MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: **BROJ INDEKSA:**

Kod kojeg nastavnika želite ustmeni?

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + y''(t) = \cos t, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0.$$

2. Zadan je P paraboloid $x^2 + y^2 = 5z$, $z \leq 5$. Izračunati $\iint_P \mathrm{d}S$? 20

3. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_S x + y \,\mathrm{d}x\mathrm{d}y$. 20

4. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_{\partial S} x + y \,\mathrm{d}x$. 20

5. Izračunati integral funkcije $f(x, y, z) = y$ u dijelu prostora omeđenog plohama $x = z^2$, $z = x$, $y = -1$ i $y = 1$. 20

Ukupno:

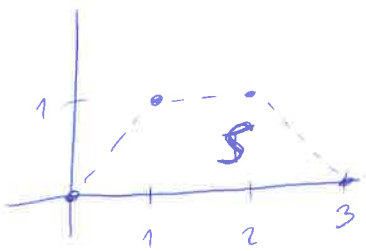
1) VIDI RADAŠ.

2) VIDI MILETIĆ

3) VIDI PREDOVAN

5) VIDI BAČINIĆ

4)



∂S JE RUB TRAPEZA, DAKLE KRIVULJA I TO ZATVORENA, TO ZNAČI DA JE $\iint_{\partial S} x+y\,dx$ POGREŠAN ZAPIS KRIVULJNOG INTEGRALA VEKTORSKE FUNKCIJE $\oint_{\partial S} \begin{pmatrix} x+y \\ 0 \end{pmatrix} \cdot d\vec{s}$. PO GREENOVOJ FORMULI:

$$\oint_{\partial S} \begin{pmatrix} x+y \\ 0 \end{pmatrix} \cdot d\vec{s} = \iint_S -1 \,dx\,dy = -\int_0^1 \int_y^{3y} 1 \,dx\,dy = -2$$

PRITOM SMO PODRAZUMIJEVALI SMJER OBILASKA SUPROTAN KAZALJKE NA SATU.

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: JOSIP FEŠTIN1 BROJ INDEKSA: 17-2-0233-2014

Kod kojeg nastavnika želite ustmeni?

POPUNJAVA NASTAVNIK Broj ↓ bodova

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Ukupno: 30

① $y'''(t) + y''(t) = \cos t \quad y(0) = 0 \quad y'(0) = 0 \quad y''(0) = 0$

~~$s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) = \frac{s}{s^2+1}$~~

~~$s^3 Y(s) - 0 - 0 - 0 = 0$~~

$s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) + s^2 Y(s) - s y(0) - y'(0) = \frac{s}{s^2+1}$

$s^3 Y(s) - 0 - 0 - 0 + s^2 Y(s) - 0 - 0 = \frac{s}{s^2+1}$

$Y s^3 + s^2 = \frac{s}{s^2+1}$

$Y(s) = \frac{\frac{s}{s^2+1}}{s^3+s^2} = \frac{s}{(s^2+1)(s^3+s^2)} = \frac{s}{(s^2+1)s^2(s+1)} = \frac{s}{s^2(s^2+1)(s+1)}$

$y(s) = \frac{s}{s^2(s+1)+(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D+Fs}{s^2+1}$ / · nazivnik

$= As(s+1)(s^2+1) + B(s+1)(s^2+1) + Cs^2(s^2+1) + (D+Fs)(s^2(s+1))$

$= (As^2+As)(s^2+1) + (Bs+B)(s^2+1) + Cs^4 + Cs^2 + Ds^2 + Fs^3(s+1)$

$= As^4 + As^2 + As^3 + As + Bs^3 + Bs + Bs^2 + B + Cs^4 + Cs^2 + Ds^3 + Ds^2 + Fs^4 + Fs^3$

$0 = A + C + F \quad 0 = B \quad B = 0$

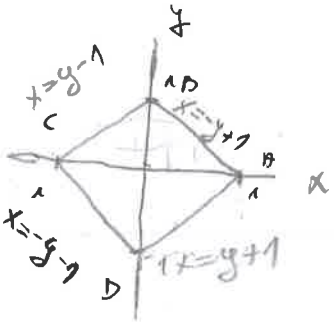
$0 = A + B + D + F \quad -A = B - 1 \quad A = 1$

$0 = A + B + C + D \quad A = B + 1$

$1 = A + B \quad A = 1$

③ $\int_S \int dx + dy \; dx \, dy$

$(y_2 - y_1)(x - x_1) = (x_2 - x_1)(y - y_1)$



A(1,0), B(0,1) C(-1,0) D(0,-1)

AB... $(1-0)(x-1) = (0-1)(y-0)$

$x - 1 = -y$

$x = -y + 1$

BC... $(0-1)(x-0) = (-1-0)(y-1)$

$-x = -y + 1$

$x = y - 1$

CD... $(-1-0)(x+1) = (0+1)(y-0)$

$-x - 1 = y$

$-x = y + 1$

$x = -y - 1$

AD... $(-1-0)(x-1) = (0-1)(y-0)$

$-x + 1 = -y$

$-x = -y - 1$

$x = y + 1$

DALJE...

JOSIP FEŠTINJI

$$P_1 = \int_0^1 \int_{y-1}^{-y+1} x+y \, dx\, dy = \int_0^1 \left| \frac{x^2}{2} + yx \right|_{y-1}^{-y+1} dy = \int_0^1 \frac{(-y+1)^2}{2} + y - \left[\frac{(y-1)^2}{2} + y \right]$$

$$= \int_0^1 \frac{y^2 + 2\cdot(-y)\cdot 1 + 1}{2} + y - \left[\frac{y^2 - 2y\cdot(-1)}{2} + 1 + y \right] = \int_0^1 \frac{y^2 - 2y + 1 + 2y - y^2 - 2y + 1 - 2y}{2}$$

$$= \int_0^1 -2y \, dy = -2\frac{y^2}{2}\Big|_0^1 = -y^2\Big|_0^1 = -1$$

$$P_2 = \int_{-1}^{0} \int_{-y-1}^{y+1} x+y \, dx\, dy = \int_{-1}^{0} \left| \frac{x^2}{2} + yx \right|_{-y-1}^{y+1} = \int_{-1}^{0} \frac{y^2+2y+1}{2} + y - \left[\frac{y^2 - 2(-y)(-1) + 1}{2} - y \right]$$

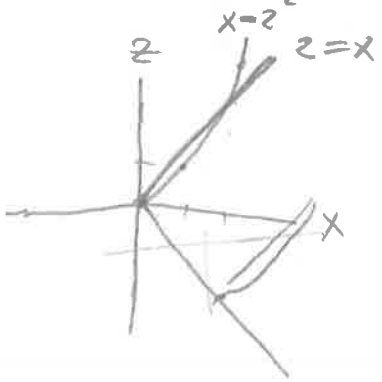
$$= \int_{-1}^{0} \frac{y^2+2y+1+2y-y^2+2y-1-2y}{2} = \int_{-1}^{0} \frac{4y}{2} = \int_{-1}^{0} 2y = \frac{2y^2}{2}\Big|_{-1}^{0} = 0 - (1)$$

$$= -1$$

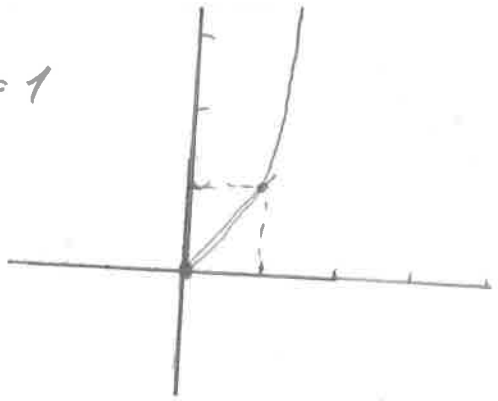
$$P_{uk} = P_1 + P_2 = -1 + (-1) = -1 - 1 = -2$$

15

(5) $f(x,y,z) = y$, $x = z^2$, $z = x$, $y = -1$ i $y = 1$



x	0	z	1
z	0	y	1



15

$$X = \int_{-1}^{1}\int_{0}^{1}\int_{z^2}^{z} y \, dx \, dz \, dy = \int_{-1}^{1}\int_{0}^{1} |xy|_{z^2}^{z} = \int_{-1}^{1}\int_{0}^{1} zy - z^2y$$

$$= \int_{-1}^{1} \left|\frac{z^2}{2}\cdot y - \frac{z^3}{3}\cdot y\right|_0^1 = \int_{-1}^{1}\left(\frac{1}{2}y - \frac{1}{3}y\right) = \int_{-1}^{1}\frac{3y-2y}{6} = \int_{-1}^{1}\frac{y}{6} = \left|\frac{1}{6}\cdot\frac{y^2}{2}\right|_{-1}^{1}$$

$$= \frac{1}{12} - \frac{1}{12} = 0$$

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IME I PREZIME: Filip Bačinić **BROJ INDEKSA:** 3718

Kod kojeg nastavnika želite ustmeni? MATE KUSOR

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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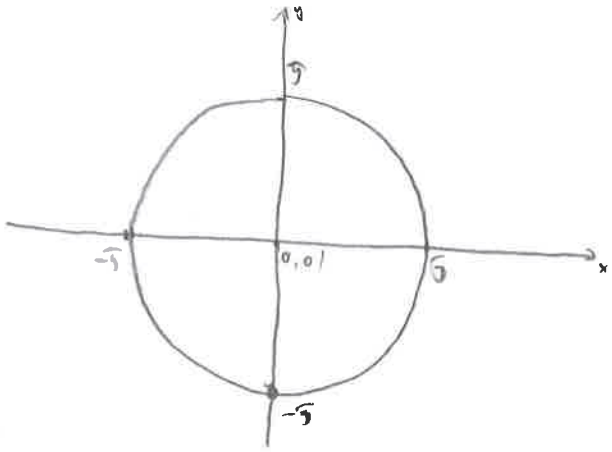
Ukupno: 20

(2) $x^2 + y^2 = 5z \qquad z \leq 5$

$x^2 + y^2 = 5 \cdot 5 = 25$

$r = \sqrt{25} = 5$



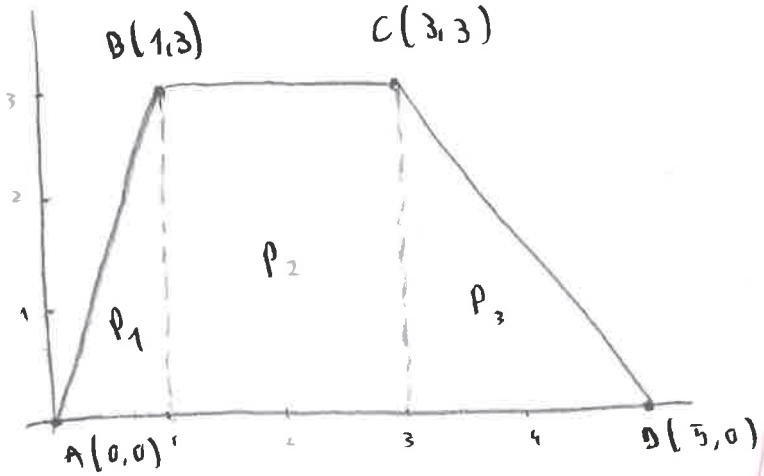
$x = r\cos\varphi = 5\cos\varphi$

$y = r\sin\varphi = 5\sin\varphi$

$\varphi = [0, 2\pi]$

$$\iint_P ds = \int_0^{2\pi}\int_0^5 r\,dr\,d\varphi = \int_0^{2\pi} d\varphi \cdot \frac{r^2}{2}\Big|_0^5 = \int_0^{2\pi} \frac{25}{2} d\varphi = \frac{25}{2}\int_0^{2\pi} d\varphi = \frac{25}{2} \cdot 2\pi = 25\pi$$

(3)



$$\iint (x+y)\,dx\,dy =$$

$$= \iint x\,dx\,dy + \iint y\,dx\,dy$$

LAKŠE:

$$\int_0^3 \int_{AD} xy \; dx\,dy$$

$P_1(AB)$ ... $y = 3x$ $P_2(BC) = \dots$ $y = 3$ $P_3(CD) = \dots$ $y = -\frac{3}{2}x$ ✗

$P = P_1 + P_2 + P_3 = \int_0^1 \int_0^{3x} (x+y)\,dx\,dy + \int_1^3 \int_0^3 (x+y)\,dx\,dy + \int_3^5 \int_0^{-\frac{3}{2}x} (x+y)\,dx\,dy$ ✗

$= \int_0^1 dx \int_0^{3x} y\,dy + \int_0^1 x\,dx \int_0^{3x} dy + \int_1^3 x\,dx \int_0^3 dy + \int_1^3 dx \int_0^3 y\,dy + \int_3^5 x\,dx \int_0^{-\frac{3}{2}x} dy +$

$+ \int_3^5 dx \int_0^{-\frac{3}{2}x} y\,dy = \int_0^1 dx \cdot \frac{y^2}{2}\Big|_0^{3x} + \int_0^1 x\,dx \cdot y\Big|_0^{3x} + \int_1^3 x\,dx \cdot y\Big|_0^3 +$

$+ \int_1^3 dx \cdot \frac{y^2}{2}\Big|_0^3 + \int_3^5 x\,dx \cdot y\Big|_0^{-\frac{3}{2}x} + \int_3^5 dx \cdot \frac{y^2}{2}\Big|_0^{-\frac{3}{2}x} =$

$= \int_0^1 \frac{9x^2}{2}\,dx + \int_0^1 3x^2\,dx + \int_1^3 3x\,dx + \int_1^3 \frac{9}{2}\,dx + \int_3^5 x\,dx \cdot -\frac{3}{2}x + \int_3^5 dx \cdot \frac{\frac{9}{4}x^2}{2} =$

$= \frac{9x^3}{6}\Big|_0^1 + \frac{3x^3}{3}\Big|_0^1 + \frac{3x^2}{2}\Big|_1^3 + \frac{9}{2}x\Big|_1^3 + -\frac{1}{2}\cdot\frac{x^3}{3}\Big|_3^5 + \frac{9^3}{24_8}x^3\Big|_3^6 =$

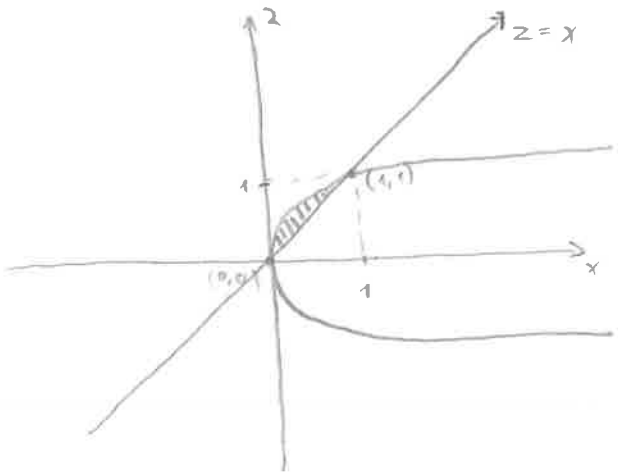
$= \frac{3}{2} + 1 + \frac{27}{2} - \frac{3}{2} + \frac{27}{2} - \frac{9}{2} - \frac{125}{2} + \frac{27}{2} + \frac{375}{8} - \frac{81}{8} =$

$= \frac{45}{4}$

⑤ $f(x,y,z) = y$ $x = z^2$ $z = x$ $y = -1$ $y = 1$

$z = \sqrt{x}$ $x = 0$ $z = 0$

$x = 1$ $z = 1$



$\iiint f(x,y,z)\,dx\,dy\,dz = \iiint y\,dx\,dy\,dz =$

$= \int_0^1 dx \int_{-1}^1 y\,dy \int_x^{\sqrt{x}} dz$

Filip Bačinić

(5.) NASTAVNIK

$$\int_0^1 dx \int_0^1 y\,dy \cdot z\Big|_x^{\sqrt{x}} = \int_0^1 (\sqrt{x} - x)\,dx \int_0^1 y\,dy = \frac{y^2}{2}\Big|_0^1 \cdot \int_0^1 (\sqrt{x} - x)\,dx =$$

$$= \frac{1}{2} \cdot \left[\int_0^1 \sqrt{x}\,dx + \int_0^1 -x\,dx \right] = \frac{1}{2}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Big|_0^1 - \frac{x^2}{2}\Big|_0^1 \right] = \frac{1}{2} \cdot \left[\frac{2}{3} - \frac{1}{2} \right] =$$

$$= \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12}$$ ✓

(1) $y'''(t) + y''(t) = \cos t \quad y(0)=0 \quad y'(0)=0 \quad y''(0)=0$

$$\mathcal{L}[y'''(t)] + \mathcal{L}[y''(t)] + \mathcal{L}[y'(t)]$$

$$\underbrace{\left[s^3 y(s) - s^2 \underbrace{y''(0)}_{=0} - s \underbrace{y(0)}_{=0} - \underbrace{y'(0)}_{=0} \right]}_{=0} + \underbrace{\left[s^2 y(s) - s y(0) - \underbrace{y'(0)}_{=0} \right]}_{=0} + \left[s\,y(s) - \underbrace{y(0)}_{=0} \right] = \frac{s}{s^2+1}$$

$$s^3 y(s) = \frac{s}{s^2+1} \quad / : s^2$$

$$y(s) = \frac{1}{s^2(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C+Ds}{s^2+1} = \frac{As^3 + As + Bs^2 + B + Cs^2 + Ds^3}{s^2(s^2+1)}$$

$$1 = (A+D)s^3 + (B+C)s^2 + As + B$$

$$\begin{cases} A + D = 0 \\ B + C = 0 \\ A = 0 \\ B = 0 \end{cases}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: DONATO PLEDOVAN **BROJ INDEKSA:** 0036461512

Kod kojeg nastavnika želite ustmeni? Kod prof. Vlašića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 20

1) $y'(t) \rightarrow s^2 Y(s) - s y(0) - y'(0)$

$y'''(t) \rightarrow s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0)$

$\cos t \rightarrow \frac{s}{s^2+1}$

$s^3 Y(s) + s^2 Y(s) = \frac{s}{s^2+1}$

$Y(s) \cdot (s^3 + s^2) = \frac{s}{s^2+1}$

$Y(s) \cdot s^2(s+1) = \frac{s}{s^2+1} \quad / : s^2(s+1)$

$Y(s) = \frac{s}{(s^2+1) \cdot s^2 (s+1)}$

RASTAV NA PARC. RAZ.

$$\frac{s}{(s^2+1) \cdot s^2 \cdot (s+1)} = \frac{As+B}{s^2+1} + \frac{C}{s} + \frac{D}{s^2} + \frac{E}{s+1} \quad / \cdot (s^2+1) \cdot s^2 (s+1)$$

$s = s^2 \cdot (s+1)(As+B) + (s^2+1) \cdot s(s+1) \cdot C + (s^2+1)(s+1) \cdot D + (s^2+1) \cdot s^2 \cdot E$

$s = (s^3+s^2)(As+B) + (s^2+1)(s^2+s) \cdot C + (s^3+s^2+s+1) \cdot D + (s^4+s^2) \cdot E$

$s = As^4 + Bs^3 + As^3 + Bs^2 + Cs^4 + Cs^3 + Cs^2 + Cs + Ds^3 + Ds^2 + Ds + D + Es^4 + Es^2$

$s = s^4(A+C+E) + s^3(B+A+C+D) + s^2(B+C+D+E) + s(C+D) + D$

$s^4 \ldots \; 0 = A + C + E \qquad 0 = D \qquad \boxed{D = 0} \qquad 1 = C + 0$

$s^3 \ldots \; 0 = B + A + C + D \qquad \boxed{C = 1}$

$s^2 \ldots \; 0 = B + C + D + E$

$s \ldots \; 1 = C + D$

$0 = A+C+E$

$0 = B+A+C+D$

$0 = B+C+D+E$

$D = 0$

$C = 1$

$B+A = -C-D$

$B+A = -1-0$

$B+A = -1$

$A = -1-B$

$1 = -A-E$

$1 = 1+B-E$

$-E+1+B = 1$

$-E = 1-1-B$

$-E = -B$

$E = B$

$B+A+C+D = 0$

$B+E+C+D = 0 \;/\cdot(-1)$

$A = 0$

$-E = 0$]+

$A-E = 0$

$-1-E-E = 0$

$-2E = 1$

$E = -\frac{1}{2}$

$B = -\frac{1}{2}$

$A = -1+\frac{1}{2}$

$A = -\frac{1}{2}$

$$Y(s) = \frac{-\frac{1}{2}s - \frac{1}{2}}{s^2+1} + \frac{1}{s} + \frac{0}{s^2} + \frac{-\frac{1}{2}}{s+1}$$

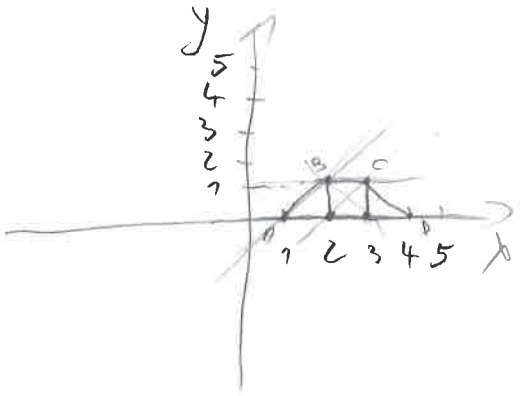
$$Y(s) = -\frac{1}{2}\cdot\frac{s}{s^2+1} - \frac{1}{2}\cdot\frac{1}{s^2+1} + 1 - \frac{1}{2}\cdot\frac{1}{s+1}$$

$$y(s) = -\frac{1}{2}\cos t - \frac{1}{2}\sin t + 1 - \frac{1}{2}e^{-t}$$

✓

DONATO PREDOVAN

3.) $\iint_S x + y \, dx \, dy$



A(1,0), B(2,1), C(3,1), D(4,0)

AB

$y - 0 = \frac{1-0}{2-1}(x-1)$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

AB ... $y = x - 1$

BC

$y - 1 = \frac{1-1}{3-2}(x-2)$

$y_1 = x - 1$

dolja

$x = y + 1$

$y - 1 = 0$

BC ... $y_2 = 1$

gornja

CD

$y - 1 = \frac{0-1}{4-3}(x-3)$

$y - 1 = -\frac{1}{1}(x-3)$

$y - 1 = 3 - x$

CD ... $y = 4 - x$

gornja

$x = 4 - y$

DA

$y - 0 = \frac{0-0}{1-4}(x-4)$

DA ... $y = 0$

dolja

$$\int_{y_1=x-1}^{y_2=1} \int_{x_1=0}^{x_2=4-y} x + y \, dx \, dy =$$

$$\iint_S x+y \, dx \, dy = \int_0^1 \int_{y+1}^{4-y} x + y \, dx \, dy = \int_0^1 \left[\frac{x^2}{2} + xy \right]_{x=y+1}^{x=4-y} dy =$$

$$= \int_0^1 \frac{(4-y)^2}{2} + (4-y)y - \frac{(y+1)^2}{2} - (y+1)y \, dy = \dots$$

5. $x = z^2$

$z = x$

$y = -1$

$y = 1$

$f(x,y,z) = y$

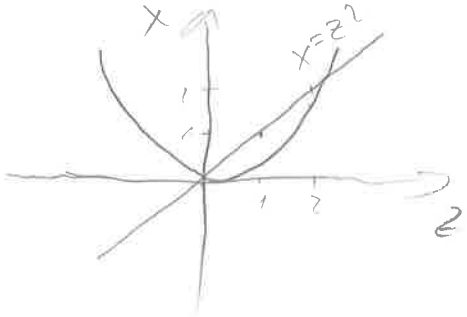
$z^2 = z$

$z^2 - z = 0$

$z(z-1) = 0$

$z_1 = 0$ - donji

$z_2 = 1$ - gornji



x	0	1
z	0	1

$x = z^2$ - donja

$x = z$ - gornja

$$\int_{x_1=z^2}^{x_2=z} \int_{-1}^{1} \int_{0}^{1} y \, dz\,dy\,dx =$$

2. $x^2 + y^2 = 5z \quad , \quad z \leq 5$

$5z = 1$

$z = \frac{1}{5}$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: LUKA RADAŠ **BROJ INDEKSA:** 57662

Kod kojeg nastavnika želite ustmeni?

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + y''(t) = \cos t, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0.$$

2. Zadan je P-paraboloid $x^2 + y^2 = 5z$, $z \leq 5$. Izračunati $\iint_P \mathrm{d}S$? 20

3. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_S x + y \,\mathrm{d}x\mathrm{d}y$. 20

4. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_{\partial S} x + y \,\mathrm{d}x$. 20

5. Izračunati integral funkcije $f(x, y, z) = y$ u dijelu prostora omeđenog plohama $x = z^2$, $z = x$, $y = -1$ i $y = 1$. 20

Ukupno: 100

$y'''(t) + y''(t) = \cos t \qquad y(0)=0 \quad y'(0)=0 \quad y''(0)=0$

$s^3F(s) - s^2\underbrace{f(0)}_{0} - s\underbrace{f'(0)}_{0} - \underbrace{f''(0)}_{0} + s^2F(s) - s\underbrace{f(0)}_{0} - \underbrace{f'(0)}_{0} = \frac{s}{s^2+1}$

$F(s)\left(s^3+s^2\right) = \frac{s}{s^2+1} \quad /:(s^3+s^2) = \frac{s}{s^2(s+1)(s^2+1)}$

$(s^2+1)(s^3+s^2)$

$s^5+s^4+s^3+s^2$

$s^3(s^2+1)+s^2(s^2+1)$

$(s^3+s^2)(s^2+1)$

$s^2(s+1)(s^2+1)$

$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1}$

$= As(s+1)(s^2+1) + B(s+1)(s^2+1) + Cs^2(s^2+1) + Ds^3(s+1) + Es^2(s+1)$

$= As^4 + As^2 + As^3 + As + Bs^3 + Bs + Bs^2 + B + Cs^4 + Cs^2 + Ds^4 + Ds^3 + Es^3 + Es^2$

$s^4: \quad 0 = A + C + D \qquad C+D=-1 \qquad -1-E-1-E=-1 \qquad -1=2E \qquad E=-\frac{1}{2}$

$s^3: \quad 0 = A + B + D + E \qquad D+E=-1 \qquad D=-1-E \qquad D=-\frac{1}{2}$

$s^2: \quad 0 = A + B + C + E \qquad C+E=-1 \qquad C=-1-E \qquad C=-\frac{1}{2}$

$s^1: \quad 1 = A + B$

$0 = B \qquad A = 1$

$F(s) = \frac{1}{s} + 0 - \frac{1}{2}e^{-t} - \frac{1}{2}\frac{s}{s^2+1} - \frac{1}{2}\frac{1}{s^2+1}$

$f(s) = 1 - \frac{1}{2}e^{-t} - \frac{1}{2}\cos(t) - \frac{1}{2}\sin(t)$ ✓

$f(0) = 1 - \frac{1}{2} - \frac{1}{2} - 0 = 0$ ✓

$f'(0) = \frac{1}{2}e^{-t} + \frac{1}{2}\sin(t) - \frac{1}{2}\cos(t)$

$= \frac{1}{2} + 0 - \frac{1}{2} = 0$ ✓

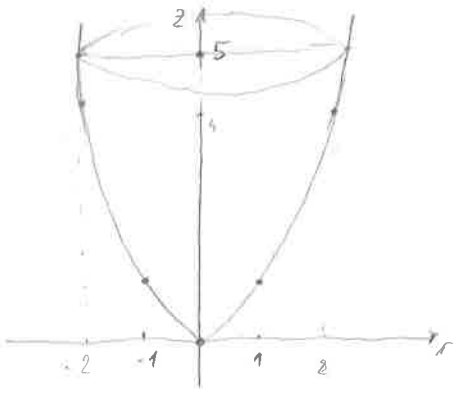
$f''(0) = -\frac{1}{2}e^{-t} + \frac{1}{2}\cos(t) + \frac{1}{2}\sin(t)$

$= -\frac{1}{2} + \frac{1}{2} = 0$ ✓

2.) PARABOLOID

$x^2 + y^2 = 5z \qquad z \leq 5$

$\iint_P ds = ?$



$x^2 + y^2 = r^2$

$r^2 = 5z$

$r = \sqrt{5}$

r	1	2	-1
z	-1	4	1

"r"

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: DENI MILETIĆ **BROJ INDEKSA:** 54143

Kod kojeg nastavnika želite ustmeni?

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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4. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_{\partial S} x + y \,\mathrm{d}x$. 20

5. Izračunati integral funkcije $f(x, y, z) = y$ u dijelu prostora omeđenog plohama $x = z^2$, $z = x$, $y = -1$ i $y = 1$. 20

Ukupno: 20

1. $y'''(t) + y''(t) = \cos t$

$y(0) = 0$
$y'(0) = 0$
$y''(0) = 0$

$s^3Y(s) + s^2y(0) - sy'(0) - y''(0) + s^2Y(s) - sy(0) - y'(0) = \frac{s}{s^2+1}$

$Y(s)(s^3 + s^2) = \frac{s}{s^2+1} \;/:(s^3+s^2)$

$Y(s) = \frac{s}{(s^3+s^2)(s^2+1)} = \frac{s}{s^2(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1} \;/\cdot s^2(s+1)(s^2+1)$

$1 = As(s+1)(s^2+1) + B(s+1)(s^2+1) + Cs^2(s^2+1) + Ds^3(s+1) + Es^2(s+1)$

$s = As^4 + As^2 + As^3 + As + Bs^3 + Bs + Bs^2 + B + Cs^4 + Cs^2 + Ds^4 + Ds^3 + Es^3 + Es^2$

za s^4; $0 = A + C + D \Rightarrow 1 = 2D \Rightarrow D = \frac{1}{2}$

za s^3; $0 = A + B + D + E \Rightarrow E = -D \Rightarrow E = -\frac{1}{2}$

za s^2; $0 = A + B + C + E \Rightarrow C = -E \Rightarrow D \Rightarrow C = \frac{1}{2}$

za s; $0 = A + B \Rightarrow A = -1$

za 1; $B = 1$

$Y(s) = -\frac{1}{s} + \frac{1}{s^2} + \frac{1}{2}\frac{1}{s+1} - \frac{1}{2}\frac{s}{s^2+1} - \frac{1}{2}\frac{1}{s^2+1}$

$y(t) = -1 + t + \frac{1}{2}e^{-t} + \frac{1}{2}\cos t - \frac{1}{2}\sin t$; $y(0)$; $-1 + \frac{1}{2} + \frac{1}{2} = 0$ ✓

$y'(t) = 1 - \frac{1}{2}e^{-t} - \frac{1}{2}\sin t - \frac{1}{2}\cos t$; $y'(0)$; $1 - \frac{1}{2} - \frac{1}{2} = 0$ ✓

$y''(t) = \frac{1}{2}e^{-t} - \frac{1}{2}\cos t + \frac{1}{2}\sin t$; $y''(0) = \frac{1}{2} - \frac{1}{2} = 0$ ✓

UVRSTI U $y'''(t) + y''(t)$ I NEĆE ISPASTI $= \cos t$

2. paraboloid $x^2+y^2=5z$, $z\le 5$ $\quad \iint_P dS$

$\Rightarrow x^2+y^2=5z$

$r^2=5z \to r=5$

$z=\frac{r^2}{5}$

$r\in[0,5]$

$\varphi\in[0,2\pi]$

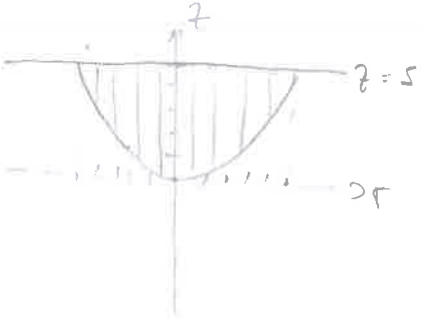
$5z\,dz=x^2dx \qquad \frac{dz}{dx}=\frac{2x}{5}$

$5z\,dz=y^2dy \qquad \frac{dz}{dy}=\frac{2y}{5}$

$\sqrt{1+\left(\frac{2}{5}x\right)^2+\left(\frac{2}{5}y\right)^2}=\sqrt{1+\frac{4}{25}x^2+\frac{4}{25}y^2}$

$=\sqrt{1+\frac{4}{25}r^2}$

$=\int_0^{2\pi}\int_0^5\left(\sqrt{1+\frac{4}{25}r^2}\right)\cdot r\,dr\,d\varphi=\left\{\begin{matrix} u=1+\frac{4}{25}r^2,\ du=\frac{16}{25}r\,dr \\ r\,dr=\frac{25}{16}du\end{matrix}\right\}$



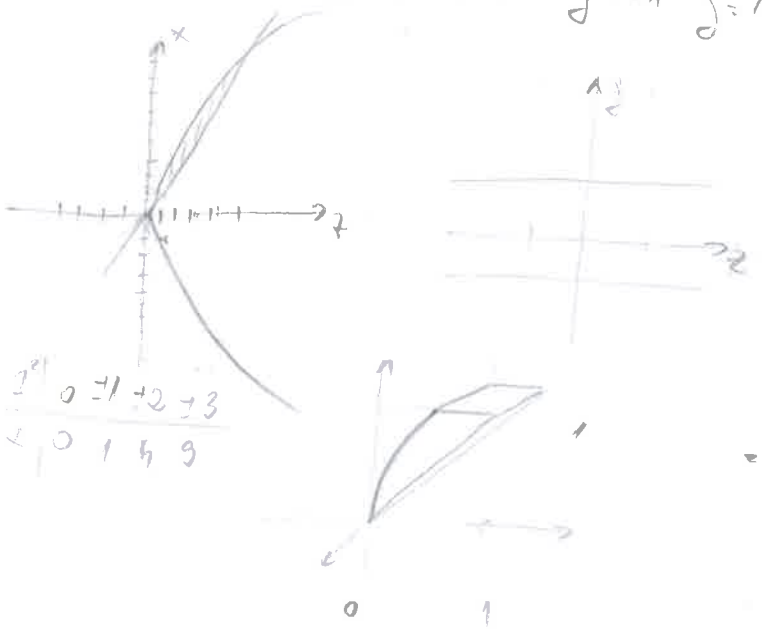
$=\int_0^{2\pi}\int_0^5\frac{25}{16}\sqrt{u}\,du\,d\varphi=\frac{25}{16}\int_0^{2\pi}\frac{2}{3}\cdot\sqrt{\left(1+\frac{4}{25}r^2\right)^3}\Big|_0^5 d\varphi=$

$=\frac{25}{16}\cdot\frac{2}{3}\int_0^{2\pi}\sqrt{\left(1+\frac{4}{25}\cdot 25\right)^3}\,d\varphi=\frac{25}{24}\int_0^{2\pi}\sqrt{5^3}\,d\varphi=\frac{25}{24}\sqrt{27}\int_0^{2\pi}$

$=\frac{25}{24}\sqrt{27}\cdot 2\pi=\frac{25}{8}\sqrt{3}\cdot 2\pi$ ✓

$\frac{r}{5}$	0	5	$-\frac{5}{5}$
z	0	5	5

5. $f(x,y,z)=y$ $\quad t=x^2$ $\quad x=y^2$ $\quad z=x$ $\quad y=-1$ $\quad y=1$



y^2	0	1	2	3
x	0	1	4	9

$\int_0^1\int_0^1\int_z^{z^2} y\,dx\,dz\,dy=\int_0^1\int_0^1 y(z^2-z)\,dz\,dy$

$=\int_0^1\int_0^1 y\left(\frac{z^3}{3}-\frac{z^2}{2}\right)\Big|_0^1 dy=\int_0^1\left(\frac{1}{3}-\frac{1}{2}\right)dy=-\frac{1}{6}\int_0^1 y\,dy$

$=-\frac{1}{6}\cdot\frac{y^2}{2}\Big|_0^1=-\frac{1}{12}$ ✗

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: GABRIELA JORDAN **BROJ INDEKSA:** 17-2-0118-20M

Kod kojeg nastavnika želite ustmeni? M. UGLEŠIĆ

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + y''(t) = \cos t, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0.$$

2. Zadan je P paraboloid $x^2 + y^2 = 5z$, $z \leq 5$. Izračunati $\iint_P \mathrm{d}S$? 20

3. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_S x + y \,\mathrm{d}x\mathrm{d}y$. 20

4. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_{\partial S} x + y \,\mathrm{d}x$. 20

5. Izračunati integral funkcije $f(x, y, z) = y$ u dijelu prostora omeđenog plohama $x = z^2$, $z = x$, $y = -1$ i $y = 1$. 20

Ukupno: 20

1. $y'''(t) + y''(t) = \cos(t)$ $\quad y(0)=0 \quad y'(0)=0 \quad y''(0)=0$

$$s^3 y(s) + s^2 y(s) = \frac{s}{s^2+1}$$

$$y(s)(s^2+s^2) = \frac{s}{s^2+1}$$

$$y(s) = \frac{s}{s^2(s+1)(s^2+1)} = \frac{1}{s(s+1)(s^2+1)}$$

$$= \frac{A}{s} + \frac{B}{s+1} + \frac{s+1}{(s^2+1)}$$

$$A(s+1)(s+1) + B(s)(s^2+1) + (Cs+D)(s)(s+1) = 1$$

$s^3(A+B+C)=0 \quad B+D=-1$

$s^2(A+C+D)=0 \quad C+D=-1$

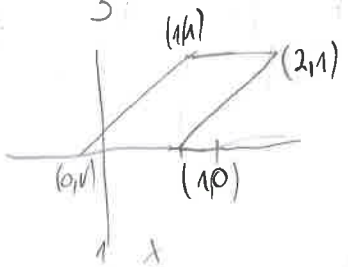
$s(A+B+D)=0 \quad B+C=-1$

$A=1 \quad D=0 \quad D=-\frac{1}{2}$

$$y(s) = \frac{1}{s} + \frac{1}{2}\,\frac{1}{s+1} - \frac{1}{2}\left(\frac{s+1}{s^2+1}\right)$$

$$y(t) = 1 - \frac{1}{2}e^{-t} - \frac{1}{2}\left(\frac{s}{s^2+1} + \frac{1}{s^2+1}\right) = 1 - \frac{1}{2}e^{-t} - \frac{1}{2}\cos t - \frac{1}{2}\sin t$$

3. $\iint_S (x+y)\,dx\,dy$



$y = x$

$y = (x+1)$

$$\int_0^1 \int_0^x 2y\,dx\,dy + \int_1^2 \int_1^{y-1} (2x+1)\,dx\,dy$$

$$= \int_0^1 x^2\,dx + \int_0^1 \left(\left[(y-1)^2 + (y-1) \right] - 2 \right) dy = 1 + \frac{(y-1)^3}{3}\Big|_0^1 + \frac{(y+1)^2}{2}\Big|_0^1 - 2y\Big|_0^1$$

$$= 1 + \frac{1}{3} - \frac{1}{2} - 2 = \frac{6+2-3-12}{6} = \frac{-7}{6}$$

2. $x^2+y^2+z^2 = 5^2$

$z \le 5$

$\left(x, y, \frac{x^2+y^2}{5} \right)$

$r = \sqrt{5}$

$r < 5$

$r < 25$

$$\frac{\partial}{\partial x} = \begin{pmatrix} 1 \\ 0 \\ \frac{2x}{5} \end{pmatrix} \qquad \frac{\partial}{\partial y} = \begin{pmatrix} 0 \\ 1 \\ \frac{2y}{5} \end{pmatrix}$$

$$\vec{n} = \begin{bmatrix} 1 \\ 0 \\ \frac{2x}{5} \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ \frac{2y}{5} \end{bmatrix} = \begin{bmatrix} -\frac{2x}{5} \\ -\frac{2y}{5} \\ 1 \end{bmatrix}$$

$$P = \iint \vec{n} = \iint \frac{4}{2^3 7} \sqrt{(x^2+y^2)+1}\,dx\,dy$$

$x = r\cos\varphi$

$y = \sin\varphi$

$$= \int_0^{2\pi} \int_0^5 \frac{4}{25} \sqrt{(5^2\cos^2\varphi + 5^2\sin^2\varphi) + 1}\; r\,dr\,d\varphi$$

$$= \int_0^{2\pi} \int_0^5 \sqrt{\frac{4}{25} r^2 + 1}\; r\,dr\,d\varphi$$

$$= \frac{2\pi}{5} \int_0^5 \sqrt{4r^2+25}\; r\,dr$$

$t = 8r\,dr$

$$= \frac{\pi}{20} \int_{25}^{125} \sqrt{t}\,dt = \frac{\pi}{20} \cdot \frac{3}{2} \sqrt{t^3}\Big|_{25}^{125}$$

$$= \frac{3\pi}{30} \left(\sqrt{25^3} - \sqrt{25^3} \right)$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: Antonijo Knežević BROJ INDEKSA: 57672.

Kod kojeg nastavnika želite ustmeni?

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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Ukupno:

1. $y'''(t) + y''(t) = \cos t, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0$

2. $x^2 + y^2 = 5z, \quad z \leq 5$

Izračunati $\iint_P dS$

$x^2 + y^2 = 5z$

$x^2 + y^2 = r^2$

$r^2 = 5 \cdot 5$

$r^2 = 25$

$r = 5$

$r \in [0, 5]$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: MARTIN JEDNAK **BROJ INDEKSA:**

Kod kojeg nastavnika želite ustmeni?

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Broj ↓ bodova

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Ukupno:

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: IVAN VELEHIR **BROJ INDEKSA:** 17-2-0067-2010

Kod kojeg nastavnika želite ustmeni?

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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Ukupno:

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: DENIS ILIĆ **BROJ INDEKSA:** 56154-2008

Kod kojeg nastavnika želite ustmeni?

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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5. Izračunati integral funkcije $f(x, y, z) = y$ u dijelu prostora omeđenog plohama $x = z^2$, $z = x$, $y = -1$ i $y = 1$. 20

Ukupno: 0

1. $y'''(t) + y''(t) = \cos t, \; y(0)=0, \; y'(0)=0, \; y''(0)=0$

$\frac{s}{s^2+a^2}$ $f'''(t) = s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

$s^3y(0) - s^2y(0) - sy(0) - y(0)$

3. $\iint_S x+y \,dx\,dy$

5. Izr. integral funkc.: $f(x,y,z) = y$

$x = z^2, \; z = x, \; y = -1, \; y = 1$

$f(z^2, 1, x) = -1$

(2.) paraboloid

$$x^2+y^2=.5z,\quad z\le 5,$$

$$\iint_P dS =$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: BRANIMIR ŠIMIĆ BROJ INDEKSA: 17-2-0086-2017

Kod kojeg nastavnika želite ustmeni?

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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2. Zadan je P paraboloid $x^2 + y^2 = 5z$, $z \leq 5$. Izračunati $\iint_P \mathrm{d}S$? 20

3. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_S x + y \,\mathrm{d}x\mathrm{d}y$. 20

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5. Izračunati integral funkcije $f(x, y, z) = y$ u dijelu prostora omeđenog plohama $x = z^2$, $z = x$, $y = -1$ i $y = 1$. 20

Ukupno: 0

2) $x^2+y^2=5z \quad z\leq 5$ Izračunati $\iint_T ds$

$x^2+y^2=r^2$

$r^2=5z$

$r^2=5\cdot 5$

$r^2=\sqrt{25}$

$r=5$

$\int_0^{2\pi}\int_0^5 r\,dz\,d\varphi \Rightarrow \int_0^{2\pi}\frac{r^2}{2}\Big|_0^5 d\varphi$

$\int_0^{2\pi}\frac{5^2}{2}-\frac{0}{2}\,d\varphi = \int_0^{2\pi}\frac{25}{2}\,d\varphi = \frac{25}{2}\varphi\Big|_0^{2\pi}$

$\frac{25}{2}\cdot 2\pi-\left(\frac{25}{2}\cdot 0\right)=\frac{50\pi}{2}$ //

5) $f(x,y,z)=y \quad x=z^2 \quad z=x \quad y=-1 \quad y=1$

$y\in[-1,1]$

$x\in[z\ \ z^2]$

$z\in[0\ \ 1]$

$z^2=z$

$z^2-z=0$

$z(z-1)=0$

$z=0 \vee z=1$

$\int_{-1}^{1}\int_0^1 yx\Big|_z^{z^2} dz\,dy$

$\int_{-1}^{1}\int_0^1\int_z^{z^2} y\,dx\,dz\,dy$

$\int_{-1}^{1}\int_0^1 y\frac{z}{z}^2$

⑤ $f(x,y,z) = y$

$x = z^2 \quad z = x \quad y = -1 \quad y = 1$

$z^2 = z \Rightarrow z^2 - z = 0 \Rightarrow z(z-1)$

$z = 0$

$z_2 = z - 1 = 0 \Rightarrow z = 1$

$y \in [-1, 1]$

$z \in [0, 1]$

$x \in [z \; z^2]$

$$\int_0^1 \int_z^{z^2} \int_{-1}^{1} y \, dy \, dx \, dz$$

$$\int_0^1 \int_z^{z^2} \frac{y^2}{2} \Big|_{-1}^{1} dx \, dz = \int_0^1 \int_z^{z^2} \left(\frac{1}{2} - \frac{1}{2}\right) dx \, dz$$

$$\int_0^1 \int_z^{z^2} dx \, dz = \int_0^1 x \Big|_z^{z^2} dz = \int_0^1 (z^2 - z) \, dz$$

$$\frac{z^3}{3} - \frac{z^2}{2} \Big|_0^1 = \left(\frac{1}{3} - \frac{1}{2}\right) - \left(\frac{(0)^3}{3} - \frac{(0)^2}{2}\right) = -\frac{1}{6}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: Ivan Rubelj BROJ INDEKSA: 17-2-0085-2011

Kod kojeg nastavnika želite ustmeni?

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + y''(t) = \cos t, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0.$$

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Ukupno: 0

(1.)

$y'''(t) + y''(t) = \cos t$

$y(0) = 0,\ y'(0) = 0,\ y''(0) = 0$

(2.)

P

$x^2 + y^2 = 5z$,

$z \leq 5$

$\iint_S x + y \,dx\,dy$

(3.)

$$\iint_S x + y \, dx \, dy$$

S

(4.)

$$\iint_{\partial S} x + y \, dx$$

(5.)

$f(x, y, z) = y$

$x = z^2$,

$z = x$,

$y = -1$,

$y = 1$.

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: LUKA MILIN **BROJ INDEKSA:** 0269066346

Kod kojeg nastavnika želite ustmeni? NIKICA UGLEŠIĆ

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 0

(1) $y'''(t) + y''(t) = \cos t \qquad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0$

$\cos(t) = \frac{s}{s^2 - a^2}$

$f''(t) = s^2 F(s) - sf(0) - f'(0)$

$f'''(t) = s^3 F(s) - sf(0) - sf'(0) - f''(0)$

$s^3 F(s) - s^2 f(0) - sf'(0) - f''(0) + s^2 F(s) - sF(0) - f'(0)$

$s^3 F(s) - s^2(0) - s(0) - 0 + s^2 F(s) - sF(0) - 0$

$s^3 F(s) + s^2 F(s) = s^2(0) - sf(0) + s^2 F(0) + \cos(t)$

(2) $x^2 + y^2 = 5z \qquad z \leq 5$

POLARNE KOORDINATE:

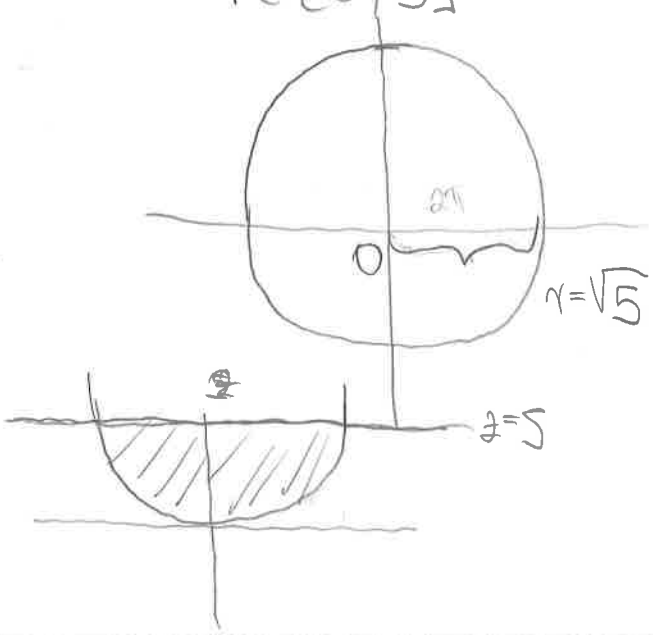
$x = r\cos\varphi$
$y = r\sin\varphi$

$r^2 = 5$
$r = \sqrt{5}$

$r^2\cos^2\varphi + r^2\sin^2\varphi = 5z$

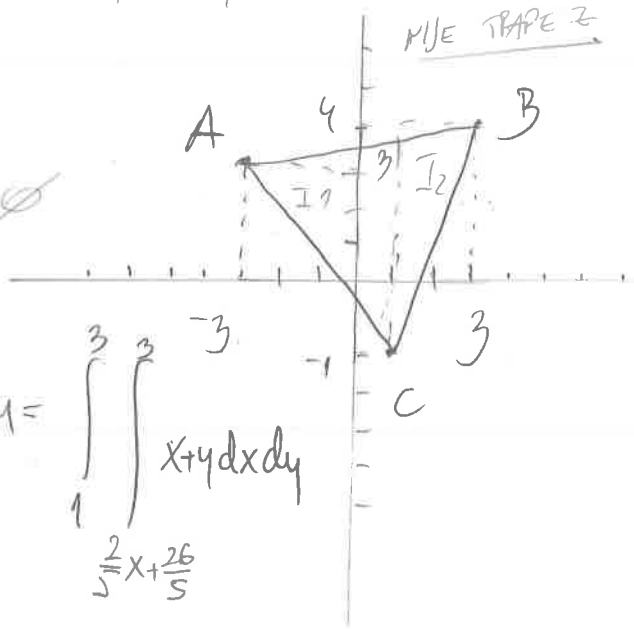
$\varphi \in [0, 2\pi], z \in \left[\frac{1}{5}, 5\right]$

$r \in [0, 5]$



$$\iint_T dS = \int_0^{\frac{12}{5}} \int_0^{\frac{2x}{5}+\frac{26}{5}} \int_{\frac{12}{5}}^{5} dS = \phi$$

③ $\iint_S x+y\,dxdy$ $A(-3,3)$ $B(3,4)$, $C(1,-1)$



$$I_1 = \int_{-3}^{1} \int_{AC}^{AB} x+y\,dxdy = \int_{-3}^{1} \int_{-x}^{3} x+y\,dxdy$$

JED. PRAVCA KROZ 2 TOČKE:

$$y-y_2 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

AB AC BC

$$I_2 = \int_1^3 \int_{CB}^{AB} x+y\,dxdy = \int_1^3 \int_{\frac{2}{5}x+\frac{26}{5}}^{3} x+y\,dxdy$$

$P = I_1 + I_2$

$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1) \Rightarrow$ $A(\overset{x_1}{-3},\overset{y_1}{3})$ $B(\overset{x_2}{3},\overset{y_2}{4})$

$y-3 = \frac{4-3}{3-3}(x+3)$

$y-3 = 0$

$y = 3$

AC $A(-3,3)$, $C(1,-1)$

$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-3 = \frac{-1-3}{1+3}(x+3)$

$y-3 = \frac{-4}{4}(x+3)$

$y-3 = -x-3$

$y = -x$

BC $B(3,4)$, $C(1,-1)$

$y-y_1 = \frac{x_2-x_1}{y_2-y_1}(x-x_1)$

$y-4 = \frac{1-3}{-1-4}(x-3)$

$y-4 = \frac{-2}{-5}(x-3)$

$y-4 = \frac{2}{5}(x-3)$

$y-4 = \frac{2}{5}x-\frac{6}{5}$

$y = \frac{2}{5}x-\frac{6}{5}+4$

$y = \frac{2}{5}x+\frac{26}{5}$

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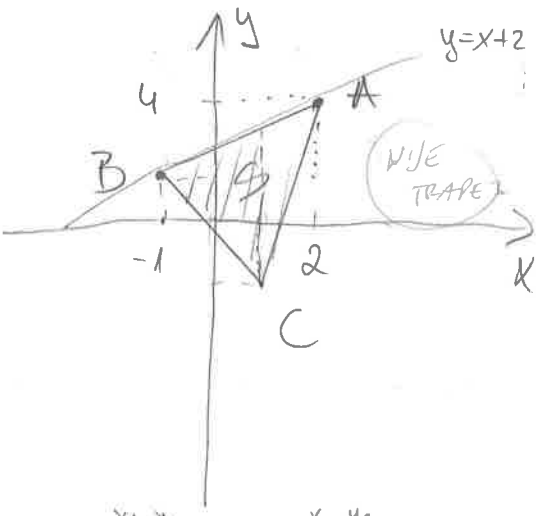

④ $A(2,4)$, $B(-1,1)$, $C(1,-1)$

$$I_1 = \int_{-1}^{1} \int_{BC}^{BA} x+y\,dx = \int_{-1}^{1} \int_{-x-2}^{x+2} x+y\,dx$$

$$I_2 = \int_{1}^{2} \int_{CA}^{BA} x+y\,dx = \int_{1}^{2} \int_{5x-6}^{x+2} x+y\,dx$$

$P = I_1 + I_2$

BC $B(-1,1)$, $C(1,-1)$ AB, $A(2,4)$, $B(-1,1)$ (x_1, y_1; x_2, y_2)

$y+1 = \frac{-1-1}{1+1}(x+1)$ $\quad y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y+1 = -x-1$ $\quad y-4 = \frac{1-4}{-1-2}(x-2)$

$y = -x-2$

AC $A(2,4)$, $C(1,-1)$ $\quad y-4 = 1(x-2)$

$y+1 = \frac{-1-4}{1-2}(x-1)$ $\quad y-4 = x-2$

$y+1 = 5(x-1)$ $\quad y = x-2+4$

$y+1 = 5x-5$ $\quad y = x+2$

$y = 5x-6$

⑤ $f(x,y,z) = y$ $\quad x = z^2$, $z = x$, $y = -1$, $y = 1$

$$\iiint x\,y\,z \;\; dz\,dy\,dx = ?$$

$x = z^2$

$z = x$