

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

C2

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- Riješiti jednačbu: $z^4 - (4+i)^2 = 0$. Prikaži rješenja u kompleksnoj ravni! 12+3
- Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2 + 2}$. 5+15+5
- Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln(4+x^2)$. 5+5+10
- Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \frac{x^2 + 2x - (2+1)}{x^2 - 1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
- Gausovom metodom riješiti matricni sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x + 2y - z + u &= 2 \\ 2x + 5y - z + 2u &= 4 \\ 3x - y - 2z + u &= 3 \\ x - y + 3z - 5u &= 4 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem: $\lim_{x \rightarrow \infty} \frac{\ln x}{x}$.

5

Ukupno:

~~10~~ 15

① $z^4 - (4+i)^2 = 0$

⑤

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 2 & 5 & -1 & 2 & 4 \\ 3 & -1 & 2 & 1 & 3 \\ 1 & -1 & 3 & -5 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & -1 & 2 \\ 0 & 3 & -6 & -4 & -2 \\ 0 & 0 & -2 & 12 & -3 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -7 & 5 & -2 & -3 \\ 0 & 3 & -4 & -6 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & -1 & 2 \\ 0 & 3 & -6 & -4 & -2 \\ 0 & 0 & -2 & 12 & -3 \\ 0 & 0 & 12 & -3/2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 0 & 3 & -4 & 5 & -2 \\ 0 & -7 & 5 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & -1 & 2 \\ 0 & 3 & -6 & -4 & -2 \\ 0 & 0 & -2 & 12 & -3 \\ 0 & 0 & 0 & 3/2 & 1/4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 0 & 3 & -4 & 5 & -2 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -7 & 5 & -2 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & -1 & 2 \\ 0 & 3 & -6 & -4 & -2 \\ 0 & 0 & -2 & 12 & -3 \\ 0 & 0 & 12 & -3/2 & 1/4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 0 & 3 & -4 & 5 & -2 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & -2 & -3 \end{bmatrix}$$

$$\frac{3}{2} u = \frac{1}{4} \Rightarrow u = \frac{1}{6} \quad u = \frac{1}{6}$$

$$-2z + 12u = -3 \Rightarrow z = 4$$

$$3y - 6z - 4u = -2$$

$$y = \frac{68}{9}$$

$$x + 2y + z - u = 2$$

⇓

$$x = \frac{-305}{18}$$

$$\textcircled{6} \lim_{x \rightarrow \infty} \frac{\ln x}{x} = \frac{\infty}{\infty} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{\frac{1}{1}} = \lim_{x \rightarrow \infty} \frac{1}{x} = 0 \quad \checkmark$$

$$\lim_{x \rightarrow \infty} \frac{\ln 1}{1} = 0 \quad ?$$

$$\textcircled{2} f(x) = x - \sqrt{x^2 + 2}$$

$$Dz = 1 \quad \checkmark$$

$$f(x) = x - (x^2 + 2)^{\frac{1}{2}}$$

$$f'(x) = 1 - \frac{1}{2} (x^2 + 2)^{-\frac{1}{2}} \cdot (2x)$$

$$f'(x) = 1 - \frac{1}{\sqrt{x^2 + 2}} \cdot 2x \quad \checkmark$$

$$f''(x) = \frac{1}{4} (x^2 + 2)^{-\frac{3}{2}} \cdot 2x - \frac{1}{2} (x^2 + 2)^{-\frac{1}{2}} \cdot 2x$$

$$f''(x) = \frac{2x}{4 \cdot \sqrt{(x^2 + 2)^3}} - \frac{1}{\sqrt{x^2 + 2}} \quad \times$$

$$f(x) = 0$$

$$x - \sqrt{x^2 + 2} = 0$$

$$x - (x^2 + 2)^{\frac{1}{2}} = 0$$

$$x_1 = 0$$

$$(x^2 + 2)^{\frac{1}{2}} = 0 \quad |^2$$

$$x^2 + 2 = 0$$

$$x^2 = -2$$

$$x_2 = \sqrt{-2}$$

$$H.A. \quad x = x_0$$

$$H.A. \quad H.A.$$

$$\lim_{x \rightarrow \infty} x - \sqrt{x^2 + 2} = -\frac{1}{2}$$

$$V.A. \quad y = y_0$$

$$\lim_{x \rightarrow \infty} x - \sqrt{x^2 + 2} = \lim_{x \rightarrow \infty} x - (x^2 + 2)^{\frac{1}{2}} =$$

$$\textcircled{3} \quad g(x) = \ln(4 + x^2)$$

$$4 + x^2 > 0$$

$$x^2 > -4$$

$$x > \sqrt{-4}$$

$$D_f = \mathbb{R} \checkmark$$

$$\textcircled{4} \quad h(x) = \frac{x^2 + 2x - (2+1)}{x^2 - 1}$$

$$h(x) = \frac{x^2 + 2x - 3}{x^2 - 1} = \frac{x^2 + 2x + 1 - 4}{x^2 - 1}$$

$$h(x) = \frac{(x+1)^2 - 4}{x^2 - 1}$$

$$D_f = \mathbb{R} \setminus \{-1, 1\}$$

