MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xoo

IME I PREZIME: Radovčić Rikardo **BROJ INDEKSA:** 17-2-0228-2012

RR

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi diferencijalnu jednadžbu $xyy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$. 15
2. Odredi ekstreme funkcije $f(x,y) = x^2 - y^2$. 15
3. Za funkciju $f(x,y) = \dfrac{x}{y}$ odrediti domenu, kodomenu, razinske krivulje i limes u ishodištu (ako postoji). ~~15~~ 8
4. $\displaystyle\int_0^{\pi} \sin^2 x \cos^3 x \, dx = ?$ 20
5. $\int_0^3 x^2 \ln x \, dx = ?$ 15
6. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$. ~~20~~

Ukupno: 73

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

5.

$$\int_0^3 x^2 \ln x\,dx = \lim_{b\to 0^+}\int_b^3 x^2 \ln x\,dx = \left[\begin{matrix} u=\ln x & dv = x^2\,dx \\ du = \frac{1}{x}\,dx & v = \frac{x^3}{3}\end{matrix}\right]$$

$$= \lim_{b\to 0^+}\left(\frac{x^3}{3}\ln x\Big|_b^3 - \frac{1}{3}\int_b^3 x^2\,dx\right)$$

$$= \lim_{b\to 0^+}\left(\frac{x^3}{3}\ln x\Big|_b^3 - \frac{x^3}{9}\Big|_b^3\right) = \lim_{b\to 0^+}\left(9\ln 3 - \frac{b^3}{3}\ln b - \frac{x^3}{9}\Big|_b^3\right)$$

$$= 9\ln 3 - \lim_{b\to 0^+}\frac{1}{9}(27-b) = 9\ln 3 - 3$$

$$\lim_{x\to 0^+} x^3 \ln x = \lim_{x\to 0^+}\frac{\ln x}{\frac{1}{x^3}} = \frac{\infty}{\infty} = L'H = \lim_{x\to 0^+}\frac{\frac{1}{x}}{-3\cdot\frac{1}{x^4}}$$

$$= -\frac{1}{3}\lim_{x\to 0^+} x^3 = 0$$

① $f(x,y) = x^2 - y^2$

$2x = 0$

$-2y = 0$

$x = 0 \quad y = 0$

$f'_x = 2x$

$f'_y = -2y$

$f''_{xx} = 2$

$f''_{xy} = 0$

$f''_{yy} = -2$

$$\begin{vmatrix} 2 & 0 \\ 0 & -2 \end{vmatrix} = -4 < 0$$

Nema ekstrema ✓

Točka $T(0,0)$ je stacionarna točka funkcije.

---

④ $$\int_0^{\pi} \sin^2 x \cos^3 x \, dx = \begin{bmatrix} t = \sin x & dx = \frac{dt}{\cos x} \\ dt = \cos x dx & \end{bmatrix}$$

$$\int_0^{\pi} t^2 \cdot \cos^{\cancel{3}\,2} x \frac{dt}{\cancel{\cos x}} = \int_0^{\pi} t^2 \cdot (1-t^2)\, dt = \int_0^{\pi} t^2 dt - \int_0^{\pi} t^4 dt =$$

$$= \frac{t^3}{3} \Big|_0^{\pi} - \frac{t^5}{5} \Big|_0^{\pi} = \frac{\sin^2 x}{3} \Big|_0^{\pi} - \frac{\sin^5 x}{5} \Big|_0^{\pi} = 0 \quad ✓$$

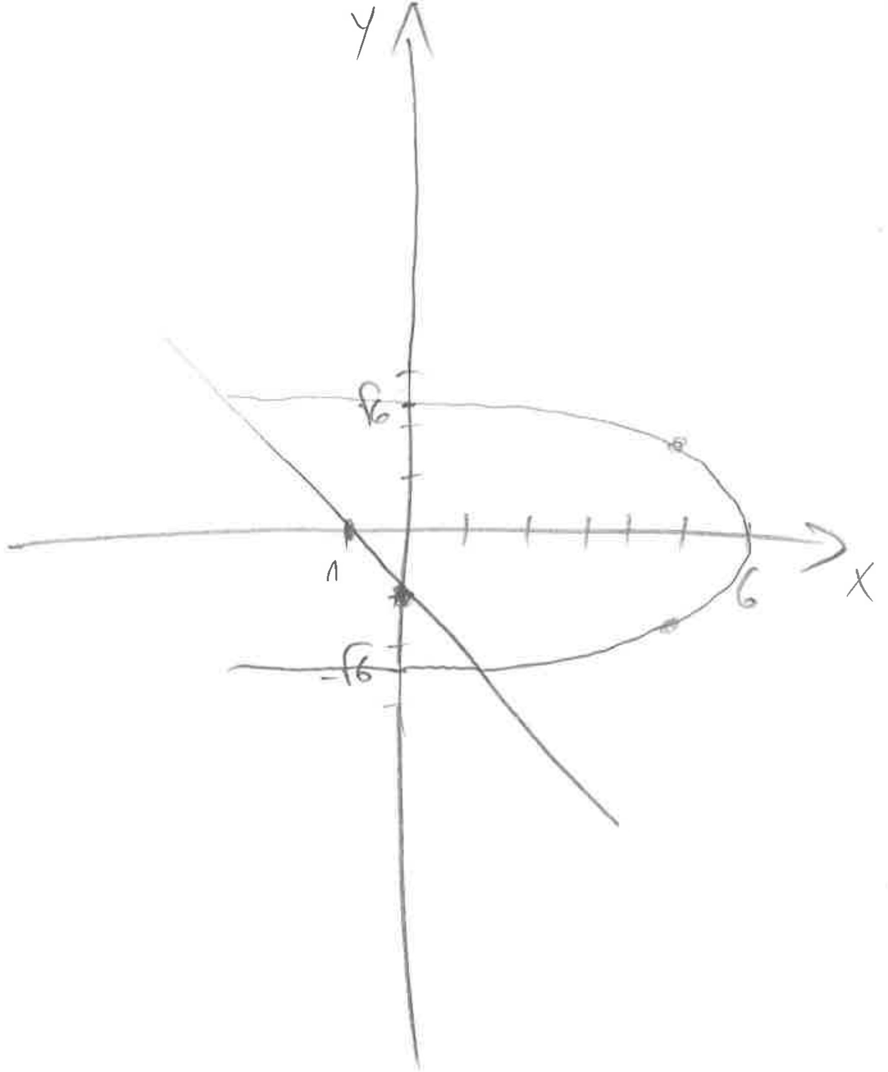
(6) $x + y^2 = 6$

$x + y + 1 = 0$

$y^2 = -x + 6$

$y = -x - 1$

x	0	-1
y	-1	0



$(-x-1)^2 = -x + 6$

$x^2 + 2x + 1 + x - 6 = 0$

$x^2 + 3x - 5 = 0$

$x_{1,2} = \frac{-3 \pm \sqrt{9 + 20}}{2}$

$x_{1,2} = \frac{-3 \pm \sqrt{29}}{2}$

$P = \int_{\frac{-3-\sqrt{29}}{2}}^{\frac{-3\pm\sqrt{29}}{2}} \left(\sqrt{-x+6} - (-x-1)\right) dx + 2\int_{\frac{-3+\sqrt{29}}{2}}^{6} \sqrt{6-x}\, dx$

$P = \int_{\frac{-3-\sqrt{29}}{2}}^{\frac{-3+\sqrt{29}}{2}} \left(\sqrt{6-x} + x + 1\right) dx + 2\int_{\frac{-3+\sqrt{29}}{2}}^{6} \sqrt{6-x}\, dx = \begin{bmatrix} t = 6-x \\ dt = -dx \end{bmatrix}$

$$P = \int_{\frac{-3-\sqrt{29}}{2}}^{\frac{-3+\sqrt{29}}{2}} -t^{\frac{1}{2}}\,dt + \left(\frac{x^2}{2}+x\right)\Bigg|_{\frac{-3-\sqrt{29}}{2}}^{\frac{-3+\sqrt{29}}{2}} - 2\int_{\frac{-3+\sqrt{29}}{2}}^{6} t^{\frac{1}{2}}\,dt$$

$=$. ~ KALKULATOR . .

Radovčić Rikardo

RR

① $xyy' = 1 - x^2$ $\quad y(1)=1$

Radovčić Rikardo

$$y\,dy = \frac{1-x^2}{x}\,dx \Big/ \int$$

$$\int y\,dy = \int\left(\frac{1}{x} - x\right)dx$$

$$\frac{y^2}{2} = \ln x - \frac{x^2}{2} + C$$

$$\boxed{y = \sqrt{2\ln x - x^2 + 2C}}$$

$y(1) = 1$

$1 = \sqrt{0 - 1 + 2C} \Big/ ^2$

$1 = 2C - 1$

$\boxed{C = 1}$

$$\boxed{y = \sqrt{2\ln x - x^2 + 2}}$$ ✓

---

③ $f(x,y) = \frac{x}{y}$ $\quad Df = \left\{\begin{matrix} x \in \mathbb{R} \\ y \in \mathbb{R} \setminus \{0\} \end{matrix}\right\}$

SKICA RAZ. KRIVULJA

$C_1 = 1$	$C_2 = 2$	$C_3 = 3$	$C_4 = 4$
$1 = \frac{x}{y}$	$2 = \frac{x}{y}$	$3 = \frac{x}{y}$	$4 = \frac{x}{y}$
$y = x$	$y = \frac{x}{2}$	$y = \frac{x}{3}$	$y = \frac{x}{4}$

Kodomena $= \mathbb{R}$ ✓ $\quad \lim\limits_{\substack{x \to 0 \\ y \to 0}} \frac{x}{y} =$ Ne postoji

ZAŠTO?

$Df\ \{(x,y) \in \mathbb{R}^2 : y \neq 0\}$ ✓

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xoo

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: Šimun Zdnlić

BROJ INDEKSA: 17-2-0297-2013

1. Riješi diferencijalnu jednadžbu $xyy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$. 15
2. Odredi ekstreme funkcije $f(x, y) = x^2 - y^2$. 15
3. Za funkciju $f(x, y) = \dfrac{x}{y}$ odrediti domenu, kodomenu, razinske krivulje i limes u ishodištu (ako postoji). 15
4. $\displaystyle\int_0^{\pi} \sin^2 x \cos^3 x \, dx = ?$ 20
5. $\int_0^3 x^2 \ln x \, dx = ?$ ~~15~~
6. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$. 20

Ukupno: 50

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
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Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\, dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
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(2) $f(x,y) = x^2 - y^2$

$\partial x = (x^2 - y^2)' = 2x$

$\partial y = (x^2 - y^2) = -2y$

$\partial xx = (2x)' = 2$

$\partial xy = (2x)' = 2x = 2 \cdot 0 = 0$

$\partial yy = (-2y)' = -2$

$2x = 0 \qquad -2y = 0$

$x = 0 \qquad y = 0$

$T(0,0) \to$ kandidat

$$\begin{bmatrix} \partial xx & \partial xy \\ \partial yx & \partial yy \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

$2 > 0$

$-4 - 0 = -4$

$-4 < 0$

$T(0,0) \Rightarrow$ sedlo ✓

⑥ $x+y^2=6$

$x+y+1=0 \Rightarrow$

$y=6-x^2$

$-y=-x-1$

$6-x^2=-x-1$

$-x^2+x+6+1=0 \quad / \cdot(-1)$

$x^2+x+7=0$

$x_{1,2}=\frac{-1\pm\sqrt{29}}{2}=$

$x_2=\frac{-1-\sqrt{29}}{2}$

$x_1=\frac{-1+\sqrt{29}}{2}$

$$\int_{x_2}^{x_1} -x^2+x+7\,dx=\int_{x_2}^{x_1} x^2dx+\int_{x_2}^{x_1} x\,dx+\int_{x_2}^{x_1} 7\,dx$$

$$=-\frac{1}{3}\Big|_{x_2}^{x_1}x^3+\frac{1}{2}\Big|_{x_2}^{x_1}x^2+7\Big|_{x_2}^{x_1}x$$

$$=-\frac{1}{3}\left(10{,}54-(-32{,}540)\right)+\frac{1}{2}\left(4{,}80-10{,}19\right)$$

$$+7\left(2{,}19-(-3{,}19)\right)$$

$$=-\frac{1}{3}\left(43{,}08\right)+\frac{1}{2}\left(-5{,}39\right)+7\cdot 5{,}38$$

$$=-14{,}36-4{,}89+37{,}66$$

$$=18{,}41 \checkmark$$

Šimun Zdrilić

① $xyy' = 1 - x^2$

$$y\frac{dy}{dx} = \frac{1-x^2}{x} \,/\cdot dx$$

$$\int y\,dy = \underbrace{\int \frac{1-x^2}{x}\,dx}$$

$$(-x^2 + 1) : x = -x + \frac{1}{x}$$

$$\frac{-x^2}{1}$$

$$\frac{1}{2}y^2 = -\int x\,dx + \frac{dx}{x}$$

$$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + \ln x + C$$

$$y(1=1)$$

$$\frac{1}{2}\cdot 1^2 = -\frac{1}{2}\cdot 1^2 + \ln 1 + C$$

$$\frac{1}{2} = -\frac{1}{2} + C \,/\cdot 2$$

$$1 = -1 + C$$

$$C = 2$$

$$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + \ln x + 2$$ ✓

Šimun Zdrilić

5. $\int_0^3 x^2 \ln x \, dx = \begin{bmatrix} u = \ln x & dv = x^2 \\ du = \frac{1}{x} & v = \frac{x^3}{3} \end{bmatrix}$

$= \frac{1}{3} \Big| _{0^+}^{3} x^3 \cdot \ln x - \int_0^3 \frac{x^3}{3} \cdot \frac{1}{x}$

$= \frac{1}{3} \Big| _{0^+}^{3} x^3 \cdot \ln x - \frac{1}{3} \int_0^3 x^2$

$= \frac{1}{3} \Big| _{0^+}^{3} x^3 \cdot \ln x - \frac{1}{9} \Big| _0^3 x^3$

$= \frac{1}{3} (27 \cdot \ln 3 + (0 \cdot \infty)) - 3 = N/P$ ×

$\to$ nepravi integral

Šimun Zdrilić

(4) $\int_0^{\pi} \sin^2 x \cdot \cos^3 x \, dx$

$= \frac{1}{8} \Big|_0^{\pi} \sin x - \frac{1}{48} \Big|_0^{\pi} \sin 3x - \frac{1}{80} \Big|_0^{\pi} \sin 5x$

?

$= \frac{1}{8} \cdot 0 - \frac{1}{48} \cdot 0 - \frac{1}{80} \cdot 0 = 0$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xoo

IME I PREZIME: FLAVIO ABATINOVIĆ **BROJ INDEKSA:** 17-2-0162-2012

POPUNJAVA NASTAVNIK

	Broj ↓ bodova
1. Riješi diferencijalnu jednadžbu $xyy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$.	15
2. Odredi ekstreme funkcije $f(x, y) = x^2 - y^2$.	15
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4. $\int\limits_0^{\pi} \sin^2 x \cos^3 x \, dx = ?$	20
5. $\int_0^3 x^2 \ln x \, dx = ?$	~~15~~
6. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$.	20
Ukupno:	35

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
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Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha \, dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln \lvert x \rvert + C$	$\int \cot x \, dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int e^x \, dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
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$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

2.) $f(x,y) = x^2 - y^2$

$\frac{df}{dx} = 2x \Rightarrow 2x = 0 \ / \left(\frac{1}{2}\right) \quad \boxed{x = 0}$

$\frac{df}{dy} = -2y \Rightarrow -2y = 0 \ / \left(\frac{1}{2}\right) \quad \boxed{y = 0}$

$\frac{d^2 f}{dx^2} = 2 = A$

$\frac{d^2 f}{dxy} = 0 = B$

$\frac{d^2 f}{dy^2} = -2 = C$

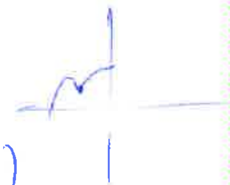
$T(0,0)$

$A = 2 > 0 \to$ MINIMUM

$\Delta = A \cdot C - B^2$

$\Delta = 2 \cdot (-2) - 0^2$

$\Delta = -4 < 0 \to$ (SEDLO)



SEDLASTA FUNKCIJA

NEMA EKSTREMA

5.) $\int_0^3 \underbrace{x^2}_{dv} \underbrace{\ln x}_{u} \, dx = \left[\begin{matrix} u = \ln x & dv = x^2 dx \,/\int \\ \frac{1}{x} dx = du & v = \int x^2 dx \\ du = \frac{1}{x} dx & v = \frac{x^3}{3} \end{matrix} \right]$ $\int u \cdot dv = u \cdot v - \int v du$

$= \ln x \cdot \left(\frac{x^3}{3}\right) - \int_0^3 \frac{\cancel{x^3}}{3} \cdot \frac{1}{\cancel{x}} dx = \ln x \cdot \left(\frac{x^3}{3}\right) - \int_0^3 \frac{x^2}{3} dx$

$\cancel{= \ln x \left(\frac{x^3}{3}\right) - \int \frac{x^3}{3x} dx}$

$= \ln x \left(\frac{x^3}{3}\right) - \frac{1}{3} \int_0^3 x^2 dx$

$= \ln x \left(\frac{x^3}{3}\right) \Big|_0^3 - \frac{1}{3} \cdot \frac{x^3}{3} \Big|_0^3$

$= \ln x \left(\frac{x^3}{3}\right) \Big|_0^3 - \frac{x^3}{9} \Big|_0^3$

$\cancel{= \ln x \left(\frac{x^3}{3}\right) + \frac{3^3}{9} - \left(-\frac{0}{9}\right)}$

$\cancel{= \ln x \left(\frac{x^3}{3}\right)}$

$= \ln 3 \left(\frac{3^3}{3}\right) - \frac{3^3}{9} - \ln \cancel{0} \left(\frac{0^3}{3}\right) - \frac{0^3}{9}$

$= \ln|3| \cdot 9 - 3 - 0 - 0$

$= \ln|3| \cdot 9 - 3$

$\approx 6{,}888$

$\boxed{\ln 0 \cdot \frac{0^3}{3} = ?}$

3.) $f(x,y) = \frac{x}{y}$

$y \neq 0 \quad D_{f(y)} = \langle -\infty, 0 \rangle, \langle 0, +\infty \rangle$ ×

4.) $\int_0^{\pi} \sin^2 x \cos^3 x \, dx = \int_0^{\pi} \sin^2 x \cdot \cos^2 x \cdot \cos x$ $\quad \cos^2 x = 1 - \sin x$

$= \int_0^{\pi} \sin^2 x \cdot (1 - \sin x) \cdot \cancel{\cos x} = \left[\begin{array}{l} t = \sin x \\ \cos x \, dx = dt \\ dx = \frac{dt}{\cancel{\cos x}} \end{array} \right]$

$= \int_0^{\pi} t^2 \cdot (1 - t) \cdot \cancel{\cos x} \cdot \frac{dt}{\cancel{\cos x}}$

$= \int_0^{\pi} t^2 - t^3 \, dt = \int_0^{\pi} t^2 dt - \int_0^{\pi} t^3 dt$

$= \frac{t^3}{3} - \frac{t^4}{4} \Big|_0^{\pi}$

$= \frac{\sin^3 x}{3} - \frac{\sin^4 x}{4} \Big|_0^{\pi}$

$= \cancel{\frac{\sin^3 \pi}{3}}^{=0} - \cancel{\frac{\sin^3 0}{3}} - \frac{\sin^4 \pi}{4}^{=0} - \frac{\sin^4 0}{4}^{=0}$

$= 0$

$\int_0^{\pi} \sin^2 x \cos^3 x \, dx = 0$

FLAVIO ABATINOVIĆ

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xoo

IME I PREZIME: SEBASTIJAN KOŠTA **BROJ INDEKSA:** 17-2-0094-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi diferencijalnu jednadžbu $xyy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$. 15
2. Odredi ekstreme funkcije $f(x, y) = x^2 - y^2$. 15
3. Za funkciju $f(x, y) = \dfrac{x}{y}$ odrediti domenu, kodomenu, razinske krivulje i limes u ishodištu (ako postoji). 15
4. $\displaystyle\int_0^{\pi} \sin^2 x \cos^3 x \, dx = ?$ 20
5. $\int_0^3 x^2 \ln x \, dx = ?$ ~~15~~
6. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$. 20

Ukupno: 0

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

$\int u\,dv = uv - \int v\,du$

5. $$\int_0^3 x^2 \ln x \, dx = \left| \begin{matrix} u = \ln x & dv = x^2 dx \\ du = \frac{1}{x}dx & v = \frac{x^3}{3} \end{matrix} \right|$$

$$= \ln x \cdot \frac{x^3}{3} - \int_0^3 \frac{x^3}{3} \cdot \frac{1}{x} \, dx = \ln x \cdot \frac{x^3}{3} - \int_0^3 \frac{x^2}{3} \, dx$$

$$= \ln x \cdot \frac{x^3}{3} - \frac{x^3}{3x}\Bigg|_0^3 = \ln 3 \cdot \frac{3^3}{3} - \frac{3^3}{3 \cdot 3} - \ln 0 \cdot \frac{0^3}{3} - \frac{0^3}{3 \cdot 0}$$

?

?

$$= \ln 3 \cdot 9 - 3 = 9.88751 - 3 = 6.88751$$

NIJE ISPITANO: $\boxed{\ln 0 \cdot \frac{0^3}{3} = ?}$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xoo

POPUNJAVA NASTAVNIK

IME I PREZIME: IVAN GAĆINA **BROJ INDEKSA:**

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$\boxed{5}$ $\int_0^3 x^2 \ln|x| \, dx$, $\left[u = \ln x \quad du = \frac{1}{x} dx \quad dv = x^2 dx \quad v = \frac{x^3}{3} \right]$

$= \ln|x| \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} dx \Rightarrow -\frac{1}{3} \int \frac{x^{\cancel{3}}}{\cancel{x}} dx$

$-\frac{1}{3} \int x^2 dx \Rightarrow -\frac{1}{3} \cdot \frac{x^3}{3}$

$\left[\ln|x| \cdot \frac{x^3}{3} - \frac{x^3}{9} \right]_0^3 = \left(\ln|3| \cdot \frac{3^3}{3} - \frac{3^3}{9} \right) - \left(\ln|0| \cdot \frac{0^3}{3} - \frac{0^3}{9} \right)$

$\cancel{\ln|0|}$ *

NEPRAVI INTEGRAL

$= 1{,}0986 \cdot 9 - 3 = 6{,}8874$ //

$\boxed{\ln 0 \cdot \frac{0^3}{3} = ?}$

$$xyy' = 1 - x^2 \qquad y(1) = 1 \;\Rightarrow\; x = 1,\; y = 1$$

(1)

$$y\frac{dy}{dx} = \frac{1-x^2}{x}$$

$$y\,dy = \frac{1-x^2}{x}\,dx \quad /\int$$

$$\boxed{\int y\,dy = \int \frac{1-x^2}{x}\,dx}$$

$$\int \frac{1}{x}\,dx - \int \frac{x^2}{x}\,dx$$

$$\ln|x| - \frac{x^2}{2} + C$$

$$\frac{y^2}{2} = \ln x - \frac{x^2}{2} \quad /\cdot 2$$

$$y^2 = 2\ln x - x^2 \quad /\sqrt{\ }$$

$$\boxed{y = \sqrt{2\ln x - x^2 + c}}$$

$$1 = \sqrt{2\ln(1) - 1} + c$$

$$-c = \sqrt{2\ln(1)} - 1 - 1$$

$$-c = -2$$

$$c = 2$$

$$1 = 0 - 1 + 2$$

$$\boxed{1 = 1}$$

$$y' = \frac{1}{2\sqrt{2\ln x}} \cdot \frac{2}{x}$$

$$x\left(\sqrt{2\ln|x|} - x\right)\cdot\left(\frac{2}{2x\sqrt{2\ln x}}\right) = 1 - x^2$$

$$x\sqrt{2\ln|x|} - x^2 \cdot \frac{1}{x\sqrt{2\ln x}} = 1 - x^2$$

$$x\sqrt{2\ln|x|} - \frac{x}{\sqrt{2\ln x}} = 1 - x^2$$

$$\frac{x\sqrt{2\ln x}\cdot\sqrt{2\ln x} - x}{\sqrt{2\ln x}} = 1 - x^2$$

$$= 1 - x$$

IVAN GACINIK

$\boxed{4}$

$$\int_0^{\pi} \sin^2 x \, \cos^3 x \, dx \quad \left\{ \begin{array}{l} \cos^3 x = t \\ 3\sin^2 x \, dx = dt \\ dx = \dfrac{dt}{3\sin^2 x} \end{array} \right\}$$

$dt = 3\cos^2 x \cdot \sin x$

$$\cancel{\sin^2 x} \cdot t \cdot \frac{dt}{3\cancel{\sin^2 x}} = \frac{1}{3}\int t \, dt$$

$$= \frac{1}{3} \frac{t^2}{2} = \frac{t^2}{6}$$

$$\left[-\frac{(\cos^3 x)^2}{6} \right]_0^{\pi} \Rightarrow \left(-\frac{(\cos^3 \pi)^2}{6} \right) + \left(-\frac{(\cos^3 0)^2}{6} \right) = -\frac{1}{6} - \frac{1}{6}$$

$$= -\frac{1}{3}$$

$\boxed{2}$

$$f(x,y) = \frac{x}{y} \qquad D_f \{ \mathbb{R} \mid y \neq 0 \} , \quad KD_f = \mathbb{R}$$

$C_1 = 1 \Rightarrow$

$C_2 = 2$

$C_3 = 3$

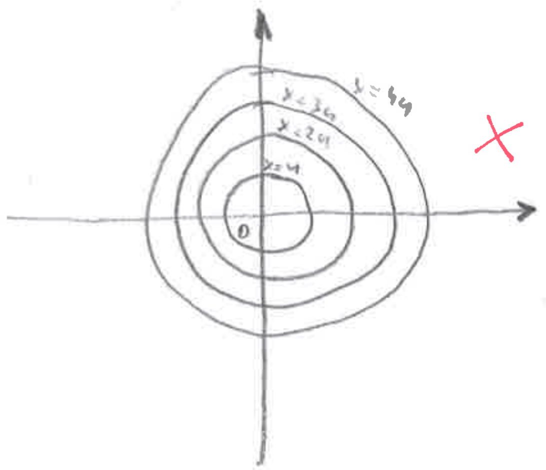
$C_4 = 4$

$C_1 \Rightarrow x = y$

$x = 2y$

$x = 3y$

$x = 4y$



"0" nije u domeni, limes ne postoji