

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

D1

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **ANDREJ ARACIĆ**

BROJ INDEKSA: **0269082268**

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 + z - 4 = 0$. *Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem!* Dalje izracunaj: $\left(\frac{z_1 - z_2}{z_2 + 4}\right)$.

4+3+8

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

10+5

$$\begin{array}{rccccrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 & = & -4 \\ & & x_2 & - & x_3 & + & x_4 & = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 & = & -3 \\ & & - & 7x_2 & + & 3x_3 & + & x_4 & = & 1 \end{array}$$

3. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(4 - x^2) + \cos(4 - 2x)$.

5+15

4. Odrediti tok funkcije $f(x) = x^2 + \frac{1}{x}$.

15(graf)

5. Odrediti i provjeriti uvrštavanjem: $\lim_{x \rightarrow 1} \frac{\ln x}{\arccos x} =$

4+1

6. Odredi derivaciju funkcije $f(x) = \frac{3}{\cos(3x)}$

10

7. Odrediti tangentu na funkciju $f(x) = \sin x$ tamo gdje je $x = -\frac{\pi}{4}$. Nacrtati graf funkcije i nacrtati izračunatu tangentu.

15+3+2

Ukupno:

45

$$3. f(x) = \ln(4-x^2) + \cos(4-2x)$$

$$4-x^2 > 0 \quad \infty > 4-2x > -\infty$$

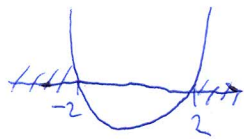
$$4-x^2 = 0 \quad x \in \mathbb{R}$$

$$4 = x^2$$

$$x = \sqrt{4}$$

$$x_1 = 2$$

$$x_2 = -2$$



$$x \in (-\infty, -2) \cup (2, \infty)$$

$$f'(x) = \frac{1}{4-x^2} \cdot (-2x) + ((-\sin(4-2x)) \cdot (-2))$$

$$f'(x) = -\frac{2x}{4-x^2} - 2\sin(4-2x)$$

$$f''(x) = \frac{-2x - 8\sin(4-2x) - 2x^2\sin(4-2x)}{4-x^2}$$

$$f''(x) = -\frac{2(x - 4\sin(4-2x) - x^2\sin(4-2x))}{4-x^2}$$

$$5. \lim_{x \rightarrow 1} \frac{\ln x}{\sin \cos x} = \lim_{x \rightarrow 1} \frac{0}{0}$$

$$\lim_{x \rightarrow 1} \frac{\frac{1}{x}}{-\frac{1}{\sqrt{1-x^2}}} = \frac{1}{-\frac{1}{0}} = -\frac{1}{0} = -\infty$$

$$\frac{0}{0} = 0$$

$$6. f(x) = \frac{3}{\cos(3x)}$$

$$f'(x) = \frac{0 - (3)(-\sin(3x) \cdot (3))}{(\cos(3x))^2} = \frac{0 - (-9\sin(3x))}{(\cos(3x))^2} = \frac{9\sin(3x)}{(\cos(3x))^2}$$

$$7. f(x) = \sin x \quad t = -\frac{\pi}{4}$$

$$y - y_1 = y'(x - x_1)$$

$$y_1 = \sin\left(-\frac{\pi}{4}\right) = -0.707$$

$$x_1 = -0.785$$

$$f'(x) = \cos x$$

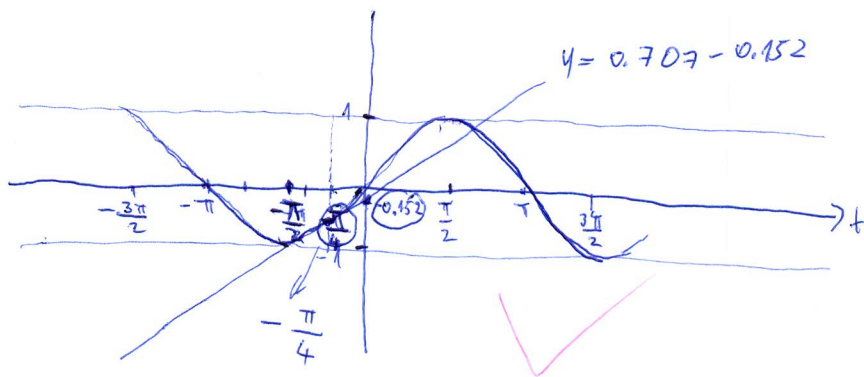
$$f'\left(-\frac{\pi}{4}\right) = \cos\left(-\frac{\pi}{4}\right) = 0.707$$

$$y + 0.707 = 0.707(x + 0.785)$$

$$y + 0.707 = 0.707x + 0.555$$

$$y = 0.707x + 0.555 - 0.707$$

$$y = 0.707x - 0.152$$



$$4. f(x) = x^2 + \frac{1}{x} \Rightarrow \frac{x^3 + 1}{x}$$

DOMENA

$$\frac{x^3 + 1}{x}$$

$$x \neq 0$$

$$x \in \mathbb{R} \setminus \{0\}$$

NULTOČKE

$$x^3 + 1 = 0$$

~~$$x^3 = -1$$~~

$$x^3 = -1$$

$$x = \sqrt[3]{-1}$$

~~$$x = -1$$~~

$$x = -1$$

VERT. ASIMPTOTE

$$x = 0$$

(y os)

KOSA ASIMPTOTA

~~$$k = \lim_{x \rightarrow \infty} \frac{x^3 + 1}{x^2} = \lim_{x \rightarrow \infty} \frac{x^3}{x^2} = \lim_{x \rightarrow \infty} x = \infty$$~~

$$k = \lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{x^3 + 1}{x^2} = \lim_{x \rightarrow \infty} \frac{x^3}{x^2} = \lim_{x \rightarrow \infty} x = \infty$$

NEMA KOSE ASIMPTOTE

$$L = \lim_{x \rightarrow \infty} [f(x) - kx] = \lim_{x \rightarrow \infty} \left[x^2 - \frac{1}{x} - \infty x \right]$$

NEMA NI HORIZ. ASIMPTOTE

STACIONARNE TOČKE

$$f'(x) = 2x - \frac{1}{x^2} = \frac{2x^3 - 1}{x^2}$$

$$f'(x) = 0 \Rightarrow 2x^3 - 1 = 0$$

$$2x^3 = 1$$

$$2x = \sqrt[3]{1}$$

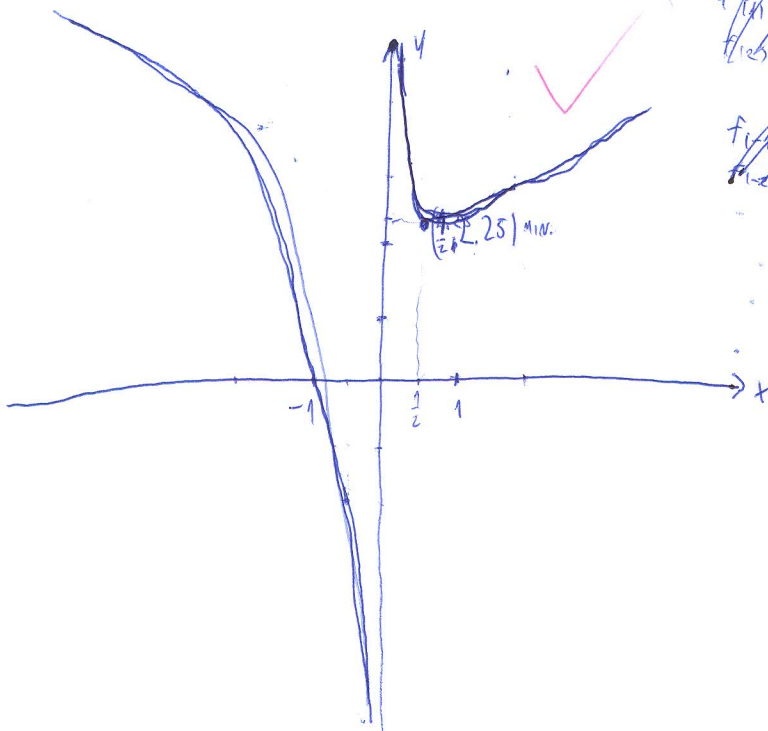
$$2x = 1$$

$$x = \frac{1}{2}$$

$$f''(x) = \frac{(6x^2)(x^2) - (2x^3 - 1)(2x)}{x^4} = \frac{6x^4 - (4x^4 - 2x)}{x^4} = \frac{2x^4 + 2x}{x^4} = \frac{2x(x^3 + 1)}{x^4}$$

$$f''\left(\frac{1}{2}\right) = \frac{4(0.125 + 1)}{0.0625} = \frac{1.125}{0.0625} = 18 > 0 \text{ MINIMUM}$$

$$f\left(\frac{1}{2}\right) = 0.25 + \frac{1}{0.5} = 2.25$$



~~$$f(x) = x^2 + \frac{1}{x}$$~~

~~$$f(x) = x^2 + \frac{1}{x}$$~~

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D1

IME I PREZIME: *Karlo Vucetic*

BROJ INDEKSA: *17-2-0400-2014*

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 + z - 4 = 0$. *Prikaži ih u kompleksnoj ravlini i provjeri uvrštavanjem!* Dalje izracunaj: $\left(\frac{z_1 - z_2}{z_2 + 4}\right)$.

4+3+8

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

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$$\begin{array}{rccccrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 & = & -4 \\ & & x_2 & - & x_3 & + & x_4 & = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 & = & -3 \\ & & - & 7x_2 & + & 3x_3 & + & x_4 & = & 1 \end{array}$$

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15(graf)

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7. Odrediti tangentu na funkciju $f(x) = \sin x$ tamo gdje je $x = -\frac{\pi}{4}$. Nacrtati graf funkcije i nacrtati izračunatu tangentu.

15+3+2

Ukupno:

35

6. $f(x) = \frac{3}{\cos(3x)}$

~~XXXXXXXXXX~~

$(3/\cos 3x)' = (3' - 3^x (\cos 3x)')$ / ~~X~~

$\cos 2(3x) = 9 \sin 3x / \cos 3x$

$\cos 2(3x) = 9 / (\sin(3x))^2$

4. $D = \mathbb{R} \setminus \{0\} = \langle -\infty, 0 \rangle \cup \langle 0, +\infty \rangle$

$\lim_{x \rightarrow 0^+} \frac{x^2+1}{x} = \text{~~XXXXXXXXXX~~ } \lim_{x \rightarrow \infty} (x + \frac{1}{x}) = -\infty$

$\lim_{x \rightarrow 0^-} \frac{x^2+1}{x} = \frac{1}{0^-} = \text{~~XXXXXXXXXX~~ } -\infty$

$\lim_{x \rightarrow 0^-} \frac{x^2+1}{x} = \lim_{x \rightarrow -\infty} (x + \frac{1}{x}) = +\infty$

$f'(x) = 1 - \frac{1}{x^2}$

$f'(x) = 0$

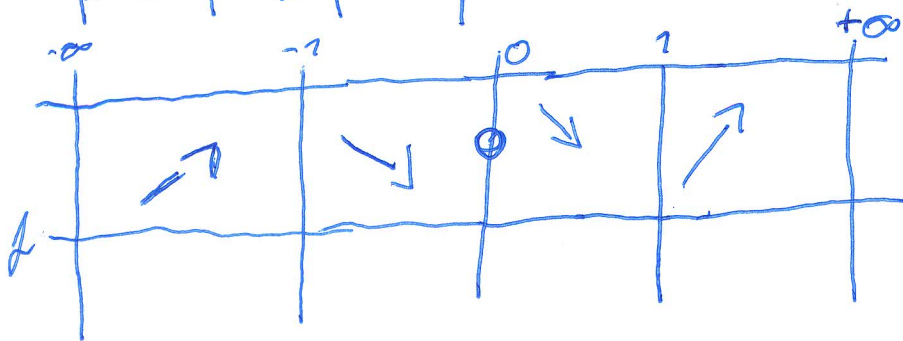
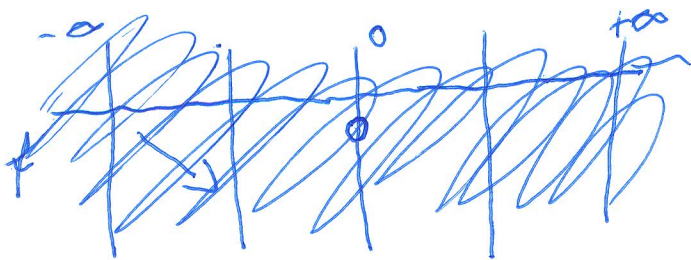
$f''(x) = \frac{2}{x^3}$

$x_1 = -1 \quad x_2 = 1$

$f'(-1) = 2 \quad (\cap)$

$f''(1) = 2 \quad (\cup)$

~~o~~



③ $f(x) = \ln(4-x^2) + \cos(4-2x)$

$\ln a \Rightarrow a > 0$ $4-x^2 > 0$
 $x^2 < 4$
 $-2 < x < 2$

$D_f = (-2, 2)$ ✓

$f'(x) = \frac{1}{4-x^2} \cdot (-2x) + (-\sin(4-2x)) \cdot (-2)$ ✓

$f'(x) = \frac{2x}{x^2-4} + 2\sin(4-2x)$

$f(-\frac{\pi}{4}) = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

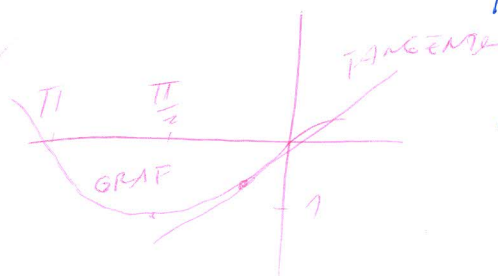
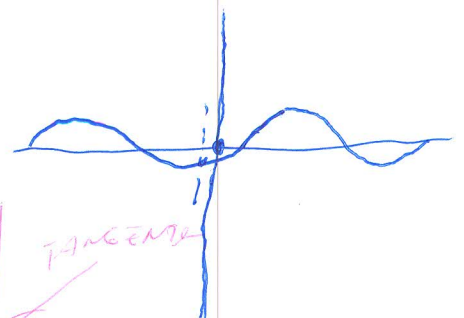
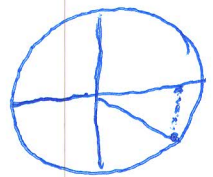
⑦ $f(x) = \sin x$ ~~.....~~ $f(x) = \sin x$
 $x = -\frac{\pi}{4}$ $f(x) = -\cos x$

$f'(-\frac{\pi}{4}) = \cos(-\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$-\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \cdot (-\frac{\pi}{4}) + l$ $y = kx + l$

$l = \frac{\sqrt{2}}{2} (\frac{\pi}{4} - 1)$ $y = \frac{\sqrt{2}}{2} x + l$

$y = \frac{\sqrt{2}}{2} x + \frac{\sqrt{2}}{2} (\frac{\pi}{4} - 1)$ ✓



odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

D1

NASTAVNIK

IME I PREZIME: **MARIO NEMEŠ**

BROJ INDEKSA:

Broj ↓
bodova

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 + z - 4 = 0$. Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem! Dalje izracunaj: $\left(\frac{z_1 - z_2}{z_2 + 4}\right)$.

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15+3+2

Ukupno:

15

4. $f(x) = x^2 + \frac{1}{x} \quad x \neq 0$
 $D = \mathbb{R} \setminus \{0\}$

V.A.

$$\lim_{x \rightarrow 0} x^2 + \frac{1}{x} = 0^2 + \frac{1}{0} \rightarrow \text{NEMA V.A.}$$

H.A.

$$\lim_{x \rightarrow \infty} \infty^2 + \frac{1}{\infty} = \infty \rightarrow \text{NEMA H.A.}$$

$(-1, 0)$

$$f'(x) = 2x + \frac{1}{x^2}$$

$$f''(x) = 2 + \frac{2x}{x^3}$$

$$f'''(x) = 2 + \frac{2}{x^3}$$

$$f'(x) = 0$$

$$2x + \frac{1}{x^2} = 0 \quad | \cdot x^2$$

$$2x^3 + 1 = 0 \quad x = -0.8$$

$$\begin{array}{l} 2x^3 = -1 \\ x^3 = -\frac{1}{2} \end{array}$$

$$f''(x) = 0$$

$$2 + \frac{2}{x^3} = 0 \quad | \cdot x^3$$

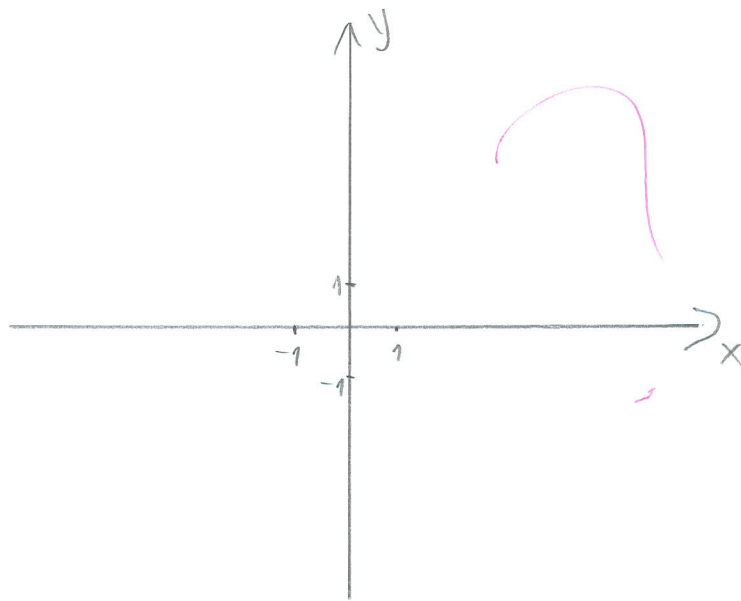
$$2x^3 + 2 = 0$$

$$2x^3 = -2$$

$$x^3 = -1$$

$$-\infty \quad -2 \quad -1 \quad x = -0.9 \quad -0.8 \quad -0.5 \quad 0 \quad 1 \quad +\infty$$

$f'(x)$	-	-	+	+
$f''(x)$	+	-	-	+
$f(x)$	∪	∩	∪	∩



$$6. \quad f(x) = \frac{3}{\cos(3x)}$$

$$f'(x) = \frac{-3 \cdot (-\sin 3x) \cdot 3}{\cos^2(3x)}$$

$$f'(x) = \frac{-6 \sin(3x)}{\cos^2(3x)} \quad \times$$

$$3. \quad f(x) = \ln(4-x^2) + \cos(4-2x)$$

$$4-x^2 \neq 0$$

$$x^2 = 4$$

$$x_1 = 2$$

$$x_2 = -2$$

$$D = \mathbb{R} \setminus \{-2, 2\} \quad \times$$

$$f'(x) = \frac{1}{4-x^2} \cdot (-2x) - \sin(4-2x) \cdot (-2) \quad \checkmark$$

$$f'(x) = -\frac{2x}{4-x^2} + 2\sin(4-2x) \quad \checkmark$$

MARIO NEMES

1. $z^2 + z - 4 = 0$

$$z_{1,2} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-4)}}{2}$$

$$z_{1,2} = -1 \pm \sqrt{1+16}$$

$$z_{1,2} = \frac{-1 \pm \sqrt{17}}{2}$$

$$z_1 = \frac{-1 + \sqrt{17}}{2} \rightarrow -\frac{1}{2} + \frac{\sqrt{17}}{2} i \quad \times$$

$$z_2 = \frac{-1 - \sqrt{17}}{2} \rightarrow -\frac{1}{2} - \frac{\sqrt{17}}{2} i \quad \times$$

$$\frac{-\frac{1}{2} + \frac{\sqrt{17}}{2} i - (-\frac{1}{2} - \frac{\sqrt{17}}{2} i)}{2} =$$

$$-\frac{1}{2} - \frac{\sqrt{17}}{2} + 4$$

$$= \frac{17}{16} + \frac{7\sqrt{17}}{16} i$$

$$= \frac{17}{16} - \frac{7\sqrt{17}}{16} i$$

$$5. \lim_{x \rightarrow 1} \frac{\ln x}{\arccos x} \Big|_{:x} = \left[\frac{0}{0} \right] \rightarrow \frac{\frac{\ln x}{x}}{\frac{\arccos x}{x}} \xrightarrow{\lim_{x \rightarrow \infty}} \frac{\ln \infty}{\infty} \Bigg/ \frac{\arccos 0}{\infty}$$

2.

$$\begin{array}{cccc|c} 1 & -2 & 3 & -4 & -4 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & -3 \\ 0 & -7 & 3 & 0 & 1 \end{array}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME:

Mate Salama

BROJ INDEKSA:

0263086727

D1

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 + z - 4 = 0$. Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem! Dalje izracunaj: $\left(\frac{z_1 - z_2}{z_2 + 4}\right)$.

4+3+8

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

10+5

$$\begin{array}{r} x_1 - 2x_2 + 3x_3 - 4x_4 = -4 \\ x_2 - x_3 + x_4 = 1 \\ x_1 + 3x_2 - 3x_4 = -3 \\ - 7x_2 + 3x_3 + x_4 = 1 \end{array}$$

3. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(4 - x^2) + \cos(4 - 2x)$.

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15+3+2

1.) ~~$z^2 + z - 4 = 0$~~

6.) $f(x) = \frac{3}{\cos(3x)}$

$$\begin{aligned} f'(x) &= -3(\cos(3x))^{-2} \cdot (-\sin(3x)) \cdot 3 \\ &= \frac{9 \sin(3x)}{\cos^2(3x)} \quad \checkmark \end{aligned}$$

3.) $f(x) = \ln(4 - x^2) + \cos(4 - 2x)$

$$= 4 - x^2 \geq 0$$

$$-x^2 \geq -4$$

$$D(f) = \mathbb{R} \quad \times$$

$$f'(x) = \frac{-2x}{4-x^2} - \sin(4-2x) \quad \times$$

Ukupno:

10

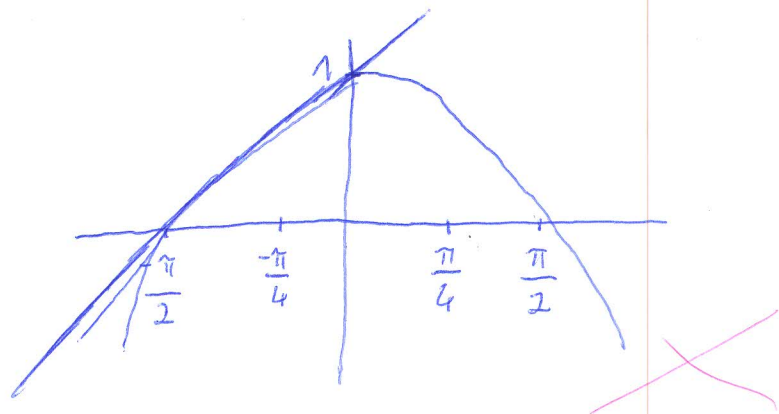
$$7.) f(x) = \sin x$$

$$f'(x) = \cos x$$

$$f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$H = y + \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \left(x + \frac{\pi}{4}\right)$$

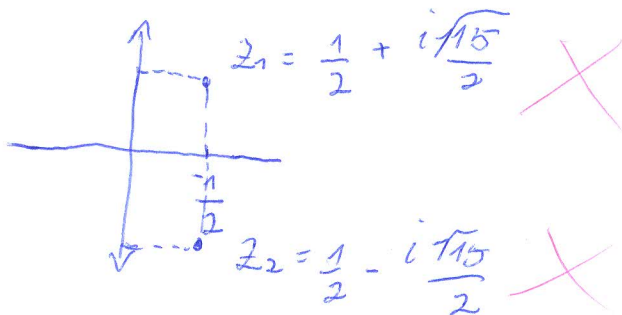
$$y = \frac{\sqrt{2}}{2}x + \frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2}$$



$$1.) z^2 + z - 4 = 0$$

$$z_1 = \frac{1}{2} + i \frac{\sqrt{15}}{2}$$

$$z_2 = \frac{1}{2} - i \frac{\sqrt{15}}{2}$$



Provjera:

$$\left(\frac{1}{2} + i \frac{\sqrt{15}}{2}\right)^2 - \left(\frac{1}{2} - i \frac{\sqrt{15}}{2}\right) - 4 = \frac{1}{4} + i \frac{\sqrt{15}}{2} - \frac{15}{4} - \frac{1}{2} - i \frac{\sqrt{15}}{2} - 4 =$$

$$= \frac{1 - 15 - 2 + 16}{4} = \frac{0}{4} = 0 \text{ dakle } z_1 = \frac{1}{2} + i \frac{\sqrt{15}}{2} \text{ je}$$

rjesenje

rjesenje:

$$\left(\frac{1}{2} - i \frac{\sqrt{15}}{2}\right)^2 - \frac{1}{2} - i \frac{\sqrt{15}}{2} - 4 = \frac{1}{4} - i \frac{\sqrt{15}}{2} - \frac{15}{4} - \frac{1}{2} + i \frac{\sqrt{15}}{2} - 4 =$$

$$= \frac{1 - 15 - 2 + 16}{4} = \frac{0}{4} = z_2 = \frac{1}{2} - i \frac{\sqrt{15}}{2} \text{ je rjesenje}$$

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

D1

IME I PREZIME: *Petra Deladić*

BROJ INDEKSA: *0269080611*

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$$\begin{array}{rccccrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 & = & -4 \\ & & x_2 & - & x_3 & + & x_4 & = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 & = & -3 \\ & & -7x_2 & + & 3x_3 & + & x_4 & = & 1 \end{array}$$

*5 PAMJENS NA 5 BODOVA
POVEĆANO NA 7.5*

5+15
15(graf)
4+1

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Ukupno:
5

① $z^2 + z - 4 = 0$

$a=1 \quad b=1 \quad c=-4$

$x_{1,2} = \frac{-1 \pm \sqrt{1+16}}{2}$

$x_{1,2} = \frac{-1 \pm \sqrt{17}}{2} \quad \sqrt{17} = 4.12$

$x_1 = \frac{-1 - \sqrt{17}}{2} = \frac{-1 - 4.12}{2} \quad x_1 = -2.56$

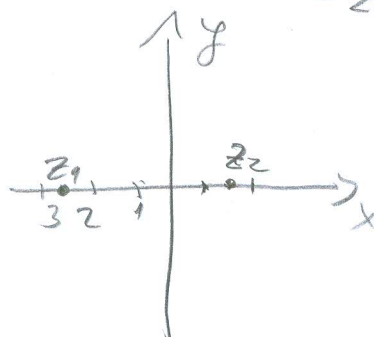
$x_2 = \frac{-1 + \sqrt{17}}{2} = \frac{-1 + 4.12}{2} \quad x_2 = 1.56$

$z = x + yi$

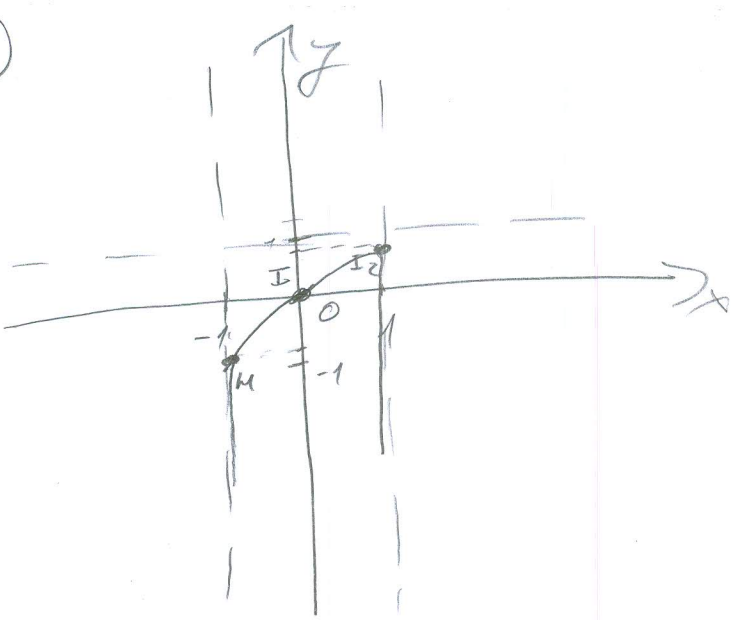
$z_1 = -2.56$

$z_2 = 1.56$

$\left(\frac{z_1 + z_2}{z_2 + 4}\right) = \frac{z_1 - z_2}{z_2 - 4} = \frac{-2.56 - 1.56}{1.56 + 4} = -0.74$



(7)



$$\textcircled{2} \begin{bmatrix} 1 & -2 & 3 & -4 & -4 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & 3 & -3 \\ 0 & -7 & 3 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 3 & -4 & -4 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & -5 & 3 & -7 & -1 \\ 0 & -7 & 3 & 1 & 1 \end{bmatrix} \cdot 5 \left[\begin{array}{l} + \\ + \\ + \\ + \end{array} \right]$$

$$\begin{bmatrix} 1 & -2 & 3 & -4 & -4 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & -2 & -2 & -1 \\ 0 & -7 & 3 & 1 & 1 \end{bmatrix} \cdot 7 \left[\begin{array}{l} + \\ + \\ + \\ + \end{array} \right]$$

$$\begin{bmatrix} 1 & -2 & 3 & -4 & -4 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & -2 & -2 & -1 \\ 0 & 0 & -4 & 8 & 8 \end{bmatrix} \cdot 2 \left[\begin{array}{l} + \\ + \\ + \\ + \end{array} \right]$$

$$\begin{bmatrix} 1 & -2 & 3 & -4 & -4 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & -2 & -2 & -1 \\ 0 & 0 & 0 & 4 & 8 \end{bmatrix}$$

$$4x_4 = 8 \quad | :4$$

$$\boxed{x_4 = 2}$$

$$-2x_3 + 2x_4 = -1$$

$$-2x_3 + 4 = -1$$

$$-2x_3 = -4 - 1$$

$$-2x_3 = -5 \quad | :(-2)$$

$$\boxed{x_3 = \frac{5}{2}}$$

$$x_2 - x_3 + x_4 = 1$$

$$x_2 - \frac{5}{2} + 2 = 1$$

$$x_2 = 1 - 2 + \frac{5}{2}$$

$$\boxed{x_2 = \frac{3}{2}}$$

$$x_1 - 2x_2 + 3x_3 - 4x_4 = -4$$

$$x_1 - 3 + \frac{15}{2} - 8 = -4$$

$$x_1 = -4 + 3 - \frac{15}{2} + 8$$

$$\boxed{x_1 = -\frac{1}{2}}$$

PROBLEMA

$$-\frac{1}{2} - 2 \cdot \frac{3}{2} + 3 \cdot \frac{5}{2} - 4 \cdot 2 = -4$$

$$\boxed{-4 = -4} \checkmark$$

$$\frac{3}{2} - \frac{5}{2} + 2 = 1$$

$$\boxed{1 = 1} \checkmark$$

$$\textcircled{5} \lim_{x \rightarrow 1} \frac{\ln x}{\arccos x} \stackrel{L'H}{=} \lim_{x \rightarrow 1} \frac{(\ln x)'}{(\arccos x)'} = \lim_{x \rightarrow 1} \frac{\frac{1}{x}}{-\frac{1}{\sqrt{1-x^2}}} = \frac{1}{-1} = -1$$

$$\textcircled{6} f(x) = \frac{3}{\cos(3x)} = \frac{3' \cdot (\cos 3x) - 3 \cdot (\cos 3x)'}{(\cos(3x))^2}$$

$$= \frac{-3 \cdot (\sin 3) \cdot 3}{(\cos(3x))^2} = \frac{3 \sin 3}{(\cos(3x))^2}$$

$$x_1 - 3 + \frac{15}{2} - 8 = -4$$

$$x_1 = -4 + 3 - \frac{15}{2} + 8$$

$$\boxed{x_1 = -\frac{1}{2}}$$

④ $f(x) = x^2 + \frac{1}{x}$

① DOMENA $x \neq 0$ $D_f = x \in \mathbb{R} \setminus \{0\}$

$\frac{x^2}{1} + \frac{1}{x} = \frac{x^3+1}{x}$

② NIL TOČKA

$x^3+1=0$
 $x^3 = -1 \sqrt[3]{-1} = -1$
 $x = -1$

③ ASIMPTOTE

V.A.

$\lim_{x \rightarrow 0^-} x^2 + \frac{1}{x} = -\infty$

$x \rightarrow 0^-$
-0.1

$x=0$ / \rightarrow vertikalna asimptota

$\lim_{x \rightarrow 0^+} x^2 + \frac{1}{x} = +\infty$

H.A.

$\lim_{x \rightarrow \pm\infty} x^2 + \frac{1}{x} = \frac{x^2}{x^2} + \frac{1}{x^1} = 1 \sqrt{y=1}$

nema ose

④ PARNOST / NEPARNOST

$f(x) = x^2 + \frac{1}{x}$
 $f(-x) = (-x)^2 + \frac{1}{-x} = x^2 - \frac{1}{x}$ funkcija je neparna

⑤ STACIONARNE TOČKE (I DERIVACIJA)

$f(x) = x^2 + \frac{1}{x} = \frac{x^3+1}{x}$ $f'(x) = \frac{(x^3+1)' \cdot x - (x^3+1) \cdot x'}{x^2} = \frac{3x \cdot x - x^3 - 1}{x^2}$

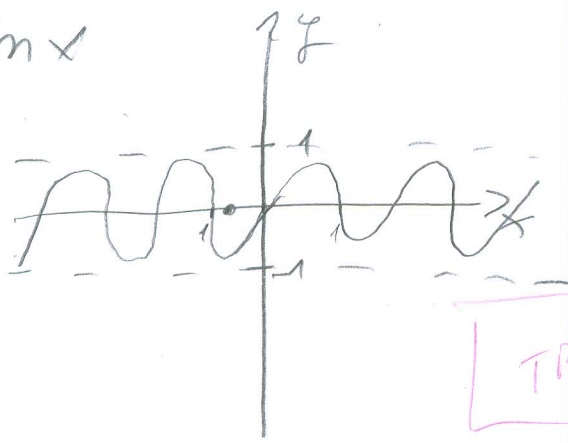
Sign chart for $f'(x) = \frac{3x^2 - x^3 - 1}{x^2}$

$x < -1$	-1	0.5	2	3	$+\infty$
f'	$-$	$-$	$+$	$-$	
f	\searrow	\searrow	\nearrow	\searrow	

$3x^2 - x^3 - 1 = 0$
 $3x^2 - x^3 = 1$
 $x^2(3-x) = 1$ $3-x=0 \rightarrow x=3$
 $x^2 = 1 \sqrt[3]{1} = 1$
 $x = 1$

$\min(1, 2) \left| \begin{array}{l} y(1) = 1^2 + \frac{1}{1} = 2 \\ y(3) = 9 - \frac{1}{3} = 8\frac{2}{3} \end{array} \right.$

⑦ $f(x) = \sin x$



$x = -\frac{\pi}{4}$ $f(x) = \sin(-\frac{\pi}{4})$

$= -0.707$

$\tilde{x} = 0.7$

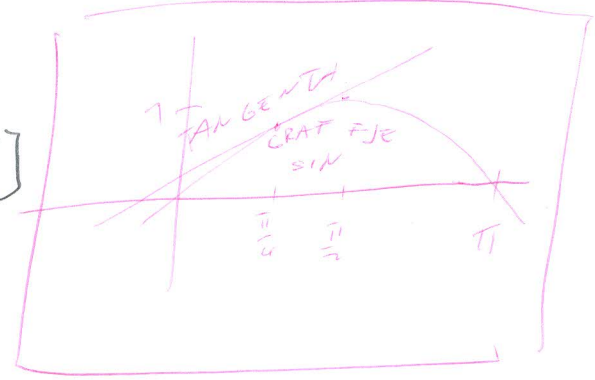
TRAŽI SE TANGENTA

GRAF

$f(x) = \sin x$

① DOMENA

$D_f x \in \mathbb{R} [-1, 1]$



② $\lim_{x \rightarrow 0} \frac{\sin x - 0}{x - 0}$ d.t. (0,0)

③ ASIMPTOTA

v.A. $\lim_{x \rightarrow -1^-} \sin x = -\infty$

$\lim_{x \rightarrow 1^-} \sin x = +\infty$

$x \rightarrow -1^-$
-0.9
 $\lim_{x \rightarrow -1^-} \sin x = -\infty$

$x \rightarrow 1^-$
0.9

$\lim_{x \rightarrow 1^+} \sin x = +\infty$

$x = 1$ \rightarrow leve. itosur

H.A.

$\lim_{x \rightarrow \pm\infty} \sin x = 1$ $\boxed{y=1}$ nenaj kase

④ I DER. (STACION. TOČKE)

$f(x) = \sin x$ $f'(x) = -\cos x$ $-\cos x = 0$
 $\boxed{x = 1}$

	$-\infty$	-1	0	1	$+\infty$
f	+	-	=	-	
f	↗	↘	↘	↘	

⑤ II DERIVACIJA (TOČKE INF.)

$f'(x) = -\cos x$ $f''(x) = \sin x$ $\sin x = 0$
 $x = 0$

$\boxed{M(-1, -0.84)}$
 $y = f(-1) = \sin x$

	$-\infty$	-1	0	1	$+\infty$
f''	-	-	+	-	
f	∩	∩	∪	∩	

$I(0,0)$
 $I(1,0.8)$ $y = \sin x$
 $y = 0.8$

⑥ TOOKA INVERSE (IF DER.)

REKA = TERAKU

$$f(x) = \frac{3x^2 - x^3 - 1}{x^2} = \frac{(3x^2 - x^3 - 1)' \cdot x^2 - (3x^2 - x^3 - 1) \cdot x'}{x^4} = \frac{(6x - 2x^2) \cdot x^2 - 3x^2 + x^3 + 1}{x^4}$$

$$= \frac{6x^3 - 2x^4 - 3x^2 + x^3 + 1}{x^4} = \frac{-2x^4 + 7x^3 - 3x^2 + 1}{x^4}$$

$$-2x^4 + 7x^3 - 3x^2 + 1 = 0$$

$$-2x^4 + 7x^3 - 3x^2 = -1$$

$$x^2(-2x^2 + 7x - 3) = -1$$

$$x^2 = -1/x \quad -2x^2 + 7x - 3 = 0$$

$$a=1 \quad b=7 \quad c=-3$$

$$x_{1,2} = \frac{-7 \pm \sqrt{49+12}}{2} = \frac{-7 \pm \sqrt{61}}{2}$$

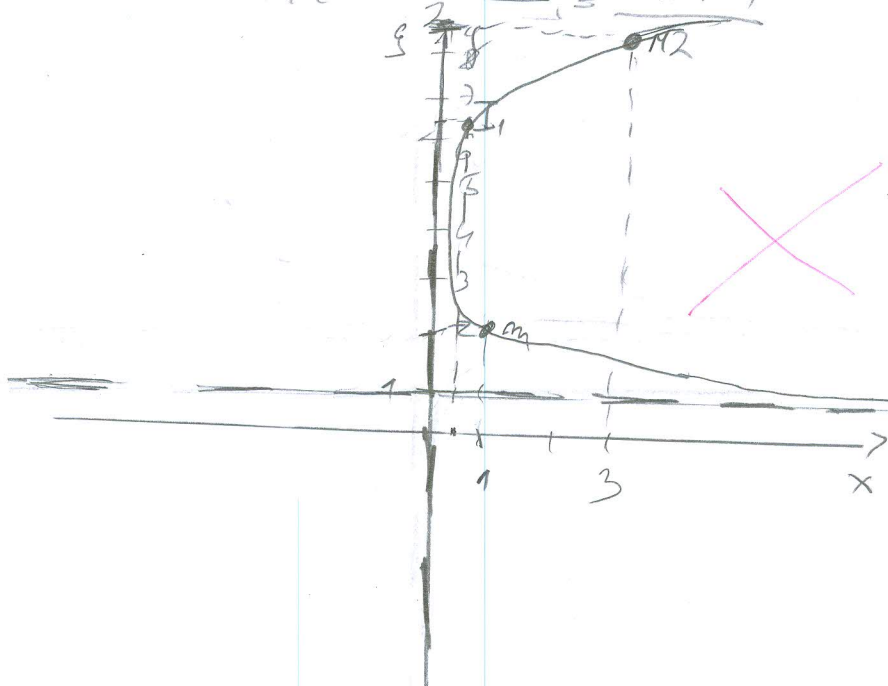
$$x_1 = \frac{-7 - 7,8}{2} = -7,4$$

$$x_2 = \frac{-7 + 7,8}{2} = 0,4$$

	$-\infty$	-8	$-7,4$	0	$0,4$	5	$+\infty$
f''		+	+	-	-		
f		U	U	U	n		

$$I(0,4; 6,4)$$

$$g(0,4) = x^2 + \frac{1}{x} = 6,4$$



③ a) DERIVATIVA

$$f(x) = \ln(4-x^2) + \cos(4-2x)$$

$$f'(x) = (\ln(4-x^2))' + (\cos(4-2x))'$$

$$= \frac{1}{4-x^2} \cdot (4-x^2)' + (-\sin(4-2x) \cdot (4-2x)')$$

$$= \frac{-2x}{4-x^2} - \sin(4-2x)(-2x)$$

$$= \frac{-2x}{4-x^2} + 2x(\sin(4-2x))$$

b) DOMAINE

$$\ln -1 \leq 0 \geq 1$$

$$-1 \leq 4-x^2 \geq 1$$

$$-1 \leq 4-x^2 \quad 4-x^2 \geq 1$$

$$-1-4 \leq x^2 \quad -x^2 \geq 1-4$$

$$-5 \leq x^2 \quad -x^2 \geq -3$$

$$D \dots x \in \mathbb{R} \setminus \{0\} \quad x^2 \geq 3$$

$$x \geq \sqrt{3}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: LUKA ŽILIC

BROJ INDEKSA: 17-2-0208-2012

D1

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 + z - 4 = 0$. *Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem!* Dalje izracunaj: $\left(\frac{z_1 - z_2}{z_2 + 4}\right)$.

4+3+8

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

10+5

$$\begin{array}{rccccrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 & = & -4 \\ & & x_2 & - & x_3 & + & x_4 & = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 & = & -3 \\ & & -7x_2 & + & 3x_3 & + & x_4 & = & 1 \end{array}$$

3. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(4 - x^2) + \cos(4 - 2x)$.

5+15

4. Odrediti tok funkcije $f(x) = x^2 + \frac{1}{x}$.

15(graf)

5. Odrediti i provjeriti uvrštavanjem: $\lim_{x \rightarrow 1} \frac{\ln x}{\arccos x} =$

4+1

6. Odredi derivaciju funkcije $f(x) = \frac{3}{\cos(3x)}$

10

7. Odrediti tangentu na funkciju $f(x) = \sin x$ tamo gdje je $x = -\frac{\pi}{4}$. Nacrtati graf funkcije i nacrtati izračunatu tangentu.

15+3+2

Ukupno:

2.



2.

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & -4 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & -3 \\ 1 & -7 & 3 & 1 & 1 \end{array} \right]$$

zilić

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **KARLO TERZIĆ**

BROJ INDEKSA: **17-2-0361-2014**

D1

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 + z - 4 = 0$. *Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem!* Dalje izracunaj: $\left(\frac{z_1 - z_2}{z_2 + 4}\right)$.

4+3+8

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

10+5

$$\begin{array}{rccccrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 & = & -4 \\ & & & & x_2 & - & x_3 & + & x_4 & = & 1 \\ x_1 & + & 3x_2 & & & & - & 3x_4 & = & -3 \\ & & - & 7x_2 & + & 3x_3 & + & x_4 & = & 1 \end{array}$$

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15+3+2

Ukupno:



IME I PREZIME: KARLO TERZIĆ

BROJ INDEKSA: 17-2-0361-2014

1. $z^2 + z - 4 = 0$

$$z_{1,2} = \frac{-1 \pm \sqrt{1+16}}{2}$$

$$z_{1,2} = \frac{-1 \pm \sqrt{17}}{2}$$

$$z_1 = -1.56$$

$$z_2 = 1.56$$

$$\left(\frac{1.56 + 1.56}{-1.56 + 4} \right) = \frac{3.12}{2.44} = 1.28$$

5. NEMA RIJEŠENJA

$$\lim_{x \rightarrow 1} \frac{\ln x}{\cos x} = \frac{\ln 1}{\cos 1} = \frac{0}{\cos 1} = \text{NEMA RIJEŠENJA}$$



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

D1

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: ANTONIO ŠARIN

BROJ INDEKSA: 0269080809

1. Neka su z_1 i z_2 rješenja kvadratne jednadžbe $z^2 + z - 4 = 0$. Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem! Dalje izračunaj: $\left(\frac{z_1 - z_2}{z_2 + 4}\right)$.

4+3+8

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

10+5

$$\begin{array}{rccccrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 & = & -4 \\ & & x_2 & - & x_3 & + & x_4 & = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 & = & -3 \\ & & -7x_2 & + & 3x_3 & + & x_4 & = & 1 \end{array}$$

3. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(4 - x^2) + \cos(4 - 2x)$.

5+15

4. Odrediti tok funkcije $f(x) = x^2 + \frac{1}{x}$.

15(graf)

5. Odrediti i provjeriti uvrštavanjem: $\lim_{x \rightarrow 1} \frac{\ln x}{\arccos x} =$

4+1

6. Odredi derivaciju funkcije $f(x) = \frac{3}{\cos(3x)}$

10

7. Odrediti tangentu na funkciju $f(x) = \frac{1}{\sqrt{2}} \sin x$ tamo gdje je $x = -\frac{\pi}{4}$. Nacrtati graf funkcije i nacrtati izračunatu tangentu.

15+3+2

Ukupno:

~~0~~

