

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME: **VINKO ŠPAR**

BROJ INDEKSA: **17-2-0054-2010**

D3

1. Među kompleksnim brojevima odrediti rješenja jednadžbe  $z^3 = -(i)^{935}$ . Prikazati rješenja u kompleksnoj ravnini!

12  
15+5

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

15+5

$$\begin{aligned} 5x + 4z + 2t &= 3 \\ x - y + 2z + t &= 0 \\ 4x + y + 2z &= 1 \\ x + y + z + t &= -1 \end{aligned}$$

3. Za funkciju  $f(x) = \frac{\cos(2x)}{x}$  odrediti koliko iznosi  $f'(\pi)$ .

13+2

4. Za funkciju:  $f(x) = \sqrt{x^2 - 3x + 6}$  treba:

(a) pronaći drugu derivaciju

10

(b) na temelju ispitivanja toka funkcije skicirati graf

20(graf)

5. Izračunati i obavezno uvrštavanjem provjeriti koliko iznosi  $\lim_{x \rightarrow -1} \left( \frac{\frac{\pi}{2} + \arcsin x}{x^2 - 1} \right)$ .

8+2

Ukupno:

27

$$\textcircled{1} z^3 = -(i)^{935} = -(i^{233 \cdot 4 + 3}) = -(1 \cdot i^3) = -(-i) = i$$

$$z^3 = i \quad z = \sqrt[3]{i}$$

$$w = i \Rightarrow \begin{aligned} x = \operatorname{Re} w &= 0 \\ y = \operatorname{Im} w &= 1 \end{aligned} \Rightarrow \begin{aligned} r = |w| &= \sqrt{x^2 + y^2} = 1 \\ \operatorname{tg} \varphi = \frac{y}{x} &= \frac{1}{0} = \infty \Rightarrow \varphi = \frac{\pi}{2} \checkmark \end{aligned}$$

$$m = 3 \Rightarrow k = 0, 1, 2$$

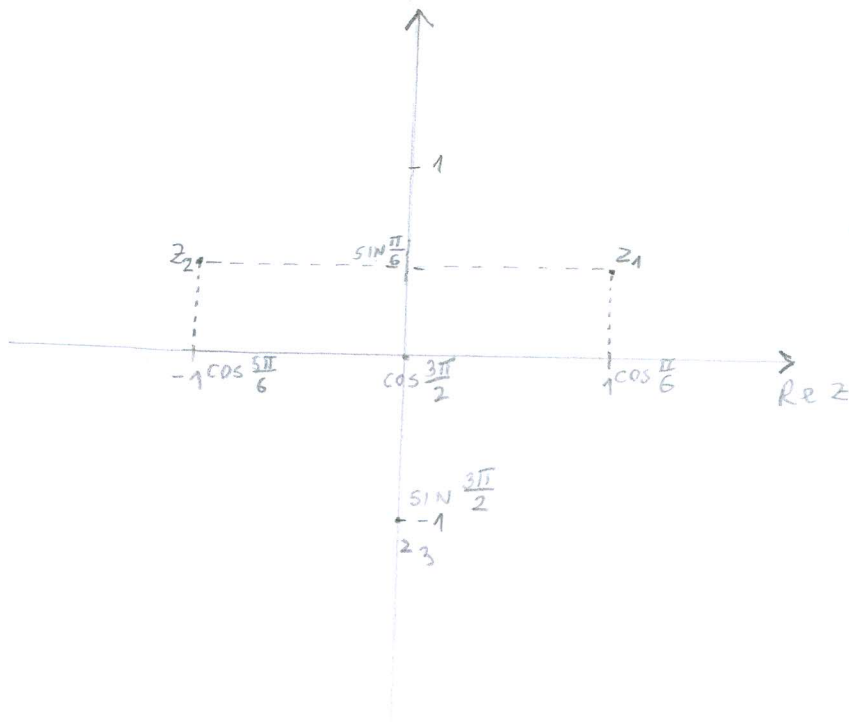
$$\sqrt[m]{w} = \sqrt[m]{r} \left( \cos \frac{\varphi + 2\pi k}{m} + i \cdot \sin \frac{\varphi + 2\pi k}{m} \right), k = 0, 1, 2, \dots, (m-1)$$

$$\begin{aligned} 1) k=0 : z_1 &= \sqrt[3]{1} \left( \cos \frac{\frac{\pi}{2} + 2\pi \cdot 0}{3} + i \cdot \sin \frac{\frac{\pi}{2} + 2\pi \cdot 0}{3} \right) = \\ &= \cos \frac{\pi}{6} + i \cdot \sin \frac{\pi}{6} = \text{IZRAČUNATI} \end{aligned}$$

$$\begin{aligned} 2) k=1 : z_2 &= \sqrt[3]{1} \left( \cos \frac{\frac{\pi}{2} + 2\pi \cdot 1}{3} + i \cdot \sin \frac{\frac{\pi}{2} + 2\pi \cdot 1}{3} \right) = \\ &= \cos \frac{5\pi}{6} + i \cdot \sin \frac{5\pi}{6} = \end{aligned}$$

$$\begin{aligned} 3) k=2 : z_3 &= \sqrt[3]{1} \left( \cos \frac{\frac{\pi}{2} + 2\pi \cdot 2}{3} + i \cdot \sin \frac{\frac{\pi}{2} + 2\pi \cdot 2}{3} \right) = \\ &= \cos \frac{3\pi}{2} + i \cdot \sin \frac{3\pi}{2} = \end{aligned}$$

$$|m| = \uparrow$$



②

$$\begin{cases} 5x + 4z + 2t = 3 \\ x - y + 2z + t = 0 \\ 4x + y + 2z = 1 \\ x + y + z + t = -1 \end{cases}$$

$$\left[ \begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 0 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & -1 \end{array} \right] =$$

$$= \left[ \begin{array}{cccc|c} 1 & 1 & 1 & 1 & -1 \\ 1 & -2 & 2 & 1 & 0 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array} \right] \begin{array}{l} \leftarrow + \\ \leftarrow + \\ \leftarrow + \end{array} =$$

$$= \left[ \begin{array}{cccc|c} 1 & 1 & 1 & 1 & -1 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -1 & 5 \\ 0 & -5 & -1 & -3 & 8 \end{array} \right] \begin{array}{l} \leftarrow + \\ \leftarrow + \\ \leftarrow + \end{array} =$$

$$= \left[ \begin{array}{cccc|c} 1 & 0 & \frac{3}{2} & 1 & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & -\frac{5}{2} & -1 & \frac{9}{2} \\ 0 & 0 & -\frac{1}{2} & -3 & \frac{11}{2} \end{array} \right] \begin{array}{l} \leftarrow + \\ \leftarrow + \\ \leftarrow + \end{array} =$$

$$= \left[ \begin{array}{cccc|c} 1 & 0 & 0 & -\frac{5}{4} & \frac{1}{4} \\ 0 & 1 & 0 & \frac{4}{4} & -1 \\ 0 & 0 & 1 & \frac{8}{4} & -1 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right] \begin{array}{l} \leftarrow + \\ \leftarrow + \\ \leftarrow + \end{array}$$

$$= \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{19}{4} \\ 0 & 1 & 0 & 0 & -\frac{15}{4} \\ 0 & 0 & 1 & 0 & -\frac{23}{4} \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

$$\Rightarrow x = \frac{19}{4} \quad y = -\frac{15}{4} \quad z = -\frac{23}{4}$$

$t = 2$

② PROJERA

$$5x + 4z + 2t = 3$$

$$5 \cdot \frac{17}{7} + 4 \cdot \frac{-23}{7} + 2 \cdot 2 = 3$$

$$\frac{85}{7} + \frac{-92}{7} + 4 = 3$$

$$\frac{3}{7} = 3$$

✓

$$x - y + 2z + t = 0$$

$$\frac{17}{7} - \left(-\frac{15}{7}\right) + 2 \cdot \frac{-23}{7} + 2 = 0$$

$$-\frac{32}{7} + \frac{-46}{7} + 2 = 0$$

$$-\frac{14}{7} + 2 = 0$$

$$-2 + 2 = 0$$

$$0 = 0$$

✓

$$4x + y + 2z = 1$$

$$4 \cdot \frac{17}{7} + \frac{-15}{7} + 2 \cdot \frac{-23}{7} = 1$$

$$9 \cdot 7 + \frac{-15}{7} + (-6 \cdot 6) = 1$$

$$9 \cdot 7 - 2 \cdot 1 - 6 \cdot 6 = 1$$

$$7 \cdot 6 - 6 \cdot 6 = 1$$

$$\frac{1}{7} = 1$$

✓

$$x + y + z + t = -1$$

$$\frac{17}{7} + \frac{-15}{7} + \frac{-23}{7} + 2 = -1$$

$$\frac{2}{7} + \frac{-23}{7} + 2 = -1$$

$$-\frac{21}{7} + 2 = -1$$

$$-3 + 2 = -1$$

$$\frac{-1}{7} = -1$$

✓



