MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox

IME I PREZIME: Stipe Rebić **BROJ INDEKSA:** 17-2-0226-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - y' = e^x + 1$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = x^3 - 3xy - y^3$. 15
3. Pronaći ravninu koja dira graf funkcije $f(x,y) = y\sqrt{x} - y^2 - x + 6y$ povučenu u točki $(4, 1, z_0)$ tog grafa. 15
4. Numeričkom integracijom procijeniti vrijednost $\int_0^1 \frac{dx}{1+\sqrt{x}}$, a zatim isti integral riješiti egzaktno Newton Leibnitzovom formulom. 10+5
5. $\int_2^3 \frac{2x^2+x+2}{x^2-1}\mathrm{d}x =$ 20
6. $\int_0^2 \frac{x-1}{x^2+x-2}\mathrm{d}x = ?$ ~~20~~

Ukupno: 65

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, \mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, \mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, \mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, \mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, \mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\, \mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

6. $\int_0^2 \frac{x-1}{x^2+x-2}dx = \int_0^2 \frac{x+1-2}{x^2+x-2} = \left\{\begin{matrix} x^2+x-1 \\ (2x+1)dx = dt \end{matrix}\right\}$

$\int_0^2 \frac{\frac{1}{2}(2x+1)-\frac{1}{2}-2}{x^2+x-2} = \frac{1}{2}\int_0^2 \frac{dt}{t-2} - \int_0^2 \frac{\frac{3}{2}}{t-2}dt$ ✗

$\left\{\begin{matrix} x^2+x-2=t \\ (2x+1)dx = dt \end{matrix}\right\}$

$= \frac{1}{2}\ln|t-2|\Big|_0^2 - \frac{3}{2}\ln|t-2|\Big|_0^2$

$= \frac{1}{2}\ln|x^2+x-2|\Big|_0^2 - \frac{3}{2}\ln|x^2+x-2|\Big|_0^2$

$= \frac{1}{2}\ln 4 - \frac{3}{2}\ln 4 = -1{,}386$

1. $y'' - y' = e^x + 1$

$y_H = C_1 + C_2 e^x$ $\quad m_1 = 0$, $m_2 = -1$

$f_1(x) = e^x \quad n=0 \quad m=1 \quad (P_n(x)\, a^{mx})$

$y_{p1} = (a_1 x + a_0) e^x - e^x$

$y_{p1}' = a_1 e^x + e^x (a_1 x + a_0)$

$y_{p1}'' = a_1 e^x + a_1 e^x + e^x (a_1 x + a_0)$

$a_1 e^x + a_1 e^x + e^x (a_1 x + a_0) - a_1 e^x - e^x (a_1 + a_0) = e^x$

$a_1 e^x = e^x \qquad \boxed{y_{p1} = x e^x}$

$a_1 = 1$

$f_2(x) = 1 \quad n=0 \quad m=0$

$y_{p2} = a_0 x \qquad -a_0 = 1 \qquad y_{p2} = -x$

$y_{p2}' = a_0 \qquad a_0 = -1$

y_{p2}''

$y = y_H + y_{p1} + y_{p2}$

$y = C_1 + C_2 e^x + x e^x - x$ ✓

3. $f(x,y) = y\sqrt{x} - y^2 - x + 6y$

$T(4, 1, z_0)$

$z_0 = 1 \cdot \sqrt{4} - 1 - 4 + 6$

$= 2 - 1 - 4 + 6$

$= 3$

$T(4,1,3)$

$z - z_0 = \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y - y_0)$

$\frac{\partial f}{\partial x} = y \cdot \frac{1}{2\sqrt{x}} - 1 = \frac{y}{2\sqrt{x}} - 1 \Rightarrow \frac{\partial f}{\partial x}(4,1) = -\frac{3}{4}$

$\frac{\partial f}{\partial y} = \sqrt{x} - 2y + 6 \Rightarrow \frac{\partial f}{\partial y}(4,1) = 6$

$z - 3 = -\frac{3}{4}(x - 4) + 6(y - 1)$

$z - 3 = -\frac{3}{4}x + 3 + 6y - 6$

$\boxed{z = -\frac{3}{4}x + 6y}$ ✓

5. $\int_2^3 \frac{2x^2+x+2}{x^2-1}\,dx =$ Stipe Rebić

$(2x^2+x+2):(x^2-1)=2$

$\mp 2x^2 \pm 2$

$x+4$

$$= \underbrace{\int_2^3 2\,dx}_{I_1} + \underbrace{\int_2^3 \frac{x+4}{x^2-1}\,dx}_{I_2} = 2 + (-32,01) = 30,01$$

$$I_1 = 2\int_2^3 dx = 2x \Big|_2^3 = 2(3-2) = 2$$

$$I_2 = \int_2^3 \frac{x+4}{x^2-1}\,dx = \int_2^3 \frac{x}{x^2-1} + 4\int_2^3 \frac{dx}{x^2-1} = 0,49 - 32,5 = 32,01$$

$$I_3 = \int_2^3 \frac{x}{x^2-1}\,dx = \left\{ \begin{array}{l} x^2-1=t \\ 2x\,dx = dt \\ dx = \frac{dt}{2x} \end{array} \right\} = \frac{1}{2}\int_2^3 \frac{dt^2}{t}$$

$$= \frac{1}{2}\ln(x^2-1)\Big|_2^3 = 0,49$$

$$I_4 = 4\int_2^3 \frac{dx}{x^2-1} = -4\,\mathrm{arctg}(x)\Big|_2^3 = -32,52$$

$$I_2 = I_3 + I_4 = -32,03$$

$$I = I_1 + I_2 = 2 + (-32,03)$$

$$\boxed{I = -30,03}$$

2. $f(x,y) = x^3 - 3xy - y^3$

$$\frac{\partial f}{\partial x} = 3x^2 - 3y$$

$$\frac{\partial f}{\partial x} = -3x - 3y^2$$

$$\begin{aligned} 3x^2 - 3y &= 0 \\ -3x - 3y^2 &= 0 \end{aligned} \quad \Big| :3$$

$x^2 - y = 0 \quad \Rightarrow \quad y = x^2$

$-x - y^2 = 0$

$-x - x^4 = 0$

$-x(1 + x^3) = 0$

$x = 0$

$x^3 = -1$

$x = \sqrt[3]{-1} = -1$

$$\boxed{\begin{matrix} x = 0 & y = 0 \\ x = -1 & y = 1 \end{matrix}}$$

$S_1(0,0)$

$S_2(-1,1)$

$$\frac{\partial^2 f}{\partial x^2} = 6x \qquad \frac{\partial^2 f}{\partial x \partial y} = -3 \qquad \frac{\partial^2 f}{\partial y^2} = -6y$$

$$\frac{\partial^2 f}{\partial x^2}(0,0) = 0 \qquad \frac{\partial^2 f}{\partial x \partial y}(0,0) = -3 \qquad \frac{\partial^2 f}{\partial y^2}(0,0) = 0$$

$$\frac{\partial^2 f}{\partial x^2}(-1,1) = -6 \qquad \frac{\partial^2 f}{\partial x \partial y}(-1,1) = -3 \qquad \frac{\partial^2 f}{\partial y^2}(-1,1) = -6$$

$$\Delta_1(0,0) = \begin{vmatrix} 0 & -3 \\ -3 & 0 \end{vmatrix} = 0 - 9 = -9$$

$\Delta_1 < 0$ NEMA EKSTREMA.

$$\Delta_2(-1,1) = \begin{vmatrix} -6 & -3 \\ -3 & -6 \end{vmatrix} = 36 - 9 = 27$$

$\Delta_2 > 0 \qquad \frac{\partial^2 f}{\partial x^2} < 0 \qquad$ LOKALNI MAX $S_2(-1,1)$

$z_{maks.} = f(-1,1) = 1$

MAKSIMUM $T(-1,1,1)$

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IME I PREZIME: MARIN MATEK **BROJ INDEKSA:**	

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Ukupno: 40

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$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, \mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, \mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$y'' - y' = e^x + 1$

$r^2 - r = 0$

$r(r-1) = 0$

$r_1 = 0 \quad r_2 = 1$

$y_H = c_1 e^{0x} + c_2 e^x$

$y_H = C_1 + C_2 e^x$

$$e^{x} = e^{\alpha x}\left(P_m(x)\cos(\beta x) + Q_n(x)\sin(\beta x)\right)$$

$$\alpha = 1,\ \beta = 0,\ \alpha + \beta i = 1 = r_2 \neq r_1 \Rightarrow \boxed{k = 1}$$

$$e^{x} = e^{x}\left(P_m(x)\right) \qquad Q_n(x) = 0$$

$$n = N/P$$

$$P_m(x) = 0$$

$$m = N/P$$

$$y_{p_1} = x^{k} e^{\alpha x}\left(T_N(x)\cos(\beta x) + Q_N(x)\sin(\beta x)\right)$$

$$y_{p_1} = x e^{x}\left(T_\alpha(x)\right)$$

$$y_{p_1} = x e^{x}$$

$$1 = e^{\alpha x}\left(P_m(x)\cos(\beta x) + Q_n(x)\sin(\beta x)\right)$$

$$\alpha = 0,\ \beta = 0 \quad \alpha + \beta i = 0 = r_1 \neq r_2 \Rightarrow \boxed{k = 1}$$

$$1 = 1\cdot\left(P_m(x)\right) \qquad Q_n(x) = 0$$

$$P_m(x) \qquad n = N/P$$

$$m = 0$$

$$y_{p_2} = x^{k} e^{\alpha x}\left(T_\alpha(x)\cos(\beta x) + S_\alpha(x)\sin(\beta x)\right)$$

$$y_{p_2} = x\left(T_\alpha(x)\right) \qquad -A = 1$$

$$A = -1$$

$$y_{p_2} = Ax \qquad y' = A \qquad y'' = 0 \qquad y_{p_2} = -x$$

6. $f(x,y) = x^3 - 3xy - y^3$

$\partial_x f = 3x^2 - 3y$

$\partial_{xx} f = 6x$

$\partial_{xy} f = -3$

$\partial_y f = -3x - 3y^2$

$\partial_{yy} f = -6y$

$$\Delta = \begin{vmatrix} 6x & -3 \\ -3 & -6y \end{vmatrix} = 0 - (9) = -9$$

$\Downarrow$

NEMA EKSTREMA

STAC. TOČKE ?

$\partial_x f = 0$

$\partial_y f = 0$

$3x^2 - 3y = 0 \Rightarrow$

$-3x - 3y^2 = 0$

$-3\sqrt{y} - 3y^2 = 0$

2

$\cancel{y} = c_1 + c_2 e^x + xe^x - x$ ✓

$y' = c_2 e^x + e^x + xe^x - 1$

$y'' = c_2 e^x + e^x + e^x + xe^x$

$c_2 e^x + 2e^x + xe^x - (c_2 e^x + e^x + xe^x - 1) = e^x + 1$

$\cancel{c_2 e^x} + 2e^x + \cancel{xe^x} - \cancel{c_2 e^x} - e^x - \cancel{xe^x} + 1 = e^x + 1$

$2e^x - e^x + 1 = e^x + 1$

$e^x + 1 = e^x + 1$

4. $\int_0^1 \frac{dx}{1+\sqrt{x}}$

	l_0	l_1	l_2
l	0	1	2
x	0	$\frac{1}{2}$	1
$f(x)$	1	$2-\sqrt{2}$	$\frac{1}{2}$

$= \frac{1}{6}\left(1 + 4(2-\sqrt{2}) + \frac{1}{2}\right) = 0{,}6405$ ✓

$= \int \frac{dx}{1+\sqrt{x}} =$

3. $f(x,y) = y\sqrt{x} - y^2 - x + 6y$ $\quad (4, 1, z_0)$

$z_0 = \sqrt{4} - 1^2 - 4 + 6$

$z_0 = 2 - 1 - 4 + 6$

$z_0 = 3$

$T(4, 1, 3)$

$f_x = y \cdot \frac{1}{2\sqrt{x}} - 1$

$f_y = \sqrt{x} - 2y + 6$

$f_x(T) = \frac{1}{2\sqrt{4}} - 1 = \frac{3}{4}$

$f_y(T) = \sqrt{4} - 2 + 6 = 6$

$z - z_0 = f_x(T)(x - x_0) + f_y(T)(y - y_0)$

$z - 3 = \frac{3}{4}(x - 4) + 6(y - 1)$ ✓

$z - 3 = \frac{3}{4}x - 3 + 6y - 6$

$z - 3 = \frac{3}{4}x + 6y - 9$

$z = \frac{3}{4}x + 6y - 6$ //

$$5.\ \int_2^3 \frac{2x^2+x+2}{x^2-1}\,dx =$$

$$2x-\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|-\ln|x+1|\ \Big|_2^3$$

$$=1,509$$

$$=\int 2\,dx - \int^{*} \frac{x-2}{x^2-1}\,dx$$

$$=\int 2\,dx - \int \frac{1}{x^2-1} - \int \frac{1}{x+1}$$

$$=2x-\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|-\ln|x+1|$$

$$\int 2\,dx + \int \frac{-x-2}{x^2-1}$$

$$(2x^2+x+2):(x^2-1)=2$$

$$-\underline{2x^2+2x+4}$$

$$-x-2$$

$$\frac{x-1-1}{x^2-1}$$

$$\frac{-x-2+1-1}{x^2-1}$$

$$\int \frac{x-1}{x^2-1} - \int \frac{1}{x^2-1}$$

$$\int \frac{\cancel{x-1}}{(\cancel{x-1})(x+1)}$$

$$\int$$

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IME I PREZIME: MIHOVIL PEDIŠIĆ BROJ INDEKSA: 17 - 2-0253-2012	

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Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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2. $f(x,y) = x^3 - 3xy - y^3$

$\frac{\delta f}{\delta x} = 3x^2 - 3y \rightarrow$ $3x^2 - 3y = -3x - 3y^2$

$\frac{\delta f}{\delta y} = -3x - 3y^2 \rightarrow$ $3x^2 + 3x = -3y^2 - 3y$

$x = -y$

$\boxed{T(-1,1) - \text{maksimum}}$?

KAKO.

KAKO DO $T(-1,1)$?

$\frac{\delta f^2}{\delta x^2} = 6x = -6 < 0$ - maksimum

$\Delta = \begin{vmatrix} -6 & -3 \\ -3 & -6 \end{vmatrix} = 36 - 9 = 27 > 0$ -> ekstrem

$\frac{\delta f^2}{\delta y^2} = -6y$

$\frac{\delta f^2}{\delta x \delta y} = -3$

3. $f(x,y) = y\sqrt{x} - y^2 - x + 6y$ $\qquad (4,1,z_0)$

$y \cdot x^{\frac{1}{2}} = \; -y \cdot \frac{x^{-\frac{1}{2}}}{2}$

$z_0 = \sqrt{4} - 1 - 4 + 6$

$z_0 = 3$

$\frac{\delta f}{\delta x} = {+}y \cdot \frac{1}{2} x^{-\frac{1}{2}} - 1 \Rightarrow -1 \cdot \frac{1}{4} - 1 = -\frac{1}{4} - 1 = -\frac{5}{4}$ X

$\frac{\delta f}{\delta y} = \sqrt{x} - 2y + 6 \Rightarrow 2 - 2 + 6 \Rightarrow 6$

$R_t \ldots \; z - 3 = -\frac{5}{4}(x-4) + 6(y-1)$

$\boxed{z - 3 = -\frac{5}{4}x + 5 + 6y - 6}$ X

6. $\int_0^2 \frac{x-1}{x^2+x-2}\,dx = \int_0^2 \frac{dx}{x+2} = \left(\begin{array}{l} t = x+2 \\ dt = dx \end{array} \right)$

$= \ln|x+2| \Big|_0^2 = \boxed{\ln 4 - \ln 2}$ ✓

$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2}$

$-\frac{4}{2} = -2$

$\frac{2}{2} = 1$

$x_1 = -2$

$x_2 = 1$

$\frac{A}{(x+2)} \quad \frac{B}{(x-1)} = \frac{x-1}{x^2+x-2}$

$Ax - A + Bx + 2B = x - 1$

$A + B = 1 \Rightarrow B = 1 - A$

$-A + 2B = -1$

$A - 2B = 1$

$A = 1 + 2B$

$A = 1 + 2(1-A)$

$A = 1 + 2 - 2A$

$+3A = 3$

$A = 1$

$B = 0$

$x^2 + x - 2 = (x+2)(x-1)$

NULTOČKE $x_1 = -2,\; x_2 = 1$

$x_2 = 1$ ~~JE~~ NIJE SINGULARITET OD $\frac{\cancel{x-1}}{(x+2)\cancel{(x-1)}} \to \frac{1}{x+2}$

MIHOVIL PEPIŠIĆ
17-2-0253-2012

5. $\int_2^3 \frac{2x^2+x+2}{x^2-1}\,dx$

$= \underbrace{\int_2^3 \frac{2x^2}{x^2-1}\,dx}_{3\#} + \underbrace{\int_2^3 \frac{x}{x^2-1}}_{1\#} + \underbrace{2\int_2^3 \frac{dx}{x^2-1}}_{2\#} = 2{,}4055 + 0{,}4904 + 0{,}202 = \boxed{3{,}0979}$

1# $\int_2^3 \frac{x}{x^2-1}\,dx = \left\{ \begin{array}{l} x^2-1 = t \\ dt = 2x\,dx \\ \frac{dt}{2} = x\,dx \end{array} \right\} = \frac{1}{2}\int_2^3 \frac{dt}{t} = \frac{1}{2}\ln|t|\Big|_2^3 = \frac{1}{2}\ln|x^2-1|\Big|_2^3 = (1{,}0397 - 0{,}5493) =$
$= \boxed{0{,}4904}$

2# $\int_2^3 \frac{dx}{x^2-1} = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|\Bigg|_2^3 = (-0{,}347 + 0{,}549) = \boxed{0{,}202}$

3# $\int_2^3 \frac{2x^2}{x^2-1}\,dx = 2\int_2^3 \frac{x^2-1+1}{x^2-1}\,dx = 2\int_2^3 \frac{x^2-1}{x^2-1}\,dx + 2\int_2^3 \frac{dx}{x^2-1} = \left(2x + \ln\left|\frac{x-1}{x+1}\right|\right)\Bigg|_2^3 =$

$= 5{,}3069 - 2{,}9014 = \boxed{2{,}4055}$

1. $y'' - y' = e^x + 1$

H.

$\lambda^2 - \lambda = 0$

$\lambda(\lambda - 1) = 0$

$\lambda_1 = 0$

$\lambda_2 = 1$

$y_H = C_1 e^0 + C_2 e^x$

$y_H = C_1 + C_2 e^x$

P.

$f(x) = e^x$ $\left\}\begin{array}{l} n=0 \\ a=1 \end{array}\right.$

$f(x) = 1$ $\left\}\begin{array}{l} n=0 \\ a=1 \end{array}\right.$

$y_{P_1} = e^x$

$y_{P_2} = A \cdot e^{0x}$

$(x^2A)' = 0$

$(2Ax)'' = 0A$

$$\boxed{y = C_1 + C_2 e^x + e^x + 1}$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox
IME I PREZIME: NANA FURLAN **BROJ INDEKSA:** 17-2-0173-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - y' = e^x + 1$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = x^3 - 3xy - y^3$. 15
3. Pronaći ravninu koja dira graf funkcije $f(x,y) = y\sqrt{x} - y^2 - x + 6y$ povučenu u točki $(4, 1, z_0)$ tog grafa. 15
4. Numeričkom integracijom procijeniti vrijednost $\int_0^1 \frac{dx}{1+\sqrt{x}}$, a zatim isti integral riješiti egzaktno Newton Leibnitzovom formulom. 10+5
5. $\int_2^3 \frac{2x^2+x+2}{x^2-1}\mathrm{d}x =$ 20
6. $\int_0^2 \frac{x-1}{x^2+x-2}\mathrm{d}x = ?$ 20

Ukupno:
15

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

1. 4

3. $f(x,y) = 4\sqrt{x} - y^2 - x + 6y$

$z_0 = 1\sqrt{4} - 1 - 4 + 6 = 2 - 1 - 4 + 6 = 3$

$z - z_1 = \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y - y_0)$

$\frac{\partial f}{\partial x} = 4 \frac{1}{2\sqrt{x}} - 1 = \frac{4}{2\sqrt{x}} - 1 \Rightarrow \frac{\partial f}{\partial x}(4,1) = -\frac{3}{4}$

$\frac{\partial f}{\partial y} = \sqrt{x} - 2y + 6 \Rightarrow \frac{\partial f}{\partial}(4,1) = 6$

$z - 3 = -\frac{3}{4}x + 3 + 6y - 6$

$z = -\frac{3}{4}x + 6y$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox **IME I PREZIME:** Anamaria Jozić **BROJ INDEKSA:** 17-2-0104-2011	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - y' = e^x + 1$. — 15
2. Odrediti lokalne ekstreme funkcije $f(x, y) = x^3 - 3xy - y^3$. — 15
3. Pronaći ravninu koja dira graf funkcije $f(x, y) = y\sqrt{x} - y^2 - x + 6y$ povučenu u točki $(4, 1, z_0)$ tog grafa. — 15
4. Numeričkom integracijom procijeniti vrijednost $\int_0^1 \frac{dx}{1+\sqrt{x}}$, a zatim isti integral riješiti egzaktno Newton Leibnitzovom formulom. — 10+5
5. $\int_2^3 \frac{2x^2 + x + 2}{x^2 - 1} dx =$ — ~~20~~
6. $\int_0^2 \frac{x-1}{x^2 + x - 2} dx = ?$ — 20

Ukupno: 5

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

1. $y'' - y' = e^x + 1$

2. $f(x,y) = x^3 - 3xy - y^3$

3. /

→

4. $\int_0^1 \frac{dx}{1+\sqrt{x}} = \left[\begin{array}{ll} x = t^2 \;/' & \begin{array}{ccc} t = \sqrt{x} & & \\ x & 0 & 1 \\ t & 0 & 1 \end{array} \\ dx = 2t\,dt & \end{array} \right]$

$= 2\int_0^1 \frac{t+1}{1+t}\,dt + 2\int_0^1 \frac{-1}{1+t}\,dt = 2\int_0^1 dt - 2\int_0^1 \frac{1}{1+t}\,dt$

$= (2t - 2\ln|1+t|\Big|_0^1 = \underline{2-2\ln 2}$ ✓

5. $\int_2^3 \frac{2x^2+x+2}{x^2-1} = \left(\begin{array}{l} 2x^2 + x + 2 \\ x^2 \quad -1 \\ \hline 2 + \frac{x+2}{x^2-1} \end{array} \right)$

$= \int_2^3 2\,dx + \int_2^3 \frac{x^2+2}{x^2-1}\,dx = 2x\Big|_2^3 + \int_2^3 \frac{2}{x^2-1}\,dx + \int_2^3 \frac{x}{x^2-1}\,dx$

$= (6-4 + 2\left(\frac{1}{2}\ln\left|\frac{1-x}{x+1}\right|\right)\Big|_2^3 + \int_2^3 \frac{x}{x^2-1}\,dx$

$= 2 + \ln\frac{1}{2} - \ln\frac{1}{3} + \int_2^3 \frac{x}{x^2-1}\,dx$

$= \int_2^3 \frac{x}{x^2-1}\,dx = \left[\begin{array}{l} x^2-1 = t \;/' \\ 2x\,dx = dt \\ x\,dx = \frac{1}{2}\,dt \end{array} \right] \rightarrow \begin{array}{cc} x & t \\ 2 & 3 \\ 3 & 8 \end{array}$

$= \int_3^8 \frac{\frac{1}{2}\,dt}{t} = \frac{1}{2}\ln|t|\Big|_3^8 = \frac{1}{2}\ln|8| - \frac{1}{2}\ln|3| = 0{,}5$

$\rightarrow = 2 + \ln\frac{1}{2} - \ln\frac{1}{3} + 0{,}5 = \boxed{2{,}9}$

6. $\int_0^2 \frac{x-1}{x^2+x-2}\,dx =$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Ná snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: IVAN KELAVA BROJ INDEKSA: 17-1-0084-11	

1. Riješiti diferencijalnu jednadžbu: $y'' - y' = e^x + 1$. 15
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5. $\int_2^3 \frac{2x^2 + x + 2}{x^2 - 1} dx =$ ~~20~~
6. $\int_0^2 \frac{x-1}{x^2 + x - 2} dx = ?$ ~~20~~

Ukupno: $\emptyset$

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
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$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

5. $\int_2^3 \frac{2x^2+x+2}{x^2-1} dx =$

=

⑥ $\int_0^2 \frac{x-1}{x^2+x-2}dx = \int_0^2 \frac{x-1}{x(x+1-2)} = \int_0^2 \frac{x-1}{x(x-1)}dx = \int_0^2 \frac{dx}{x} = \ln|x| + C$

$= \ln|2| - \ln|0| = 0.69314$