$

$$\int_k (1-3x)\,dx\,dy$$

?

$x = r\cos t$

$y = r\sin t + 2$

?

$$ds = \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$$

$$ds = \sqrt{4\sin^2 t + 4\cos^2 t}\,dt$$

$$ds = \sqrt{4\underbrace{(\sin^2 t + \cos^2 t)}_{1}}\,dt$$

$$ds = \sqrt{4}\,dt$$

$$ds = 2\,dt$$

$$\int_k (1-3x)\,dx\,dy = \int_0^4 (1-3r\cos t)\cdot 2\,dt$$

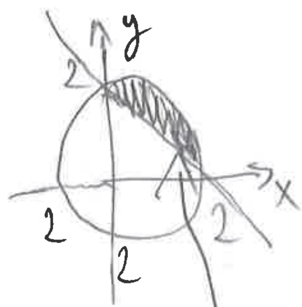
NASTAVAK

$$= \int_0^4 2\,dt - \int_0^4 3r\cos t\cdot 2\,dt = 2\int_0^4 dt - 6r\int_0^4 \cos t\,dt =$$

$$= 2t\Big|_0^4 - 6r\sin t\Big|_0^4 = 2\cdot 4 - 6r\cdot\sin 4 =$$

$$= 8 - 6r\cdot 0.06975 = 8 - 24\cdot 0.06975 =$$

$$= 8 - 1.674 = 6.3258$$



2.) $z = \frac{x^2}{3} + \frac{y^2}{3}$

$x^2 + y^2 \leq 4$

$x = \sqrt{3}\,r\cos\varphi$

$y = \sqrt{3}\,r\sin\varphi$

$J = abr$

$$I = \int_0^{2\pi} d\varphi \int_0^2 3r\,dr \cdot$$ (površina paraboloida s kružnice što sječe)

$$I = \int_0^{2\pi} d\varphi \cdot 3\int_0^2 r\,dr \cdot \int_{2-y}^{x^2+y^2=4} \left(\frac{x^2}{3} + \frac{y^2}{3}\right) dz = \text{NASTAVAK}$$

→

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: ALEN BURA **BROJ INDEKSA**: 17-2-0035-2011	Broj ↓ bodova
Kod kojeg nastavnika želite ustmeni?	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) - x(t) = t, \quad x(0) = x'(0) = 1, \; x''(0) = 2.$$

2. Neka je S gornja polusfera radijusa $r = 3$ sa centrom u ishodištu ($z \geq 0$) orijentirana prema van. Izračunati $\iint_S 3dxdy$? Eventualno može pomoći rot $(3x\mathbf{j}) = 3\mathbf{k}$. 20

3. Neka je kvadrat $K = \{(x, y, z) : x = 3, \, y \in [-2, 2], \, z \in [-2, 2]\}$. Kako preko potencijalnog polja izračunati $\iint_{\partial K} ydy$? 20

4. Izračunati volumen područja unutar cilindra $x^2 + y^2 = 3$, koji je između plašta stošca $z = \sqrt{x^2 + y^2}$ i ravnine $x + y + z = 8$. 20

5. Izračunati volumen tijela omeđenog ravninama $x = 0$, $x = 1$, $y = 0$, $z = 0$, $z = 1 - y$.? 20

Ukupno:

5.

$x \in [0, 1]$

$y \in [0, 1]$

$z \in [0, 1-y]$

$$\int_0^1 \int_0^1 \int_0^{1-y} dz\,dy\,dx = \int_0^1 \int_0^1 z \Big|_0^{1-y} dy\,dx = \int_0^1 \int_0^1 1-y \; dy\,dx$$

$$= \int_0^1 y - \frac{y^2}{2} \Big|_0^1 dx = \int_0^1 \frac{1}{2} dx = \frac{1}{2} x \Big|_0^1 = \frac{1}{2}$$

$z = 1-y$

$0 = 1-y$

$y = 1$

4. $x^2+y^2=3$

$r^2 = 3$

$z = \sqrt{x^2+y^2} \,/\, ^2 \quad x+y+z = 8$

$z^2 = x^2+y^2$

$z^2 = r^2$

$z^2 = 3 \,/\sqrt{\ }$

$z = \sqrt{3}$

?

1. $x'''(t) - x(t) = 1$

$x(0) = x'(0) = 1$

$x''(0) = 2$

$x(t) \to x(s)$ $\qquad t = \frac{1}{s^2}$

$x''(t) \to s^2 x(s) - s\,x(0) - x(0)$

$x'''(t) \to s^3 x(s) - s^2 (x)0 - s x'(0) - x''(0)$

$s^3 x(s) - s^2 x(0) - s x'(0) - x'(0) - x(s) = \frac{1}{s^2}$

$x(s)\,(s^3 - 1) = \frac{1}{s^2} + s^2 + s + 2$

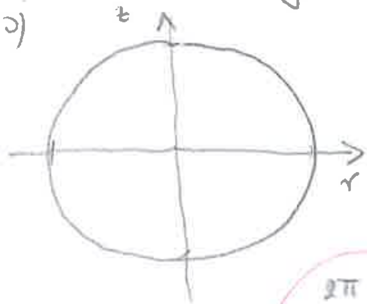
$x(s) = \frac{s^4 + s^3 + 2s^2 + 1}{s^2(s^3-1)} = \frac{s^2+1}{s-1}$

$= \frac{s^2}{s-1} + \frac{1}{s-1} \to \delta(t) + \delta'(t) + e^t$

$x(t) = \delta(t) + \delta'(t) + e^t$?

2. $r = 3$ $\quad z \geq 0$ $\quad \iint_S 3\,dx\,dy$

$T(0,0,0)$



$x = r\cos\varphi$

$y = r\sin\varphi$

$dx\,dy = r\,dr\,d\varphi$

$\varphi \in [0, 2\pi]$

$\int_0^{2\pi}\int_0^3 3r\,dr\,d\varphi = \int_0^{2\pi} 3 \cdot \frac{r^2}{2}\Big|_0^3 d\varphi = \int_0^{2\pi} \frac{27}{2}\,d\varphi = \frac{27}{2}\varphi\Big|_0^{2\pi} = \frac{27}{\cancel{2}}\cancel{2}\pi = 27\pi$ ✗

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: JOSIP MARIĆ BROJ INDEKSA: 17-2-0227-2012	Broj ↓ bodova
Kod kojeg nastavnika želite ustmeni? NIKICA UGLEŠIĆ	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) - x(t) = t, \quad x(0) = x'(0) = 1, \, x''(0) = 2.$$

2. Neka je S gornja polusfera radijusa $r = 3$ sa centrom u ishodištu ($z \geq 0$) orijentirana prema van. Izračunati $\iint\limits_S 3dxdy$? Eventualno može pomoći rot $(3x\mathbf{j}) = 3\mathbf{k}$. 20

3. Neka je kvadrat $K = \{(x, y, z) : x = 3, \, y \in [-2, 2], \, z \in [-2, 2]\}$. Kako preko potencijalnog polja izračunati $\iint\limits_{\partial K} ydy$? 20

4. Izračunati volumen područja unutar cilindra $x^2 + y^2 = 3$, koji je između plašta stošca $z = \sqrt{x^2 + y^2}$ i ravnine $x + y + z = 8$. 20

5. Izračunati volumen tijela omeđenog ravninama $x = 0$, $x = 1$, $y = 0$, $z = 0$, $z = 1 - y$.? 20

Ukupno: Ø

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: ŠIME-BORNA MAGAŠ BROJ INDEKSA: 17-2-0108-2011	Broj ↓ bodova
Kod kojeg nastavnika želite ustmeni?	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) - x(t) = t, \quad x(0) = x'(0) = 1,\ x''(0) = 2.$$

2. Neka je S gornja polusfera radijusa $r = 3$ sa centrom u ishodištu ($z \geq 0$) orijentirana prema van. Izračunati $\iint\limits_S 3\mathrm{d}x\mathrm{d}y$? Eventualno može pomoći $\operatorname{rot}(3x\mathbf{j}) = 3\mathbf{k}$. 20

3. Neka je kvadrat $K = \{(x,y,z) : x = 3,\ y \in [-2,2],\ z \in [-2,2]\}$. Kako preko potencijalnog polja izračunati $\iint\limits_{\partial K} y\mathrm{d}y$? 20

4. Izračunati volumen područja unutar cilindra $x^2 + y^2 = 3$, koji je između plašta stošca $z = \sqrt{x^2+y^2}$ i ravnine $x + y + z = 8$. 20

5. Izračunati volumen tijela omeđenog ravninama $x = 0$, $x = 1$, $y = 0$, $z = 0$, $z = 1 - y$.? 20

Ukupno: 0

1

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: MARINO ŽUBČIĆ

BROJ INDEKSA: 17-2-0216-2012

Kod kojeg nastavnika želite ustmeni? prof. Uglešić

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) - x(t) = t, \quad x(0) = x'(0) = 1,\ x''(0) = 2.$$

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3. Neka je kvadrat $K = \{(x, y, z) : x = 3,\ y \in [-2, 2],\ z \in [-2, 2]\}$. Kako preko potencijalnog polja izračunati $\iint\limits_{\partial K} y dy$? 20

4. Izračunati volumen područja unutar cilindra $x^2 + y^2 = 3$, koji je između plašta stošca $z = \sqrt{x^2 + y^2}$ i ravnine $x + y + z = 8$. 20

5. Izračunati volumen tijela omeđenog ravninama $x = 0$, $x = 1$, $y = 0$, $z = 0$, $z = 1 - y$.? 20

Ukupno:

∅

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: ANTON RAŠIĆ BROJ INDEKSA: 17-2-0084-2011	Broj ↓ bodova
Kod kojeg nastavnika želite ustmeni? prof. UGLEŠIĆ	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: **20**

$$x'''(t) - x(t) = t, \quad x(0) = x'(0) = 1, \; x''(0) = 2.$$

2. Neka je S gornja polusfera radijusa $r = 3$ sa centrom u ishodištu ($z \geq 0$) orijentirana prema van. Izračunati $\iint_S 3dxdy$? Eventualno može pomoći rot $(3x\mathbf{j}) = 3\mathbf{k}$. **20**

3. Neka je kvadrat $K = \{(x, y, z) : x = 3, \; y \in [-2, 2], \; z \in [-2, 2]\}$. Kako preko potencijalnog polja izračunati $\iint_{\partial K} ydy$? **20**

4. Izračunati volumen područja unutar cilindra $x^2 + y^2 = 3$, koji je između plašta stošca $z = \sqrt{x^2 + y^2}$ i ravnine $x + y + z = 8$. **20**

5. Izračunati volumen tijela omeđenog ravninama $x = 0$, $x = 1$, $y = 0$, $z = 0$, $z = 1 - y$.? **20**

Ukupno: $\cancel{0}$

(1) $x'''(t) - x(t) = t,$

$x''(0) = 2,$

$x(0) = x'(0) = 1$