

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod! B2

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME: Foko Šimurinu

BROJ INDEKSA:

1. Riješi jednadžbu među kompleksnim brojevima:  $z^3 - 6 + 7i = 0$ . Prikaži rješenja u kompleksnoj ravnini! 12+3

2. Koji su globalni ekstremi funkcije  $g(x) = \sqrt{x^2 + 7}$  10/8

3. Ispitati asimptote funkcije:  $h(x) = \sqrt{x^2 + 2x}$ . Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote)  
20(graf)

4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat 7+2

$$(a) \lim_{x \rightarrow 0} \left( \frac{\sqrt{7+x} - \sqrt{7}}{x} \right) =$$

$$(b) \lim_{n \rightarrow \infty} \left( \frac{x^2 + 6}{x^2} \right) =$$

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rclcl} 4x & - & y & + & z & + 2u = -1 \\ 2x & + & y & & - 3u & = 4 \\ x & - & y & + & 2z & + u = 2 \\ 2x & + & y & + & z & - 4u = 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije:  $f(x) = \ln(\cos(2x^2 - 1))$ . 10/10

Ukupno:

~~53~~ 59

2.

$$g(x) = \sqrt{x^2 + 7}$$

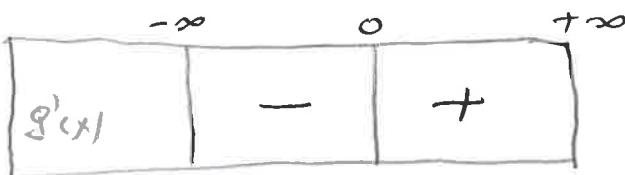
$$x^2 + 7 \geq 0$$

$$x^2 \geq -7$$

$$D(g) = \mathbb{R}$$

$$g'(x) = \frac{2x}{2\sqrt{x^2 + 7}} = \frac{x}{\sqrt{x^2 + 7}}$$

$$g'(x) = 0 \Rightarrow x = 0$$



$$g(0) = \sqrt{7} \Rightarrow \min(0, \sqrt{7})$$

GLOB MAX?

8

$$6. \quad f(x) = \ln(\cos(2x^2 - 1))$$

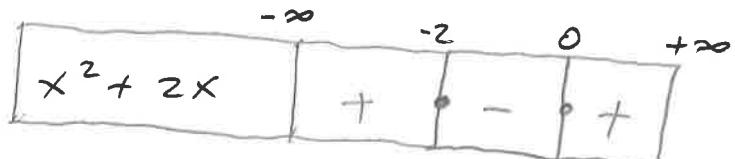
$$f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot (-\sin(2x^2 - 1)) \cdot 4x \quad \checkmark$$

$$= -4x \cdot \operatorname{tg}(2x^2 - 1)$$

$$3. \quad h(x) = \sqrt{x^2 + 2x}$$

$$\begin{aligned} x^2 + 2x &\geq 0 \\ x(x+2) &\geq 0 \end{aligned}$$

$$x_1 = 0 \quad x_2 = -2$$



$$D(h) = (-\infty, -2] \cup [0, +\infty)$$

$$\begin{aligned} \lim_{x \rightarrow -2^-} h(x) &= 0 & \lim_{x \rightarrow 0^+} h(x) &= 0 \\ \text{Nema V.A.} \end{aligned}$$

$$k = \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 2x}}{x} = \lim_{x \rightarrow -\infty} -\sqrt{1 + \frac{2}{x}} = -1$$

$$L = \lim_{x \rightarrow -\infty} \left( \sqrt{x^2 + 2x} + x \right) \cdot \frac{\sqrt{x^2 + 2x} - x}{\sqrt{x^2 + 2x} - x}$$

$$= \lim_{x \rightarrow -\infty} \frac{x^2 + 2x - x^2}{\sqrt{x^2 + 2x} - x} \stackrel{H \ddot{o} l}{=} \lim_{x \rightarrow -\infty} \frac{2}{-\sqrt{1 + \frac{2}{x}} - 1} = -1 \quad \checkmark$$

$$\angle K A \dots y = -x - 1 \quad \checkmark$$

D. K.A. - ?

$$3. \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2+2x}}{x} = \lim_{x \rightarrow +\infty} \sqrt{1+\frac{2}{x}} = 1$$

$$L = \lim_{x \rightarrow +\infty} (\sqrt{x^2+2x} - x) \cdot \frac{\sqrt{x^2+2x} + x}{\sqrt{x^2+2x} + x}$$

$$= \lim_{x \rightarrow +\infty} \frac{x^2+2x-x^2}{\sqrt{x^2+2x} + x} = \lim_{x \rightarrow +\infty} \frac{2}{\sqrt{1+\frac{2}{x}}+1} = 1$$

DkA ...  $y = x+1$  ✓

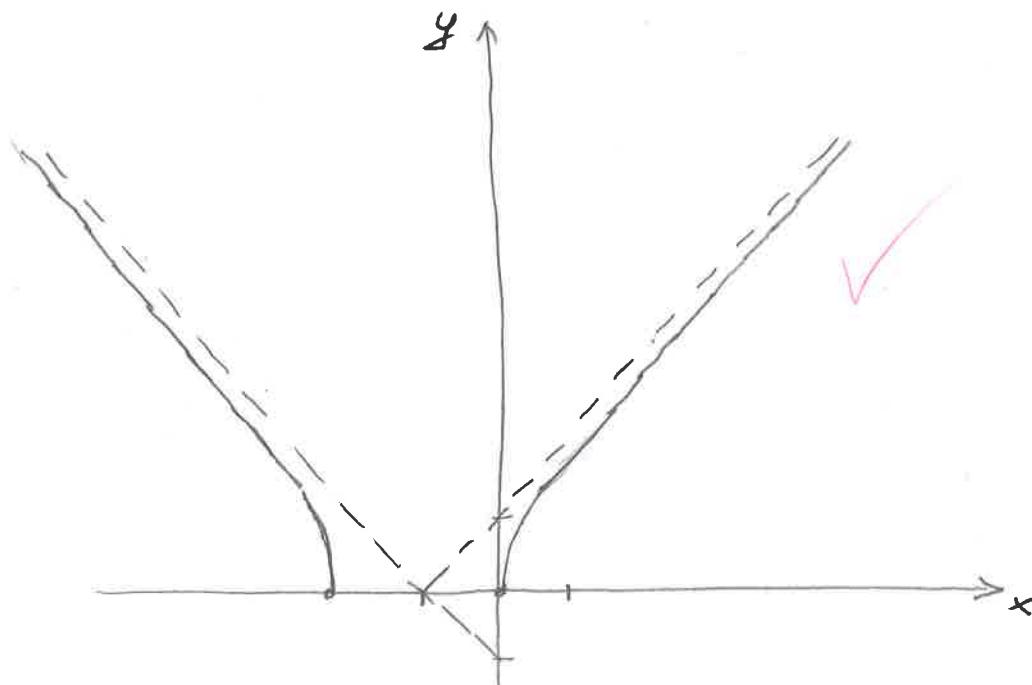
10

$$h'(x) = \frac{2x+2}{2\sqrt{x^2+2x}}$$

$$h'(x) = 0 \Rightarrow x = -1 \quad (\text{Nije u domeni})$$

$h'(x)$	-		+

$$\min(-2, 0) \quad \downarrow \quad \uparrow \quad \min(0, 0)$$



4.

$$a) \lim_{x \rightarrow 0} \left( \frac{\sqrt{7+x} - \sqrt{7}}{x} \right) = \left[ \frac{0}{0} \right] \stackrel{L'H}{=} \underline{\underline{}}$$

$$= \lim_{x \rightarrow 0} \frac{1}{2\sqrt{7+x}} = \frac{1}{2\sqrt{7}} \checkmark$$

$$b) \lim_{n \rightarrow \infty} \left( \frac{x^2+6}{x^2} \right) = \lim_{n \rightarrow \infty} \left( 1 + \frac{6}{x^2} \right) = 1 \quad \checkmark$$

IME I PREZIME: Sefsa Šakanović

BROJ INDEKSA: 17-2-0386-2014

1. Riješi jednadžbu među kompleksnim brojevima:  $z^3 - 6 + 7i = 0$ . Prikaži rješenja u kompleksnoj ravni! 12+3

2. Koji su globalni ekstremi funkcije  $g(x) = \sqrt{x^2 + 7}$  10

3. Ispitati asimptote funkcije:  $h(x) = \sqrt{x^2 + 2x}$ . Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote)

4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat 20(graf)

$$(a) \lim_{x \rightarrow 0} \left( \frac{\sqrt{7+x} - \sqrt{7}}{x} \right) =$$

$$(b) \lim_{n \rightarrow \infty} \left( \frac{x^2 + 6}{x^2} \right) =$$

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rclcl} 4x & - & y & + & z & + 2u = -1 \\ 2x & + & y & & - 3u & = 4 \\ x & - & y & + & 2z & + u = 2 \\ 2x & + & y & + & z & - 4u = 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije:  $f(x) = \ln(\cos(2x^2 - 1))$ . 10

$$1. z^3 - 6 + 7i = 0$$

$$z^3 = 6 - 7i$$

$$z = \sqrt[3]{6 - 7i} \quad (w)$$

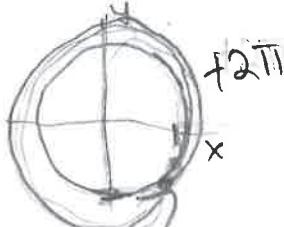
$$w = 6 - 7i$$

$$x = 6 \quad y = -7$$

$$|w| = \sqrt{6^2 + 7^2} = \sqrt{85}$$

$$\operatorname{tg} \varphi = -\frac{7}{6} = -0.862 + 2\pi = 5.42$$

$$w = \sqrt{85} (\cos 5.42 + i \sin 5.42)$$



$$k=0 \rightarrow z_1 = \sqrt[3]{\sqrt{85}} \cdot \left( \cos \frac{5.42 + 2\pi \cdot 0 \cdot \frac{1}{3}}{3} + i \sin \frac{5.42 + 2\pi \cdot 0 \cdot \frac{1}{3}}{3} \right)$$

$$= \sqrt[3]{\sqrt{85}} \cdot (-0.23 + 0.97i)$$

$$= -0.48 + 2.03i$$

$$k=1 \rightarrow z_2 = \sqrt[3]{\sqrt{85}} \cdot \left( \cos \frac{5.42 + 2\pi \cdot 1 \cdot \frac{1}{3}}{3} + i \sin \frac{5.42 + 2\pi \cdot 1 \cdot \frac{1}{3}}{3} \right)$$

$$= \sqrt[3]{\sqrt{85}} \cdot (-0.73 + (-0.69i))$$

$$= -1.53 - 1.45i$$

$$k=2 \rightarrow z_3 = \sqrt[3]{\sqrt{85}} \cdot \left( \cos \frac{5.42 + 2 \cdot 2 \cdot \pi}{3} + i \sin \frac{5.42 + 2 \cdot 2 \cdot \pi}{3} \right)$$

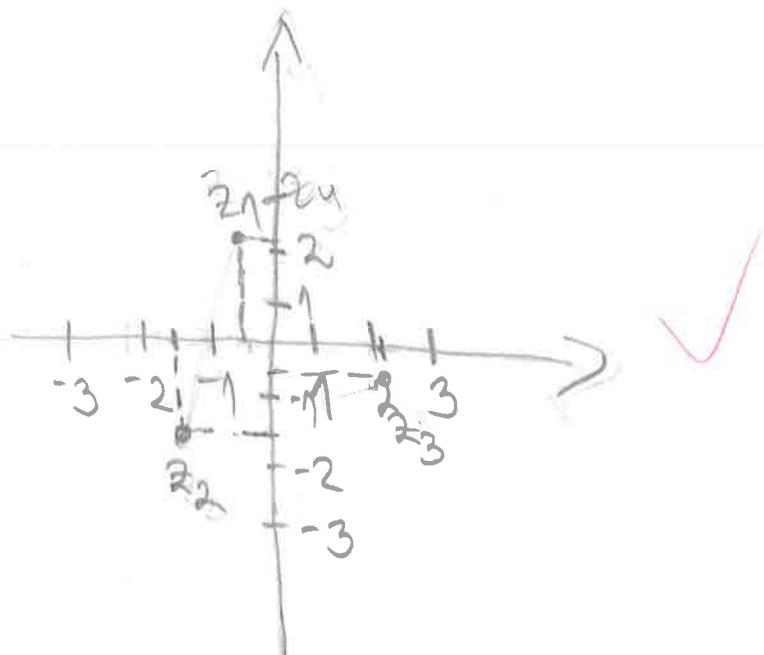
$$= \sqrt[3]{\sqrt{85}} \cdot (0.96 + (-0.28i))$$

$$= 2.01 - 0.59i$$

$$k=3 \rightarrow z_4 = \sqrt[3]{\sqrt{85}} \cdot \left( \cos \frac{5.42 + 2 \cdot 3 \cdot \pi}{3} + i \sin \frac{5.42 + 2 \cdot 3 \cdot \pi}{3} \right)$$

$$= \sqrt[3]{\sqrt{85}} \cdot (-0.23 + 0.97i)$$

$$= -0.48 + 2.03i$$



4.

a)  $\lim_{x \rightarrow 0} \frac{(\sqrt{7+x} - \sqrt{7})}{x} = \lim_{x \rightarrow 0} \frac{(\sqrt{7+0} - \sqrt{7})}{0} = \frac{\sqrt{7} - \sqrt{7}}{0} = \frac{0}{0}$

b)  $\lim_{x \rightarrow \infty} \frac{x^2 + 6}{x^2} = LH = \lim_{x \rightarrow \infty} \frac{(x^2 + 6)/x^2}{x^2/x^2} = \lim_{x \rightarrow \infty} 1 + \frac{6/x^2}{1}$

= 1 ✓

③ NASTAVAK 3. zadatka

$$f(x) = \sqrt{x^2 + 2x}$$

$$f'(x) = \frac{1}{2\sqrt{x^2 + 2x}} \cdot (2x + 2) = \frac{2x + 2}{2\sqrt{x^2 + 2x}}$$

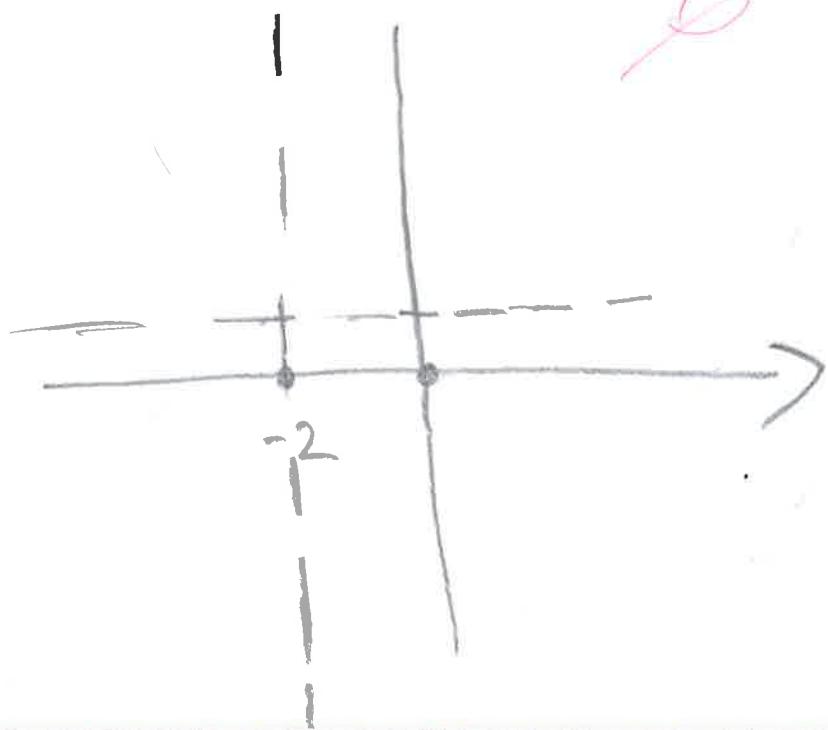
$$f'(x) = 0$$

$$2x + 2 = 0$$

$$2x = -2$$

$$\boxed{x = -1}$$

$$S(-1, 0) \quad \frac{f'(x)}{f}$$



IME I PREZIME:

Šeila Šakanović

BROJ INDEKSA: 17 - 2 - 0386 - 2014

$$(2.) \quad g(x) = \sqrt{x^2 + 7}$$

$$g'(x) = \frac{1}{2\sqrt{x^2+7}} \cdot 2x = \frac{2x}{2\sqrt{x^2+7}} = \frac{x}{\sqrt{x^2+7}}$$

$$g'(x) = 0 \quad (0, \sqrt{7}) \text{ - min } \checkmark$$

$$x = 0$$

vertikalna asimptota

$$h(x) = \sqrt{x^2 + 2x}$$

$$x^2 + 2x \geq 0$$

$$x(x+2) \geq 0$$

$$x \geq 0 \quad x \geq -2$$

$$\begin{cases} \lim_{x \rightarrow -2^-} \sqrt{x^2 + 2x} = -\infty \\ \lim_{x \rightarrow -2^+} \sqrt{x^2 + 2x} = \infty \end{cases}$$

OVA - 2

~~x =~~

$$\begin{cases} \lim_{x \rightarrow 0^+} \sqrt{x^2 + 2x} = 0 \end{cases}$$

$$\lim_{x \rightarrow 0^-} \sqrt{x^2 + 2x} = 0$$

H.A.S.M.

$$\lim_{x \rightarrow \pm\infty} \sqrt{x^2 + 2x} = \lim_{x \rightarrow \pm\infty} \sqrt{\frac{x^2}{x^2} + \frac{2x}{x^2}} = \sqrt{1 + \frac{2}{x^2}} = \sqrt{1 + 0} = \sqrt{1} = 1 \times$$

O.H.A.  $y=1$ 

$$f(x) = 0$$

NT(0,0)

$$x^2 + 2x = 0$$

(-2,0)

$$x(x+2) = 0$$

$$x=0 \quad x+2=0$$

$$x=-2$$

⑥.  $f(x) = \ln(\cos(2x^2 - 1))$

$$f'(x) = \ln'(\cos(2x^2 - 1)) \cdot (\cos(2x^2 - 1))'$$
$$= \frac{1}{\cos(2x^2 - 1)} \cdot (-\sin(2x^2 - 1)) \cdot 2x \checkmark$$
$$= -2x \frac{\sin(2x^2 - 1)}{\cos(2x^2 - 1)}$$
$$= -2x \operatorname{tg}(2x^2 - 1)$$

IME I PREZIME: Šefka Šakanović

BROJ INDEKSA: 17-2-0386-2014

$$\textcircled{5} \quad \left[ \begin{array}{cccc|c} 4 & -1 & 1 & 2 & 1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \sim \left[ \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \xrightarrow{\cdot(-2) \quad \cdot(-4) \quad \cdot(-2)} \left[ \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \xrightarrow{+ \quad + \quad +} \sim$$

$$\sim \left[ \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right] \xrightarrow{:(3)} \left[ \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right] \xrightarrow{\cdot 1 \quad \cdot(-3)} \sim$$

$$- \left[ \begin{array}{cccc|c} 1 & 0 & \frac{2}{3} & -\frac{2}{3} & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 0 & -3 & 3 & -9 \\ 0 & 0 & 1 & -1 & 3 \end{array} \right] \xrightarrow{(-3)} \left[ \begin{array}{cccc|c} 1 & 0 & \frac{2}{3} & -\frac{2}{3} & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -1 & 3 \end{array} \right] \xrightarrow{\cdot(-1) \quad \cdot \frac{4}{3} \quad \cdot \frac{2}{3}} \sim$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -3 & 4 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad \left[ \begin{array}{l} x=0 \\ y=4 \\ z=3 \\ u=0 \end{array} \right] \rightarrow \text{rješenje Gausovom metodom}$$

BESKONACNO RJEŠENJE  
 $x=0$   
 $y = 4 + 3u$   
 $z = 3 + u$

$$\left[ \begin{array}{c} x \\ y \\ z \\ u \end{array} \right] = \left[ \begin{array}{c} 0 \\ 4 \\ 3 \\ 0 \end{array} \right] + \left[ \begin{array}{c} 0 \\ 3u \\ u \\ 1 \end{array} \right], \lambda \in \mathbb{R}$$

PROVJERA:  
 $\frac{4 \cdot 0 - 4 + 3 + 2 \cdot 0}{4} = -1$   
 $0 + 1 + 1 = -1$

$$0 - 4 + 2 \cdot 3 + 0 = 2$$

$$2 = 2$$

$$2 \cdot 0 + 4 - 3 \cdot 0 = 4$$

$$4 = 4$$

$$2 \cdot 0 + 4 + 3 - 4 \cdot 0 = 7$$

$$7 = 7$$

0001

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod! B2

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME:

STIPE  
PREĐORAN

BROJ INDEKSA: 0269082252

- ✓ 1. Riješi jednadžbu među kompleksnim brojevima:  $z^3 - 6 + 7i = 0$ . Prikaži rješenja u kompleksnoj ravnini! 12+3
2. Koji su globalni ekstremi funkcije  $g(x) = \sqrt{x^2 + 7}$  10
3. Ispitati asimptote funkcije:  $h(x) = \sqrt{x^2 + 2x}$ . Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote)  
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- ✓ 4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

(a)  $\lim_{x \rightarrow 0} \left( \frac{\sqrt{7+x} - \sqrt{7}}{x} \right) =$  7+2

(b)  $\lim_{n \rightarrow \infty} \left( \frac{x^2 + 6}{x^2} \right) =$  4+2

- ✓ 5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 5/10

$$\begin{array}{rclcl} 4x & - & y & + & z & + 2u = -1 \\ 2x & + & y & & - & 3u = 4 \\ x & - & y & + & 2z & + u = 2 \\ 2x & + & y & + & z & - 4u = 7 \end{array}$$

- ✓ 6. Odrediti prvu derivaciju funkcije:  $f(x) = \ln(\cos(2x^2 - 1))$ . 10

Ukupno:

24



STIPE

PREDOVAN

VNREDNI

IME I PREZIME:

BROJ INDEKSA:

5) Rješi sustav gauševom metodom i obavezno napiši rješenje.

$$4x - y + z + 2u = -1$$

$$2x + y - 3u = 4$$

$$x - y + 2z + u = 2$$

$$2x + y + z - 4u = 7$$

$$\left[ \begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \xrightarrow{\cdot(-2)} \left[ \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -2 & -5 & -4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \xrightarrow{\cdot(-4)} \left[ \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -8 & -20 & -16 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \xrightarrow{\cdot(-1)} \left[ \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -8 & -20 & -16 \\ 0 & 9 & -7 & -18 & -17 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -7 & -2 & -16 \\ 0 & 9 & -7 & -18 & -17 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \xrightarrow{:3} \left[ \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -\frac{7}{3} & -\frac{2}{3} & -\frac{16}{3} \\ 0 & 3 & -7 & -18 & -17 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -\frac{7}{3} & -\frac{2}{3} & -\frac{16}{3} \\ 0 & 3 & -7 & -18 & -17 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right] \xrightarrow{\cdot(-1)} \left[ \begin{array}{cccc|c} 1 & 0 & \frac{2}{3} & -\frac{2}{3} & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 0 & 3 & 9 & -9 \\ 0 & 0 & 1 & -1 & 3 \end{array} \right] \xrightarrow{:(-3)} \left[ \begin{array}{cccc|c} 1 & 0 & \frac{2}{3} & -\frac{2}{3} & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -1 & 3 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & \frac{2}{3} & -\frac{2}{3} & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -1 & 3 \end{array} \right] \xrightarrow{\cdot(-1)} \left[ \begin{array}{cccc|c} 1 & 0 & \frac{2}{3} & -\frac{2}{3} & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & 1 & -3 \end{array} \right] \xrightarrow{\cdot(-\frac{2}{3})} \left[ \begin{array}{cccc|c} 1 & 0 & 0 & -\frac{2}{3} & 2 \\ 0 & 1 & 0 & -\frac{5}{3} & 0 \\ 0 & 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & 1 & -3 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & -\frac{2}{3} & 2 \\ 0 & 1 & 0 & -\frac{5}{3} & 0 \\ 0 & 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & 1 & -3 \end{array} \right] \xrightarrow{\cdot(-1)} \left[ \begin{array}{cccc|c} 1 & 0 & 0 & \frac{2}{3} & 2 \\ 0 & 1 & 0 & \frac{5}{3} & 0 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -1 & 3 \end{array} \right] \xrightarrow{\cdot(\frac{1}{3})} \left[ \begin{array}{cccc|c} 1 & 0 & 0 & \frac{2}{3} & 2 \\ 0 & 1 & 0 & \frac{5}{3} & 0 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -1 & 3 \end{array} \right] \xrightarrow{\text{Gauševom metodom}} \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right] \xrightarrow{\text{Lijevo množenje sa jedinicom}} \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

Sustav ima beskonačno mnogo rješenja a mi moramo neličiti jedno od rješenja i umanjiti

$$\begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \\ 1 \\ 1 \end{bmatrix} \lambda, \lambda \in \mathbb{R}$$

10

$$x=0$$

$$z=k$$

$$y-u=4$$

$$u-z=-3 \cdot k$$

$$k=\mathbb{R}$$

$$x=0$$

$$y=4+zu$$

$$u=-3 \cdot 4+z$$

$$x=$$

$$y=$$

$$z=$$

$$u=$$

$$x=$$

$$y=$$

$$z=$$

$$u=$$

$$x=$$

$$y=$$

$$z=$$

$$u=$$

$$x=0$$

$$y-3u=4$$

$$z-u=-3 \cdot 4$$

$$u=k$$

$$k=\mathbb{R}$$

$$x=0$$

$$y=4+3u$$

$$z=-3 \cdot 4+u$$

$$x=0$$

$$y=4$$

$$z=-3$$

$$u=0$$

$$x=0$$

$$y-3u=4$$

$$z+u$$

$$x=0$$

$$y=4+3 \cdot k$$

$$z=-3 \cdot 4+k$$

$$u=0$$

$$4 \cdot 0 - 4 + 3 + 2 \cdot 0 = -1$$

$$0 \vee 0 \quad \text{DE}$$

PREVJERA

$$2 \cdot 0 + 4 - 3 \cdot 0 = 4$$

$$0 - 4 + 2 \cdot 3 + 0 = 2$$

$$2 \cdot 0 + 4 + 3 - 4 \cdot 0 = 7$$

$$z_1 = \sqrt[3]{9.22} \cdot \left[ \cos \left( \frac{5.42 + 2 \cdot 1 \cdot \pi}{3} \right) + i \cdot \sin \left( \frac{5.42 + 2 \cdot 1 \cdot \pi}{3} \right) \right]$$

$$z_1 = \sqrt[3]{9.22} \cdot [-0.72 - 0.68i]$$

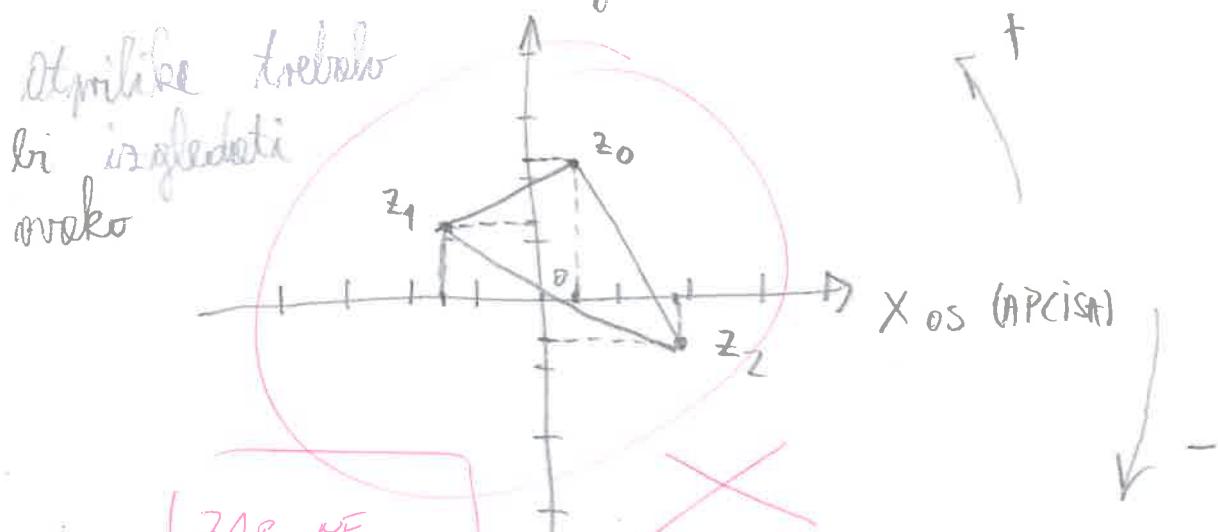
$$z_1 = -1.51 + 1.43i$$

$$z_2 = \sqrt[3]{9.22} \cdot \left[ \cos \left( \frac{5.42 + 2 \cdot 2 \cdot \pi}{3} \right) + i \cdot \sin \left( \frac{5.42 + 2 \cdot 2 \cdot \pi}{3} \right) \right]$$

$$z_2 = \sqrt[3]{9.22} \cdot [0.95 + i \cdot (-0.28)]$$

$$z_2 = 1.99 - 0.58i$$

y os (ordinata)



ZAR NE  
VIDITE SA  
SKICE DA  
NIJE DOBRO?

TREBALI BI VIDJETI  
ZNAK MERCEDESA!

IME I PREZIME: STIPE PREDOVAN

BROJ INDEKSA: 0269082252

1) Reši jednacu među kompleksnim brojevima  $z^3 - 6 + 7i = 0$ .  
Rešenje se piše u kompleksnoj ravni.

$$\textcircled{1} \quad z^3 = 6 + 7i = 0$$

$$z^3 = 6 - 7i / \sqrt[3]{-7}$$

$$z = \sqrt[3]{6 - 7i}$$

$$x = 6 \quad y = -7$$

$$\theta = \arctan \frac{y}{x}$$

$$\theta = \arctan \frac{-7}{6}$$

$$\theta = -0.86 \text{ rad}$$

KUT NE SMI BITI  
NEGATIVAN

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{6^2 + (-7)^2}$$

$$r = 9.22$$

Zad je  $x$  pozitivno i  $y$  negativno tako da je  $\theta$  u trećem kvadrantu pa je  $\theta$  pravi odjeli kota  $2\pi + \theta \rightarrow$  kosi imamo

$$\theta_{\text{pravi}} = 2 \cdot 3.14 + (-0.86)$$

$$\theta_{\text{pravi}} = 5.42$$

Tako je  $z^3$  to smije da smo imati 3 kompleksna rešenja

Koristimo formulu:

$$z_k = \sqrt[n]{r} \cdot \left[ \cos \left( \frac{\theta + 2k\pi}{n} \right) + i \sin \left( \frac{\theta + 2k\pi}{n} \right) \right], \quad k = 0, 1, 2$$

$$z_0 = \sqrt[3]{9.22} \cdot \left[ \cos \left( \frac{5.42 + 2 \cdot 0 \cdot \pi}{3} \right) + i \sin \left( \frac{5.42 + 2 \cdot 0 \cdot \pi}{3} \right) \right]$$

$$z_0 = \sqrt[3]{9.22} \cdot [-0.23 + i \cdot 0.97]$$

$$z_0 = -0.48 + 1.03i \checkmark$$

4) pod b)

$$\lim_{n \rightarrow \infty} \left( \frac{x^2 + 6}{x^2} \right) \stackrel{1}{=} x^2$$

$$\lim_{n \rightarrow \infty} \left( \frac{1 + \frac{6}{x^2}}{1} \right)$$

$$\lim_{n \rightarrow \infty} \left( \frac{1 + 0}{1} \right)$$

$$\lim_{n \rightarrow \infty} \frac{1}{1} = 1 \quad \checkmark$$

4) pod a)

$$\lim_{x \rightarrow 0} \left( \frac{\sqrt{7+x} - \sqrt{7}}{x} \right)$$

$$\lim_{x \rightarrow 0} \left( \frac{\boxed{\sqrt{7+x} - \sqrt{7}}}{x} \right)$$

$$\lim_{x \rightarrow 0} \left( \frac{\left(\sqrt{7+x}\right)^{\frac{1}{2}} - \sqrt{7}}{x} \right)$$

$$\lim_{x \rightarrow 0} \left( \frac{\frac{1}{2} \cdot \sqrt{7+x}^{\frac{1}{2}-1} \cdot (7+x)^{\frac{1}{2}} - 0}{x} \right)$$

$$\lim_{x \rightarrow 0} \left( \frac{\frac{1}{2} \cdot \sqrt{7+x}^{\frac{1}{2}-1} \cdot (7+x)^{\frac{1}{2}}}{1} \right) = ?$$

Kada limes doći u beskonačno neki razlomak obliku neki broj kroz nulto je nula.

$$\frac{x^2}{x^{2-1}}$$

doći

STIPE PREPORAN  
VANREDNI

IME I PREZIME:

BROJ INDEKSA:

6) Odrediti prvu derivaciju funkcije

$$f(x) = \ln(\cos(2x^2 - 1))$$

$$f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot (\cos(2x^2 - 1))'$$

$$f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot (-\sin x \cdot (2x^2 - 1) \cdot (2x^2 - 1)')$$

$$f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot (-\sin x \cdot (2x^2 - 1) \cdot (2 \cdot 2x^{2-1} - 0))$$

$$f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot \frac{(-\sin x \cdot (2x^2 - 1) \cdot 4x)}{1} \quad \checkmark$$

$$f'(x) = \frac{-\sin x \cdot (2x^2 - 1) \cdot 4x}{\cos(2x^2 - 1)}$$

$$\frac{\sin x}{\cos x} = \tan x$$

$$f'(x) = 4x \cdot \tan(2x^2 - 1)$$

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod! B2

IME I PREZIME: JURE ŠUŠĆ

BROJ INDEKSA: 17-1-0259-2014

0269087878

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

1. Riješi jednadžbu među kompleksnim brojevima:  $z^3 - 6 + 7i = 0$ . Prikaži rješenja u kompleksnoj ravnini! 12+3

2. Koji su globalni ekstremi funkcije  $g(x) = \sqrt{x^2 + 7}$  10

3. Ispitati asimptote funkcije:  $h(x) = \sqrt{x^2 + 2x}$ . Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote)  
20(graf)

4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat 7+2

$$(a) \lim_{x \rightarrow 0} \left( \frac{\sqrt{7+x} - \sqrt{7}}{x} \right) =$$

$$(b) \lim_{n \rightarrow \infty} \left( \frac{x^2 + 6}{x^2} \right) =$$

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 4+2

$$\begin{array}{rclll} 4x & - & y & + & z & + 2u = -1 \\ 2x & + & y & & - 3u & = 4 \\ x & - & y & + & 2z & + u = 2 \\ 2x & + & y & + & z & - 4u = 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije:  $f(x) = \ln(\cos(2x^2 - 1))$ . 10

15+5

10

Ukupno:

21

4.) a)  $\lim_{x \rightarrow 0} \left( \frac{\sqrt{7+x} - \sqrt{7}}{x} \right) = \frac{\sqrt{7} - \sqrt{7}}{0} = \frac{0}{0} = 0 \quad VA \dots x=0$

b)  $\lim_{n \rightarrow \infty} \left( \frac{x^2 + 6/x^2}{x^2/x^2} \right) = \lim_{n \rightarrow \infty} \frac{1 + \frac{6}{x^2}}{1} = \frac{1}{1} = 1 \quad HA \dots y=1$

6.)  $f(x) = \ln(\cos(2x^2 - 1))$

$$f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot -2\sin(2x^2 - 1) \cdot 4x = \frac{-2\sin(2x^2 - 1) + 4x}{\cos(2x^2 - 1)}$$



$$5.) \begin{array}{c} x \ y \ z \ u \\ \left[ \begin{array}{ccccc|c} 4 & -1 & 1 & 2 & 1 & -1 \\ 2 & 1 & 0 & -3 & 1 & 4 \\ 1 & -1 & 2 & 1 & 1 & 2 \\ 2 & 1 & 1 & -4 & 1 & 7 \end{array} \right] \cdot \frac{1}{4} \end{array} \quad \left[ \begin{array}{ccccc|c} 1 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 1 & -\frac{1}{4} \\ 2 & 1 & 0 & -3 & 1 & 4 \\ 1 & -1 & 2 & 1 & 1 & 2 \\ 2 & 1 & 1 & -4 & 1 & 7 \end{array} \right] \xrightarrow{\begin{array}{l} +1 \\ + \\ + \end{array}} \left[ \begin{array}{ccccc|c} 1 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 1 & -\frac{1}{4} \\ 0 & 2 & 1 & -3 & 2 & 7 \\ 1 & -1 & 2 & 1 & 1 & 2 \\ 2 & 1 & 1 & -4 & 1 & 7 \end{array} \right] \cdot (-2) \cdot (-1) \cdot (-2)$$

$$\left[ \begin{array}{ccccc|c} 1 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 1 & -\frac{1}{4} \\ 0 & \frac{3}{2} & -\frac{1}{2} & -4 & \frac{9}{2} & \frac{9}{2} \\ 0 & -\frac{3}{4} & \frac{1}{4} & \frac{1}{2} & \frac{9}{4} & \frac{9}{4} \\ 0 & \frac{3}{2} & \frac{1}{2} & -5 & \frac{15}{2} & \frac{15}{2} \end{array} \right] \cdot \frac{2}{3} \quad \left[ \begin{array}{ccccc|c} 1 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 1 & -\frac{1}{4} \\ 0 & 1 & -\frac{1}{3} & -\frac{8}{3} & 3 & 3 \\ 0 & -\frac{3}{4} & \frac{1}{4} & \frac{1}{2} & \frac{9}{4} & \frac{9}{4} \\ 0 & \frac{3}{2} & \frac{1}{2} & -5 & \frac{15}{2} & \frac{15}{2} \end{array} \right] \xrightarrow{\begin{array}{l} + \\ + \\ + \end{array}} \left[ \begin{array}{ccccc|c} 1 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 1 & -\frac{1}{4} \\ 0 & 1 & -\frac{1}{3} & -\frac{8}{3} & 3 & 3 \\ 0 & 0 & 1 & -1 & 3 & 3 \\ 0 & 0 & 1 & -1 & 3 & 3 \end{array} \right] \cdot \left( \frac{3}{4} \right) \cdot \left( \frac{1}{4} \right) \cdot \left( -\frac{3}{2} \right)$$

$$\left[ \begin{array}{ccccc|c} 1 & 0 & \frac{1}{6} & -\frac{1}{6} & 1 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{3} & -\frac{8}{3} & 1 & 3 \\ 0 & 0 & \frac{3}{2} & -\frac{3}{2} & \frac{9}{2} & \frac{9}{2} \\ 0 & 0 & 1 & -1 & 3 & 3 \end{array} \right] \cdot \frac{2}{3} \quad \left[ \begin{array}{ccccc|c} 1 & 0 & \frac{1}{6} & -\frac{1}{6} & 1 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{3} & -\frac{8}{3} & 1 & 3 \\ 0 & 0 & 1 & -1 & 3 & 3 \\ 0 & 0 & 1 & -1 & 3 & 3 \end{array} \right] \xrightarrow{\begin{array}{l} + \\ + \\ + \end{array}} \left[ \begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -3 & 1 & 4 \\ 0 & 0 & 1 & -1 & 3 & 3 \\ 0 & 0 & 1 & -1 & 3 & 3 \end{array} \right] \cdot (-1) \cdot \left( \frac{1}{3} \right) \cdot \left( -\frac{1}{6} \right)$$

$$\left[ \begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -3 & 1 & 4 \\ 0 & 0 & 1 & -1 & 3 & 3 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{\begin{array}{l} + \\ + \\ + \\ + \end{array}} \left[ \begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -3 & 1 & 4 \\ 0 & 0 & 1 & -1 & 3 & 3 \\ 0 & 0 & 0 & 1 & -3 & 3 \end{array} \right] \xrightarrow{\begin{array}{l} + \\ + \\ + \\ + \end{array}} \left[ \begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -5 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -3 \end{array} \right]$$

$$\begin{array}{l} x=0 \\ y=-5 \\ z=0 \\ u=-3 \end{array} \quad \begin{array}{l} 4 \cdot 0 + 5 + 0 + 2 \cdot (-3) = -1 \checkmark \\ 2 \cdot 0 - 5 - 3 \cdot (-3) = 4 \checkmark \\ 0 + 5 + 2 \cdot 0 - 3 = 2 \checkmark \\ 2 \cdot 0 - 5 + 0 - 4 \cdot (-3) = 7 \checkmark \end{array}$$

✓

PJEŠEME JE:

$$x = 0$$

$$y = 4 + 3\lambda$$

$$z = 3 + \lambda$$

$$u = \lambda$$

$$\lambda \in \mathbb{R}$$

PROVJERITE DETERMINANTU MATRICE SUSTAVA!

IME I PREZIME: JURE ŠUSTIĆ

BROJ INDEKSA: 14-1-0259-2014  
0269087878

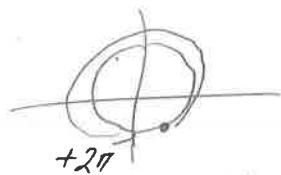
$$1.) z^3 - 6 + 4i = 0$$

$$z^3 = 6 - 4i$$

$$W_1 = 6 - 4i$$

$$|W_1| = \sqrt{6^2 + (-4)^2} = \sqrt{85}$$

$$\operatorname{tg} \varphi_1 = \frac{-4}{6} = -0,8621400547 \\ \varphi_1 = 5,421015253$$



$$W_1 = \sqrt{85} (\cos(\varphi_1) + i \sin(\varphi_1))$$

$$W_1 = \sqrt{85} (\cos(5,421015253) + i \sin(5,421015253))$$

$$W_1 = 6 - 4i$$

$$z^3 = 6 - 4i$$

$$z = \sqrt[3]{\sqrt{85}} \cdot \left( \cos \frac{\varphi_1 + 2k\pi}{3} + i \sin \frac{\varphi_1 + 2k\pi}{3} \right) \quad k=0, 1, 2$$

$$\varphi_2 = \varphi_1 = -0,8621400547 + 2\pi$$

$$\varphi_2 = 5,421015253$$



$$k=0 \Rightarrow \sqrt[3]{\sqrt{85}} \cdot \left( \cos \frac{5,42 + 2 \cdot 0 \cdot \pi}{3} + i \sin \frac{5,42 + 2 \cdot 0 \cdot \pi}{3} \right)$$

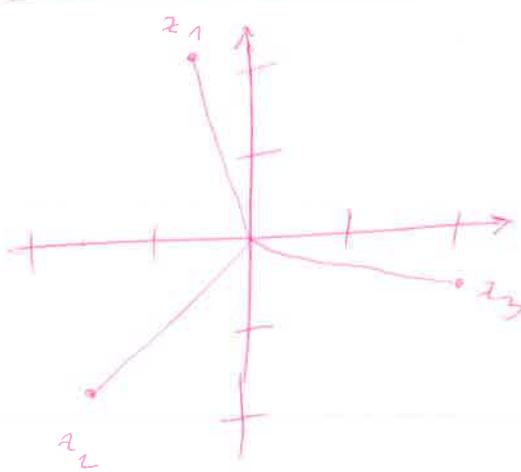
$$k=0 \Rightarrow -0,49 + 2,04i$$

$$k=1 \Rightarrow \sqrt[3]{\sqrt{85}} \cdot \left( \cos \frac{5,42 + 2 \cdot 1 \cdot \pi}{3} + i \sin \frac{5,42 + 2 \cdot 1 \cdot \pi}{3} \right)$$

$$k=1 \Rightarrow -1,52 - 1,44i$$

$$k=2 \Rightarrow \sqrt[3]{\sqrt{85}} \cdot \left( \cos \frac{5,42 + 2 \cdot 2 \cdot \pi}{3} + i \sin \frac{5,42 + 2 \cdot 2 \cdot \pi}{3} \right)$$

$$k=2 \Rightarrow 2,01 - 0,59i$$



$$2.) g(x) = \sqrt{x^2 + 4}$$

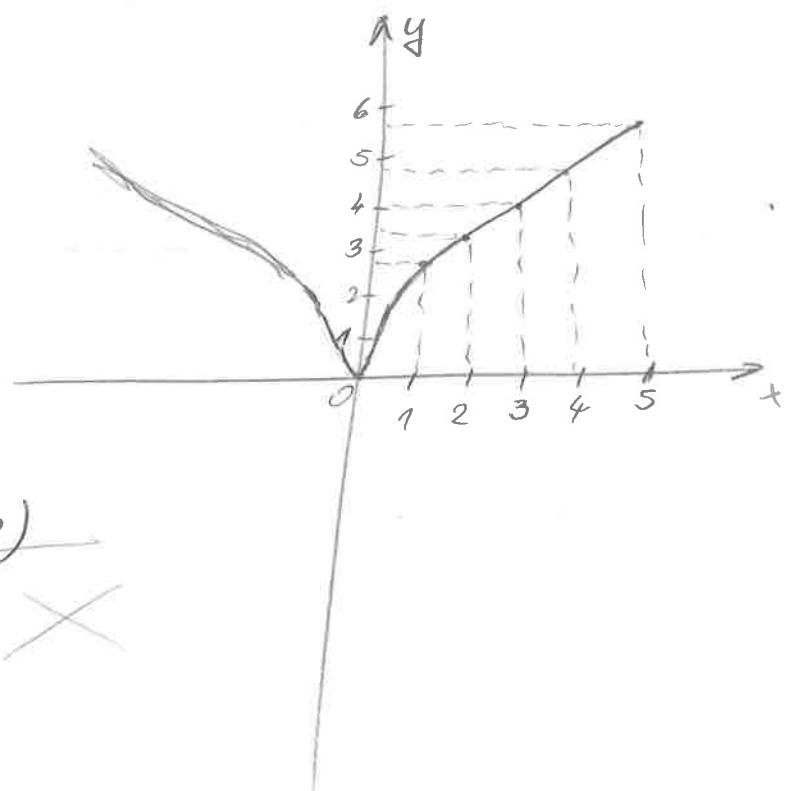
$$g'(x) = \frac{1}{2\sqrt{x^2+4}} + 2x = \frac{2x}{2\sqrt{x^2+4}}$$

$$g''(x) = \frac{\frac{1}{2} \cdot 2\sqrt{x^2+4} - 2x \cdot \frac{1}{2\sqrt{x^2+4}} \cdot 2x}{(2\sqrt{x^2+4})^2} = \frac{4\sqrt{x^2+4} - \frac{4x}{\sqrt{x^2+4}}}{4(x^2+4)}$$

$$g''(x) = \frac{4\sqrt{x^2+4} - \frac{x}{\sqrt{x^2+4}}}{4(x^2+4)}$$

$$g'(x) = 0 \quad 2x = 0 \quad x = 0$$

$$g''(x) = 0$$



$-\infty$	0	$+\infty$
-		+
$\frac{g'(x)}{g'(x)}$		
$g(x)$	↓	↗

$\min(0, 0)$



$$3.) h(x) = \sqrt{x^2 + 2x}$$

$$x^2 + 2x \geq 0$$

$$x^2 + 2x + 1 \geq 0$$

$$x_{1,2} = \frac{-2 \pm \sqrt{4 - 4}}{2}$$

$$x_{1,2} = \frac{-2 \pm 0}{2} = -1$$

$$D(f) = \mathbb{R} \setminus \{-1\}$$

VA ...  $x = -1$

HA ...

$$KA = \frac{h(x)}{x} = \frac{\sqrt{x^2 + 2x}}{x} + 1^2 = \frac{x^2 + 2x/x^2}{x^2/x^2} = \frac{1 + \frac{2}{x}}{1} = 1$$

$$y_1 = 1 = x_1$$

$$y_2 = 2 = x_2$$

$$d = h(x) - ax = \sqrt{x^2 + 2x} - x^2 = x^2 + 2x - x^2 = 2x = 2$$

$$h'(x) = \frac{1}{2\sqrt{x^2+2x}} + x^2 \cdot 2 = \frac{2x^2}{2\sqrt{x^2+2x}} = \frac{x^2}{\sqrt{x^2+2x}}$$

$$h''(x) = \frac{4x \cdot (2\sqrt{x^2+2x}) - 2x^2 \cdot \frac{1}{2\sqrt{x^2+2x}} \cdot 2x + 2}{4(x^2+2x)} = \frac{8x\sqrt{x^2+2x} - x^2 + 2x + 2}{4x^2 + 8x}$$

$$= \frac{x^2 + 6x + 2}{4x^2 + 8x}$$

$$h'(x=0) \quad x^2 = 0 \Rightarrow x=0$$

$$h''(x)=0 \quad x^2 + 6x + 2 = 0$$

$$x_{1,2} = \frac{-6 \pm \sqrt{36 - 8}}{2}$$

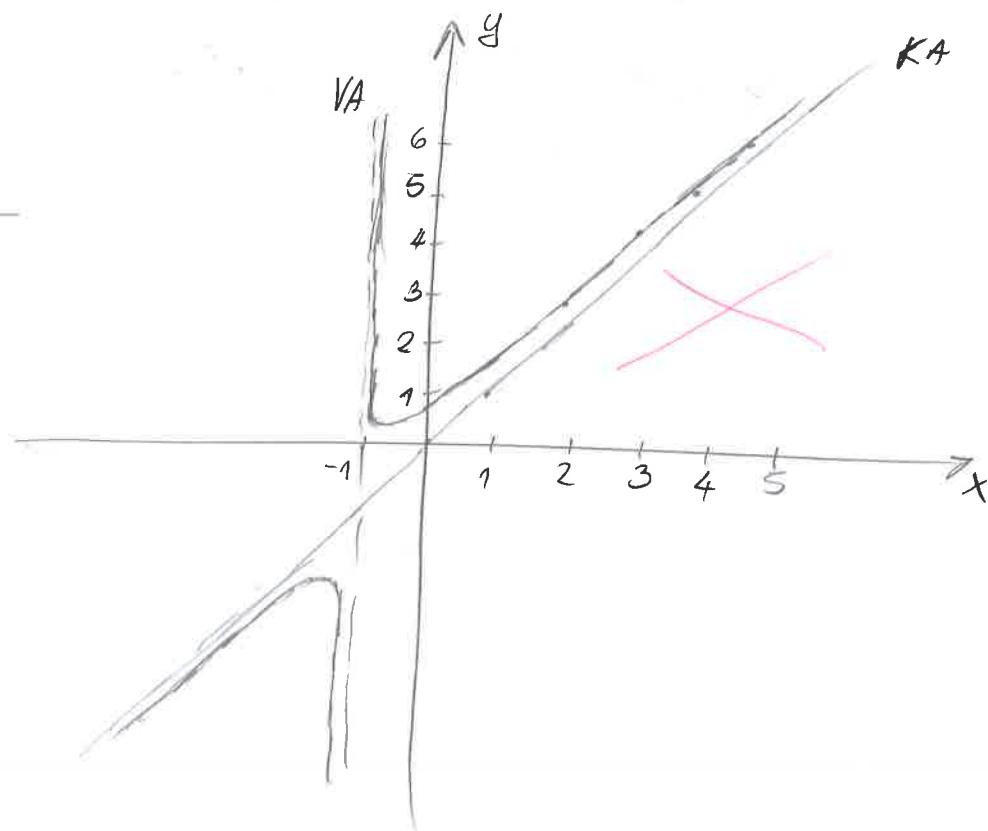
$$x_{1,2} = \frac{-6 \pm 2\sqrt{7}}{2}$$

$$x_1 = -0,35$$

$$x_2 = -5,65$$

$$-\infty \quad -5,65 \quad -0,35 \quad 0 \quad \infty$$

$h'(x)$	+			
$h''(x)$				
$h(x)$				



- Riješi jednadžbu među kompleksnim brojevima:  $z^3 - 6 + 7i = 0$ . Prikaži rješenja u kompleksnoj ravnini! 12+3
- Koji su globalni ekstremi funkcije  $g(x) = \sqrt{x^2 + 7}$  10
- Ispitati asimptote funkcije:  $h(x) = \sqrt{x^2 + 2x}$ . Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote)  
20(graf)
- Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

$$(a) \lim_{x \rightarrow 0} \left( \frac{\sqrt{7+x} - \sqrt{7}}{x} \right) =$$

$$(b) \lim_{n \rightarrow \infty} \left( \frac{x^2 + 6}{x^2} \right) =$$

- Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

$$\begin{array}{rrrrrr} 4x & - & y & + & z & + & 2u = -1 \\ 2x & + & y & & - & 3u = & 4 \\ x & - & y & + & 2z & + & u = 2 \\ 2x & + & y & + & z & - & 4u = 7 \end{array}$$

- Odrediti prvu derivaciju funkcije:  $f(x) = \ln(\cos(2x^2 - 1))$ .

10

Ukupno:

(14)

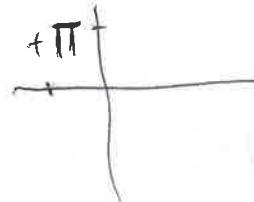
$$1.) z^3 - 6 + 7i = 0$$

$$\underline{z^3 = (-6 + 7i)} \quad \times$$

$$\underline{z^3 =}$$

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ r &= \sqrt{-6^2 + 7^2} \\ r &= \sqrt{13} \end{aligned}$$

$$\begin{aligned} \operatorname{tg} \ell &= \frac{y}{x} \\ \operatorname{tg} \ell &= \frac{7}{-6} \\ \operatorname{tg} \ell &= 4,00 \end{aligned}$$



$$\sqrt[n]{z} = \sqrt[n]{r} \left( \cos \frac{\ell + 2k\pi}{n} + i \sin \frac{\ell + 2k\pi}{n} \right)$$

$$\sqrt[3]{z} = \sqrt[3]{\sqrt{13}} \left( \cos \frac{\ell + 2k\pi}{3} + i \sin \frac{\ell + 2k\pi}{3} \right)$$

$$z_1 = \Rightarrow 1.533 (0.2352 + 0.9719i)$$

$$z_1 = k_0 \Rightarrow 0.3605 + 1.4899i$$

$$k_1 \Rightarrow 1.533 (-0.9593 + (-0.28229)i)$$

$$k_1 \Rightarrow -1.47060 + (-0.43275)i$$

$$k_2 = 1.11 + (-1.03729)i$$

$$5.) \left[ \begin{array}{ccccccc} 4 & -1 & 1 & 2 & 1 & -1 \\ 2 & 1 & 0 & -3 & 1 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 1 & 7 \end{array} \right] \sim \left[ \begin{array}{cccccc} 1 & -1 & 2 & 1 & 1 & 2 \\ 2 & 1 & 0 & -3 & 1 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 1 & 7 \end{array} \right] / \cdot (-2) + II / \cdot (-4) + III / \cdot (-2)$$

$$\left[ \begin{array}{ccccccc} 1 & -1 & 2 & 1 & 1 & 2 \\ 0 & 3 & -4 & -5 & 1 & 0 \\ 0 & 3 & -7 & -2 & 1 & 6 \\ 0 & 3 & -3 & -7 & 1 & 3 \end{array} \right] \sim \left[ \begin{array}{cccccc} 1 & -1 & 2 & 1 & 1 & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 1 & 0 \\ 0 & 3 & -7 & -2 & 1 & 6 \\ 0 & 3 & -3 & -7 & 1 & 3 \end{array} \right] / \cdot 3 \sim \left[ \begin{array}{cccccc} 1 & -1 & 2 & 1 & 1 & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 1 & 0 \\ 0 & 3 & -7 & -2 & 1 & 6 \\ 0 & 3 & -3 & -7 & 1 & 3 \end{array} \right] / \cdot 1 + I / \cdot (-3) + III / \cdot (-3) + IV$$

$$\left[ \begin{array}{ccccccc} 1 & 0 & \frac{2}{3} & -\frac{2}{3} & 1 & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 1 & 0 \\ 0 & 0 & -3 & \frac{2}{3} & 1 & 6 \\ 0 & 0 & 1 & -2 & 1 & 3 \end{array} \right] \sim \left[ \begin{array}{cccccc} 1 & 0 & \frac{2}{3} & -\frac{2}{3} & 1 & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 & 3 \\ 0 & 0 & -3 & \frac{2}{5} & 1 & 6 \end{array} \right] / \cdot (-\frac{2}{3}) + I / \cdot (\frac{4}{3}) + II / \cdot 3 + IV$$

$$\left[ \begin{array}{ccccccc} 1 & 0 & 0 & \frac{2}{3} & 1 & 0 \\ 0 & 1 & 0 & -\frac{10}{3} & 1 & 4 \\ 0 & 0 & 1 & -2 & 1 & 3 \\ 0 & 0 & 0 & -\frac{28}{5} & 1 & 15 \end{array} \right] \sim \left[ \begin{array}{cccccc} 1 & 0 & 0 & \frac{2}{3} & 1 & 0 \\ 0 & 1 & 0 & -\frac{10}{3} & 1 & 4 \\ 0 & 0 & 1 & 2 & 1 & 3 \\ 0 & 0 & 0 & 1 & 1 & -\frac{28}{75} \end{array} \right] / \cdot -\frac{28}{5} \sim \left[ \begin{array}{cccccc} 1 & 0 & 0 & \frac{2}{3} & 1 & 0 \\ 0 & 1 & 0 & -\frac{10}{3} & 1 & 4 \\ 0 & 0 & 1 & 2 & 1 & 3 \\ 0 & 0 & 0 & 1 & 1 & -\frac{28}{75} \end{array} \right] / \cdot \left( -\frac{2}{3} \right) + II / \frac{10}{3} + II$$

$$\sim \left[ \begin{array}{cccccc} 1 & 0 & 0 & 0 & 1 & \frac{56}{225} \\ 0 & 1 & 0 & 0 & 1 & \frac{234}{75} \\ 0 & 0 & 1 & 0 & 1 & \frac{28}{75} \\ 0 & 0 & 0 & 1 & 1 & -\frac{28}{75} \end{array} \right]$$

PROBLEMA?

$$2.) g(x) = \sqrt{x^2 + 7}$$

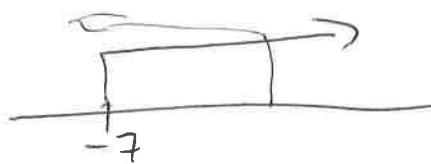
$$x^2 + 7 \neq 0$$

$$x^2 = -7$$

$$x = \pm \sqrt{7}$$

$$x^2 + 7 \geq 0$$

$$x^2 \geq -7$$



$$x \in [-7, 7]$$

?

~~Δx ≠ 0~~

~~Δx ≠ ±~~

6.)  $\ln(\cos(2x-1))$

$$f'(x) = \ln'(\cos(2x^2-1)) \cdot (\cos(2x^2-1))' \cdot (2x^2-1)$$

$$= \frac{1}{\cos(2x^2-1)} \cdot [\sin(2x^2-1)] \cdot 2x \quad \checkmark$$

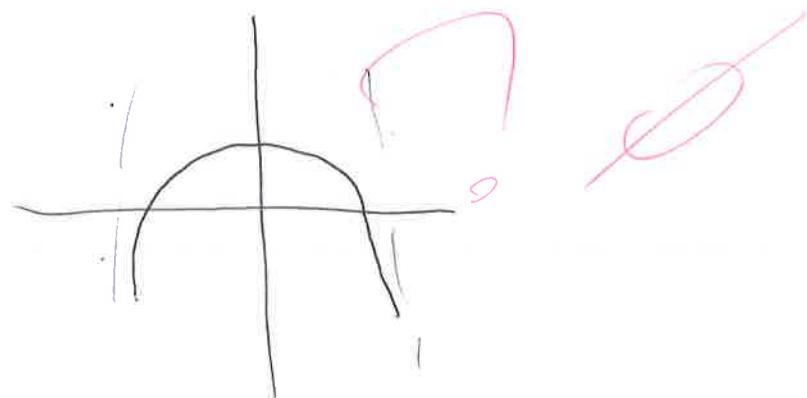
$$= -2x \operatorname{tg}(2x^2-1)$$

4.)  $\lim_{n \rightarrow \infty} \left( \frac{x^2+6}{x^2} \right) = \frac{x^2+6/x^2}{x^2/x^2} = \frac{\cancel{x^2}^1 + \cancel{6}^0}{\cancel{x^2}^1} = \frac{1}{1} = 1 = \text{NE}$

a.)  $\lim_{x \rightarrow \infty} \left( \frac{\sqrt{7+x} - \sqrt{7}}{x} \right) = \frac{\sqrt{7+x} - \sqrt{7}}{x} \cdot \frac{y^2}{y^2} = \frac{7+x^2 - 7}{x} \cdot \frac{1/x^2}{1/x^2} = \frac{\cancel{7+x^2}^1 - \cancel{7}^0}{\cancel{x}^1} = \frac{1}{\cancel{x}^1} = \frac{1}{0} = \infty$

3.)  $n(x) = \sqrt{x^2+2x}$

$$\sqrt{x^2+2x}$$



IME I PREZIME: Aute Jerolimov

BROJ INDEKSA: 47-2-0122-2011

- Riješi jednadžbu među kompleksnim brojevima:  $z^3 - 6 + 7i = 0$ . Prikaži rješenja u kompleksnoj ravni! 12+3
- Koji su globalni ekstremi funkcije  $g(x) = \sqrt{x^2 + 7}$  10
- Ispitati asimptote funkcije:  $h(x) = \sqrt{x^2 + 2x}$ . Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote)  
20(graf)
- Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

$$(a) \lim_{x \rightarrow 0} \left( \frac{\sqrt{7+x} - \sqrt{7}}{x} \right) =$$

$$(b) \lim_{n \rightarrow \infty} \left( \frac{x^2 + 6}{x^2} \right) =$$

- Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rclcl} 4x & - & y & + & z & + 2u = -1 \\ 2x & + & y & - & 3u & = 4 \\ x & - & y & + & 2z & + u = 2 \\ 2x & + & y & + & z & - 4u = 7 \end{array}$$

- Odrediti prvu derivaciju funkcije:  $f(x) = \ln(\cos(2x^2 - 1))$ . 10

$$⑥ f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot (-\sin(2x^2 - 1)) \cdot 4x$$

Ukupno:  
14

$$③ h(x) = \sqrt{x^2 + 2x}$$

$$x^2 + 2x \geq 0$$

$$x_{1,2} = \frac{-2 \pm \sqrt{4 - 4 \cdot 1}}{2}$$

$$x_1 = 1$$

$$\begin{aligned} Df &= \mathbb{R} \setminus \{1\} \\ &= (-\infty, 1) \cup (1, +\infty) \\ &\quad \text{rubovi otvoreni} \end{aligned}$$

$$\text{VA: } \lim_{x \rightarrow 1^+} \sqrt{x^2 + 2x} = \sqrt{1+2} = \sqrt{3} \rightarrow \text{Nije VA}$$

$$\lim_{x \rightarrow 1^-} \sqrt{x^2 + 2x} = \sqrt{1_- + 2_-} = \sqrt{3}$$

$$\begin{aligned} \text{HA: } \lim_{x \rightarrow +\infty} \sqrt{x^2 + 2x} &= \sqrt{\infty + \infty} = \infty + \infty = \lim_{x \rightarrow +\infty} \sqrt{x^2 + 2x} \cdot \frac{\sqrt{x^2 - 2x}}{\sqrt{x^2 - 2x}} \\ &= \lim_{x \rightarrow +\infty} \frac{x^2 + 2x + x^2 - 2x}{\sqrt{x^2 - 2x}} \Big|_{x^2} = \lim_{x \rightarrow +\infty} \frac{2x^2}{\sqrt{x^2 - 2x}} = \lim_{x \rightarrow +\infty} \frac{2x^2}{\sqrt{x^2(1 - \frac{2}{x^2})}} = \lim_{x \rightarrow +\infty} \frac{2x^2}{x\sqrt{1 - \frac{2}{x^2}}} = \lim_{x \rightarrow +\infty} \frac{2x^2}{x\sqrt{1 - \frac{2}{x^2}}} = \frac{2}{\sqrt{1 - \frac{2}{x^2}}} = 2 \end{aligned}$$

LHA:  $\lim_{x \rightarrow -\infty} \sqrt{x^2 + 2x} = \begin{cases} x \rightarrow -x & \\ -\infty \rightarrow +\infty & \end{cases} Y = \lim_{x \rightarrow +\infty} \sqrt{(-x)^2 + 2 \cdot (-x)} = \infty \rightarrow \infty$  Neutra  
~~Nul tocke~~  $x^2 + 2x = 0$   
 $x_1 = 1$

KOSE Neutra ✓

Prva derivacija:

=

⑤  $\left[ \begin{array}{ccccc} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \xrightarrow{\text{RREF}} \left[ \begin{array}{ccccc} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \xrightarrow{2R_2 - 2R_1} \left[ \begin{array}{ccccc} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \xrightarrow{R_3 - 4R_1} \left[ \begin{array}{ccccc} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -7 & -21 & -9 \\ 0 & 3 & -7 & -6 & 3 \end{array} \right] \xrightarrow{(1/3)} \left[ \begin{array}{ccccc} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -\frac{7}{3} & -7 & -3 \\ 0 & 1 & -7 & -21 & -9 \end{array} \right]$

$$\left[ \begin{array}{ccccc} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -7 & -6 & 3 \end{array} \right] \xrightarrow{R_1 + R_2} \left[ \begin{array}{ccccc} 1 & 0 & -\frac{2}{3} & -\frac{2}{3} & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 0 & -\frac{12}{3} & \frac{13}{3} & -9 \\ 0 & 0 & -\frac{12}{3} & -\frac{9}{3} & 3 \end{array} \right] \xrightarrow{(-1/12)} \left[ \begin{array}{ccccc} 1 & 0 & -\frac{2}{3} & -\frac{2}{3} & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 0 & 1 & -\frac{29}{12} & -\frac{27}{4} \\ 0 & 0 & -1 & -\frac{12}{3} & 1 \end{array} \right] \xrightarrow{(-1/12)} \left[ \begin{array}{ccccc} 1 & 0 & -\frac{2}{3} & -\frac{2}{3} & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 0 & 1 & -\frac{29}{12} & -\frac{27}{4} \\ 0 & 0 & -1 & -1 & 1 \end{array} \right]$$

$$\textcircled{4} \lim_{x \rightarrow 0} \left( \frac{\sqrt{7+x} - \sqrt{7}}{x} \right) = \lim_{x \rightarrow 0} \left( \frac{\sqrt{7+0} - \sqrt{7}}{0} \right) = \frac{\sqrt{7} - \sqrt{7}}{0} = \frac{0}{0} \text{ L'H}$$

L'H  
 $\lim_{x \rightarrow 0} \left( \frac{\frac{1}{\sqrt{7+x}}}{1} \right) = \frac{1}{\sqrt{7+0}} = \frac{1}{\sqrt{7}}$  Auté zero liniov

$$(\sqrt{7+x})' = \frac{1}{2\sqrt{7+x}}$$

$$\textcircled{5} \lim_{n \rightarrow \infty} \left( \frac{x^2 + 6}{x^2} \right) = \left( \frac{\infty}{\infty} \right) \stackrel{\text{L'H}}{=} \frac{2x}{2x} = 1 \checkmark$$

$$\textcircled{1} z^3 - 6 + 7i = 0 \\ z^3 = 6 - 7i$$

$$r = \sqrt{x^2 + y^2} \\ = \sqrt{36 - 49} \\ = \sqrt{15} \quad \times$$

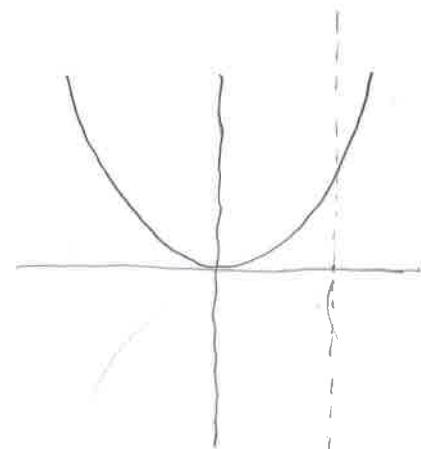
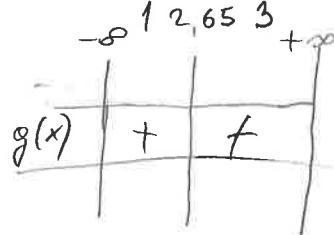
$$\textcircled{2} g(x) = \sqrt{x^2 + 7}$$

$$DP: x^2 + 7 \geq 0$$

$$x_{1,2} = \frac{0 \pm \sqrt{0 - 28}}{2}$$

$$x_{1,2} = \frac{5\sqrt{3}}{2} \quad x_2 = -\frac{5\sqrt{3}}{2}$$

$$x_1 = 2,65 \quad x_2 = \cancel{-2,65} \text{ množstvo bude veči lbi položko nuli}$$



VA  
 $\lim_{x \rightarrow 2,65^+} \sqrt{(2,65)^2 + x} = \sqrt{14,0225} = 3,7 \text{ Nije VA.}$

Funkcija je periodična jer ne sadrži (sin, cos, tan, cot.)

GLOBALNI EXTREMI?



IME I PREZIME: SANDRO-LOVŠIĆ BROJ INDEKSA: 17-1-0240-14  
0269083867

- ✓ 1. Riješi jednadžbu među kompleksnim brojevima:  $z^3 - 6 + 7i = 0$ . Prikaži rješenja u kompleksnoj ravni! 12+3
- ✓ 2. Koji su globalni ekstremi funkcije  $g(x) = \sqrt{x^2 + 7}$  10
3. Ispitati asimptote funkcije:  $h(x) = \sqrt{x^2 + 2x}$ . Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote)  
20(graf)
4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

(a)  $\lim_{x \rightarrow 0} \left( \frac{\sqrt{7+x} - \sqrt{7}}{x} \right) =$  7+2

(b)  $\lim_{n \rightarrow \infty} \left( \frac{x^2 + 6}{x^2} \right) =$  4+2

- ✓ 5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rrrrrr} 4x & - & y & + & z & + & 2u = -1 \\ 2x & + & y & & - & 3u = & 4 \\ x & - & y & + & 2z & + & u = & 2 \\ 2x & + & y & + & z & - & 4u = & 7 \end{array}$$

- ✓ 6. Odrediti prvu derivaciju funkcije:  $f(x) = \ln(\cos(2x^2 - 1))$ . 10

Ukupno:

17

1.)  $z^3 = 6 + 7i$  /  $13\sqrt{}$

$r = \sqrt{6^2 + 7^2}$

$\theta = \sqrt{85}$

$\text{tg } \varphi = \frac{7}{6}$

$\varphi = 0,8622$

$\varphi$

$\sqrt[3]{z} = \sqrt[3]{r} \left( \cos \left( \frac{1+2k\pi}{3} \right) + i \cdot \sin \left( \frac{1+2k\pi}{3} \right) \right)$

$k=0,1,2$   
 $b=0 \Rightarrow \sqrt[3]{\sqrt{85}} \left( \cos \left( \frac{0,8622+2 \cdot 0 \cdot \pi}{3} \right) + i \cdot \sin \left( \frac{0,8622+2 \cdot 0 \cdot \pi}{3} \right) \right)$   
 $= 2,01 + 0,59i \quad \checkmark$

$b=1 \Rightarrow \sqrt[3]{\sqrt{85}} \left( \cos \left( \frac{0,8622+2 \cdot 1 \cdot \pi}{3} \right) + i \cdot \sin \left( \frac{0,8622+2 \cdot 1 \cdot \pi}{3} \right) \right)$   
 $= -0,49 + 2,04i \quad \checkmark$

$b=2 \Rightarrow \sqrt[3]{\sqrt{85}} \left( \cos \left( \frac{0,8622+2 \cdot 2 \cdot \pi}{3} \right) + i \cdot \sin \left( \frac{0,8622+2 \cdot 2 \cdot \pi}{3} \right) \right)$   
 $= -1,84 - 0,49i \quad \checkmark$

$$2. \quad g(x) = \sqrt{x^2 + 2}$$

$$g'(x) = \frac{1}{2\sqrt{x^2+2}} \cdot (x^2+2)$$

$$= \frac{1}{2\sqrt{x^2+2}} \cdot 2x$$

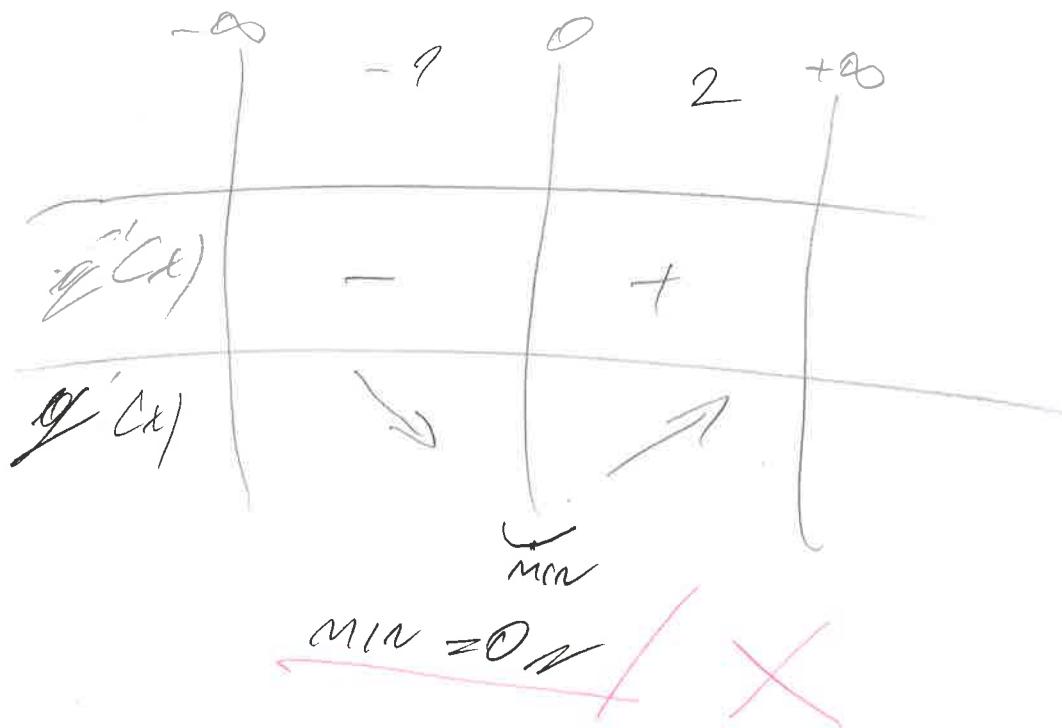
$$= \frac{2x}{2\sqrt{x^2+2}}$$

$$g'(x) = 0$$

$$\frac{2x}{2\sqrt{x^2+2}} = 0 \quad | : 2\sqrt{x^2+2}$$

$$2x = 0$$

$$x = 0$$



SANDRO - LTON BVLGARI

$$\left[ \begin{array}{cccc} 1 & 0 & \frac{2}{3} & 12 \\ 0 & 1 & -\frac{4}{3} & 10 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & -3 & -9 \end{array} \right] \quad \left. \begin{array}{l} 1 \cdot (-\frac{2}{3}) + 0 \\ 1 \cdot \frac{4}{3} + 10 \\ 1 \cdot 3 + 12 \\ 1 \cdot (-3) + (-9) \end{array} \right\}$$

$$\left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 13 \end{array} \right] \quad \text{MARICA nema RIESTEDE}$$

VHL ŠAKANOVIC

6.)  $f(x) = \ln(\cos(2x^2 - 1))$

$$f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot (-\sin)(2x^2 - 1)$$

$$f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot (-\sin) \cdot 4x$$

$$f'(x) = \frac{-4 \sin x}{\cos(2x^2 - 1)}$$

7.) ASIMTOZE VERDHOČITI nema je R DFLR

D.H.A  $\lim_{x \rightarrow \infty} \sqrt{x^2 + x} = \infty$

L.H.A  $\lim_{x \rightarrow -\infty} \sqrt{x^2 + x} = \infty$

L.A  $b(x) = \frac{f(x)}{x}$   $\ell = \lim_{x \rightarrow \infty} [f(x) - b(x)]$

$$b(x) = \frac{\sqrt{x^2 + x}}{x}$$

## SANDRO-LEON BUSTAMANTE

3)  $h(x) = \sqrt{x^2 + 2x}$

$x^2 + 2x \geq 0$

$2x \geq -2$

$x^3 \geq 2$  for

$x \geq \sqrt[3]{2}$

5)  $\left[ \begin{array}{rrrrr|rr} 4 & -1 & 1 & 2 & 1 & -7 \\ 2 & 1 & 0 & -3 & 4 & \\ 1 & -1 & 2 & 1 & 2 & \\ 2 & 1 & 1 & -4 & 2 & \end{array} \right] \sim \left[ \begin{array}{rrrrr|rr} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & -4 & 2 \\ 2 & 1 & 1 & -4 & 2 \end{array} \right]$

$\left[ \begin{array}{rrrr|rr} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & -4 & 2 \\ 2 & 1 & 1 & -4 & 2 \end{array} \right] \xrightarrow{\text{I} \cdot (-2) + \text{II}, \text{I} \cdot (-4) + \text{III}, \text{I} \cdot (-2) + \text{IV}} \left[ \begin{array}{rrrr|rr} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -2 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right]$

$\left[ \begin{array}{rrrr|rr} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -2 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right] \xrightarrow{\text{R3} : 3} \left[ \begin{array}{rrrr|rr} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 1 & -\frac{2}{3} & -\frac{2}{3} & -9 \\ 0 & 1 & -1 & -2 & 1 \end{array} \right] \xrightarrow{\text{R1} + \text{R2}, \text{R3} + \text{R4}} \left[ \begin{array}{rrrr|rr} 1 & 0 & \frac{2}{3} & -\frac{2}{3} & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 0 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 0 & -1 & -1 & 3 \end{array} \right]$

$\left[ \begin{array}{rrrr|rr} 1 & 0 & \frac{2}{3} & -\frac{2}{3} & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 0 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 0 & -1 & -1 & 3 \end{array} \right] \sim \left[ \begin{array}{rrrr|rr} 1 & 0 & \frac{2}{3} & -\frac{2}{3} & 2 \\ 0 & 1 & -1 & -1 & 3 \\ 0 & 0 & -1 & -1 & 3 \\ 0 & 0 & -3 & 3 & -9 \end{array} \right]$

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod! B2

**IME I PREZIME:** Ante Špoljarić

**BROJ INDEKSA:** 0269076958

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

1. Riješi jednadžbu među kompleksnim brojevima:  $z^3 - 6 + 7i = 0$ . Prikaži rješenja u kompleksnoj ravnini! 12+3
2. Koji su globalni ekstremi funkcije  $g(x) = \sqrt{x^2 + 7}$  10
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(a)  $\lim_{x \rightarrow 0} \left( \frac{\sqrt{7+x} - \sqrt{7}}{x} \right) =$

$\cancel{7+2}$

(b)  $\lim_{n \rightarrow \infty} \left( \frac{x^2 + 6}{x^2} \right) =$

4+2

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rclclcl} 4x & - & y & + & z & + & 2u = -1 \\ 2x & + & y & & & - & 3u = 4 \\ x & - & y & + & 2z & + & u = 2 \\ 2x & + & y & + & z & - & 4u = 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije:  $f(x) = \ln(\cos(2x^2 - 1))$ . 10

Ukupno:

~~15~~ 15



6

Antonio Jordi

$$4x - y + z + 2u = -1$$

$$2x + y - 3y = 4$$

$$x - y + 2z + 4 = 2$$

$$2x + y + z - 4y = 7$$

$$\left[ \begin{array}{ccccc} 4 & -1 & 1 & 2 & -1 \\ 2 & -1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \sim \left[ \begin{array}{ccccc} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right]$$

$$\sim \left[ \begin{array}{ccccc|c} 1 & -1 & 2 & 1 & 1 & 2 \\ 0 & 0 & -1 & 1 & 1 & -3 \\ 0 & -1 & 1 & 2 & 1 & -1 \\ 2 & 1 & 1 & -4 & 1 & 7 \end{array} \right] \xrightarrow{(-2)R_4 + R_1} \sim \left[ \begin{array}{ccccc|c} 1 & -1 & 2 & 1 & 1 & 2 \\ 0 & 0 & -1 & 1 & 1 & -3 \\ 0 & -3 & -1 & 8 & 1 & -15 \\ 2 & 1 & 1 & -5 & 1 & 7 \end{array} \right] \xrightarrow{(-1)R_1 + R_2} \sim \left[ \begin{array}{ccccc|c} 1 & -1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & -6 \\ 0 & -3 & -1 & 8 & 1 & -15 \\ 0 & 3 & 3 & -6 & 1 & 3 \end{array} \right] \sim$$

$$\left[ \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & -4 & 2 & -12 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right] \xrightarrow{(\cdot:-2)} \sim \left[ \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & 2 & -1 & 6 \\ 0 & 1 & -1 & -2 & 1 \end{array} \right] \xrightarrow{\text{IV} + \text{II}} \sim \left[ \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -2 & -1 & -2 \\ 0 & 0 & 2 & -1 & 6 \\ 0 & 1 & -1 & -2 & 1 \end{array} \right] \xrightarrow{\text{III} + \text{II}}$$

$$\left[ \begin{array}{ccccc|c} 1 & 0 & 1 & -1 & 3 \\ 0 & 1 & -2 & -1 & -2 \\ 0 & 0 & 2 & -1 & 6 \\ 0 & 1 & -1 & -2 & 1 \end{array} \right] \sim \left[ \begin{array}{ccccc|c} 1 & 0 & 1 & -1 & 3 \\ 0 & 1 & 0 & -2 & 4 \\ 0 & 0 & 2 & -1 & 6 \\ 0 & 1 & -1 & -2 & 1 \end{array} \right] \xrightarrow{(-1)+IV} \left[ \begin{array}{ccccc|c} 1 & 0 & 1 & -1 & 3 \\ 0 & 1 & 0 & -2 & 4 \\ 0 & 0 & 2 & -1 & 6 \\ 0 & 0 & 0 & 1 & -3 \end{array} \right] \sim \left[ \begin{array}{ccccc|c} 1 & 0 & 1 & -1 & 3 \\ 0 & 1 & 0 & -2 & 4 \\ 0 & 0 & 2 & -1 & 6 \\ 0 & 0 & -1 & 0 & -3 \end{array} \right]$$

$$\sim \left[ \begin{array}{ccccc|c} 1 & 0 & 1 & -1 & 1 & 3 \\ 0 & 1 & 0 & -2 & 1 & 4 \\ 0 & 0 & -1 & 0 & 1 & -3 \\ 0 & 0 & 2 & -1 & 1 & 6 \end{array} \right] \xrightarrow{+} \left[ \begin{array}{ccccc|c} 1 & 0 & 1 & -1 & 1 & 3 \\ 0 & 1 & 0 & -2 & 1 & 4 \\ 0 & 0 & -1 & 0 & 1 & -3 \\ 0 & 0 & 1 & -1 & 1 & 3 \end{array} \right] \xrightarrow{(-1)} \left[ \begin{array}{ccccc|c} 1 & 0 & 1 & -1 & 1 & 3 \\ 0 & 1 & 0 & -2 & 1 & 4 \\ 0 & 0 & -1 & 0 & 1 & -3 \\ 0 & 0 & 1 & -1 & 1 & 3 \end{array} \right] \xrightarrow{(-1)} \left[ \begin{array}{ccccc|c} 1 & 0 & 1 & -1 & 1 & 3 \\ 0 & 1 & 0 & -2 & 1 & 4 \\ 0 & 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & -1 & 1 & 1 & -3 \end{array} \right] \xrightarrow{+}$$

$$\sim \left[ \begin{array}{ccccc} 1 & 0 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & -2 & | & 4 \\ 0 & 0 & 1 & 0 & | & 3 \\ 0 & 0 & -1 & 1 & | & -3 \end{array} \right] \sim \left[ \begin{array}{ccccc} 1 & 0 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & -2 & | & 4 \\ 0 & 0 & 1 & 0 & | & 3 \\ 0 & 0 & 0 & 1 & | & 0 \end{array} \right] \xrightarrow{(r2)+r1} \left[ \begin{array}{ccccc} 1 & 0 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & 0 & | & 4 \\ 0 & 0 & 1 & 0 & | & 3 \\ 0 & 0 & 0 & 1 & | & 0 \end{array} \right]$$

$$\begin{bmatrix} x \\ y \\ z \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 3 \\ 0 \end{bmatrix}$$

PROVJERI:

$$x=0 \\ y=4 \\ z=3 \\ v=0$$

$$4x - y + z + 2v = -1 \\ 4 \cdot 0 - 4 + 3 + 2 \cdot 0 = -1 \\ -1 = -1$$

Matrica imala  
njegova sve  
njegova množenja!

$$2x + y - 3v = 4 \quad x - y + 2z + v = 2 \\ 0 + 4 - 0 = 4 \quad 0 - 4 + 6 + 0 = 2 \\ 4 = 4 \quad 2 = 2$$

$$2x + y + z - 4v = 2 \\ 0 + 4 + 3 - 0 = 2 \\ 2 = 2$$

PROVJERITE  
DETERMINANTU  
PLASICE SUSTAVU  
⇒ PRAVILA RJEŠENJA!  
JESTE RJEŠENJA!

4.

a)  $\lim_{x \rightarrow 0} \left( \frac{\sqrt{7+x} - \sqrt{7}}{x} \right)$

PROVJERI:

$$\lim \left( \frac{\sqrt{7+0,01} - \sqrt{7}}{0,01} \right) = 0,189$$

$$\lim_{x \rightarrow 0^+} \left( \frac{\sqrt{7+x^+} - \sqrt{7}}{0^+} \right) = +\infty$$

$$\lim_{x \rightarrow 0^-} \left( \frac{\sqrt{7+x^-} - \sqrt{7}}{0^-} \right) = 0,189$$

$$\lim_{x \rightarrow 0^-} \left( \frac{\sqrt{7+x^-} - \sqrt{7}}{0^-} \right) = +\infty$$

Nije učudno.

3.

$$h(x) = \sqrt{x^2 + 2x}$$

$$\sqrt{x^2 + 2x} \geq 0 \quad Df \in (-\infty, -2] \cup [0, +\infty)$$

$$\sqrt{x^2 + 2x} = 0 / 2$$

$$x^2 + 2x = 0$$

$$x(x+2) = 0$$

$$x_1 = 0$$

$$x_2 = -2$$

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod! B2

**IME I PREZIME:** JURE IVICOVIC

**BROJ INDEKSA:** 17-2-0387-2014

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

1. Riješi jednadžbu među kompleksnim brojevima:  $z^3 - 6 + 7i = 0$ . Prikaži rješenja u kompleksnoj ravnini! 12+3
2. Koji su globalni ekstremi funkcije  $g(x) = \sqrt{x^2 + 7}$  10
3. Ispitati asimptote funkcije:  $h(x) = \sqrt{x^2 + 2x}$ . Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote)  
20(graf)
4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

(a)  $\lim_{x \rightarrow 0} \left( \frac{\sqrt{7+x} - \sqrt{7}}{x} \right) =$  7+2

(b)  $\lim_{n \rightarrow \infty} \left( \frac{x^2 + 6}{x^2} \right) =$  4+2

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje! 15+5

$$\begin{array}{rrrrrr} 4x & - & y & + & z & + & 2u = -1 \\ 2x & + & y & & & - & 3u = 4 \\ x & - & y & + & 2z & + & u = 2 \\ 2x & + & y & + & z & - & 4u = 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije:  $f(x) = \ln(\cos(2x^2 - 1))$ . 10

Ukupno:



**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod! B2

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME:

BROJ INDEKSA:

MATEJ BERETIN

001245

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IME I PREZIME: MARKO ČUDINA

BROJ INDEKSA: 57664-2009

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

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Ukupno:

0

$$\begin{aligned} 5. \quad & 4x - y + z + 2u = -1 \\ & 2x + y - 3u = 4 \\ & x - y + 2z + u = 2 \\ & 2x + y + z - 4u = 7 \end{aligned}$$

$$\left[ \begin{array}{ccccc} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \rightsquigarrow \sim \left[ \begin{array}{ccccc} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \xrightarrow{\text{II}-\text{IV}} \sim \left[ \begin{array}{ccccc} 1 & -1 & 2 & 1 & 2 \\ 0 & 0 & -1 & 1 & -3 \\ 4 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right]$$

