MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: RJEŠENJA BROJ INDEKSA:	
Kod kojeg nastavnika želite ustmeni?	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) + 3x'(t) = t, \quad x'(0) = x''(0) = 0,\ x(0) = 1.$$

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3. Neka je kvadrat $K = \{(x, y, z) : x = 4,\ y \in [-2, 2],\ z \in [-2, 2]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} y ds$? 20

4. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2 + y^2 + z^2 = 9$ za koji vrijedi $z \ge 2$. 20

5. Neka je K krug radijusa $r = 4$ sa centrom u točki $T(0, 2)$. Izračunati $\int\limits_K (1 - 3x)\, dxdy$. 20

Ukupno:

1) BRAJKOVIĆ

2)

$$r(x,y) = \begin{pmatrix} x \\ y \\ \frac{x^2}{3} + \frac{y^2}{3} \end{pmatrix}, \quad \partial_x r = \begin{pmatrix} 1 \\ 0 \\ \frac{2x}{3} \end{pmatrix}, \quad \partial_y r = \begin{pmatrix} 0 \\ 1 \\ \frac{2y}{3} \end{pmatrix}$$

$$n(x,y) = \partial_x r \times \partial_y r = \begin{pmatrix} 1 \\ 0 \\ \frac{2}{3}x \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ \frac{2}{3}y \end{pmatrix} = \begin{pmatrix} -\frac{2}{3}x \\ -\frac{2}{3}y \\ 1 \end{pmatrix}$$

$$\|n(x,y)\| = \sqrt{\frac{4}{9}x^2 + \frac{4}{9}y^2 + 1}$$

$$P = \iint\limits_D \|n(x,y)\|\, dxdy = \int_0^{2\pi}\int_0^2 \sqrt{\frac{4}{9}r^2 + 1} \cdot r\, dr d\varphi = 2\pi \int_0^2 r\sqrt{1 + \frac{4}{9}r^2}\, dr$$

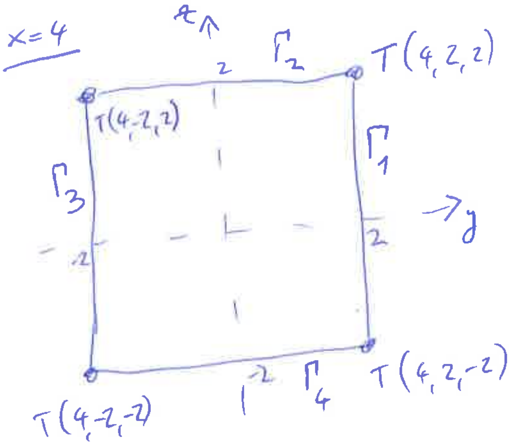
$$= \left\{ \begin{array}{l} t = 1 + \frac{4}{9}r^2 \\ dt = \frac{8}{9} r dr \end{array} \right\} = 2\pi \int_1^{\frac{25}{9}} \sqrt{t} \cdot \frac{9}{8} dt = \dots$$

$r dr = \frac{9}{8} dt$

$r = 0 \Rightarrow t = 1$

$r = 2 \Rightarrow t = 1 + \frac{16}{9} = \frac{25}{9}$

3) $x=4$



PARAMETRIZIRATI SVAKU OD 4 STRANICE KVADRATA

$$\Gamma_1 : \ r_1(t) = \begin{pmatrix} 4 \\ 2 \\ t \end{pmatrix}, \quad t \in [-2,2], \quad \sqrt{r_1'(t)^2} = 1$$

$$\Gamma_2 : \ r_2(t) = \begin{pmatrix} 4 \\ t \\ 2 \end{pmatrix}, \quad t \in [-2,2], \quad \sqrt{r_2'(t)^2} = 1$$

$$\Gamma_3 : \ r_3(t) = \begin{pmatrix} 4 \\ -2 \\ t \end{pmatrix}, \quad t \in [-2,2], \quad \sqrt{r_3'(t)^2} = 1$$

$$\Gamma_4 : \ r_4(t) = \begin{pmatrix} 4 \\ t \\ -2 \end{pmatrix}, \quad t \in [-2,2], \quad \sqrt{r_4'(t)^2} = 1$$

$$\iint_{\partial K} y\,ds = \int_{\partial K} y\,ds = \int_{\Gamma_1} y\,ds + \int_{\Gamma_2} y\,ds + \int_{\Gamma_3} y\,ds + \int_{\Gamma_4} y\,ds$$

$$= \underbrace{\int_{-2}^{2} 2\cdot 1\,dt}_{=8} + \underbrace{\int_{-2}^{2} t\cdot 1\,dt}_{=0} + \underbrace{\int_{-2}^{2} (-2)\cdot 1\,dt}_{=-8} + \underbrace{\int_{-2}^{2} t\cdot 1\,dt}_{=0} = 0$$

DVOSTRUKI INTEGRAL JE POGREŠNA OZNAKA. ∂K JE RUB KVADRATA, TO JE KRIVULJA. ds SE JAVLJA KOD KRIVULJNIH INTEGRALA SKALARNE FJE.

OVO JE DOBRA OZNAKA

4) VIDI BRAJKOVIĆ

5) VIDI BRAJKOVIĆ

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: TINO BRAJKOVIĆ BROJ INDEKSA: A-2-0100-2011	Broj ↓ bodova
Kod kojeg nastavnika želite ustmeni? PROF. UGLEŠIĆ	

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Ukupno: 60

1. $x'''(t) + 3x'(t) \quad x'(0) = x''(0) = 0, \quad x(0) = 1 \quad / \text{Laplace}$

$x'(0) = x''(0)$

$x(0) = 1$

$$s^3 X(s) - s^2 \underbrace{x(0)}_{=1} - \underbrace{s x'(0)}_{=0} - \underbrace{x''(0)}_{=0} + 3s X(s) - 3\underbrace{x(0)}_{=1} = \frac{1}{s^2}$$

$$X(s) = \frac{1}{s^2(s^3+3s)} + \frac{s^2+3}{s^3+3s} \quad / (s^3+3s)$$

$$X(s)(s^3+3s) = \frac{1}{s^2(s^3+3s)} + \frac{\cancel{s^2+3}}{s(\cancel{s^2+3s})}$$

$$X(s) = -\frac{1}{9}\cdot\frac{1}{s} + \frac{1}{3}\cdot\frac{1}{s^3} + \frac{1}{9}\cdot\frac{s}{s^2+3} + \frac{1}{s}$$

$$X(s) = -\frac{1}{9}\cdot\frac{1}{s} + \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{2}{s^3} + \frac{1}{9}\cdot\frac{s}{s^2+3} + \frac{1}{s} \quad / \mathcal{L}^{-1}$$

$$x(t) = -\frac{1}{9} + \frac{1}{6}t^2 + \frac{1}{9}\cdot\cos(\sqrt{3}t) + 1 \quad \checkmark$$

$$x(t) = \frac{8}{9} + \frac{1}{6} + t^2 + \frac{1}{9}\cos(\sqrt{3}t)$$

$\Rightarrow$

RASTAV NA PARCIJALNE RAZLOMKE

$$\frac{1}{s^2(s^2+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3}$$

$$1 = A s^2(s^2+3) + B s(s^2+3) + C(s^2+3) + (Ds+E)(s^3)$$

$A+D=0 \Rightarrow D=\frac{1}{9}$

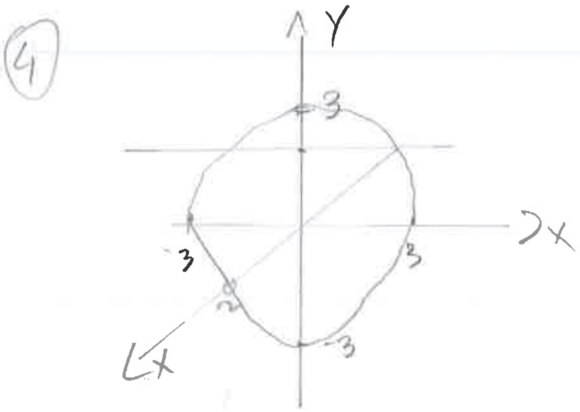
$B+E=0 \Rightarrow E=0$

$3A+C=0 \Rightarrow A=\frac{-1}{9}$

$3B=0 \Rightarrow B=0$

$3C=1 \Rightarrow C=\frac{1}{3}$

(4)



$z=2$

$x^2+y^2=9-z^2$

$x^2+y^2=9-4$

$r^2\cos^2\varphi + r^2\sin^2\varphi = 5$

$r^2=5$

$r=\sqrt{5}$

$$V=\int_0^{2\pi} d\varphi \int_0^{\sqrt{5}} dr \int_2^{\sqrt{9-r^2}} r\,dz = \int_0^{2\pi} d\varphi \int_0^{\sqrt{5}} r\left(\sqrt{9-r^2}-2\right)dr$$

$$= \int_0^{2\pi} d\varphi \left[\left(-\frac{1}{3}(9-r^2)^{3/2}\right)\Big|_0^{\sqrt{5}} - 2\frac{r^2}{2}\Big|_0^{\sqrt{5}}\right]$$

$$= \int_0^{2\pi} \frac{4}{3}\,d\varphi = 2\pi \cdot \frac{4}{3} = \frac{8\pi}{3}$$

CILINDRIČNE KOORDINATE

$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

$2 \le z \le \sqrt{9-x^2-y^2}$

$2 \le z \le \sqrt{9-r^2\cos^2\varphi - r^2\sin^2\varphi}$

$\varphi=[0,2\pi]$

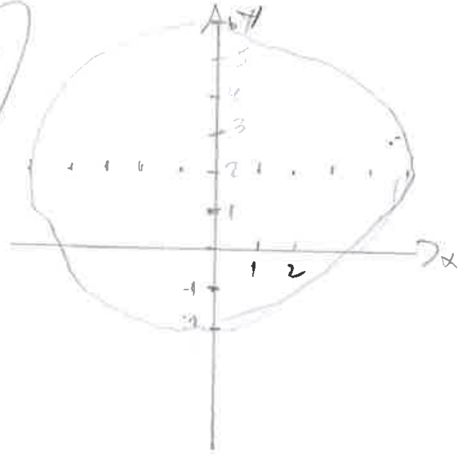
$2 \le z \le \sqrt{9-r^2}$

$\Downarrow$

$r \in [0,\sqrt{5}]$

TINO BRAŠKOVIĆ

5.



POMAKNUTE POLARNE KOORDINATE

$x = r\cos\varphi \quad r \in [0, 4]$

$y = r\sin\varphi + 2 \quad \varphi \in [0, 2\pi]$

Jakobian $= r$

$$\int_0^{2\pi} d\varphi \int_0^4 r \cdot (1 - 3r\cos\varphi)\, dr =$$

$$= \int_0^{2\pi} d\varphi \left(\frac{r^2}{2} - 3\cos\varphi \frac{r^3}{3} \right)\Big|_0^4$$

$$= \int_0^{2\pi} d\varphi \left(8 - 3\cos\varphi \cdot \frac{4^3}{3} \right) = \int_0^{2\pi} d\varphi \, (8 - 64\cos\varphi)$$

$$= 16\pi$$

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IME I PREZIME: ANĐELO ZMIRE BROJ INDEKSA:	
Kod kojeg nastavnika želite ustmeni? PROF. UGLEŠIĆ	

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Ukupno: 55

1. $x'''(t) + 3x'(t) = t \quad x'(0) = x''(0) = 0 \quad x(0) = 1$

$$s^3F(s) - s^2f(0) - sf'(0) - f''(0) + 3(sF(s) - f(0)) = \frac{1}{s^2}$$

$$s^3F(s) - s^2\underset{1}{\underset{\Downarrow}{f(0)}} - s\underset{0}{\underset{\Downarrow}{f'(0)}} - \underset{0}{\underset{\Downarrow}{f''(0)}} + 3sF(s) - 3\underset{1}{\underset{\Downarrow}{f(0)}} = \frac{1}{s^2}$$

$$s^3F(s) - s^2 + 3sF(s) - 3 = \frac{1}{s^2}$$

$$s^3F(s) + 3sF(s) = \frac{1}{s^2} + s^2 + 3$$

$$F(s)\, s(s^2+3) = \frac{1 + s^4 + 3s^2}{s^2}$$

$$F(s) = \frac{1+s^4+3s^2}{s^3(s^2+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+3} \quad / \cdot s^3(s^2+3)$$

$$1 + s^4 + 3s^2 = A\cdot(s^2(s^2+3)) + B(s(s^2+3)) + C(s^2+3) + (Ds+E)\cdot s^3$$

$$1 + s^4 + 3s^2 = A(s^4+3s^2) + B(s^3+3s) + C(s^2+3) + (Ds+E)s^3$$

$$1 + s^4 + 3s^2 = As^4 + \underbrace{3As^2} + Bs^3 + 3Bs + \underbrace{Cs^2} + \underbrace{3C} + Ds^4 + Es^3$$

$3C = 1$

$\boxed{C = \frac{1}{3}}$

$A + D = 1 \Rightarrow \frac{8}{9} + D = 1$

$3A + C = 3$

$3A + \frac{1}{3} = 3$

$3A = 3 - \frac{1}{3}$

$3A = \frac{8}{3} \ /:3$

$\boxed{A = \frac{8}{9}}$

$D = 1 - \frac{8}{9}$

$\boxed{D = \frac{1}{9}}$

$B + E = 0$

$3B = 0$

$\boxed{B = 0}$

$\boxed{E = 0}$

$$F(s)=\frac{\frac{8}{9}}{s}+\frac{\frac{1}{3}}{s^3}+\frac{\frac{1}{9}s}{s^2+3}$$

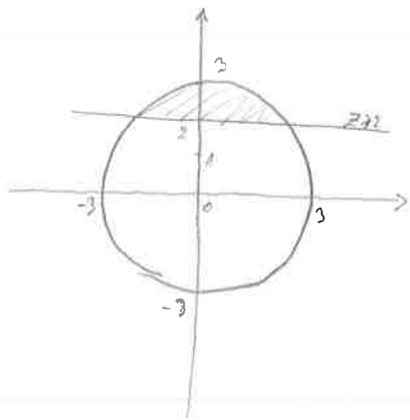
$$=\frac{8}{9}\cdot\frac{1}{s}+\frac{1}{3}\cdot\frac{1}{s^2}+\frac{1}{9}\quad\frac{s}{s^2+(\sqrt{3})^2}$$

$$=\frac{8}{9}+\frac{1}{3}t+\frac{1}{9}\cos(\sqrt{3}t)$$

$$\frac{8}{9}+0+\frac{1}{9}\cdot\cos(\sqrt{3}\cdot 0)$$

$$\frac{8}{9}+\frac{1}{9}\cdot 1=1 \checkmark$$

5. $x^2+y^2+z^2=9 \qquad z\geq 2$



$$x^2+y^2+z^2=R^2$$

$$r^2+z^2=R^2$$

$$r^2=R^2-z^2$$

$$r=\sqrt{R^2-z^2}$$

$$r=\sqrt{9-z^2}$$

$$x=r\cos\varphi$$

$$y=r\sin\varphi$$

$$z=z$$

$$dx\,dy\,dz=dr\,d\varphi\,dz$$

$$r\in[0,\sqrt{9-z^2}]$$

$$\varphi\in[0,2\pi]$$

$$z\in[2,3]$$

15

$$\int_0^{2\pi}\int_2^3\int_0^{\sqrt{9-z^2}}1\cdot r\,dr\,dz\,d\varphi=\int_0^{2\pi}\int_2^3\frac{(\sqrt{9-z^2})^2}{2}\,dz\,d\varphi=\int_0^{2\pi}\int_2^3\frac{9-z^2}{2}\,dz\,d\varphi$$

$$=\int_0^{2\pi}\int_2^3\frac{9}{2}-\frac{z^2}{2}\,dz\,d\varphi=\frac{9}{2}\int_0^{2\pi}\int_2^3 dz\,d\varphi-\frac{1}{2}\int_0^{2\pi}\int_2^3 z^2\,dz\,d\varphi=\frac{9}{2}\cdot 2\pi-\frac{1}{2}\cdot 5\cdot 2\pi=9\pi-5\pi=4\pi$$

ANĐELO ŽMIRE

5. $r = 4$

$T(0,2)$

$\int (1-3x)\,dx\,dy$

$x = 4\cos\varphi \qquad r\in[0,4]$

$y = 4\sin\varphi + 2 \qquad \varphi\in[0,2\pi]$

$$\int_0^{2\pi}\int_0^4 (1-3(4\cos\varphi))\cdot r\,dr\,d\varphi = \int_0^{2\pi}\int_0^4 r - 12r\cos\varphi\,dr\,d\varphi = \int_0^{2\pi}\int_0^4 r\,dr\,d\varphi - \int_0^{2\pi}\int_0^4 12r\cos\varphi\,dr\,d\varphi$$

$$= 16\pi - \int_0^{2\pi} 96\cos\varphi\,d\varphi = 16\pi \checkmark$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: ROKO KRALJEV BROJ INDEKSA: 17-2-0191-2012	Broj ↓ bodova
Kod kojeg nastavnika želite ustmeni?	

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$X(s)(s^3 + 3s) = \frac{1}{s^2} + s^2 + 3 \; / : (s^3 + 3s)$

$X(s) = \frac{1}{s^3(s^2+3)} + \frac{\cancel{s^2+3}}{s\cancel{(s^2+3)}} = \frac{1}{s^2(s^3+3s)} + \frac{1}{s}$

$\frac{1}{s^3(s^2+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+3} \; / \cdot s^3(s^2+3)$

$1 = As^2(s^2+3) + Bs(s^2+3) + C(s^2+3) + (Ds+E) \cdot s^3$

$A + D = 0 \qquad D = \frac{1}{9}$

$B + E = 0 \qquad E = 0$

$3A + C = 0 \qquad B = 0$

$3B = 0 \qquad C = \frac{1}{3}$

$3C = 1 \qquad A = -\frac{1}{9}$

$$x(s) = -\frac{1}{9}\cdot\frac{1}{s} + \frac{1}{3}\cdot\frac{1}{s^3} + \frac{1}{9}s\cdot\frac{1}{s^2+3} + \frac{1}{s} \; / \mathcal{L}$$

$$x(t) = -\frac{1}{9} + \frac{1}{6}t^2 + \frac{1}{9}\cos(\sqrt{3}\,t) + 1$$

$$x(t) = \frac{8}{9} + 16 \cdot t^2 + \frac{1}{9}\cos(\sqrt{3}\,t)$$

5.

$T(0,2) \qquad r = 4 \qquad \int_K (1-3x)\,dx\,dy$

$$x^2 + (y-2)^2 = 16$$

$$x = r\cos\varphi = 4\cos\varphi$$

$$y-2 = 4\sin\varphi \Rightarrow y = 4\sin\varphi - 2$$

$$\int (1-3x)\,dx\,dy = \iint (1-12\cos\varphi)\,r\,dr\,d\varphi =$$

$$= \iint r\,dr\,d\varphi - 12\iint \cos\varphi\; r\,dr\,d\varphi = \int_0^{2\pi} d\varphi \int_0^4 r\,dr -$$

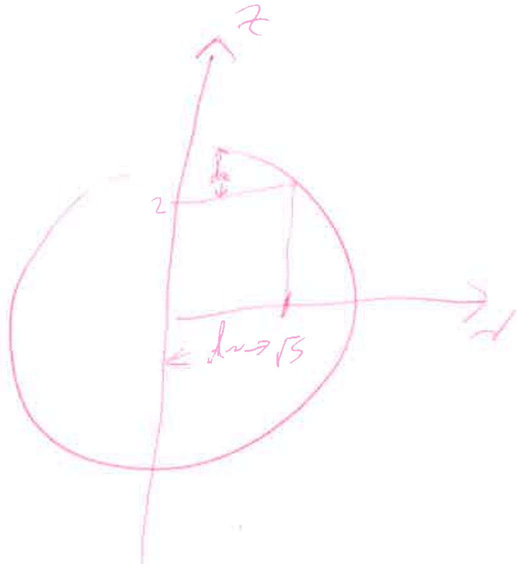
$$-12\int_0^{2\pi} \cos\varphi\,d\varphi \int_0^4 r\,dr = 2\pi\cdot\frac{r^2}{2}\Big|_0^4 - 12\int_0^{2\pi}\cos\varphi\,d\varphi\cdot\frac{r^2}{2}\Big|_0^4 =$$

$$= 16\pi - 12\int_0^{2\pi}\cos\varphi\,d\varphi\cdot 8 = 16\pi - 96\cdot\sin\varphi\Big|_0^{2\pi} =$$

$$= 16\pi - 96\cdot(0-0) = \underline{\underline{16\pi}}$$

IME I PREZIME: ROKO KRALJEV BROJ INDEKSA: 17-2-0191-2012

4. $x^2+y^2+z^2=9 \qquad z \geq 2$



$x^2+y^2+4=9$

$x^2+y^2=5 \qquad r=\sqrt{5}$

$x=r\cos\varphi$

$y=r\sin\varphi$

$z=z$

$\sqrt{9-x^2-y^2}=\sqrt{9-r^2\cos^2\varphi-r^2\sin^2\varphi}=\sqrt{9-r^2}$

$\int_0^{2\pi}\int_0^{\sqrt{5}}\int_2^{\sqrt{9-r^2}} 1\, r\, dz\, dr\, d\varphi = \dots$ ✓

15

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: IVAN RADOVIĆ **BROJ INDEKSA:** 57230	
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$$x'''(t) + 3x'(t) = t \qquad x'(0) = x''(0) = 0$$
$$x(0) = 1$$

$$s^3X(s) - s^2x(0) - sx'(0) - x''(0) + 3sX(s) - 3x(0) = \frac{1}{s^2}$$

$$s^3X(s) + 3sX(s) - s^2 - 3 = \frac{1}{s^2}$$

$$X(s)\left(s^3 + 3s\right) = \frac{s^4 + 3s^2 + 1}{s^2}$$

$$X(s) = \frac{s^4 + 3s^2 + 1}{s^2 \cdot s(s^2+3)} = \frac{s^4 + 3s^2 + 1}{s^3(s^2+3)}$$

$$\frac{s^4 + 3s^2 + 1}{s^3(s^2+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds + E}{s^2+3} \quad \Big/ \cdot s^3(s^2+3)$$

$$= As^2(s^2+3) + Bs(s^2+3) + C(s^2+3) + (Ds+E)s^3 = As^4 + 3As^2 + Bs^3 + 3Bs + Cs^2 + 3C + Ds^4 + Es^3$$

$s^4 \ldots 1 = A + D$

$s^3 \ldots 0 = B + E$

$s^2 \ldots 3 = 3A + C$

$s \ldots 0 = 3B \qquad \underline{B = 0}$

$\ldots 1 = 3C \qquad \underline{C = \frac{1}{3}}$

$$3 = 3A + \frac{1}{3}$$
$$3 - \frac{1}{3} = 3A$$
$$\frac{8}{3} = 3A \ /:3$$
$$\underline{A = \frac{8}{9}}$$

$$0 = 0 + E$$
$$\underline{E = 0}$$

$$1 = \frac{8}{9} + D$$
$$1 - \frac{8}{9} = D$$
$$\underline{\frac{1}{9} = D}$$

→

$A = \frac{8}{9}, \; B = 0, \; C = \frac{1}{3}, \; D = \frac{1}{9}, \; E = 0$

$$X(s) = \frac{8}{9} \cdot \frac{1}{s} + \frac{1}{3} \cdot \frac{1}{s^3} + \frac{1}{9} \frac{s}{s^2+3}$$

$$x(t) = \frac{8}{9} + \frac{1}{6} t^2 + \frac{1}{9} \cos(\sqrt{3}\, t)$$

$$x'(t) = \frac{1}{3} t - \frac{1}{9} \sqrt{3} \sin(\sqrt{3}\, t)$$

$$x''(t) = \frac{1}{3} - \frac{1}{9} \sqrt{3} \cdot \sqrt{3} \cos(\sqrt{3}\, t)$$

$$x(0) = \frac{8}{9} + \frac{1}{6} 0^2 + \frac{1}{9} \cos(\sqrt{3} \cdot 0) = \frac{8}{9} + \frac{1}{9} = 1 \, //$$

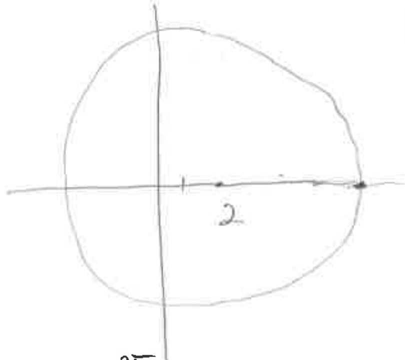
$$x'(0) = \frac{1}{3} 0 - \frac{1}{9} \cdot 0 = 0 \, //$$

$$x''(0) = \frac{1}{3} - \frac{1}{\cancel{9}_3} \cdot \cancel{3} \cdot 1 = 0 \, //$$

IVAN RADOVIĆ

(5) $r=4 \quad T(\overset{x}{0},\overset{y}{2})$

$$\int (1-3x)\,dxy$$



$r \in (0,4)$

$x = r\cos\varphi$

$y = r\sin\varphi + 2$

$\varphi = (0, 2\pi)$

$$\int_0^{2\pi}\int_0^{4} r(1-3r\cos\varphi)\,dr\,d\varphi = \int_0^{2\pi}\int r-3r^2\cos\varphi\,dr\,d\varphi$$

$$\int_0^{2\pi} \frac{r^2}{2} - 3\frac{r^3}{3}\cos\varphi\Big|_0^4 d\varphi = \int_0^{2\pi} 8-64\cos\varphi\,d\varphi = 8\varphi + 64\sin\varphi\Big|_0^{2\pi}$$

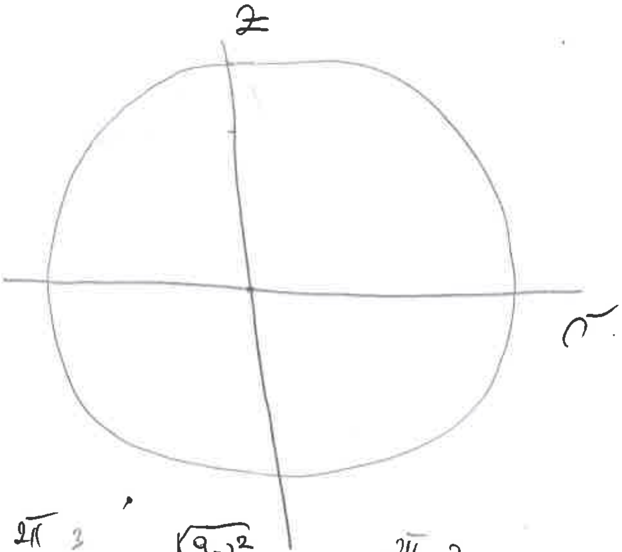
$$= 16\pi$$

(4) $x^2+y^2+z^2=9$

$r^2+z^2=9$

$r=\sqrt{9-z^2}$

$z \geq 2$



$\varphi \in (0, 2\pi)$

$r \in (z, \sqrt{9-z^2})$

$z(2,3)$

$$\int_0^{2\pi}\int_2^{3}\int_z^{\sqrt{9-z^2}} r\,dr\,dz\,d\varphi = \int_0^{2\pi}\int_2^{3} \frac{1}{2}r^2\Big|_z^{\sqrt{9-z^2}} dz\,d\varphi = \int_0^{2\pi}\int_2^{3} \frac{9}{2} - z^2\,dz\,d\varphi$$

$$\int_0^{2\pi} \frac{9}{2}z - \frac{1}{2}z^2\Big|_2^3 d\varphi = \int_0^{2\pi} \frac{27}{2} - 3^3 - 7 \quad d\varphi = \int_0^{2\pi} \frac{27-6-14}{2}\,d\varphi$$

$$\int_0^{2\pi} \frac{7}{2}\,d\varphi = 7\varphi\Big|_0^{2\pi} = 7\cdot 2\pi = 14\pi$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: ELENA BEG BROJ INDEKSA: 17-2-0181-2012	Broj ↓ bodova
Kod kojeg nastavnika želite ustmeni? Vuglesić	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) + 3\,x'(t) = t, \quad x'(0) = x''(0) = 0,\; x(0) = 1.$$

2. Koristeći plošni integral postaviti formulu za ploštinu dijela paraboloida $z = \frac{x^2}{3} + \frac{y^2}{3}$ što leži iznad područja $D \ldots x^2 + y^2 \leq 4$. Nije potrebno računati površinu baze. 20

3. Neka je kvadrat $K = \{(x,y,z) : x = 4,\ y \in [-2,2],\ z \in [-2,2]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} y ds$? 20

4. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2 = 9$ za koji vrijedi $z \geq 2$. 20

5. Neka je K krug radijusa $r = 4$ sa centrom u točki $T(0,2)$. Izračunati $\int\limits_K (1-3x)\,dxdy$. 20

Ukupno: 40

5. $r = 4 \quad T(0,2) \quad \iint\limits_K (1-3x)\,dxdy$

$x = r\cos\varphi$
$y = r\sin\varphi + 2$
$dxdy = r\,dr\,d\varphi$

$\varphi \in [0, 2\pi]$
$r \in [0, 4]$

$$\int_0^{2\pi}\int_0^4 (1-3(r\cos\varphi))r\,dr\,d\varphi = \int_0^{2\pi}\int_0^4 (1-3r\cos\varphi)\,r\,dr\,d\varphi =$$

$$= \int_0^{2\pi}\int_0^4 r - 3r^2\cos\varphi\,dr\,d\varphi = \underbrace{\int_0^{2\pi}\int_0^4 r\,dr\,d\varphi}_{\text{I.}} - \underbrace{\int_0^{2\pi}\int_0^4 3r^2\cos\varphi\,dr\,d\varphi}_{\text{II.}} =$$

I. $\displaystyle\int_0^{2\pi}\int_0^4 r\,dr\,d\varphi = \int_0^{2\pi} d\varphi \int_0^4 r\,dr = \int_0^{2\pi} d\varphi \left[\frac{r^2}{2}\right]\Big|_0^4 = \int_0^{2\pi}\left(\frac{4^2}{2} - 0\right)d\varphi$

$$= 8\int_0^{2\pi} d\varphi = 8\,[\varphi]\Big|_0^{2\pi} = 8\cdot 2\pi - 0 = 16\pi$$

II. $\displaystyle\int_0^{2\pi}\int_0^4 3r^2\cos\varphi\,dr\,d\varphi = \int_0^{2\pi}\cos\varphi\,d\varphi \int_0^4 3r^2\,dr = 3\int_0^{2\pi}\cos\varphi\,d\varphi\left[\frac{r^3}{3}\right]\Big|_0^4 =$

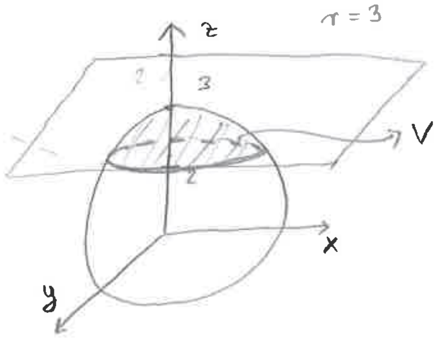
$$= 3\int_0^{2\pi}\cos\varphi\,d\varphi\left(\frac{4^3}{3} - 0\right) = \frac{64}{\cancel{3}}\cdot\cancel{3}\int_0^{2\pi}\cos\varphi\,d\varphi = 64\cdot[\sin\varphi]\Big|_0^{2\pi} = 64(\sin 2\pi - \sin 0)$$

$$= 64\cdot 0 = 0$$

$$\text{I} - \text{II} = 16\pi - 0 = 16\pi \;\checkmark$$

4. $x^2+y^2+z^2=9$ $\quad z \geq 2$

$x = r\cos\varphi$
$y = r\sin\varphi$
$z = z$
$dxdy = r\,dr\,d\varphi$



$\varphi \in [0, 2\pi]$

$r = ?$ $\quad x^2+y^2+z^2=9$

$r^2+z^2=9$

$r^2 = 9-z^2$

$r = \sqrt{9-z^2}$

$r \in [0, \sqrt{9-z^2}]$

$z \in [2,3]$

$$\int_0^{2\pi} d\varphi \int_2^3 dz \int_0^{\sqrt{9-z^2}} 1\, r\,dr = \int_0^{2\pi} d\varphi \int_2^3 dz \left[\frac{r^2}{2}\right]_0^{\sqrt{9-z^2}} = \int_0^{2\pi} d\varphi \int_2^3 \left[\frac{(\sqrt{9-z^2})^2}{2} - 0\right] dz$$

$$= \int_0^{2\pi} d\varphi \int_2^3 \frac{9-z^2}{2}\, dz = \frac{1}{2}\int_0^{2\pi} d\varphi \int_2^3 9-z^2\, dz =$$

$$= \frac{1}{2}\int_0^{2\pi} \left[9z - \frac{z^3}{3}\right]_2^3 d\varphi = \frac{1}{2}\cdot\int_0^{2\pi} \left[9\cdot 3 - \frac{3^3}{3} - \left(9\cdot 2 - \frac{2^3}{3}\right)\right] d\varphi =$$

$$= \frac{1}{2}\int_0^{2\pi} \frac{8}{3}\, d\varphi = \frac{1}{2}\cdot\frac{8}{3}\int_0^{2\pi} d\varphi = \frac{4}{3}\int_0^{2\pi} d\varphi = \frac{4}{3}\cdot[\varphi]\Big|_0^{2\pi} =$$

$$= \frac{4}{3}(2\pi - 0) = \frac{8\pi}{3}$$

3. $K = \{(x,y,z) : x = 4,\ y \in [-2,2],\ z \in [-2,2]$ $\quad \iint_{\partial K} y\,ds\ ?$

$$\iint_{\partial K} \begin{bmatrix} 4 \\ y \\ z \end{bmatrix} ds = \iiint_K \underbrace{\operatorname{div}\begin{bmatrix} 4 \\ y \\ z \end{bmatrix}}_{0} = 0$$

ELENA BEG
17-2-0181-2012

2. $z = \frac{x^2}{3} + \frac{y^2}{3}$ $D \ldots \; x^2+y^2 \leq 4$

$z = \frac{x^2+y^2}{3}$ $r^2 \leq 4$

$r \leq 2$

$r \in [0,2]$

$z = \frac{r^2}{3}$ $z \in [0, \frac{r^2}{3}]$

$$\boxed{\int_0^{2\pi} d\varphi \int_0^2 r\,dr \int_0^{\frac{r^2}{3}} dz} = \int_0^{2\pi} d\varphi \int_0^2 r\,dr\,[z]_0^{\frac{r^2}{3}} = \int_0^{2\pi} d\varphi \int_0^2 \frac{r^2}{3}\, r\,dr =$$

$$= \int_0^{2\pi} d\varphi \int_0^2 \frac{r^3}{3}\,dr = \frac{1}{3}\int_0^{2\pi} d\varphi \left[\frac{r^4}{4}\right]\Big|_0^2 = \frac{1}{3}\int_0^{2\pi} \frac{16}{4}\,d\varphi =$$

$$= \frac{4}{3}\int_0^{2\pi} d\varphi = \frac{4}{3}(\varphi)\Big|_0^{2\pi} = \frac{4}{3}\cdot(2\pi - 0) = \frac{8\pi}{3}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: MARKO ČULINA BROJ INDEKSA: 17-1-0008-2010	Broj ↓ bodova
Kod kojeg nastavnika želite ustmeni? PROF. UGLEŠIĆ	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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5. Neka je K krug radijusa $r = 4$ sa centrom u točki $T(0, 2)$. Izračunati $\int\limits_K (1 - 3x)\,dxdy$. 20

Ukupno: 40

MARKO ČULINA

17-1-0008-2010
08:50

1) $x'''(t) + 3x'(t) = t$ $\quad x'(0) = x''(0) = 0 \quad x(0) = 1$

$$s^3F(s) - s^2 \underbrace{f(0)}_{1} - s \underbrace{f'(0)}_{0} - \underbrace{f''(0)}_{0} + 3(sF(s) - \underbrace{f(0)}_{1}) = \frac{1}{s^2}$$

$$s^3F(s) - s^2 + 3sFs - 3 = \frac{1}{s^2}$$

$$Fs(s^3+3s) = \frac{1}{s^2} + s^2 + 3$$

$$Fs(s^3+3s) = \frac{1+s^4+3s^2}{s^2} \quad /(s^3+3s)$$

$$Fs = \frac{s^4+3s^2+1}{s^2(s^3+3s)} = \frac{s^4+3s+1}{s^2 \cdot s(s^2+3)}$$

$$s^3 = 0 \qquad s_{1,2} = \cancel{\frac{-0 \pm \sqrt{0^2 - 4 \cdot 1 \cdot 3}}{2}}$$

$$\frac{s^4+3s^2+1}{s^3(s^2+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{(s^2+3)} \quad /s^3(s^2+3)$$

$$s^4+3s^2+1 = As^2(s^2+3) + Bs(s^2+3) + C(s^2+3) + (Ds+E)(s^2+3)$$

$$s^4+3s^2+1 = As^4 + 3As^2 + Bs^3 + 3Bs + Cs^2 + 3C + Ds^3 + 3Ds + Es^2 + 3E$$

uz $s^4 \to \boxed{1 = A}$

uz $s^3 \to 0 = B + D \to 0 = D + D = 0 = 2D$ $\boxed{D=0}$

uz $s^2 \to 3 = 3A + C + E \to 3 = 3 - E + \frac{1}{3} + E$ $\boxed{E=0}$

uz $s \to 0 = 3B + 3D \to 3B = 3D \to B = D \to \boxed{B=0}$

SAMO $\to 1 = 3C + 3E \to 3C = 3E - 1 \ /:3 \quad C = -E + \frac{1}{3} \to C = \frac{1}{3}$

$$Fs = \frac{1}{s} + \frac{\frac{1}{3}}{s^3} = 1 + \frac{1}{3} \cdot \frac{1}{s^3} = 1 + \frac{1}{3} \cdot t^2$$

X

3)

$x = 4$

$y \in [-2,2]$

$z \in [-2,2]$

MARKO ČULINA

17-1-0008-2010
08:50

5) $r=4$ $\quad \iint_K (1-3x)\,dx\,dy$

$T(0,2)$

$r\in(0,4)$

$\varphi\in(0,2\pi)$

$x=r\cos\varphi+0$

$y=r\sin\varphi+2$

$$=\iint_K (1-3x)\,dx\,dy=\int_0^{2\pi}\int_0^4 (1-3x)\,r\,dr\,d\varphi$$

$$=\int_0^{2\pi}\int_0^4 (1-3r\cos\varphi)\,r\,dr\,d\varphi=\int_0^{2\pi}\int_0^4 r-3r^2\cos\varphi\,dr\,d\varphi$$

$$=\int_0^{2\pi}\frac{1}{2}r^2-r^3\cos\varphi\Big|_0^4\,d\varphi=\int_0^{2\pi}\frac{1}{2}4^2-4^3\cos\varphi\,d\varphi$$

$$=\int_0^{2\pi}8-64\cos\varphi\,d\varphi=8\varphi+64\sin\varphi\Big|_0^{2\pi}=8\cdot2\pi=16\pi$$

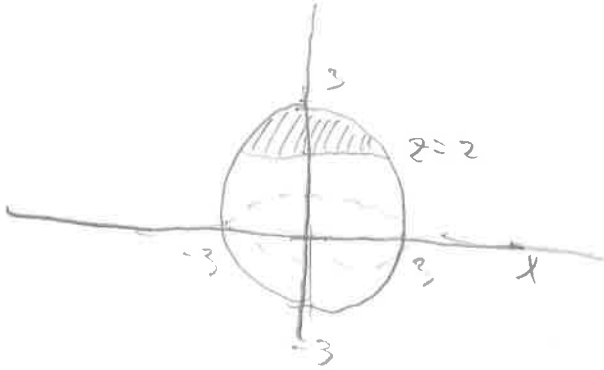
4) $x^2+y^2+z^2=9$ $\quad z\geq2$

$x^2+y^2+z^2=3^2$

$x^2+z^2=9$

$r^2=9-z^2$

$r=\sqrt{9-z^2}$



$r\in(0,\sqrt{9-z^2})$

$z\in(2,3)$

$\varphi\in(0,2\pi)$

$$=\int_0^{2\pi}\int_2^3\int_0^{\sqrt{9-z^2}} r\,dr\,dz\,d\varphi=\int_0^{2\pi}\int_2^3\frac{1}{2}r^2\Big|_0^{\sqrt{9-z^2}}dz\,d\varphi=\int_0^{2\pi}\int_2^3\frac{1}{2}\sqrt{9-z^2}^2\,dz\,d\varphi$$

$$=\int_0^{2\pi}\int_2^3\frac{1}{2}\,9-z^2\,dz\,d\varphi=\int_0^{2\pi}\int_2^3\frac{9}{2}-\frac{z^2}{2}\,dz\,d\varphi=\int_0^{2\pi}\frac{9}{2}z-\frac{1}{6}z^3\Big|_2^3\,d\varphi$$

$$=\int_0^{2\pi}\left(\frac{27}{2}-\frac{9}{2}\right)-\left(9-\frac{4}{3}\right)d\varphi=\int_0^{2\pi}\frac{4}{3}\,d\varphi=\frac{4}{3}\varphi\Big|_0^{2\pi}=\frac{4}{3}\cdot2\pi=\frac{8}{3}\pi$$

12.02.2015.

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: Rikardo Radovčić BROJ INDEKSA:

Kod kojeg nastavnika želite ustmeni?

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 40

(4)

$$x^2 + y^2 + z^2 = 9 \qquad z \ge 2$$

$$V = \int_2^3 \int_0^{2\pi} \int_0^{\sqrt{9-z^2}} r\,dr\,d\varphi\,dz = \int_2^3 \int_0^{2\pi} \left(\frac{r^2}{2}\right)\Big|_0^{\sqrt{9-z^2}} d\varphi\,dz =$$

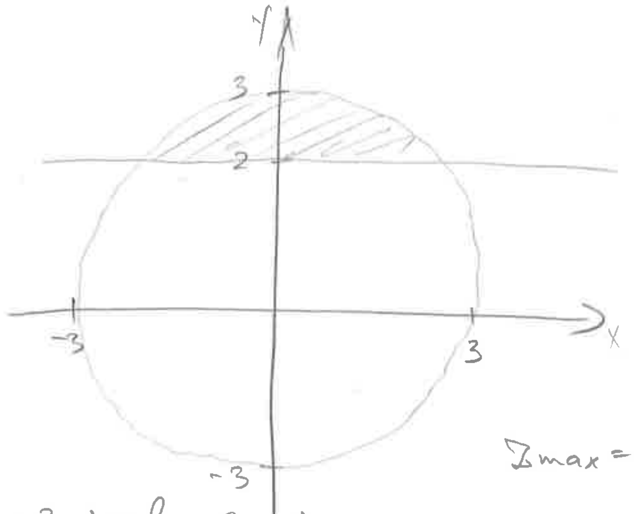
$$= \int_2^3 \int_0^{2\pi} \frac{\cancel{(\sqrt{9-z^2})^2} - 0}{2} = \int_2^3 \int_0^{2\pi} \frac{1}{2}(9 - z^2) = 2\pi \int_2^3 \frac{1}{2}(9 - z^2)\,dz$$

$$= 2\pi \cdot \frac{1}{2}\left(9 \cdot (3 - 2) - \frac{(27-8)}{3}\right)$$

$$= 2\pi \cdot \frac{1}{2}\left(9 - \frac{19}{3}\right)$$

$$= 2\pi \cdot \frac{1}{2} \cdot \frac{8}{3}$$

$$= \frac{8}{3}\pi \quad \checkmark$$



$z_{max} = 3$

$$r^2\cos^2\varphi + r^2\sin^2\varphi + z^2 = 9$$

$$r^2 + z^2 = 9$$

$$r = \sqrt{9 - z^2}$$

$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

① $x'''(t)+3x'(t)=t, \quad x'(0)=x''(0)=0, \quad x(0)=1$

$$x'''(t) \Rightarrow s^3 X(s) - s^2 x(0) - s x'(0) - x''(0)$$

$$\Rightarrow s^3 X(s) - s^2$$

$$x'(t) \Rightarrow s X(s) - 1$$

$$t \Rightarrow \frac{1}{s^2}$$

$$s^3 X(s) - s^2 + 3 s X(s) - 3 = \frac{1}{s^2}$$

$$X(s)(s^3+3s) = \frac{1}{s^2} + s^2 + 3$$

$$X(s)(s^3+3s) = \frac{1+s^4+3s^2}{s^2}$$

$$X(s) = \frac{1+s^4+3s^2}{s^3(s^2+3)}$$

$$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+3} \cdot s^3(s^2+3)$$

$$= A(s^2)(s^2+3) + B(s)(s^2+3) + C(s^2+3) + Ds+E(s^3)$$

$$= A(s^4+3s^2) + B(s^3+3) + C(s^2+3) + Ds+E(s^3)$$

$$= As^4 + 3As^2 + Bs^3 + 3B + Cs^2 + 3C + Ds^4 + Es^3$$

$s^4(A+D)$ $\quad D=\frac{1}{9} \quad B=0 \quad C=\frac{1}{3} \quad E=0$

$s^3(B+E)$

$s^2(3A+C)$ $\quad 3A+\frac{1}{3}=3$

$A+D=1$

$B+E=0$ $\quad X(s)=\frac{8}{9}\cdot\frac{1}{s}+\frac{1}{3}\cdot\frac{1}{s^3}+\frac{1}{9}\frac{s}{s^2+3}$ $\quad A=\frac{8}{9}$

$3A+C=3$ $\quad x(t)=\frac{8}{9}+\frac{1}{3}\cdot\frac{1}{2}t^2+\frac{1}{9}\cos(\sqrt{3}t)$

$3B=0$

$3C=1$

(2) $z = \frac{x^2}{3} + \frac{y^2}{3}$ $\quad D \dots \underbrace{x^2+y^2 \le 4}$ KRUG

DULJINA NORMALE :

$$\left.\begin{aligned} \partial_x z &= \frac{2x}{3} \\ \partial_y z &= \frac{2y}{3} \end{aligned}\right\} \Rightarrow \vec{m}(x,y) = \begin{pmatrix} \frac{2x}{3} \\ \frac{2y}{3} \\ \cancel{1} \end{pmatrix}, \quad \|\vec{m}\| = \sqrt{\frac{4x^2}{9} + \frac{4y^2}{9}} = \frac{2}{3}\sqrt{x^2+y^2}$$

POVRŠINA :

POLARNE KOORDINATE

$$P = \iint_D \|\vec{m}(x,y)\|\, dx\,dy = \iint_{x^2+y^2\le 4} \frac{2}{3}\sqrt{x^2+y^2}\, dx\,dy = \left\{\begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \\ r &\in [0, 4]^2 \\ \varphi &\in [0, 2\pi] \end{aligned}\right\} =$$

$$= \int_0^{2\pi}\int_0^{4} \frac{2}{3}\sqrt{(r\cos\varphi)^2 + (r\sin\varphi)^2}\cdot r \; dr\,d\varphi$$

$$= \frac{2}{3}\int_0^{2\pi}\int_0^{4} \sqrt{r^2(\cos^2\varphi + \sin^2\varphi)}\cdot r\; dr\,d\varphi$$

$$= \frac{2}{3}\int_0^{2\pi}\int_0^{4} r^2\, dr\,d\varphi = \frac{2}{3}\cdot 2\pi\cdot \left|\frac{r^3}{3}\right|_0^{4} = \frac{16\pi}{9}$$

Rikardo Radovčić $\quad$ 12.02.2015.

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: MARKO PARANČIN **BROJ INDEKSA:** 17-1,-0062-2011

Kod kojeg nastavnika želite ustmeni? Nikica Uglešić

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) + 3\,x'(t) = t, \quad x'(0) = x''(0) = 0,\ x(0) = 1.$$

2. Koristeći plošni integral postaviti formulu za ploštinu dijela paraboloida $z = \frac{x^2}{3} + \frac{y^2}{3}$ što leži iznad područja $D \ldots x^2 + y^2 \leq 4$. Nije potrebno računati površinu baze. 20

3. Neka je kvadrat $K = \{(x,y,z) : x = 4,\ y \in [-2,2],\ z \in [-2,2]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} y\,ds$? 20

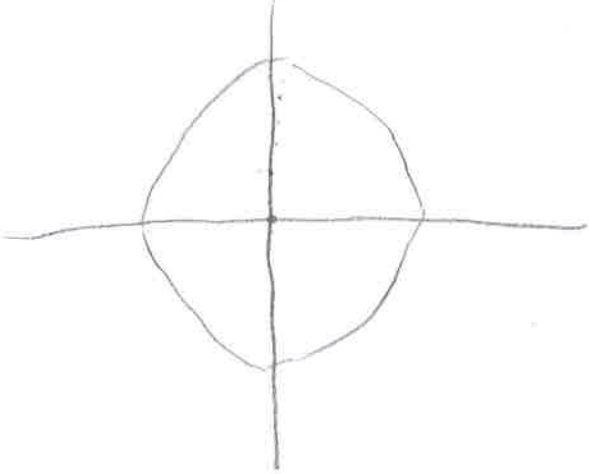
4. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2 + y^2 + z^2 = 9$ za koji vrijedi $z \geq 2$. 20 15

5. Neka je K krug radijusa $r = 4$ sa centrom u točki $T(0,2)$. Izračunati $\int\limits_K (1 - 3x)\,dx\,dy$. 20 15

Ukupno: 30

(5) $r=4$

$T=0,2$



$$\int_K (1-3x)\,dx\,dy = ?$$

$x = r\cos\varphi$

$y = r\sin\varphi$

$T(0,2)$

$x = r\cos\varphi$

$y = r\sin\varphi + 2$ $\quad dx\,dy = r\,dr\,d\varphi \quad r \in [0,4],\ \varphi \in [0,2\pi]$

$$\int_K (1-3x)\,dx\,dy = \int_0^{2\pi}\int_0^4 [1-3(r\cos\varphi)]\,r\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^4 1-3r\cos\varphi\,r\,dr\,d\varphi = \int_0^{2\pi}\int_0^4 1r - 3r^2\cos\varphi\,dr\,d\varphi = \int_0^{2\pi} 1\frac{r^2}{2} - 3\frac{r^3}{3}\cos\varphi\,d\varphi$$

$$= \int_0^{2\pi} 1\frac{4^2}{2} - 4^3\cos\varphi\,d\varphi = \int_0^{2\pi} 8 - 64\cos\varphi\,d\varphi = 8\varphi - 64\sin\varphi\Big|_0^{2\pi} = 8\pi$$

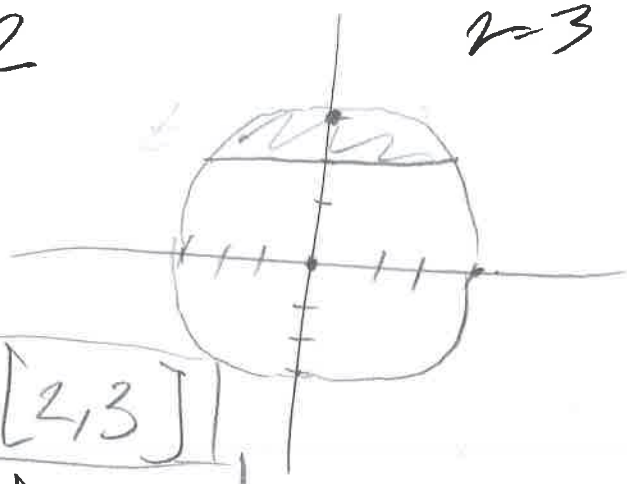
$\int_0^{2\pi} 8\cdot 4 =$

16π

15

④ $x^2+y^2+z^2=9$, $z \geq 2$ $r=3$

$x^2+y^2+z^2=r^2$



$$\boxed{y^2+z^2=9}$$
$$\boxed{r^2=9-z^2}$$
$$\boxed{r=+\sqrt{9-z^2}}$$

$$\boxed{z\in[2,3]}$$
$$\boxed{\varphi\in[0,2\pi]}$$
$$\boxed{r\in[0,\sqrt{9-z^2}]}$$

15

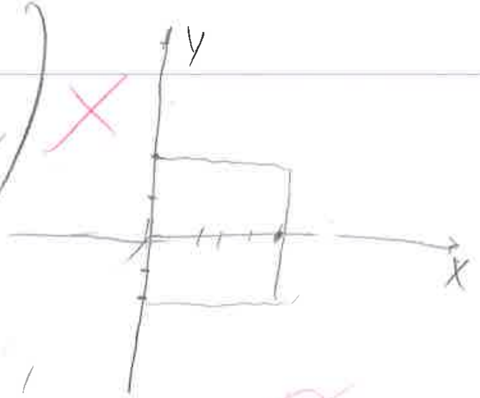
$$\iiint r\,dr\,dz\,d\varphi = \int_0^{2\pi}\int_2^3\int_0^{\sqrt{9-z^2}} r\,dr\,dz\,d\varphi =$$

$$\int_0^{2\pi}\int_2^3 \frac{r^2}{2}\Big|_0^{\sqrt{9-z^2}} dz\,d\varphi = \int_0^{2\pi}\int_2^3 \frac{(\sqrt{9-z^2})^2}{2} = \int_0^{2\pi}\int_2^3 \frac{9-z^2}{2}\,dz\,d\varphi = \int_0^{2\pi} \frac{9z-\frac{z^3}{3}}{2}\Big|_2^3 d\varphi$$

$$= \int_0^{2\pi} \left(\frac{9-\frac{3^3}{3}}{2}\right) - \left(\frac{9-\frac{2^3}{3}}{2}\right) = \int_0^{2\pi} \cancel{\left(\frac{9-9}{2}\right)} - \left(\frac{19}{6}\right) d\varphi = \int_0^{2\pi} -\frac{19}{6}\Big|_0^{2\pi} = -\frac{19}{3}\pi$$

③ $K=\{x,y,z\}: x=4,\ y\in[-2,2],\ z\in[-2,2]$ $\int_{\partial K} y\,ds$

$$r(t)=\begin{pmatrix}4\\-2,2\\-2,2\end{pmatrix} = r'(t)=\begin{pmatrix}4x\\-2y,2y\\-2z,2z\end{pmatrix}$$



$$\|r'(t)\| = \sqrt{(4x)^2+(-2y+2y)^2+(-2z+2z)^2}$$

$$=\sqrt{16x+(-4y+4y)^2+(-2z+2z)^2}$$

$$=\sqrt{16x}$$

$$\int_{-2}^{2}\int_{-2}^{2}\int_0^4 \sqrt{16x}\,dt \quad \ldots$$

0

MARKO PAKUNCIN

① $x'''(t) + 3x'(t) = t \qquad x'(0) = x''(0) = 0$

$x(0) = 1$

$s^3F(s) - s^2 f(0) - sf'(0) - f''(0) + 3(sF(s) - f(0)) = \frac{1}{s^2}$

$s^3F(s) - s^2 \cdot 1 - \cancel{s \cdot 0} - \cancel{0} + 3sF_s + 3 \cdot 0 = \frac{1}{s^2}$

$s^3F(s) - s^2 + 3sF(s) = \frac{1}{s^2}$

$F(s)(s^3 + 3s) - s^2 = \frac{1}{s^2}$

$F(s)(s^3+3s) = \frac{1}{s^2} + s^2 \Big/ \cdot \frac{1}{s^3+3s}$

$F(s) = \frac{\frac{1}{s^2} + s^2}{s^3+3s} \Rightarrow s(s^2+3)$

$F(s) = \frac{A}{s} + \frac{Bs+C}{(s^2+3)} \Rightarrow A(s^2+3) + (Bs+C) \cdot s = As^2 + 3A + Bs^2 + Cs$

za $s=0 \Rightarrow 3A = A = 3$

za $s=0$ $1 \Rightarrow C$

za $s^2 = 1 = A + B$

$= B = 3-1$

$B = 2$

$F(s) \frac{3}{s} + \frac{2s+7}{(s^2+3)} \Rightarrow$

$f(s) = 3 + \cos(3)$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: Marko Mustać **BROJ INDEKSA:** 17-2-0135-2011

Kod kojeg nastavnika želite ustmeni?

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) + 3x'(t) = t, \quad x'(0) = x''(0) = 0, \; x(0) = 1.$$

2. Koristeći plošni integral postaviti formulu za ploštinu dijela paraboloida $z = \frac{x^2}{3} + \frac{y^2}{3}$ što leži iznad područja $D \ldots x^2 + y^2 \leq 4$. Nije potrebno računati površinu baze. 20

3. Neka je kvadrat $K = \{(x,y,z) : x = 4, \, y \in [-2,2], \, z \in [-2,2]\}$. Kako preko definicije izračunati $\iint_{\partial K} y ds$? 20

4. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2 = 9$ za koji vrijedi $z \geq 2$. 20

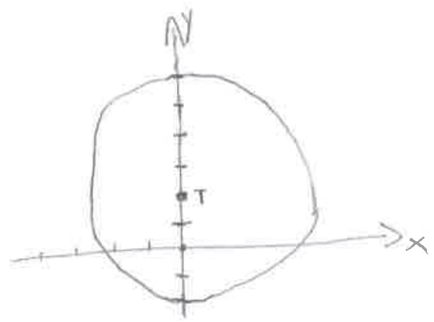
5. Neka je K krug radijusa $r = 4$ sa centrom u točki $T(0,2)$. Izračunati $\int_K (1-3x)\, dxdy$. 20

Ukupno: Ø

5.) $r = 4$

$T(0,2)$

$$\int_K (1-3x)\, dx\, dy$$



$x = r\cos\varphi + P$ $\quad x = 4\cos\varphi$

$y = r\sin\varphi + Q$ $\quad y = 4\sin\varphi + 2$

$dx\,dy = r\,dr\,d\varphi$

$$\int_K (1 - 3\cdot(4\cos\varphi)\cdot 4 \; dr\, d\varphi$$

$$\int_0^{2\pi}\int_0^4 (1 - 12\cos\varphi \cdot 4)\, dr\, d\varphi$$

$$\int_0^{2\pi}\int_0^4 (1 - 48\cos\varphi)\, dr\, d\varphi$$

$\int 1\, dr - \int 48\cos\varphi\, dr$

$1\int dr - 48\cos\varphi \int dr$

$1\cdot r\Big|_0^4 - 48\cos\varphi \cdot r\Big|_0^4$

$$\int_0^4 (1 - 48\cos\varphi)\, dr = 1\cdot r\Big|_0^4 - 48\cos\varphi \cdot r\Big|_0^4$$

$= 1\cdot 4 - 1\cdot 0 - 48\cos\varphi\cdot 4 + 48\cos\varphi\cdot 0$

$= 4 - 48\cos\varphi\cdot 4$

$$= 4 - 192 \cos \varphi$$

$$\int_0^{2\pi} (4 - 192 \cos \varphi) \, d\varphi$$

$$\int_0^{2\pi} 4 \, d\varphi - \int_0^{2\pi} 192 \cos \varphi \, d\varphi$$

$$4 \cdot \varphi \Big|_0^{2\pi} - 192 \cdot \sin \varphi \Big|_0^{2\pi}$$

$$4 \cdot 2\pi - 4 \cdot 0 - 192 \cdot \sin(2\pi) + 192 \cdot 0$$

$$= 8\pi$$

1.) $x'''(t) + 3x'(t) = t$ $\qquad x'(0) = x''(0) = 0 \quad x(0) = 1$ Marko Mustać

$$s^3F(s) - s^2x(0) - sx'(0) - x''(0) + 3\cdot\left(sF(s) - x(0)\right) = \frac{1}{s^2}$$

$$s^3F(s) - s^2\cdot 1 - s\cdot 0 - 0 + 3sF(s) - 3 = \frac{1}{s^2}$$

$$s^3F(s) - s^2 - 0 - 0 + 3sF(s) - 3 = \frac{1}{s^2}$$

$$s^3F(s) - s^2 + 3sF(s) - 3 = \frac{1}{s^2}$$

$$s^3F(s) + 3sF(s) = \frac{1}{s^2} + s^2 + 3$$

$$F(s)\cdot(s^3 + 3s) = \frac{1}{s^2} + s^2 + 3 \quad / :(s^3+3s)$$

$$F(s) = \frac{1}{s^2\cdot(s^3+3s)} + \frac{s^2+3}{s^3+3s}$$

$$\frac{1}{s^2\cdot(s^3+3s)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3+3s} \quad / \cdot \; s^2\cdot(s^3+3s)$$

$$1 = A\cdot s(s^3+3s) + B\cdot(s^3+3s) + C\cdot s^2$$

$$1 = As^4 + 3As + Bs^3 + 3Bs + Cs^2$$

$s^2\cdot s(s^2+3s)$

$s^3(s^2+3s)$

$$s_{1,2} = \frac{-3 \pm \sqrt{3^2 - 4\cdot 1\cdot 0}}{2}$$

$s_1 = 0$

$s_2 = -3$

$0 = A$

$0 = B$

$0 = C$

$0 = 3A + 3B$

$1 =$

DALJE. — ✗

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: Ivan Klanac BROJ INDEKSA: 17-2-0098-2011	Broj ↓ bodova
Kod kojeg nastavnika želite ustmeni? Prof. Uglešić	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: — ~~20~~

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4. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2 + y^2 + z^2 = 9$ za koji vrijedi $z \geq 2$. — 20
5. Neka je K krug radijusa $r = 4$ sa centrom u točki $T(0, 2)$. Izračunati $\int\limits_K (1 - 3x)\,dxdy$. — ~~20~~

Ukupno: 0

① $x'''(t) + 3x'(t) = t \quad x'(0) = x''(0) = 0 \quad x(0) = 1$

$$F(s) = \frac{1 + s^4 + s^2}{s^3(s^2+1)}$$

$$F(s) = \frac{1 + s^4 + s^2}{s^3 - (s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s^2+1}$$

$$s^4 + s^2 + 1 = As^2(s^2+1) + Bs(s^2+1) + C(s^2+1) + (Ds+E)s^3$$

$$= s^4(A+D) + s^3(B+E) + s^2(A+C) + s(B+C)$$

$A + D = 1 \Rightarrow D = 1$

$B + C = 0 \Rightarrow E = 0$

$A + C = 1 \Rightarrow A = 0$

$B = 0$

$C = 1$

$\Rightarrow$

$$F(s) \doteq \frac{1}{s^3} + \frac{s}{s^2+1}$$

$$F(s) \doteq \frac{1}{2} \cdot \frac{2}{s^3} + \frac{s}{s^2+1^2}$$

$$x(t) = \frac{1}{2} t^2 + \cos t$$

$$x(0) = \frac{1}{2} \cdot 0^2 + \cos 0 = 1$$

$$x'(0) = \left(\frac{1}{2} t^2\right)' + (\cos t)'$$

$$x'(0) = 0 - \sin 0 = 0$$

$$x''(0) = (t') - (\sin t)'$$

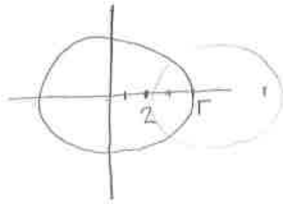
$$x''(0) = 1 - \cos 0 = 0$$

PROVJERA

$$x''' + 3x' = \sin t + 3 \cdot (t - \sin t) = 3t - 2\sin t$$

X

Ivan Klanac 17-2-0098-2011



(5) $r=4$

$c = T(0,2)$

$\int_K (1+3x)\,dx\,dy$

$x = r\cos\varphi$

$y =$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: PETAR JELAVIĆ MITROVIĆ BROJ INDEKSA: 17-2-0245-2012	Broj ↓ bodova
Kod kojeg nastavnika želite ustmeni?	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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Ukupno: Ø

1. $x'''(t) + 3x'(t) = t \qquad x'(0) = x''(0) = 0 \qquad x(0) = 1$

$x'''(t) \Rightarrow s^3 X(s) - s^2 X(0) - s\,X'(0) - X''$

$\Rightarrow s^3 X(s) - s^2$

$X'(t) \Rightarrow s X(s) - 1$

$t \Rightarrow \frac{1}{s^2}$

$s^3 X(s) - s^2 + 3s X(s) - 3 = \frac{1}{s^2}$

$X(s)(s^3 + 3s) = \frac{1}{s^2} + s^2 + 3$

$X(s) = \dfrac{1 + s^4 + 3s^2}{s^3(s^2+3)}$

$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds + E}{s^2+3} \cdot s^3(s^2+3)$

$= A(s^2)(s^2+3) + B(s)(s^2+3) + C(s^2+3)$
$+ Ds + E \quad (s^3)$

$= A(s^4 + 3s^2) + B(s^3 + 3) + C(s^2+3) + Ds + E(s^3)$

$A s^4 + 3As^2 + Bs^3 + 3B + Cs^2 + 3C + Ds^4 + Es^3$

$s^4 (A + D) \qquad D = \frac{1}{9} \qquad B = 0 \qquad C = \frac{1}{3}$

$s^3 (B + E)$

$s^2 (3A + C) \qquad E = 0 \qquad 3A + \frac{1}{3} = 3$

$A = \frac{8}{9}$

$A + D = 1$
$B + E = 0$
$3A + C = 3$
$3B = 0$
$3C = 1$

$$\frac{5}{9} \cdot \frac{1}{s} + \frac{1}{3} \cdot \frac{1}{s^3} + \frac{1}{9} \frac{s}{s^2+3}$$

$$= \frac{8}{9} + \frac{1}{3} \cdot \frac{1}{2} t^2 + \frac{1}{9} \cos(\sqrt{3}\, t)$$

2. PLOŠTINA PARABOLA

$$z = \frac{x^2}{3} + \frac{y^2}{3}$$

IZNAD $D \ldots x^2 + y^2 \le 4$

KRUG

NORMALA

$$\partial_x z = \frac{2}{3} x$$

$$\partial_y z = \frac{2y}{3}$$

$$n(x,y) = \begin{pmatrix} \frac{2x}{3} \\ \frac{2y}{3} \end{pmatrix},$$

$$\|\vec{n}\| = \sqrt{\frac{4}{9}x^2 + \frac{4}{9}y^2} = \frac{2}{3}\sqrt{x^2+y^2}$$

POVRŠINA

$$P = \iint \|\vec{n}(x,y)\|\, dx\, dy = \iint\limits_{x^2+y^2 \le 4} \frac{2}{3}\sqrt{x^2+y^2}\, dx\, dy =$$

POLARNE KOORDINATE

$x = r\cos\varphi$
$y = r\sin\varphi$
$r \in [0, 2]$
$\varphi \in [0, 2\pi]$

$$P = \int^{2\pi}\int^{2} \frac{2}{3}\sqrt{r^2(\underbrace{\cos^2\varphi + \sin^2\varphi}_{1})} \cdot r\, d\varphi \cdot dr$$

$$P = \int_0^{2\pi}\int_0^{2} \frac{2}{3} \cdot r \cdot r \cdot d\varphi \cdot dr$$

$$P = \frac{2}{3}\int^{2\pi}\int^{2} r^2\, dr\, d\varphi = \frac{2}{3} \cdot \int_0^{2\pi} \frac{1}{3} \cdot r^3 \Big|_0^2 d\varphi = \frac{2}{9}\int_0^{2\pi} (8-0)\, d\varphi$$

$$P = \frac{2}{9} \cdot 8(2\pi - 0) = \frac{32\pi}{9}$$

0

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: JELENA MALEŠ **BROJ INDEKSA:** 17-2-0103-2011	Broj ↓ bodova
Kod kojeg nastavnika želite ustmeni?	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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Ukupno: 0

1. $x'''(t) + 3x'(t) = t \qquad x'(0) = x''(0) = 0, \; x(0) = 1$

$$s^3F(s) - s^2 \underbrace{f(0)}_{1} - s\underbrace{f'(0)}_{0} - \underbrace{f''(0)}_{0} + 3\cdot(sF(s) - \underbrace{f(0)}_{1}) = \frac{1}{s^2}$$

$$s^3F(s) - s^2 + 3sF(s) - 1 = \frac{1}{s^2}$$

$$s^3F(s) + 3sF(s) = \frac{1}{s^2} + s^2 + 1$$

$$F(s)(s^3 + 3s) = \frac{1}{s^2} + s^2 + 1 \quad /:(s^3 + 3s)$$

$$F(s) = \frac{\frac{1}{s^2} + s^2 + 1}{(s^3 + 3s)}$$

$$F(s) = \frac{1}{(s^3 + 3s)(s^2 + s^2 + 1)} = \frac{1}{(s^3 + 3s)(2s^2 + 1)} = \frac{1}{s(s^2 + 3)(2s^2 + 1)}$$

PARCIJALNI RAZLOMCI:

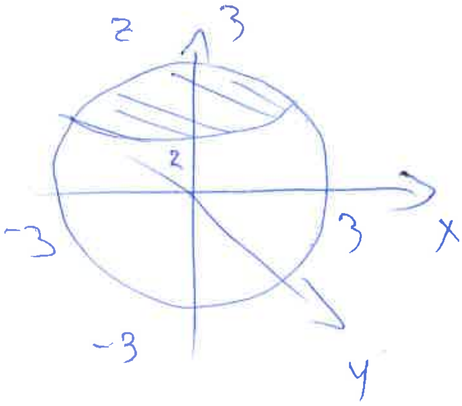
IME I PREZIME: JELENA MALEŠ **BROJ INDEKSA:** 17-2-0103-2011

4. $x^2+y^2+z^2=9$ $\quad z \geq 2$

$x = r\cos\varphi$ $\quad r^2 = 9$

$y = r\sin\varphi$ $\quad r = 3$

$z = z$



$$\int_0^{2\pi}\int_0^{3}\int_2^{3} r\,dr\,d\varphi\,dz = \int_0^{2\pi}\int_0^{3}\int_2^{3} r\cos\varphi + r\sin\varphi\,dr\,d\varphi\,dz$$

5. $r = 4$ $\quad T(\underset{x}{0}, \underset{y}{2})$ $\quad \int_K (1-3x)\,dx\,dy$

$x = r\cos\varphi + 0$

$y = r\sin\varphi + 2$

$dx\,dy = r\,dr\,d\varphi$ $\quad r\cos\varphi + r\sin\varphi = r$

$$\int_0^{2\pi} (r-3x)\,r\,dr\,d\varphi =$$

$$= \int_0^{2\pi} (1 - 3\cdot r\cos\varphi)(r\cos\varphi + r\sin\varphi)\,dr\,d\varphi =$$

$$= \int_0^{2\pi} r\cos\varphi + r\sin\varphi - 3r^2\cos\varphi + 3r\cos\varphi\sin\varphi\,dr\,d\varphi =$$

$=$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano. **IME I PREZIME:** FILIP MEŠTROVIĆ **BROJ INDEKSA:** 01711256000 Kod kojeg nastavnika želite ustmeni? prof Mate Kosor	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) + 3\,x'(t) = t, \quad x'(0) = x''(0) = 0,\ x(0) = 1.$$

2. Koristeći plošni integral postaviti formulu za ploštinu dijela paraboloida $z = \frac{x^2}{3} + \frac{y^2}{3}$ što leži iznad područja $D \ldots x^2 + y^2 \leq 4$. Nije potrebno računati površinu baze. 20

3. Neka je kvadrat $K = \{(x, y, z) : x = 4,\ y \in [-2, 2],\ z \in [-2, 2]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} y\,ds$? 20

4. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2 + y^2 + z^2 = 9$ za koji vrijedi $z \geq 2$. 20

5. Neka je K krug radijusa $r = 4$ sa centrom u točki $T(0, 2)$. Izračunati $\int\limits_K (1 - 3x)\,dx dy$. 20

Ukupno:

5 $T(0,2)$ $r=4$ $x^2+(y-2)^2=16$

$x = r\sin\varphi$

$y = r\cos\varphi$

$dx = r\,dr\,d\varphi$

$\iint (1-3r\sin\varphi)\, r\,dr\,d\varphi$

$1-3r\sin\varphi$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: KRISTIAN MARTINOVIĆ BROJ INDEKSA: 17-2-0110-2011	Broj ↓ bodova
Kod kojeg nastavnika želite ustmeni? UGLEŠIĆ	

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