MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ A2

IME I PREZIME: DINO BADŽOKA **BROJ INDEKSA:** 0269066783

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2-2}$. 5+15+5
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln(4-x^2)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-2x-(2+1)}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz kompliciira. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x+2y-z+u &= 2 \\ 2x+5y-z+2u &= 3 \\ 3x-y-2z+u &= 2 \\ x-y+3z-5u &= 2 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 45

2)

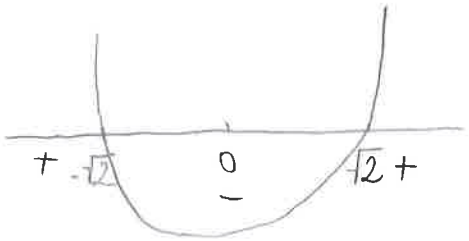
$f(x) = x - \sqrt{x^2-2}$

DOMENA

$x^2 - 2 \geq 0$

$x^2 \geq 2 \mid \sqrt{\ }$

$x \pm \sqrt{2}$



$D(f) = \langle -\infty, -\sqrt{2}] \cup [\sqrt{2}, +\infty \rangle$

DERIVACIJA

$f(x) = x - \sqrt{x^2-2}$

$f'(x) = 1 - \frac{2x}{2\sqrt{x^2-2}}$

$f'(x) = 1 - \frac{x}{\sqrt{x^2-2}}$

$f''(x) = 1 - \frac{x}{\sqrt{x^2-2}}$

$f''(x) = -\frac{1 \cdot \sqrt{x^2-2} - x \frac{2x}{2\sqrt{x^2-2}}}{x^2-2}$

$f''(x) = -\frac{\sqrt{x^2-2} - \frac{x^2}{\sqrt{x^2-2}}}{x^2-2}$

$f''(x) = -\frac{\frac{x^2-2-x^2}{\sqrt{x^2-2}}}{x^2-2}$

$f''(x) = -\frac{2}{(x^2-2)^{\frac{3}{2}}}$ ✓

ASIMPTOTE

$f(x) = x - \sqrt{x^2-2}$

1.) $z^4 - (4-i)^2 = 0$

$z^4 = (4-i)^2$

$z^4 = 16 - 8i - 1$

$z^4 = 15 - 8i$

$tg = \varphi = \frac{8}{15}$

$r = \sqrt{15^2+8^2} = 17$

$\varphi_0 = \frac{\varphi}{n} = 82°58'55''$

$\Delta\varphi = 90°$

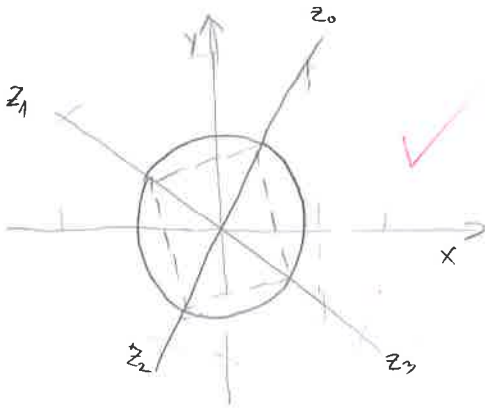
$z^4 = 17(\cos 331°55'39'' + i\sin 331°55'39'')$

$z_0 = \sqrt{17}(\cos 82°58'55'' + i\sin 82°58'55'')$

$z_1 = \sqrt{17}(\cos 172°58'55'' + i\sin 172°58'55'')$

$z_2 = \sqrt{17}(\cos 262°58'55'' + i\sin 262'58'55'')$

$z_3 = \sqrt{17}(\cos 352°58'55'' + i\sin 352°58'55'')$



3.) DOMENA

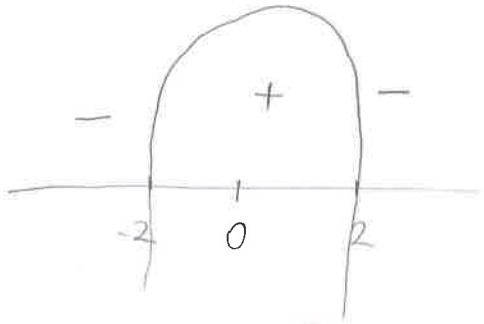
$g(x) = \ln(4-x^2)$

$4-x^2 > 0$

$+x^2 > -4 \mid (-1)$

$x^2 < 4 \mid \sqrt{\ }$

$x < 2$



$D(g) = \langle -2, 2 \rangle$ ✓

(NE) PARNOST

$g(x) = \ln(4-x^2)$

$g(-x) = \ln(4-(-x^2))$

$g(-x) = \ln(4-x^2)$

Funkcija je PARNA ✓

ZAKRIVLJENOST

$g(x) = \ln(4-x^2)$

$$g'(x) = \frac{1}{4-x^2} \cdot (-2x) = \frac{2x}{x^2-4}$$

$$g''(x) = \frac{2(x^2-4)-2x\cdot(2x)}{(x^2-4)^2} = \frac{2x-8-4x^2}{(x^2-4)^2} = \frac{-2x^2-8}{(x^2-4)^2} = \frac{-2(x^2+4)}{(x^2-4)^2} = 0$$

$x^2+4=0$

$x^2=-4$

Nema realnih rješenja

$$\left.\begin{array}{l} (x^2-4)^2 \geq 0 \quad \forall x \in \mathbb{R} \\ x^2+4>0 \quad \forall x \in \mathbb{R} \end{array}\right\} \Rightarrow -2\,\frac{(x^2+4)}{(x^2-4)^2} < 0$$

Funkcija $g(x) = \ln(4-x^2)$ je KONKAVNA na cijelom području ✓

IME I PREZIME: DINO BADŽOKA BROJ INDEKSA: 0269066783

5.) $x+2y-z+u=2$

$2x+5y-z+2u=3$

$3x-y-2z+u=2$

$x-y+3z-5u=2$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 2 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & -7 & 1 & -2 & -3 \\ 0 & -3 & 4 & -6 & 1 \end{array}\right] \begin{array}{l} \\ \text{II}-2\text{I} \\ \text{III}-3\text{I} \\ \text{IV}-\text{I} \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 8 & -2 & -17 \\ 0 & 0 & 7 & -6 & -5 \end{array}\right] \begin{array}{l} \\ \\ \text{III}+7\text{II} \\ \text{IV}+3\text{II} \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 8 & -2 & -17 \\ 0 & 0 & 56 & -48 & -40 \end{array}\right] 8\cdot\text{IV}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 8 & -2 & -17 \\ 0 & 0 & 0 & -34 & 79 \end{array}\right] \text{IV}-7\text{III}$$

$-34u=79$

$u=-\frac{79}{34}$

$8z-2u=-17$

$z=-\frac{46}{17}$

$x+2y-z+u=2$

$x=\frac{7}{34}$

$$X=\begin{bmatrix} \frac{7}{34} \\ \frac{12}{17} \\ -\frac{46}{17} \\ -\frac{79}{34} \end{bmatrix}$$

$r(A)=4$

$u=4$ $\Rightarrow$ rješenje je jedinstveno

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ A2

IME I PREZIME: MATE PARAĆ **BROJ INDEKSA:** 17-1-0179-2013

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
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6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 35

③ $g(x) = \ln(4-x^2)$

DOMENA

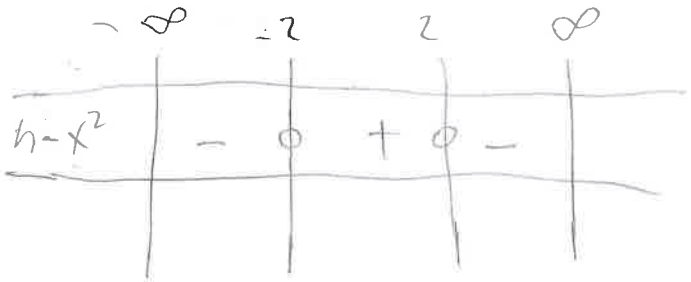
$4-x^2 > 0$

$4-x^2 = 0$

$-x^2 = -4$

$x^2 = 4$

$x = \pm 2$



$D_g : \langle -2, 2 \rangle$ ✓

PARNOST

$g(x) = \ln(4-x^2)$

$g(-x) = \ln(4-(-x)^2)$

$= \ln(4-x^2)$

$g(x) = g(-x)$

Funkcija je parna ✓

ZAKRIVLJENOST ?

$g'(x) = \dfrac{1}{4-x^2} \cdot (-2x) = \dfrac{-2x}{4-x^2}$

$g''(x) = \dfrac{2(4-x^2) - 2x(-2x)}{(4-x^2)^2} = \dfrac{8-2x^2+4x^2}{(4-x^2)^2}$

$= \dfrac{8+2x^2}{(4-x^2)^2}$

$g''(x) = -\dfrac{8+2x^2}{(4-x^2)^2}$

$g''(x) = 0$

→

$$-\frac{8+2x^2}{(4-x^2)^2}=0$$

Nema realnih rješenja

$8+2x^2=0 \mid :2$

$4+x^2=0$

$x^2=-4 \mid \sqrt{\ }$

$x=\pm\sqrt{-4}$

1. $z^4-(4-i)^2=0$

$z^4=16-8i+i^2$

$z^4=15-8i \mid \sqrt[4]{\ }$

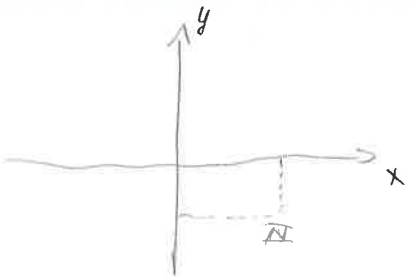
$z=\sqrt[4]{15-8i}$

$r=\sqrt{15^2+(-8)^2}$

$r=17$

$\operatorname{tg}\varphi=\frac{-8}{15}$

$\varphi=-0,48996$



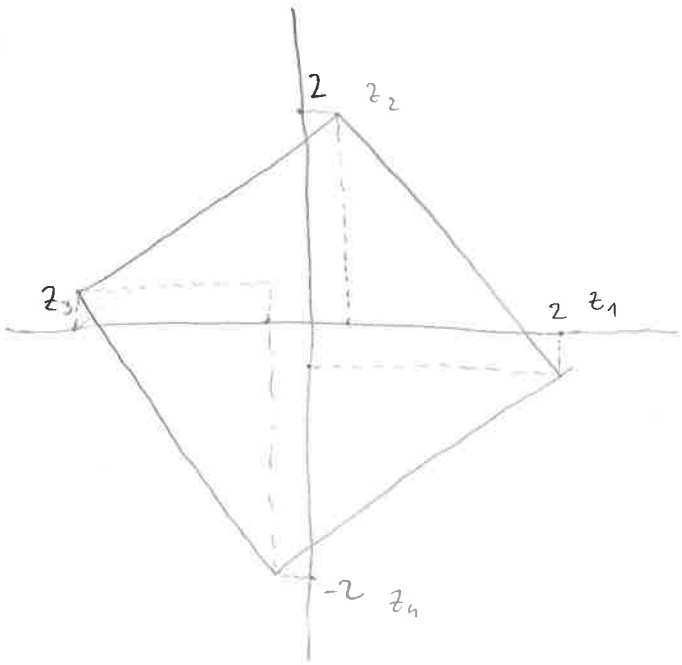
$$\sqrt[4]{15-8i}=\sqrt[4]{17}\left(\cos\frac{-0.48996+2k\pi}{4}+i\sin\frac{-0.48996-2k\pi}{4}\right)$$

$k=0 \Rightarrow z_1=2,02-0,25i$

$k=1 \Rightarrow z_2=0,25+2,02i$

$k=2 \Rightarrow z_3=-2,02+0,25i$

$k=3 \Rightarrow z_4=-0,25-2,02i$



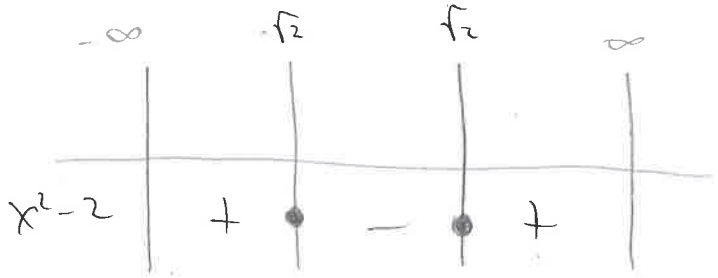
IME I PREZIME: MATE PARAĆ BROJ INDEKSA: 17-1-0179-2013

2. DOMENA

$x^2 - 2 \geq 0$

$x^2 - 2 = 0$

$x = \pm\sqrt{2}$



$D_f : \langle -\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty\rangle$ ✓

DERIVACIJA

$$f'(x) = 1 - \frac{1}{2}\left(x^2-2\right)^{\frac{1}{2}} \cdot \left(x^2-2\right)' = 1 - \frac{x}{\sqrt{x^2-2}}$$

$$f''(x) = -\frac{1\cdot\sqrt{x^2-2} - x \cdot \frac{1\cdot 2x}{2\sqrt{x^2-2}}}{x^2-2}$$

$$= \frac{-\sqrt{x^2-2} + \frac{x^2}{\sqrt{x^2-2}}}{x^2-2} \checkmark = \frac{x^2}{\sqrt{(x^2-2)^2}} - \frac{1}{\sqrt{x^2-2}}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ A2

IME I PREZIME: Luka Grbin **BROJ INDEKSA:** 17-1-0274-2019

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
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6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 21

4.

$$h(x) = \frac{x^2-2x-(2+1)}{x^2+1}$$

$$h(x) = \frac{x^2-2x-3}{x^2+1}$$

$x^2+1=0$

$x^2=-1$

$x=\pm\sqrt{-1}$

$V=(0,-3)$

$D(h)=\mathbb{R}$

$x^2-2x-3=0$

$$x_{12} = \frac{2 \pm \sqrt{(-2)^2-4\cdot 1\cdot(-3)}}{2}$$

$$x_{12} = \frac{2\pm 4}{2}$$

$$x_1 = \frac{2-4}{4} = -\frac{1}{2} \qquad x_2 = \frac{2+4}{4} = \frac{3}{2}$$

N.T

$\left(-\frac{1}{2},0\right)\left(\frac{3}{2},0\right)$

V. A.

$x_1 = -\frac{1}{2} \quad x_2 = -\frac{3}{2}$

$h(+\infty) = \lim_{x\to+\infty} (h(x)) = \lim_{x\to+\infty} \frac{x^2-2x-3}{x^2+1} \;/:x^2 \quad \lim_{x\to\infty} \frac{\frac{x^2}{x^2}-\frac{2x}{x^2}-\frac{3}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}} = \frac{1}{1} = 1$

$\lim_{x\to+\infty} \frac{h(x)}{x} = \left[\frac{\frac{x^2-2x-3}{x^2+1}}{x}\right] = \lim_{x\to\infty} \frac{x^2-2x-3}{x(x^2+1)} = \frac{x^2-2x-3}{x^3+1} \;/:x^3$

$\lim_{x\to\infty} \frac{\frac{x^2}{x^3}-\frac{2x}{x^3}-\frac{3}{x^3}}{\frac{x^3}{x^3}+\frac{1}{x^3}} = \frac{0}{1} = 0$. NEMA KOSIH ASIMPTOTA

$h(x) = \frac{x^2-2x-3}{x^2+1}$

$y' = \frac{(x^2-2x-3)'\cdot(x^2+1) - (x^2-2x-3)\cdot(x^2+1)'}{(x^2+1)^2}$

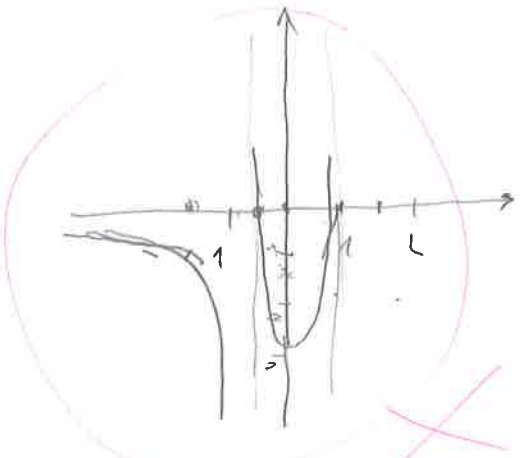
$y' = \frac{(2x-2)\cdot(x^2+1) - (x^2-2x-3)\cdot 2x}{(x^2+1)^2}$

$y' = \frac{2x^3+2x-2x^2-2-(2x^3-4x^2-6x)}{(x^2+1)^2}$

$y' = \frac{2x^3-2x-2x^2-2-2x^3+4x^2+6x}{(x^2+1)^2}$

$y' = \frac{2x^2+4x-2}{(x^2+1)^2}$

	$-\infty$		-1		$-\frac{1}{2}$		0		$+\infty$
			N.T				N.T		
$f'(x)$		$-$		$-$	0	$-$		$+$	
$f(x)$		↘		↘		↘		↗	



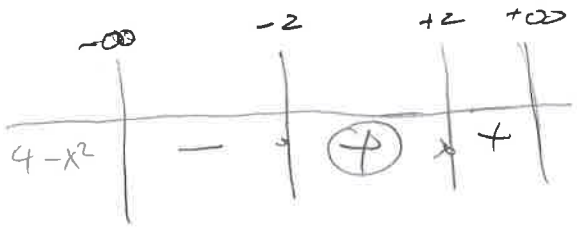
3. $g(x) = \ln(4-x^2)$

$4-x^2 > 0$

$-x^2 > -4 \quad /\cdot(-1)$

$x^2 < 4$

$x < \pm 2$



$D(g(x)) = \langle -2, 2 \rangle$ ✓

N. T.

$y' = \ln(4-x^2)$

$y' = \frac{1}{4-x^2} \cdot (4-x^2)'$

$y' = \frac{1}{4-x^2} \cdot (-2x)$

$y' = \frac{-2x}{4-x^2}$ ✓

P. N-

$g(-x) = \ln(4-(-x)^2)$

$g(-x) = \ln(4-x^2)$ ✓

$\ln(4-x^2) = 0$

$4-x^2 = e^0$

$4-x^2 = 0$

$-x^2 = -4$

$x^2 = 4$

$x_{1,2} = \pm 2$

$y'' =$

$y'' =$

$y'' =$

N.T $(-2, 0)$ $(2, 0)$

IME I PREZIME: Luka Grbin

BROJ INDEKSA: 17-1-0274-2014

5.

$$x + 2y - z + v = 2$$
$$2x + 5y - z + 2v = 3$$
$$3x - y - 2z + v = 2$$
$$x - y + 3z - 5v = 2$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 2 \end{array}\right] \begin{array}{l} /\cdot(-2) \\ /\cdot(-3) \\ /\cdot(-1) \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & -7 & 1 & -2 & -4 \\ 0 & -3 & 4 & -6 & 0 \end{array}\right] \begin{array}{l} /\cdot 7 \\ /\cdot(3) \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 8 & -2 & -11 \\ 0 & 0 & 7 & -6 & -3 \end{array}\right] \begin{array}{l} \\ \\ \\ /(-1) \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 & 14 \\ 0 & 0 & 7 & -6 & -3 \end{array}\right] /(-7)$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 & 14 \\ 0 & 0 & 0 & -34 & 101 \end{array}\right]$$

$$x + 2y - z + v = 2$$
$$y + z = -1$$
$$z + 4v = 14$$
$$-34v = 101$$

$$\boxed{v = -\frac{101}{34}}$$

$$z + 4\cdot\left(-\frac{101}{34}\right) = 14$$
$$z - \frac{202}{17} = 14$$
$$z = 14 + \frac{202}{17}$$

$$\boxed{z = \frac{440}{17}}$$

$$y + z = -1$$
$$y + \frac{440}{17} = -1$$
$$y = -1 - \frac{440}{17}$$

$$\boxed{y = -\frac{457}{17}}$$

$$x + 2\cdot\left(-\frac{457}{17}\right) - \frac{440}{17} + \left(\frac{101}{34}\right) = 2$$
$$x + \frac{914}{17} - \frac{440}{17} - \frac{101}{34} = 2$$
$$x = \frac{914}{17} + \frac{440}{17} + \frac{101}{34} + 2$$
$$x = \frac{2877}{34}$$

Luka Grbin 17-1-0274-2014

3.

$ax^2+8=0$

$a=2 \quad b=0 \quad c=8$

$x_{1,2} = \frac{0 \pm \sqrt{0^2-4\cdot 2\cdot 8}}{4}$

$y' = \frac{-2x}{4-x^2}$ ✓

$y'' = \frac{(-2x)'\cdot(4-x^2)-(-2x)\cdot(4-x^2)'}{(4-x^2)^2}$

$y'' = \frac{-2(4-x^2)-(-2x)\cdot(-2x)}{(4-x^2)^2}$

$y'' = \frac{-8+2x^2-(4x^2)}{(4-x^2)^2}$

$\boxed{y'' = \frac{-2x^2-8}{(4-x^2)^2}}$ ✓

$-2x^2-8<0$

$f''(x)$

ZAKRIVLJENOST?

IME I PREZIME: Luka Grbin

BROJ INDEKSA: 17-1-0274-2014

1.

$$z^4-(4-i)^2=0$$

$$z^4=(4-i)^2$$

$$z^4=|4|^2-2\cdot 4\cdot i+(i)^2$$

$$z^4=16-8i-1$$

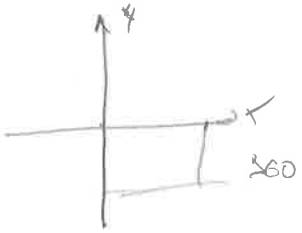
$$z^4=15-8i$$

$$z=\sqrt[4]{15-8i}$$

$x=15 \quad y=-8$

$$tg\,\varphi=\frac{y}{x}=\frac{15}{8}$$

$$\varphi=61°55'39''$$



$$\varphi=360-61°55'39''$$

$$\varphi=298°4'21''$$

$$r=\sqrt{x^2+y^2}$$

$$r=17$$

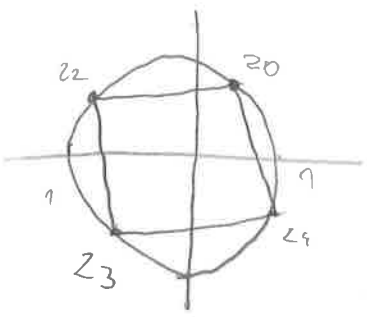
$k=0 \quad \sqrt[4]{z}=\sqrt{17}\left(\cos\frac{298°4'21''+0\cdot 360}{4}+i\sin\frac{298°4'21''+0\cdot 360}{4}\right)$

$k=0 \quad z=\sqrt{17}\left(\cos 74°31'5''+i\sin 74°31'5''\right)=?$

$k=1 \quad z=\sqrt{17}\left(\cos 164°31'5''+i\sin 164°31'5''\right)=?$

$k=2 \quad \sqrt{17}\left(\cos 254°31'5''+i\sin 254°31'5''\right)=?$

$k=3 \quad \sqrt{17}\left(\cos 344°31'5''+i\sin 344°31'5''\right)=?$



2.

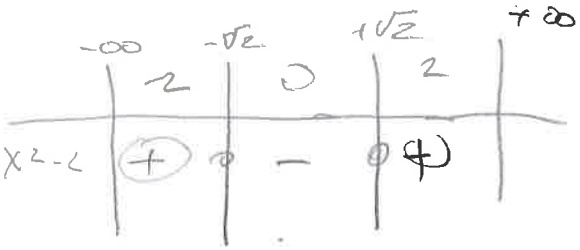
$$f(x) = x - \sqrt{x^2-2}$$

1. DOMENA

$$x^2 - 2 \geq 0$$

$$x^2 = 2 \mid \sqrt{\ }$$

$$x = \pm\sqrt{2}$$



$$D\,f(x) = \langle -\infty, -\sqrt{2}] \cup [\sqrt{2}, +\infty)$$

$$y = x - \sqrt{x^2-2}$$

$$y' = (x)' - \left(\sqrt{x^2-2}\right)'$$

$$y' = 1 - \frac{1}{2}(x^2-2)^{-\frac{1}{2}} \cdot (2x)$$

$$y' = 1 - \frac{x}{\sqrt{x^2-2}}$$

$$y'' = \frac{-1 \cdot \sqrt{x^2-2} - x \cdot \frac{1}{2}(x^2-2)^{-\frac{1}{2}} \cdot 2x}{\left(\sqrt{x^2-2}\right)^2}$$

$$y'' = \frac{-x^2+2 - \frac{x^2}{\sqrt{x^2-2}}}{\left(\sqrt{x^2-2}\right)^2}$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x - \sqrt{x^2-2}$$

$$\lim_{x\to\infty} \frac{x}{x} - \sqrt{\frac{x^2}{x^2} - \frac{2}{x^2}}$$

$$\lim = 1 - \sqrt{1}$$

$$\lim_{x\to\infty} = 0$$

$$\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x - \sqrt{x^2-2}}{x} \Big/ x = \lim_{x\to\infty} \frac{\frac{x}{x} - \sqrt{\frac{x^2}{x^2} - \frac{2}{x^2}}}{\frac{x}{x}} = \lim_{x\to\infty} = \frac{1-1}{1}$$

$$\lim_{x\to\infty} = 0$$

NEMA KOSIH ASIMP

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ A2

IME I PREZIME: DINO ŽENKO **BROJ INDEKSA:** 0269091526

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2-2}$. 5+15+5
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln(4-x^2)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-2x-(2+1)}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz kompliciira. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x+2y-z+u &= 2 \\ 2x+5y-z+2u &= 3 \\ 3x-y-2z+u &= 2 \\ x-y+3z-5u &= 2 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 18

IME I PREZIME: Dino Renko BROJ INDEKSA: 0269091526

(2.) $f(x) = x - \sqrt{x^2-2}$ domena, asimpt, $f'''(x)$

$a=1$ $b=0$ $c=-2$ $x^2-2 \geq 0$ (−1.4, 0, 1.4)

$x^2 \geq 2$

$\sqrt{8} = 2.83$ $x_{1,2} = \frac{\pm\sqrt{4 \cdot 1 \cdot (-2)}}{2}$

$x_{1,2} = \frac{\pm\sqrt{8}}{2}$

$x_1 = 1.415$ $x_2 = -1.415$

$(-\sqrt{2}) \cdot (-\sqrt{2}) = (\sqrt{2})^2 = 2$

$\sqrt{\sqrt{(\sqrt{2})^2 - 2}} = \sqrt{0} = 0$ $\sqrt{\infty - 2} = \sqrt{\infty} = \infty$

$(x-\sqrt{2})(x+\sqrt{2})$ $(a-b)(a+b)$

$x^2 + \sqrt{2}\cdot\sqrt{2} - \sqrt{2}\cdot\sqrt{2} - \sqrt{2}\cdot\sqrt{2}$ $a^2 + ab - ab - b^2$

$(x-\sqrt{x^2-2})\cdot(x+\sqrt{x^2-2}) = x^2 - (\sqrt{x^2-2})^2$

$-\sqrt{x^2-2}\cdot\sqrt{x^2-2}$ $(\sqrt{x^2-2})^2$ $(\sqrt{x^2-2})$

$D(f): \cancel{\langle -\infty, +1.41 \rangle \cup \langle 1.41, +\infty \rangle}$

$\langle -\infty, -\sqrt{2}] \cup [\sqrt{2}, +\infty \rangle$ ✓

V.A.

$\lim_{x \to -\sqrt{2}} x - \sqrt{x^2-2} = \lim_{x \to -\sqrt{2}} -\sqrt{2} - \sqrt{(-\sqrt{2})^2 - 2} = \lim_{x \to -\sqrt{2}} -\sqrt{2} - 0 = \lim_{x \to -\sqrt{2}} -\sqrt{2}$

$\lim_{x \to \sqrt{2}} x - \sqrt{x^2-2} = \lim_{x \to \sqrt{2}} \sqrt{2} - \sqrt{(\sqrt{2})^2 - 2} = \lim_{x \to \sqrt{2}} \sqrt{2}$

nema vertikalnih asimptota

KOSE?

H.A.

$\lim_{x \to \infty} x - \sqrt{x^2-2} = \lim_{x \to \infty} [\infty - \infty]$ neodredeni oblik, racionalizacija

$\lim_{x \to \infty} \frac{x - \sqrt{x^2-2}}{1} \cdot \frac{x + \sqrt{x^2-2}}{x + \sqrt{x^2-2}} = \lim_{x \to \infty} \frac{x^2 - \sqrt{x^2-2}}{x + \sqrt{x^2-2}} = \lim_{x \to \infty} \frac{-2}{x + \sqrt{x^2-2}}$

$= \lim_{x \to \infty} \frac{-2}{\infty} = 0$ D. H. A.

$\lim_{x \to -\infty} \frac{x - \sqrt{x^2-2}}{1} = \left\{ \begin{matrix} x \mapsto -\infty \\ -\infty \mapsto +\infty \end{matrix} \right\} = \lim_{x \to \infty} \frac{-x - \sqrt{x^2-2}}{1} = \lim_{x \to \infty} -\infty - \infty$

$= -\infty$

nema H. A.

$$f'(x) = \left(x - \sqrt{x^2-2}\right)' = x' - \left(\sqrt{x^2-2}\right)' = 1 - \frac{2x}{2\sqrt{x^2-2}}$$

$f(g(x))' = f'(g(x)) \cdot g'x \qquad g'(x) = 2x$

$f = \sqrt{x} = x^{\frac{1}{2}} \qquad f' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2 \cdot x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$

$g = x^2 - 2$

$$\left(\sqrt{x^2-2}\right)' = \frac{1}{2\sqrt{x^2-2}} \cdot 2x = \frac{2x}{2\sqrt{x^2-2}}$$

$$f''(x) = \left(1 - \frac{2x}{2\sqrt{x^2-2}}\right)' = 1' - \left(\frac{2x}{2\sqrt{x^2-2}}\right)'$$

$$= -\frac{2 \cdot 2\sqrt{x^2-2} - 2x \cdot 2 \cdot \frac{2x}{2\sqrt{x^2-2}}}{\left(2\sqrt{x^2-2}\right)^2} = \frac{-4\sqrt{x^2-2} - \frac{8x^2}{2\sqrt{x^2-2}}}{\left(2\sqrt{x^2-2}\right)^2}$$

$1' = 0$

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ (c·f)

$f = 2x \qquad 2x' = 2$

$g = 2\sqrt{x^2-2} \qquad \left(2\sqrt{x^2-2}\right)'$

$f(g(x)) \qquad (c \cdot f) \downarrow f(g(x))$

$$= \frac{-2\sqrt{x^2-2}\left(2 - \frac{8x^2}{1}\right)}{\left(2\sqrt{x^2-2}\right)^2} = \frac{-2 + 8x^2}{2\sqrt{x^2-2}}$$

$$f''(x) = \frac{8x^2 - 2}{2\sqrt{x^2-2}}$$

(3.) $g(x) = \ln(4 - x^2)$ | parna: $f(-x) = f(x)$ | neparna: $f(-x) = -fx$ | $\frac{1}{2 \cdot (-3)^2}$

$a = 1 \quad b = 0 \quad c = 4$

$4 - x^2 > 0$

$-x^2 > -4 \quad / \cdot (-1)$

$x^2 < 4$

$+ + + \quad - \quad +$

$-2 \quad 0 \quad 2$

$4 - x^2 = 0$

$$x_{1,2} = \frac{\pm\sqrt{-4 \cdot (-1) \cdot 4}}{2}$$

$$x_{1,2} = \frac{\pm 4}{2}$$

$x_1 = 2 \qquad x_2 = -2$

$D(g) \langle -\infty, -2 \rangle \cup \langle -2, 2 \rangle \cup \langle 2, +\infty \rangle$

$= \ln(4 - (-x)^2)$

$= \ln(4 - x^2)$

$f(-x) = f(x)$

funkcija je parna

	$-\infty$	-3	-2	$+2$	$+3$	$+\infty$
$f''(x)$		+		+	+	
		↗		↗	↗	

$f'(x) = \ln(4 - x^2)' = \frac{1}{-2x}$

$f = \ln(x) \quad g = 4 - x^2 \quad (4 - x^2)' = -2x$

$$f''(x) = \left(\frac{1}{-2x}\right)' = \frac{0 \cdot (-2x) - 1 \cdot -2}{4x^2} = \frac{1}{2x^2}$$

$\frac{f'g - fg'}{g^2} \quad f = 1 \quad g = -2x \quad (-2x)' = -2 \quad (2x^2 \neq 0)$

623449

1. $z^4 - (4-i)^2 = 0$ $\quad t = z^4$ probaj u trigonom. obliku u čijoj

$z^4 = (4-i)^2$

$z^4 = 16 - 8i + i^2 \quad (i^2 = -1)$

$z^4 = 15 - 8i$

$t = 15 - 8i$

$r = \sqrt{x^2+y^2} = \sqrt{15^2 + (-8)^2} = 17$

$x > 0, \ \varphi = \arctan \frac{y}{x} \quad \varphi = 1.08$

$z = r(\cos\varphi + i\sin\varphi)$

$w_{k+1} = r\left(\cos\frac{\varphi+2k\pi}{n} + i\sin\frac{\varphi+2k\pi}{n}\right)$

$w_0 = 17\left(\cos\frac{1.08+2\cdot 3.14}{1} + i\sin\frac{1.08+2\cdot 3.14}{1}\right)$

$w_1 = 8 + 15i$

$w_2 =$

$w_3 =$

$w_4 =$

$w_1 = 17\left(\cos\frac{\pi/3 + 2\pi}{1} + i\sin\frac{\pi/3+2\pi}{1}\right)$

~~$w_1 = 8.5 + 16.5i$~~ $\quad w_1 = 16.85 + 2.15i$ ✗

$w_2 = 17\left(\cos\frac{\pi/3+4\pi}{2} + i\sin\frac{\pi/3+4\pi}{2}\right)$

$w_2 = 16.88 + 3.85i$ ✗

$w_3 = 17\left(\cos\frac{\pi/3+6\pi}{3} + i\sin\frac{\pi/3+6\pi}{3}\right)$

$w_3 = 16.88 + 2.78i$ ✗

$w_4 = 17\left(\cos\frac{\pi/3+8\pi}{4} + i\sin\frac{\pi/3+8\pi}{4}\right)$

$w_4 = 16.88 + 2.16i$ ✗

6. $\lim_{x\to\infty} \frac{e^{x+x}}{x+x} = \lim_{x\to\infty} \frac{e^{\infty}}{\infty} = \left[\frac{\infty}{\infty}\right]$

$\lim_{x\to\infty} \frac{e^x}{\frac{x}{x}} = \lim_{x\to\infty} e^x = \lim_{x\to\infty} e^{\infty}$ ✓

$\lim_{x\to\infty} \left(1+\frac{1}{\infty}\right)^{\infty}$

$\lim_{x\to\infty} \left(1+\frac{1}{R}\right)^{R} \quad \frac{1}{e^{-\infty}}$

domena kritične točke
asimptote
svojstva
monotonost
derivacije

IME I PREZIME: Dino Renko BROJ INDEKSA: 026gcy1526

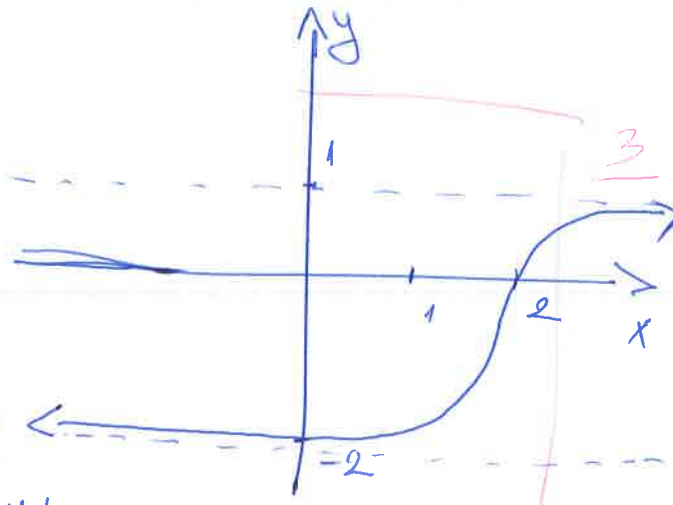
4. $h(x) = \dfrac{x^2-2x-(2+1)}{x^2+1}$

$x^2+1=0$ ($a=1, b=0, c=1$)

$(x_{1,2}$ ∄ $)$

$D(f)=\mathbb{R}$

$\langle -\infty, +\infty \rangle$



$\lim\limits_{x\to\infty} \dfrac{x^2-2x-3 \,/:x^2}{x^2+1 \,/:x^2} = \lim\limits_{x\to\infty}\left[\dfrac{\infty}{\infty}\right]$ ← neodređeno nagrađi koeficijent

$\lim\limits_{x\to\infty} \dfrac{\frac{x^2}{x^2}-\frac{2x}{x^2}-\frac{3}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}} = \lim\limits_{x\to\infty} \dfrac{1-\frac{2}{x}-\frac{3}{x^2}}{1+\frac{1}{x^2}} = \lim\limits_{x\to\infty} 1$ D.H.A.

$\lim\limits_{x\to-\infty} \dfrac{x^2-2x-3}{x^2+1} = \left\{\begin{matrix} x \mapsto -x \\ -\infty \mapsto +\infty \end{matrix}\right\} = \lim\limits_{x\to\infty} \dfrac{x^2+2x-3 \,/:x^2}{x^2+1 \,/:x^2} \left[\dfrac{\infty}{\infty}\right]$

$\lim\limits_{x\to\infty} \dfrac{\frac{x^2}{x^2}+\frac{2x}{x^2}-\frac{3}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}} = \lim\limits_{x\to\infty} \dfrac{1+\frac{2}{x}-\frac{3}{x^2}}{1+\frac{1}{x^2}} = \lim\limits_{x\to\infty} -2$ L.H.A.

$f'(x) = \left(\dfrac{x^2-2x-3}{x^2+1}\right)' = \dfrac{2x-2}{2x} = \dfrac{x-2}{x}$ $\quad x-2=0,\ x=2$

Sg-ff / g^2

$(x^2-2x-3)'(x^2+1) - (x^2+2x-3)(x^2+1)'$ / $(x^2+1)'$; $x-(x-2)$; x^2 ; $-\frac{2}{x^2}$

	$-\infty$	1	2	$3+$	∞
$f'(x)$		−	0	+	
		↘	⊙	↗	

$\dfrac{(2x-2)(x^2+1) - 2x(x^2+2x-3)}{2x} = \dfrac{2x^3+2x-2x^2-2-2x^3-4x^2+6x}{2x}$

$= \dfrac{-6x^2-4x-2}{2x} = \dfrac{2(-3x^2-2x-1)}{2x} = \dfrac{-3x^2-2x-1}{x}$

$f''(x) = \dfrac{(-6x-2)x-(-3x^2-2x-1)}{x^2} = \dfrac{-6x^2-2x+3x^2+2x+1}{x^2} = \dfrac{-3x^2+1}{x^2}$

Dino Reako $\frac{2^5}{5}$

026909526

$\lim_{x\to n}\left(1+\frac{1}{n}\right)^n$

(6) $\lim_{x\to\infty} \frac{e^x : x}{x : x} = \lim \frac{e^\infty}{\infty} = \left[\frac{\infty}{\infty}\right]$?

$\lim_{x\to\infty} \frac{\frac{e^x}{x}}{\frac{x}{1}} = \lim_{x\to\infty} \frac{e^x}{x^2}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ A2

IME I PREZIME: TONČI KATALINIĆ **BROJ INDEKSA:** 0269093441

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2-2}$. 5+15+5
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln\left(4 - x^2\right)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2 - 2x - (2+1)}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x + 2y - z + u &= 2 \\ 2x + 5y - z + 2u &= 3 \\ 3x - y - 2z + u &= 2 \\ x - y + 3z - 5u &= 2 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 0

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ A2

IME I PREZIME: Ante Šapić **BROJ INDEKSA:** 17-2-02 11-2012

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
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6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno:

5)

$$\left|\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 2 \end{array}\right| \begin{array}{l} \text{I} \\ \text{II}-2\text{I} \\ \text{III}-3\text{I} \\ \text{IV}-\text{I} \end{array}$$

$$\left|\begin{array}{cccc|c} 0 & 3 & -4 & 6 & 0 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & 1 & -2 & 1 & 2 \\ 1 & 1 & 3 & -5 & 2 \end{array}\right| \text{II}$$

$$\sim \left|\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & -7 & 1 & -2 & -4 \\ 0 & -3 & 4 & -4 & 0 \end{array}\right| \begin{array}{l} \text{I}-2\text{II} \\ \\ \text{III}+7\text{II} \\ \text{IV}+3\text{III} \end{array}$$

$$\sim \left|\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 4 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 8 & -2 & -11 \\ 0 & 0 & 7 & -4 & -3 \end{array}\right| :8$$

$$\sim \left|\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 4 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & -\frac{2}{8} & -\frac{11}{8} \\ 0 & 0 & 7 & -4 & -3 \end{array}\right| \begin{array}{l} \text{I}+3\text{III} \\ \text{II}-\text{III} \\ \\ \text{IV}-7\text{III} \end{array}$$

$$\sim \left|\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & \frac{13}{8} \\ 0 & 1 & 0 & -\frac{2}{8} & \frac{3}{8} \\ 0 & 0 & 1 & -\frac{2}{8} & -\frac{11}{8} \\ 0 & 0 & 0 & -\frac{12}{4} & \frac{53}{8} \end{array}\right| \begin{array}{l} \cdot 2 \\ \cdot 2 \\ \cdot 2 \\ \cdot 2 \end{array}$$

$$\sim \left|\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{3}{4} \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{11}{4} \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{53}{4} \end{array}\right| \begin{array}{l} \text{I} + \text{IV} \\ \text{II} - \text{IV} \\ \text{III} - \text{IV} \end{array} \sim \left|\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{55}{4} \\ 0 & 1 & 0 & 0 & -\frac{25}{2} \\ 0 & 0 & 1 & 0 & -16 \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{53}{4} \end{array}\right| \cdot(-2)$$

$$\sim \left|\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{55}{4} \\ 0 & 1 & 0 & 0 & -\frac{25}{2} \\ 0 & 0 & 1 & 0 & -16 \\ 0 & 0 & 0 & 1 & -\frac{53}{2} \end{array}\right|$$

$$\frac{55}{4} + 2\cdot\frac{25}{2} - (-16) + \left(-\frac{53}{2}\right) = 2$$

[illegible]

JOŠ . . .

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ A2

IME I PREZIME: Stjepan Štefančić

BROJ INDEKSA: 17-2-0359-2014

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
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6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 0

(2) $f(x) = x - \sqrt{x^2-2}$

$x - \sqrt{x^2-2} = 0$

$x = \sqrt{x^2-2} \,/^2$

$x^2 = x^2 - 2$

$F(x) = x - \sqrt{x^2-2} = x - (x^2-2)^{\frac{1}{2}}$

$f'(x) = -\frac{1}{2}(x^2-2)^{x^2-2}$ ×

$f''(x) = x^2-2 \cdot (-\frac{1}{2})(2x) = (x^2-2)(-x) = -x^3+2x$

$-\dfrac{1}{2\sqrt{x^2-2}} \cdot (x^2-2) = -\dfrac{x^2-2}{2\sqrt{x^2-2}} =$

$= -\dfrac{2x(2\sqrt{x^2-2}) - (x^2-2)\cdot\frac{1}{2\sqrt{x^2-2}}}{(2\sqrt{x^2-2})^2} =$

$= \dfrac{9x\sqrt{x^2-2} - \frac{x^2-2}{2\sqrt{x^2-2}}}{4\cdot(x^2-2)} =$

$= \dfrac{(9x\sqrt{x^2-2})(2\sqrt{x^2-2}) - x^2-2}{2\sqrt{x^2-2}} =$

$= 9x^2-8$

(3) $g(x) = \ln(4-x^2)$

$\ln(4-x^2) = 0 \,/e$

$e^{\ln(4-x^2)} = e^0$

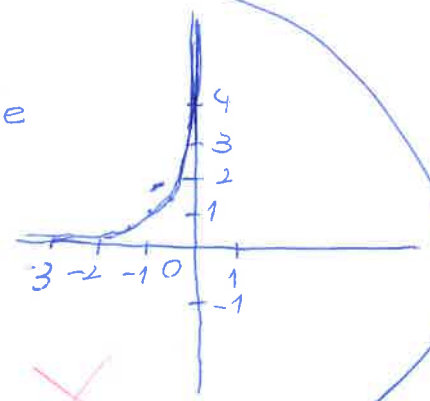
$4-x^2 = 1$

$-x^2 = 1-4$

$-x^2 = -3$

$x^2 = 3 \,/\sqrt{}$

$x_{1/2} = \pm\sqrt{3}$



×

(5) $\begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 5 & -1 & 2 \\ 3 & -1 & -2 & 1 \\ 1 & -1 & 3 & -5 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 2 \\ 2 \end{bmatrix}$

$= \dfrac{9x(x^2-2) - (x^2-2)}{2\sqrt{x^2-2}} = \dfrac{x^2-2(9x-1)}{2\sqrt{x^2-2}}$

$9x^2-8 \qquad 9x^2-8$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ A2

IME I PREZIME: MAJA ŠOGORIĆ **BROJ INDEKSA:** 02 69040783

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2-2}$. 5+15+5
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln(4-x^2)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-2x-(2+1)}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x+2y-z+u &= 2 \\ 2x+5y-z+2u &= 3 \\ 3x-y-2z+u &= 2 \\ x-y+3z-5u &= 2 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno:

5. $\left.\begin{aligned} x+2y-z+u&=2\\ 2x+5y-z+2u&=3\\ 3x-y-2z+u&=2\\ x-y+3z-5u&=2 \end{aligned}\right\}$ $A=\begin{bmatrix}1&2&-1&1\\2&5&-1&2\\3&-1&-2&1\\1&-1&3&-5\end{bmatrix}\begin{bmatrix}x\\y\\z\\u\end{bmatrix}=\begin{bmatrix}2\\3\\2\\2\end{bmatrix}$

$$\left[\begin{array}{cccc|c}1&2&-1&1&2\\2&5&-1&2&3\\3&-1&-2&1&2\\1&-1&3&-5&2\end{array}\right]\begin{matrix}\\R2-2R1\\R3-3R1\\R4-R1\end{matrix}\sim\left[\begin{array}{cccc|c}1&2&-1&1&2\\0&1&1&0&-1\\0&-7&-1&-2&-4\\0&-3&4&-6&0\end{array}\right]\begin{matrix}R1-2R2\\\\R3+7R2\\R4-3R2\end{matrix}\sim\left[\begin{array}{cccc|c}1&0&-3&1&0\\0&1&1&0&-1\\0&0&-8&-9&-6\\0&0&1&-12&0\end{array}\right]\cdot\left(-\frac{1}{8}\right)R3\sim$$

$$\left[\begin{array}{cccc|c}1&0&-3&1&0\\0&1&1&0&-1\\0&0&1&\frac{9}{8}&\frac{6}{8}\\0&0&1&-12&0\end{array}\right]\begin{matrix}R1+3\cdot R2\\\\R3-\frac{9}{8}R2\\\\\end{matrix}\left[\begin{array}{cccc|c}1&0&0&1&-3\\0&1&1&0&-1\\0&0&1&0&-\frac{3}{8}\\0&0&1&-12&0\end{array}\right]R4-R2\sim\left[\begin{array}{cccc|c}1&0&0&1&3\\0&1&1&0&-1\\0&0&1&0&-\frac{3}{8}\\0&0&0&-12&0\end{array}\right]R2-R3$$

$$\left[\begin{array}{cccc|c}1&0&0&1&3\\0&1&0&0&-1\\0&0&1&0&-\frac{3}{8}\\0&0&0&-12&0\end{array}\right]\begin{matrix}\\\\\\R4-12R1\end{matrix}\sim\left[\begin{array}{cccc|c}1&0&0&1&3\\0&1&0&0&-1\\0&0&1&0&-\frac{3}{8}\\0&0&0&1&0\end{array}\right]R1-R4\sim\left[\begin{array}{cccc|c}1&0&0&0&3\\0&1&0&0&-1\\0&0&1&0&-\frac{3}{8}\\0&0&0&1&0\end{array}\right]$$

$$X=\begin{pmatrix}x\\y\\z\\u\end{pmatrix}=\begin{pmatrix}3\\-1\\-\frac{3}{8}\\0\end{pmatrix}$$

Provjera $\quad 3+2\cdot(-1)-\left(-\frac{3}{8}\right)+0=3-2+\frac{3}{8}=1+\frac{3}{8}=\frac{8+3}{8}=\frac{11}{8}$

$$\begin{bmatrix}1&2&-1&1\\2&5&-1&2\\3&-1&-2&1\\1&-1&3&-5\end{bmatrix}\begin{bmatrix}3\\-1\\-\frac{3}{8}\\0\end{bmatrix}$$

$2\cdot3+5\cdot(-1)-\left(-\frac{3}{8}\right)+2\cdot0=6-5+\frac{3}{8}=1+\frac{3}{8}=\frac{11}{8}$

$3\cdot3-(-1)-2\cdot\left(-\frac{3}{8}\right)+0=9+1+\frac{6}{8}=10+\frac{6}{8}=\frac{80+6}{8}=\frac{86}{8}$

1) $1\cdot3+2\cdot(-1)+(-1)\cdot\left(-\frac{3}{8}\right)+1\cdot0=3-2+\frac{3}{8}=\frac{1+3}{8}=\frac{4}{8}=\frac{1}{2}$

2) $2\cdot(3)+5\cdot(-1)+(-1)\cdot\left(-\frac{3}{8}\right)+2\cdot0=6-5+\frac{3}{8}=1+\frac{3}{8}=\frac{9+3}{8}=\frac{12}{8}=\frac{3}{2}$

MAJA JOGORIĆ

1. $z^4-(4-i)^2=0$

$(z^2)^2-(4-i)^2=0$

$(z^2)^2-16-8i-i^2=0$

$(z^2)^2-16-8i-1=0$

$(z^2)^2-17-8i=0$

1) $(z_{1,2})^2=\frac{-17\pm\sqrt{17^2-4\cdot(-8)\cdot1}}{2\cdot1}=\frac{-17\pm\sqrt{499+32}}{2}=\frac{-17\pm\sqrt{531}}{2}$

$(z_1)^2=\frac{-17+\sqrt{531}}{2}$ $(z_2)^2=\frac{-17-\sqrt{531}}{2}$ ×

2) $(z^2)^2-17-8i=0$

$\left(\frac{-17+\sqrt{531}}{2}\right)-17-8i=0$

2. $f(x)=x-\sqrt{x^2-2}$

$f'(x)=1-(\sqrt{x^2-2})'$ ×

$=1-(2x-0)$ f''

$=1-2x$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 2 \end{array}\right] \begin{array}{l} \\ R2-2R1 \\ R3-3R1 \\ R4-R1 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & -7 & -5 & -2 & -2 \\ 0 & -3 & 2 & -6 & 0 \end{array}\right] \begin{array}{l} R1-2R2 \\ \\ R3+7R2 \\ R4+3R2 \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -7 & 1 & 0 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & -16 & -2 & -5 \\ 0 & 0 & -13 & -8 & 0 \end{array}\right] R3 \cdot 16 \sim \left[\begin{array}{cccc|c} 1 & 0 & -7 & 1 & 0 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & 1 & -32 & -60 \\ 0 & 0 & -13 & -8 & 0 \end{array}\right] \begin{array}{l} R1+7R3 \\ \\ \sim \\ \\ R4+13R3 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & -7 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & -4 & -32 & -60 \\ 0 & 0 & 0 & 408 & -13 \end{array}\right]$$

MAJA ŠOKORIĆ

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 2 \end{array}\right] \begin{array}{l} \\ R2-2R1 \\ R3-3R1 \\ R4-R1 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & -3 & 0 & -3 \\ 0 & -7 & -5 & -2 & -2 \\ 0 & -3 & 2 & -6 & 0 \end{array}\right] \begin{array}{l} R1-2R2 \\ \\ \sim \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -7 & 0 & -4 \\ 0 & 1 & -3 & 0 & -3 \\ 0 & -7 & -5 & -2 & -2 \\ 0 & -3 & 2 & -6 & 0 \end{array}\right] \begin{array}{l} \\ \\ R3+7R2 \\ R4+3R2 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & -7 & 0 & -4 \\ 0 & 1 & -3 & 0 & -3 \\ 0 & 0 & -30 & -2 & -2 \\ 0 & 0 & -4 & -6 & 0 \end{array}\right]$$

FAJA SOGORI