MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

OXO

IME I PREZIME: Josip Kovaček BROJ INDEKSA: 17-1-0171-2013

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $4y'' - y = x \sin x$ i na kraju provjeriti rješenje. ~~20~~
2. Riješi diferencijalnu jednadžbu $x^2 yy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$. 15
3. Skiciraj razinske krivulje funkcije $f(x, y) = \ln(x + y)$. 15
4. $\displaystyle\int_0^2 \frac{x+2}{3x^2 - 2x - 5} dx = ?$ 20
5. $\displaystyle\int_{-2}^{0} 3\sqrt{1 - 3x} dx = ?$ 15
6. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(2, 2)$, $B(0, -4)$, $C(4, 0)$. 15

Ukupno: 60

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\| \frac{a+x}{a-x} \right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\| \frac{x-a}{x+a} \right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
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$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
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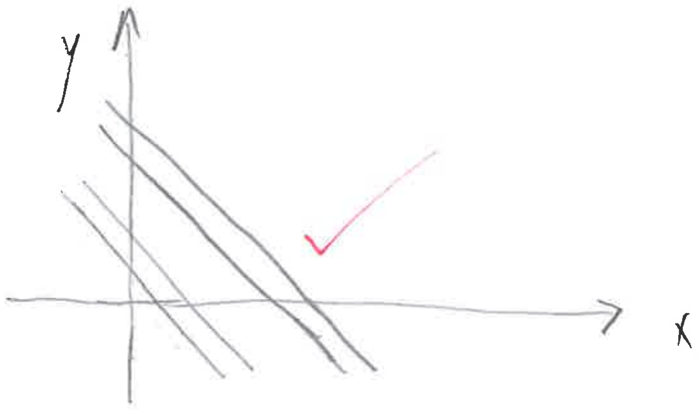
3. $z = C$

$\ln(x+y) = C$ / C

$x + y = e^C$

$y = e^C - x$ → PRAVCI PARALELNI S $y = -x$

TAKOĐER PROIZVOLJNA KONSTANTA



3.

$$\int_{-2}^{0} 3\sqrt{1-3x}\,dx = 3\int_{-2}^{0}\sqrt{1-3x}\,dx \qquad \begin{array}{l} 1-3x=u \\ dx=-\frac{1}{3}du \end{array}$$

$$= -\int_{7}^{1}\sqrt{u}\,du = -\frac{2}{3}u^{\frac{3}{2}}\Big|_{7}^{1} = 11{,}68$$

2.

$$x^2 y\,y' = 1-x^2 \qquad y(1)=1$$

$$y\,y' = \frac{1}{x^2}-1$$

$$y\,dy = \left(\frac{1}{x^2}-1\right)dx \,/\int$$

$$\frac{y^2}{2} = -\frac{1}{x} - x + C \qquad y(1)=1$$

$$y^2 = C-\frac{2}{x}-2x \qquad 1 = C-2-2 \rightarrow C=5$$

$$y^2 = 5-\frac{2}{x}-2x$$

1. PRETPOSTAVKA

$$y = Ax\sin x + Bx\cos x + C\sin x + D\cos x$$

$$y' = A\sin x + Ax\cos x + B\cos x - Bx\sin x + C\cos x - D\sin x$$

$$y'' = A\cos x + A\cos x - Ax\sin x - B\sin x - B\sin x - Bx\cos - C\sin x - D\cos x$$

$$4y'' - y = x\sin x$$

$$8A\cos x - 8B\sin x - 4Ax\sin x - 4Bx\cos - 4C\sin x - 4D\cos x -$$

$$Ax\sin x - Bx\cos - C\sin x - D\cos x = x\sin x$$

Josip Kovačić   17-1-0171-2013

nastavak

1\.

$$-5Ax\sin x - 5Bx\cos x - 5C\sin x - 5D\cos x + 8A\cos x - 8B\sin x = x\sin x$$

SLIJEDI:

$$-5A = 1 \Rightarrow A = -\frac{1}{5}$$

$$B = 0 \qquad C = 0$$

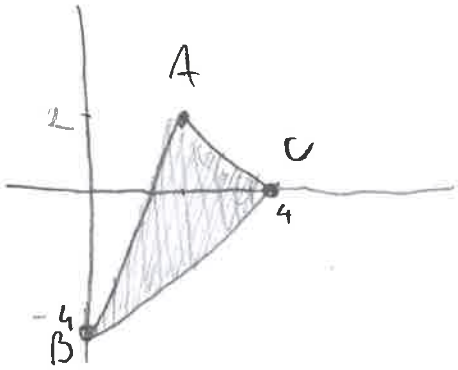
$$-5D = 8A \Rightarrow D = \frac{-8}{25}$$

DAKLE:

$$y = -\frac{1}{5}x\sin x - \frac{8}{25}\cos x$$ ✓

+ HOMOGENO RJEŠ.

6\.



PRAVAC AB

$$y - 2 = \frac{-4-2}{0-2}(x-2)$$

$$y = 3x - 4$$

PRAVAC AC

$$y = 4 - x$$

PRAVAC BC

$$y = x - 4$$

$$P = \int_{-4}^{0} (4+4) - \left(\frac{4+4}{3}\right) dy + \int_{0}^{2} (4-y) - \left(\frac{y+4}{3}\right) dy$$

$$= \left(\frac{y^2}{2} + 4y - \frac{y^2}{6} - \frac{4}{3}y\right)\Bigg|_{-4}^{0} + \left(4y - \frac{y^2}{2} - \frac{y^2}{6} - \frac{4}{3}y\right)\Bigg|_{0}^{2}$$

$$= 8$$ ✓

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POPUNJAVA NASTAVNIK

IME I PREZIME: ZORAN SUBOTIĆ **BROJ INDEKSA:** 17-1-0209-2013

	Broj ↓ bodova
1. Riješiti diferencijalnu jednadžbu: $4y'' - y = x \sin x$ i na kraju provjeriti rješenje.	20
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Ukupno:	50

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Tablica nekih integrala		
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
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1. $4y'' - y = x \sin x$

$4r^2 - 1 = 0$

$4r^2 = 1$

$r^2 = \frac{1}{4}$

$r_{1,2} = \pm\sqrt{\frac{1}{4}}$

$r_{1,2} = \pm\frac{1}{2}$

$y_{H(x)} = C_1 \cdot e^{-\frac{1}{2}x} + C_2 \cdot e^{\frac{1}{2}x}$

$x \sin x = e^{\alpha x}\left(P_{m(x)} \cos(\beta x) + Q_{n(x)} \sin(\beta x)\right)$

$\alpha = 0$

$\beta = 1$

$P_{m(x)} = 0;\ m = N/P$

$Q_{n(x)} = x;\ n = 1$

$\alpha + \beta i = 0 + 1i = 1i$

$k = 0$

$N = 1$

$\max\{N/P, 1\}$

$y_{p(x)} = x^k \cdot e^{\alpha x}\left(S_{N(x)} \cos(\beta x) + T_{N(x)} \sin(\beta x)\right)$

$y_{p(x)} = x^0 \cdot e^{0 \cdot x}\left(S_{N(x)} \cos x + T_{N(x)} \sin x\right)$

$y_{p(x)} = (A + Bx)\cos x + (C + Dx)\sin x$

$y' = B \cdot \cos x + (A+Bx)\cdot(-\sin x) + D \sin x + (C+Dx)\cdot \cos x$

$y'' = -B \sin x + B(-\sin x) - (A+Bx)\cdot\cos x + D\cos x + D\cdot\cos x +$
$+ (C+Dx)\cdot(-\sin x)$

$y'' = -2B\sin x - (A+Bx)\cos x + 2D\cos x - (C+Dx)\sin x$

$4\cdot(-2B\sin x - (A+Bx)\cos x + 2D\cos x - (C+Dx)\sin x) - (A+Bx)\cos x -$
$-(C+Dx)\sin x = x\sin x$

$-8B\sin x - 4(A+Bx)\cos x + 8D\cos x - 4(C+Dx)\sin x - (A+Bx)\cos x -$
$-(C+Dx)\sin x = x\sin x$

$\Longrightarrow$

$-8B\sin x - 5(A+Bx)\cos x - 8D\cos x - 5(C+Dx)\sin x = x\sin x$

$-8B\sin x - 5A\cos x - 5Bx\cos x - 8D\cos x - 5C\sin x - 5Dx\sin x = x\sin x$

$-5Dx\sin x - 5Bx\cos x + \sin x(-8B-5C) + \cos x(-5A-8D) = x\sin x$

$-5D = 1$ $\quad -5B = 0$ $\quad -8B - 5C = 0$ $\quad -5A - 8D = 0$

$D = -\frac{1}{5}$ $\quad B = 0$ $\quad -5C = 8B$ $\quad -5A = 8D$

$C = 0$

$A = \frac{8D}{-5} = \frac{8}{-5}\cdot\left(-\frac{1}{5}\right) = \frac{8}{25}$

$y_{p(x)} = (A+Bx)\cos x + (C+Dx)\sin x$

$y_{p(x)} = \frac{8}{25}\cos x - \frac{1}{5}x\sin x$

$y_{(x)} = y_{H(x)} + y_{p(x)}$

$y_{(x)} = C_1 \cdot e^{-\frac{1}{2}x} + C_2 \cdot e^{\frac{1}{2}x} + \frac{8}{25}\cos x - \frac{1}{5}x\sin x$

=>

List 2

4. $\int_0^2 \frac{x+2}{3x^2-2x-5}dx = \int_0^2 \frac{x}{3x^2-2x-5}dx + \int_0^2 \frac{2}{3x^2-2x-5}dx = \frac{3}{3}\int_0^2 \frac{x}{3x^2-2x-5}dx$

$+\int_0^2 \frac{2}{3x^2-2x-5}dx = \frac{1}{3}\int_0^2 \frac{3x+1-1}{3x^2-2x-5}dx + \int_0^2 \frac{2}{3x^2-2x-5}dx$

$= \frac{1}{3}\int_0^2 \frac{3x-1}{3x^2-2x-5}dx + \frac{3}{3}\int_0^2 \frac{2+1}{3x^2-2x-5}dx = .$

#1

$\frac{1}{3}\int_0^2 \frac{3x-1}{3x^2-2x-5}dx = \left[\begin{array}{l} 3x^2-2x-5=t \,/\,' \\ 6x-2\,dx = dt \\ 3x-1\,dx = \frac{dt}{2}\end{array}\right] = \frac{1}{6}\int_0^2 \frac{dt}{t} = \left[\frac{1}{6}\ln|3x^2-2x-5|\right]_0^2$

#2

$9\int_0^2 \frac{dx}{9x^2-6x-15} = 9\int_0^2 \frac{dx}{9x^2-6x+1-16} = 9\int_0^2 \frac{dx}{(3x-1)^2-16} = \left[\begin{array}{l} 3x-1=t \,/\,' \\ 3dx = dt \\ dx = \frac{dt}{3}\end{array}\right] =$

$= \frac{9}{3}\int_0^2 \frac{dt}{t^2-4^2} = \left[3\cdot\frac{1}{2\cdot 4}\ln\left|\frac{t-4}{t+4}\right|\right]_0^2 = \left[\frac{3}{8}\ln\left|\frac{3x-1-4}{3x-1+4}\right|\right]_0^2$

$= \left[\frac{1}{6}\ln|3x^2-2x-5| + \frac{3}{8}\ln\left|\frac{3x-5}{3x+3}\right|\right]_0^2 =$

$= \left(\frac{1}{6}\ln|3| + \frac{3}{8}\ln\left|\frac{1}{9}\right|\right) - \left(\frac{1}{6}\ln|-5| + \frac{3}{8}\ln\left|-\frac{5}{3}\right|\right)$

POGREŠAN ARGUMENT

OVO JE NEPRAVI INTEGRAL!

$\ln|-5| = \ln 5 = \_\ \_$

APS. VRIJEDNOST $\rightarrow$ $|-5| = 5$

ZORAN SUBOTIĆ, BR. IND.: 17-1-0209-2013

2. $x^2 y y' = 1 - x^2$; $y(1) = 1$

$$yy' = \frac{1-x^2}{x^2}$$

$$yy' = \frac{1}{x^2} - 1$$

$$y\frac{dy}{dx} = \frac{1}{x^2} - 1 \quad / dx$$

$$y\,dy = \left(\frac{1}{x^2} - 1\right)dx \quad / \int$$

$$\int y\,dy = \int\left(\frac{1}{x^2} - 1\right)dx$$

$$\frac{y^2}{2} = \int x^{-2}dx - \int dx$$

$$\frac{y^2}{2} = -\frac{1}{x} - x + C \quad / \cdot 2$$

$$y^2 = -\frac{2}{x} - 2x + 2C$$

$$y = \sqrt{-\frac{2}{x} - 2x + 2C}$$

$$1^2 = -\frac{2}{1} - 2 + 2C$$

$$2C = 1 + 2 + 2$$

$$C = \frac{5}{2}$$

$$\Rightarrow \quad y^2 = -\frac{2}{x} - 2x + 5 \quad \checkmark$$

$$y = \sqrt{-\frac{2}{x} - 2x + 5}$$

5. $$\int_{-2}^{0} 3\sqrt{1-3x}\,dx = 3\int_{-2}^{0}\sqrt{1-3x}\,dx = \left[\begin{array}{l} 1-3x = t \;/' \\ -3dx = dt \\ dx = -\frac{dt}{3} \end{array}\right] = 3\int_{-2}^{0}\sqrt{t}\left(-\frac{dt}{3}\right) =$$

$$= -\frac{3}{3}\int_{-2}^{0} t^{\frac{1}{2}}dt = \left[-\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_{-2}^{0} = \left[-\frac{2\sqrt{t^3}}{3}\right]_{-2}^{0} = \left[-\frac{2\sqrt{(1-3x)^3}}{3}\right]_{-2}^{0} =$$

$$= \left(-\frac{2\cdot\sqrt{(1-0)^3}}{3}\right) - \left(-\frac{2\sqrt{(1-3\cdot(-2))^3}}{3}\right) = -\frac{2}{3} - \left(-\frac{2\sqrt{7^3}}{3}\right) =$$

$$= \frac{-2 + 14\sqrt{7}}{3} \approx 11{,}68 \quad \checkmark$$

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IME I PREZIME: LUKA Bičić

BROJ INDEKSA: 0070036921

POPUNJAVA NASTAVNIK

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$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
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1. $4y'' - y = x \sin x$

6. $P_1 = \int_0^2 (-4 + 3x + 4 - x)\,dx = \int_0^2 2x\,dx = 4$

$P_2 = \int_2^4 4-x+4-x = 8\cdot(4-2) - x^2 \Big|_2^4$

$P_2 = 16 - (16-4) = 4$

2. $~~x^2 y y' = 1 - x^2~~$

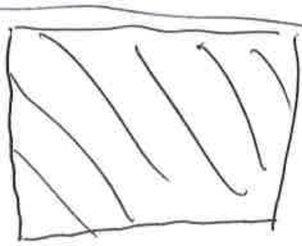
$P = P_1 + P_2 = 8$ ✓

$f(x,y) = \ln(x+y)$

3. $\ln(x+y) = c$ $\quad y = k - x$

$x + y = e^c$

$x + y = k$



④ $$\int_0^2 \frac{x+2}{3x^2-2x-5}\,dx =$$

$D_f = \mathbb{R}$

$$3x^2-2x-5=0$$

$$x = \frac{2 \pm \sqrt{4+60}}{6}$$

$$x = \frac{2 \pm 8}{6}$$

$$x_1 = \frac{5}{3}, \quad x_2 = -1$$

ovo je nepravi integral jer je unutar domene!

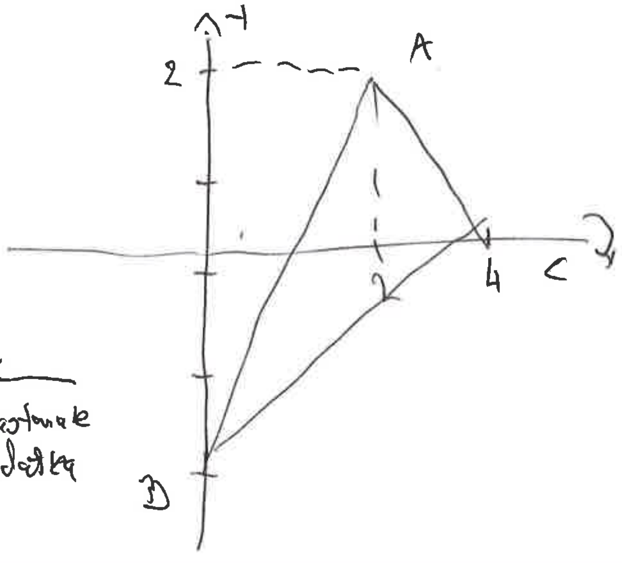
⑤ $$\int_{-7}^{0} 3\sqrt{1-3x}\,dx =$$

$$\left|\begin{array}{l} 1-3x = t \\ -3dx = dt \\ 3dx = -dt \end{array}\right|$$

$$-\int_7^1 \sqrt{t}\,dt = \int_1^7 t^{\frac{1}{2}}\,dt = \int_1^7 \frac{t^{\frac{3}{2}}}{\frac{3}{2}}\,dt$$

$$= \frac{2}{3}\int_1^7 \sqrt{t^3}\,dt = \frac{2}{3}\left(\sqrt{7^3}-1\right) = 11,68$$

⑥ Integriranjem odrediti površinu trokuta koji je zadan točkama A(2,2) B(0,-4) C(4,0)



nastavak zadatka

p_{AB} ... $y = -4+3x$

p_{AC} ... $y = 4-x$

p_{BC} ... $y = -4+x$

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p_{AC} ... $y = 4-x$

p_{BC} ... $y = -4+x$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OXO

IME I PREZIME: FILIP MEŠTROVIĆ **BROJ INDEKSA:**

	POPUNJAVA NASTAVNIK Broj ↓ bodova
1. Riješiti diferencijalnu jednadžbu: $4y'' - y = x\sin x$ i na kraju provjeriti rješenje.	20
2. Riješi diferencijalnu jednadžbu $x^2yy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$.	~~15~~
3. Skiciraj razinske krivulje funkcije $f(x,y) = \ln(x+y)$.	15
4. $\int_0^2 \frac{x+2}{3x^2-2x-5}\mathrm{d}x = ?$	~~20~~
5. $\int_{-2}^0 3\sqrt{1-3x}\mathrm{d}x = ?$	15
6. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(2,2)$, $B(0,-4)$, $C(4,0)$.	~~15~~
Ukupno:	15

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_a x\ (a > 0)$	$\frac{1}{x\ln a}$
e^x	e^x
$a^x\ (a > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

4) $\int_0^2 \frac{(x+2)dx}{3x^2-2x-5} = \int_0^2 \frac{-\frac{1}{3}(6x-2)+\left(2-\frac{2am-b^2n}{4a}\right)}{3x^2-2x-5} = \int_0^2 \frac{-\frac{1}{\cancel{3}}(6x-2)+\left(\frac{12}{6}-\frac{1}{6}\right)}{3x^2-2x-5}dx$

$\frac{mx+n}{ax^2-bx-c}$

NEPRAVI

~~0~~

$\int \frac{(x+2)}{3x^2-2x-5}dx = \int \frac{\frac{1}{6}(6x-2)+\frac{14}{6}}{3x^2-2x-5} = \frac{1}{6}\int \frac{6x-2}{3x^2-2x-5} + \frac{7}{3}\cancel{\frac{14}{6}}\int \frac{dx}{x^2-2x-5}$

$3x^2-2x-5=t$
$(6x-2)dx = dt$

$= \frac{1}{6}\int \frac{dt}{t} + \frac{7}{3}\int \frac{dx}{x^2-2x-5}$

$\frac{1}{6}\ln|3x^2-2x-5| + \frac{7}{3}\int \frac{dx}{(x-1)^2+4} \Big| = \begin{matrix} x-1=t \\ dx=dt \end{matrix}$ $(x-1)^2+1^2-5$ $x^2-2x+1+4.$

$+\frac{7}{3}\int_0^2 \frac{dt}{t^2+2^2} = \frac{7}{3}\cdot\frac{1}{2}\arctan\frac{x-1}{2}\Big|_0^2 + C =$

$= \frac{7}{6}\arctan\frac{1}{2} - \frac{7}{6}\arctan -\frac{1}{2} = 61.19$

$$\frac{1}{6}\ln|3x^2-2x-5|\Big|_0^2+\frac{7}{6}\arctan\frac{x-1}{2}\Big|_0^2=$$

$$\frac{1}{6}\ln|6-4-5|-\frac{1}{6}\ln|-5|+\frac{7}{6}\arctan\frac{1}{2}-\frac{7}{6}\arctan-\frac{1}{2}$$

$$\frac{1}{6}\ln(3)-\frac{1}{6}\ln(5)+\frac{7}{6}\arctan\frac{1}{2}-\frac{7}{6}\arctan\left(-\frac{1}{2}\right)$$

$$=\cancel{1{,}805}=0{,}997.$$

5. zadatak

$$3\int_{-2}^{0}\sqrt{1-3x}\,dx= \quad 1-3x=t$$

$$-3\,dx=dt$$

$$dx=-\frac{1}{3}dt$$

$$\frac{3}{3}\int\sqrt{t}\,dt=-\frac{1}{3}\cdot\frac{2t^{\frac{3}{2}}}{3}=-\frac{2}{3}\sqrt{(1-3x)^3}$$

$$-\frac{2}{3}\sqrt{(1-3x)^3}\Big|_{-2}^{0}=-\frac{2}{3}(1-7\sqrt{7})=-\frac{2}{3}+\frac{14\sqrt{7}}{3}$$

$$\int_{-2}^{0}\sqrt{1-3x}\,dx=-\frac{2}{3}+\frac{14\sqrt{7}}{3}$$

$x^2 y y' = 1 - x^2 / : x^2 y \quad y(1) = 1$

$y' = \frac{1-x^2}{x^2 y}$

$y' = \frac{1}{x^2 y} - \frac{1}{y}$

$y = u \cdot v$

$y' = u'v + uv'$

$u'v + uv' = \frac{1}{x^2 uv} - \frac{1}{uv}$

$uv' = \frac{1}{x^2 uv}$

$y' = \frac{1}{x^2 y} / \cdot dx \cdot y$

$\int y\,dy = \int \frac{dx}{x^2}$

$\frac{y^2}{2} = \frac{x^3}{3} / \cdot 2$

$y^2 = \frac{2}{3} x^3 / \sqrt{\ }$

$y = \sqrt{\frac{2}{3} x^3}$ ✗

FILIP NEŠTROVIĆ

6. zadatak

1 $A(2,2)$ $B(0,-4)$ $C(4,0)$ 2

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-2=\frac{0-2}{4-2}(x-2)$$

$(2,2)$ $(4,0)$

$$y=-x+2+2$$

$\overline{AC}$ $y=-x+4$

$$y-2=\frac{0-2}{4-2}(x-2)$$

$$y=-\frac{2}{2}(x-2)+2$$

$$y=-x+4$$

$(2,2)(0,-4)$

$$-x+4=3x-4$$

$$-4x=-8$$

$$x=2$$

$$y-2=\frac{-4-2}{0-2}(x-2)$$

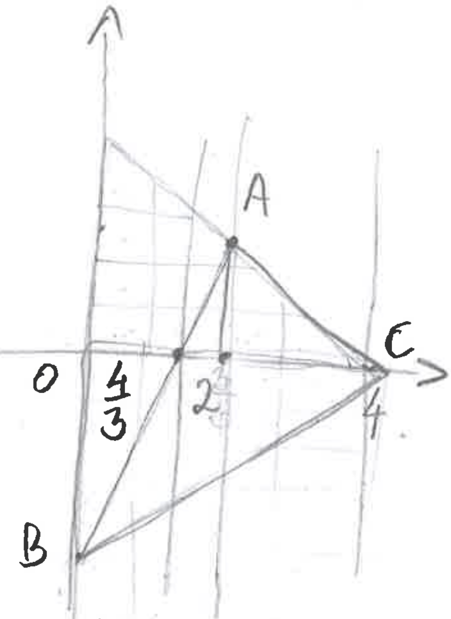
$$y-2=\frac{-6}{-2}(x-2)$$

$$3x-4=0$$

$$3x=4$$

$$x=\frac{4}{3}$$

$$y=3x-6+2$$



AB $y=3x-4$

AC $y=-x+4$

BC $y=x-4$

$$y-2=\frac{-4-2}{0-2}(x-2)$$

$\overline{AB}$

$$y=3x-6+2$$

$$3x-4=0$$

$$x=\frac{4}{3}$$

$$\int_{\frac{4}{3}}^{2}(3x-4)dx+\int_{2}^{4}(-x+4)dx+\int_{4}^{0}(x-4)dx-\int_{\frac{4}{3}}^{0}(3x-4)dx$$

$$y+4=\frac{0+4}{4-0}(x-0)$$

$\overline{BC}$

$$=\left(\frac{3x^2}{2}-4x\right)\Big|_{\frac{4}{3}}^{2}+\left(-\frac{x^2}{2}+4x\right)\Big|_{2}^{4}+\left(\frac{x^2}{2}-4x\right)\Big|_{4}^{0}-\left(\frac{3x^2}{2}-4x\right)\Big|_{\frac{4}{3}}^{0}$$

$$y=x-4$$

$$=6-8-\left(\frac{3}{2}\cdot\frac{16}{9}-\frac{8}{3}\right)+\left[-\frac{16}{2}+16-\left(-2+8\right)\right]+\left[0-\left(\frac{16}{2}-16\right)\right]$$

$$-x+4=3x-4$$

$$-4x=-8$$

$$x=2$$

$$-\left[0-\left(\frac{3}{2}\cdot\frac{16}{9}-\frac{16}{3}\right)\right]=-2+\frac{16}{6}-6+\frac{16}{2}+\frac{8}{3}=$$

$$=\frac{-12+48-36+48-16}{6}=\frac{32}{6}=$$

$$=\boxed{\frac{16}{3}}$$

$\boxed{16}$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

OXO

IME I PREZIME: ANTE STANIŠIĆ **BROJ INDEKSA:** 17-1-0066-2011

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $4y'' - y = x\sin x$ i na kraju provjeriti rješenje. 20
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Ukupno: 15

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
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$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

5) $\displaystyle\int_{-2}^{0} 3\sqrt{1-3x}\,dx$ $\qquad \sqrt{1-3x} = t$ $\qquad x_1 = -2 \quad x_2 = 0$

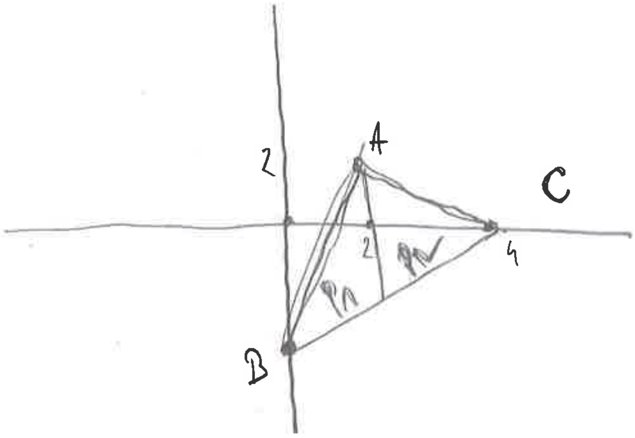
$-3dx = dt \qquad t_1 = 7 \quad t_2 = 1$

$\displaystyle 3\int_{-2}^{0} \sqrt{1-3x}\,dx$

$\displaystyle -9\int_{7}^{1} \sqrt{t}\,dt = -9\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}} = -9\cdot\frac{2}{3}t^{\frac{3}{2}}\Big|_7^1 = -6\cdot 1^{\frac{3}{2}} - \left(-6\cdot 7^{\frac{3}{2}}\right)$

$= 12{,}6$

$A(2,2) \quad B(0,-4) \quad C(4,0)$



$A(\underset{x_1}{2},\underset{y_1}{2}) \quad C(\underset{x_2}{4},\underset{y_2}{0})$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-2=\frac{0-2}{4-2}(x-2)$$

$$y-2=\frac{-2}{2}(x-2)$$

$$y=2+2-x \Rightarrow y=-x+4$$

$A(\underset{x_1}{2},\underset{y_1}{2}) \quad B(\underset{x_2}{0},\underset{y_2}{-4})$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-2=\frac{-4-2}{0-2}(x-2)$$

$$y=2+3x-6 \Rightarrow \boxed{y=3x-4}$$

$B(\underset{x_1}{0},\underset{y_1}{-4}) \quad C(\underset{x_2}{4},\underset{y_2}{0})$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y+4=\frac{0+4}{4-0}(x-0)$$

$$y=-4+x \Rightarrow \boxed{y=x-4}$$

$$P_1=\int_0^2((3x-4)-(x-4))dx=\int_0^2(3x-\cancel{4}-x+\cancel{4})dx=\int_0^2(2x)dx=\cancel{2}\cdot\frac{x^2}{\cancel{2}}\Big|_0^2=4$$

$$P_2=\int_2^4((-x+4)-(x-4))dx=\int_2^4((-x+4-x+4))dx=\int_2^4(-2x+8)dx=-\cancel{2}\frac{x^2}{\cancel{2}}+8x\Big|_2^4=-16+8\cdot4-(-4+8\cdot2)$$

$=16-12=$

$$P_1+P_2=P \Rightarrow 4+4=8$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OXO

POPUNJAVA NASTAVNIK

IME I PREZIME: DINO CVITAN **BROJ INDEKSA:** 17-2-0068

	Broj ↓ bodova
1. Riješiti diferencijalnu jednadžbu: $4y'' - y = x\sin x$ i na kraju provjeriti rješenje.	20
2. Riješi diferencijalnu jednadžbu $x^2yy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$.	15
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Ukupno:	0

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$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
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$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

4. $$\int \frac{x+2}{3x^2-2x-5}dx = \frac{1}{3}\int \frac{x+2}{x^2 - \frac{2}{3}x - \frac{5}{3}}dx = \left[\begin{array}{l}\left(x^2 - \frac{2}{3}x - \frac{5}{3}\right)' \\ = 2x - \frac{2}{3}\end{array}\right] = \frac{1}{3}\int \frac{\frac{1}{2}\left(2x - \frac{2}{3}\right) + \frac{7}{3}}{x^2 - \frac{2}{3}x - \frac{5}{3}}dx$$

NEPRAVO

$$= \frac{1}{3}\int \frac{\frac{1}{2}\left(2x - \frac{2}{3}\right) + \frac{7}{3}}{\left(2x - \frac{1}{3}\right)^2 - \frac{16}{9}} \quad \left[\begin{array}{l} 2x - \frac{1}{3} = t \quad a^2 = \frac{16}{9} \\ a = \frac{4}{3}\end{array}\right]$$

$(3 - 1) - 2$

$$= \frac{1}{3}\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert = \frac{1}{24}\ln\left\lvert\frac{\left(2x - \frac{1}{3}\right) - \frac{4}{3}}{\left(2x - \frac{1}{3}\right) + \frac{4}{3}}\right\rvert = \frac{1}{24}\ln\left\lvert\frac{2x - \frac{5}{3}}{2x+1}\right\rvert + C \Bigg|_0^2$$

$$= \frac{1}{24}\ln\left\lvert\frac{2\cdot 2 - \frac{5}{3}}{2\cdot 2 + 1}\right\rvert - \frac{1}{24}\ln\left\lvert\frac{2\cdot 0 - \frac{5}{3}}{2\cdot 0 + 1}\right\rvert = \frac{1}{24}\ln\left\lvert\frac{7}{15}\right\rvert - \frac{1}{24}\ln\left\lvert -\frac{5}{3}\right\rvert$$

$= -0{,}05304023649$

5. $\int_{-2}^{0} 3\sqrt{1-3x}\,dx = ?$

$\int 3\sqrt{1-3x}\,dx = 3\int \sqrt{1-3x}\,dx = \left[\begin{array}{l} 1-3x = t \\ -3dx = dt \\ dx = \frac{dt}{3} \end{array}\right]$

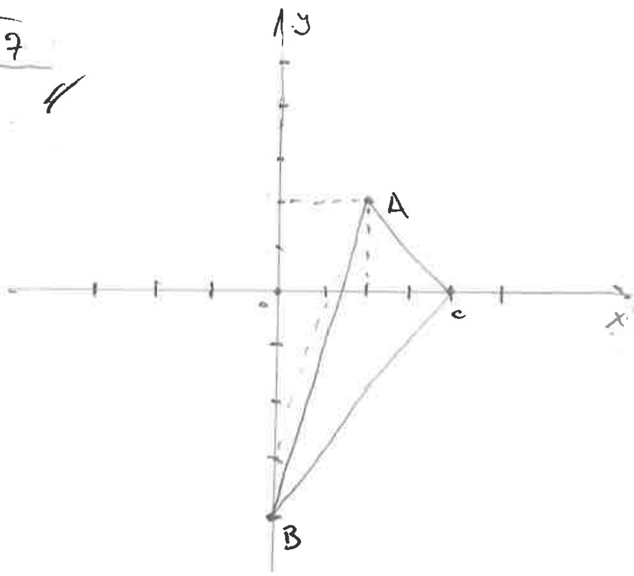
ZABORAVLJENI —

$= 3\int \sqrt{t}\,\frac{dt}{3} = \int t^{\frac{1}{2}}\,dt = \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3}\sqrt{t^3}$

$= \frac{2}{3}\sqrt{(1-3x)^3}\Big|_{-2}^{0} = \frac{2}{3}\sqrt{(1-3(0))^3} - \frac{2}{3}\sqrt{(1-3(-2))^3}$

$= \frac{2}{3}\sqrt{1} - \frac{2}{3}\sqrt{7} = \frac{2-2\sqrt{7}}{3}$ //

6. $A(2,2)$, $B(0,-4)$, $C(4,0)$



$\overline{AB}$, $\overline{BC}$, $\overline{CA}$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\overline{AB} = y-2 = \frac{-4-2}{0-2}(x-2) = y-2 = 3x-6 = y = 3x-4$

$\overline{BC} = y+4 = \frac{0+4}{4-0}(x-0) = y+4 \quad y = x-4$

$\overline{CA} = y-0 = \frac{2-0}{2-4}(x-4) = y-0 = -1x+4 = y = -1x+4$

$\int_{2}^{4} -1x+4 - \int_{0}^{2} 3x-4 - \int_{-4}^{0} x-4$

P_1 $-4+4-(-2)+4=6$

P_2 $3\cdot 2-4-3\cdot 0-4=-2$

P_3 $-4-(-8)=4$

$P = P_1 - P_2 - P_3 = 6-(2)-4 = 4$ jed. X