

**MATEMATIKA 2:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME:

Anamaria Jazić

BROJ INDEKSA:

17-2-0104-2011

1. Odrediti integracijom (analitički):  $\int_{-4}^{-1} x^2 e^x dx$  .

15

2. Izračunati  $\int_0^2 \frac{2x^2 + x + 2}{x^2 - 1} dx$

15

3. Zadane su točke  $A(2, 2, 4)$ ,  $B(0, -1, 3)$  i  $C(-1, 0, 2)$ . Koliki je kut između pravaca  $AB$  i  $AC$ ? Integracijom izračunati površinu trokuta  $ABC$ .

5+10

4. Ispitati domenu i ekstreme funkcije  $f(x, y) = x^2 + y^2 + \frac{2}{xy}$ .

20 15

5. Riješiti:  $y' + 2xy = x - 3$ .

20

6. Riješiti:  $y'' - 4y' + 4y = 4$ .

15

Ukupno:

4.5

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln  \cos x $	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int \frac{dx}{x} = \ln  x  + C$	$\int \cot x dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left  x + \sqrt{x^2 \pm a^2} \right  + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[ x \sqrt{x^2 \pm a^2} \pm a^2 \ln \left( x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[ x \sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$

①.

$$\int_{-4}^{-1} x^2 e^x dx = \left. \begin{array}{l} u = x^2 \quad du = 2x dx \\ dv = \int e^x dx \\ v = e^x \end{array} \right\}$$

$$= x^2 e^x \Big|_{-4}^{-1} - 2 \int_{-4}^{-1} x e^x dx = \left. \begin{array}{l} u = x \quad du = dx \\ dv = \int e^x dx \\ v = e^x \end{array} \right\}$$

$$= \left[ (-1)^2 e^{-1} - (-4)^2 e^{-4} \right] - 2 \cdot \left[ x e^x \Big|_{-4}^{-1} - \int_{-4}^{-1} e^x dx \right]$$

$$= e^{-1} - 16 e^{-4} - 2 \left[ (-1) \cdot e^{-1} - (-4) \cdot e^{-4} - e^x \Big|_{-4}^{-1} \right]$$

$$= e^{-1} - 16 e^{-4} + 2 e^{-1} - 8 e^{-4} + 2 e^{-1} - 2 e^{-4}$$

$$= 5 e^{-1} - 26 e^{-4} = 1.363$$

$$\textcircled{2} \int_0^2 \frac{2x^2 + x + 2}{x^2 - 1} dx =$$

$$x^2 - 1 = 0 \Rightarrow x \neq 1$$

podintegralna funkcija nije definirana  
u  $x=1$

NEPRAVILNI INTEGRAL

$$= \lim_{\epsilon_1 \rightarrow 1} \int_0^{\epsilon_1} \frac{2x^2 + x + 2}{x^2 - 1} dx + \lim_{\epsilon_2 \rightarrow 1} \int_{\epsilon_2}^2 \frac{2x^2 + x + 2}{x^2 - 1} dx \quad ; \quad (2x^2 + x + 2) : |x^2 - 1| = 2$$

$$= \lim_{\epsilon_1 \rightarrow 1} \int_0^{\epsilon_1} \left( 2 + \frac{x+4}{x^2-1} \right) dx + \lim_{\epsilon_2 \rightarrow 1} \int_{\epsilon_2}^2 \left( 2 + \frac{x+4}{x^2-1} \right) dx$$

$$= \lim_{\epsilon_1 \rightarrow 1} \left[ 2x \Big|_0^{\epsilon_1} + \int_0^{\epsilon_1} \frac{x}{x^2-1} dx + 4 \int_0^{\epsilon_1} \frac{dx}{x^2-1} \right] + \lim_{\epsilon_2 \rightarrow 1} \left[ 2x \Big|_{\epsilon_2}^2 + \int_{\epsilon_2}^2 \frac{x}{x^2-1} dx + 4 \int_{\epsilon_2}^2 \frac{dx}{x^2-1} \right]$$

$$= \left\{ \begin{array}{l} x^2 - 1 = t \\ 2x dx = dt \\ x dx = \frac{1}{2} dt \end{array} \right.$$

$$= \lim_{\epsilon_1 \rightarrow 1} \left[ 2\epsilon_1 + \frac{1}{2} \ln|x^2-1| \Big|_0^{\epsilon_1} + \frac{4}{2} \ln \left| \frac{x-1}{x+1} \right| \Big|_0^{\epsilon_1} \right] + \lim_{\epsilon_2 \rightarrow 1} \left[ 2(2-\epsilon_2) + \frac{1}{2} \ln|x^2-1| \Big|_{\epsilon_2}^2 + \frac{4}{2} \ln \left| \frac{x-1}{x+1} \right| \Big|_{\epsilon_2}^2 \right]$$

$$= 2 + \frac{1}{2} \lim_{\epsilon_1 \rightarrow 1} \left[ \ln|\epsilon_1^2 - 1| - \ln|1-1| \right] + 2 \lim_{\epsilon_1 \rightarrow 1} \left[ \ln \left| \frac{\epsilon_1 - 1}{\epsilon_1 + 1} \right| - \ln \left| \frac{-1}{1} \right| \right]$$

$$+ 2 + \frac{1}{2} \lim_{\epsilon_2 \rightarrow 1} \left[ \ln|5| - \ln|\epsilon_2^2 - 1| \right] + 2 \lim_{\epsilon_2 \rightarrow 1} \left[ \ln \left| \frac{1}{3} \right| - \ln \left| \frac{\epsilon_2 - 1}{\epsilon_2 + 1} \right| \right]$$

$$= 4 + \frac{1}{3} \lim_{\epsilon_2 \rightarrow 1} \ln|\epsilon_2^2 - 1| - \ln 1 + 2 \lim_{\epsilon_1 \rightarrow 1} \left| \frac{\epsilon_1 - 1}{\epsilon_1 + 1} \right| - \ln 1 + \frac{1}{2} \ln 5 - \frac{1}{2} \lim_{\epsilon_2 \rightarrow 1} \ln|\epsilon_2^2 - 1|$$

$$2 \lim_{\epsilon_1 \rightarrow 1} \left| \frac{\epsilon_1 - 1}{\epsilon_1 + 1} \right| = 4 + 0.8 - 2 \cdot 1.4 = 2.6$$

③.

OVAJ INTEGRAL DIVERGIRA

7BOG

$$\lim_{\epsilon_1 \rightarrow 1} \ln|\epsilon_1^2 - 1| = \lim_{t \rightarrow 1} \ln|t-1|$$

$$= \lim_{u \rightarrow 0} \ln|u| = \lim_{u \rightarrow 0^+} \ln u$$

$$= -\infty$$

6.  $y'' - 4y' + 4y = 4$

$x^2 - 4r + 4 = 0$

$r_{1/2} = \frac{4 \pm \sqrt{16 - 16}}{2} = 2$

$y_H = C_1 e^{2x} + C_2 x e^{2x} \quad C_1, C_2 \in \mathbb{R}$

PARTIKULARNI

$y_p = A$   
 $y_p' = 0$   
 $y_p'' = 0$

$4A = 4$   
 $A = 1$

$y_p = 1$

$y = y_H + y_p = C_1 e^{2x} + C_2 x e^{2x} + 1$

PROVERA

$y' = 2C_1 e^{2x} + C_2 e^{2x} + 2C_2 x e^{2x}$

~~$y'' = 4C_1 e^{2x} + 2C_2 e^{2x} + 2C_2 x e^{2x}$~~   
 $= (2C_1 + C_2) e^{2x} + 2C_2 x e^{2x}$

$y'' = (4C_1 + 2C_2) e^{2x} + 2C_2 e^{2x} + 4C_2 x e^{2x}$   
 $= (4C_1 + 4C_2) e^{2x} + 4C_2 x e^{2x}$

$y'' - 4y' + 4y = e^{2x} \left( \frac{4C_1 + 4C_2 - 8C_1 - 4C_2 + 4C_1}{x} + 4C_2 \right) + 4 = 4$   
 $x e^{2x} \left( \frac{4C_2 - 8C_2 + 4C_2}{x} \right) + 4 = 4$

4)  $(x, y) = x^2 + y^2 + \frac{2}{xy}$

DOMENA?

$\frac{\partial f}{\partial x} = 2x - \frac{2}{y x^2}$

$2x - \frac{2}{y x^2} = 0 \Rightarrow 2x = \frac{2}{y x^2} \quad | \cdot \frac{y}{x} \quad | : 2$

$\frac{\partial f}{\partial y} = 2y - \frac{2}{x y^2}$

$2y - \frac{2}{x y^2} = 0 \quad y = \frac{1}{x^3}$

$\frac{2}{x^3} = \frac{2}{x \cdot \frac{1}{x^6}} \Rightarrow x^3 = x^5$   
 $y^5 - x^3 = 0$   
 $x^3(x^2 - 1) = 0$

$x_1 = 0 \quad x_2 = 1 \quad x_3 = -1$   
 $y_1 = \infty \quad y_2 = 1 \quad y_3 = -1$

$\frac{\partial^2 f}{\partial x^2} (x, y) = 2 + \frac{4}{y x^3}$

$\frac{\partial^2 f}{\partial y^2} (x, y) = 2 + \frac{4}{x y^3}$

$\frac{\partial^2 f}{\partial x^2 \partial x} (x, y) = \frac{2}{y^2 x^2}$

$T_1(1, 1)$	$T_1 = 6$	$6$
$T_2(-1, -1)$	$6$	$6$
	$2$	$2$

$\Delta \begin{vmatrix} 6 & 2 \\ 2 & 6 \end{vmatrix} = 36 - 4 = 32 > 0$

$u \quad T_1 \quad T_2 \quad \text{MINIMUM}$

15



odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: VALENTINO ŠARE

BROJ INDEKSA: 17-2-0149-2011

1. Odrediti integracijom (analitički):  $\int_{-4}^{-1} x^2 e^x dx$ . 15
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Ukupno:

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$$1) \int_{-4}^{-1} x^2 \cdot e^x dx = \left[ \begin{array}{l} u = x^2 \quad du = 2x dx \\ dv = e^x \quad v = e^x \end{array} \right]$$

$$= u \cdot v - \int v du$$

$$= x^2 e^x - \int e^x 2x dx$$

$$= x^2 e^x - 2e^x \Big|_{-4}^{-1} = (-1)^2 e^{-1} - 2e^{-1} - ((-4)^2 e^{-4} - 2e^{-4}) = 0,624298 \cdot 1 - 0,624298 = 0,624298$$

$$\int_0^2 \frac{2x^2 + x + 2}{x^2 - 1} dx =$$

VALENTINO SARE

$$5) y' + 2xy = x - 3$$

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**MATEMATIKA 2:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

POPUNJAVA  
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Broj ↓  
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IME I PREZIME: **ANTONIO PRIBIL**

BROJ INDEKSA: **57666**

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ANTONIO PRIBIL

57666

MATEMATIKA 2

$$2 \int \frac{x^2 + x + 2}{x^2 - 1} dx = 2 \int \frac{\frac{x^2+1}{2+1} + x}{x^2+1 - 1} = 2 \int \frac{\frac{x^3+x}{3}}{x^3-1} = 2 \int \frac{3x^3+x}{3x^3-1} =$$
$$= 2 \int \frac{3 \cdot 2^3 + 2}{3 \cdot 6^3 - 1} = 2 \int \frac{2 \cdot 8 + 2}{-1} = 2 \cdot \int \frac{26}{-1}$$



**MATEMATIKA 2:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME: **ROKO DUŠEVIĆ**

BROJ INDEKSA: **57351-2009**

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2.  $\int_0^2 \frac{2x^2 + x + 2}{x^2 - 1} dx =$

$$3. A(2, 2, 4), B(0, -1, 3), C(-1, 0, 2)$$

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IME I PREZIME:

Lovre Kresović

BROJ INDEKSA:

56640-2008

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IME I PREZIME: ANDRO KLARIN

BROJ INDEKSA:

1. Odrediti integracijom (analitički):  $\int_{-4}^{-1} x^2 e^x dx$  . 15
2. Izračunati  $\int_0^2 \frac{2x^2 + x + 2}{x^2 - 1} dx$  15
3. Zadane su točke  $A(2, 2, 4)$ ,  $B(0, -1, 3)$  i  $C(-1, 0, 2)$ . Koliki je kut između pravaca  $AB$  i  $AC$ ? Integracijom izračunati površinu trokuta  $ABC$ . 5+10
4. Ispitati domenu i ekstreme funkcije  $f(x, y) = x^2 + y^2 + \frac{2}{xy}$ . 20
5. Riješiti:  $y' + 2xy = x - 3$ . 20
6. Riješiti:  $y'' - 4y' + 4y = 4$ . 15

Ukupno:

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln  \cos x $	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int \frac{dx}{x} = \ln  x  + C$	$\int \cot x dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left  x + \sqrt{x^2 \pm a^2} \right  + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[ x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left( x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[ x\sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$

