MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** oox

IME I PREZIME: ŠIMURINA FRANKO **BROJ INDEKSA:** 17-1-0222-2013

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' + 2y' = 1$, uz $y(0) = 0$ i $y'(0) = 0$. Na kraju provjeri rješenje. 15

2. Nađi implicitno rješenje jednadžbe $\frac{y'}{x} = \frac{\sin x}{y}$. 15

3. Za funkciju $f(x, y) = \ln\left(\frac{y}{x}\right)$ odrediti domenu, kodomenu, razinske krivulje i limes u ishodištu (ako postoji). ~~20~~ 16

4. $\int\limits_0^2 \frac{x-1}{x^2+3x+2}\,dx = ?$ 15

5. Zadana je funkcija $f(x) = \sqrt{x}$. Traži se površina ispod grafa funkcije (do osi apcise) na segmentu $[0, 4]$. Podijeliti segment na nekoliko dijelova i preko trapezne formule procijeniti traženu površinu. Skicirati graf funkcije, površinu koja je dobivena procjenom i vizualno ocijeniti grešku numeričkog postupka. ~~20~~

6. Integriranjem izračunati površinu trokuta zadanog točkama $A(1, -2)$, $B(2, 0)$, $C(-1, 1)$. ~~15~~

Ukupno: 61

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_a x$ $(a > 0)$	$\frac{1}{x \ln a}$
e^x	e^x
a^x $(a > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4) $\int\limits_0^2 \frac{x-1}{x^2+3x+2}\,dx = ?$

$x_{1,2} = \frac{-3 \pm 1}{2}$ $\quad x_1 = -2 \quad x_2 = -1$

$\frac{x-1}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$

$x - 1 = A(x+1) + B(x+2)$

$x = -2 \longrightarrow -3 = -A$
$A = 3$

$x = -1 \longrightarrow -2 = B$

$I = \int\limits_0^2 \left(\frac{3}{x+2} - \frac{2}{x+1}\right) dx$

$= \left(3\ln\lvert x+2\rvert - 2\ln\lvert x+1\rvert\right)\Big|_0^2$

$= 3\ln 4 - 2\ln 3 - 3\ln 2 + 2\ln 1$

$= -0.118$

(2)

$$\frac{y'}{x} = \frac{\sin x}{y} \Big/ \cdot xy$$

$$y \cdot \frac{dy}{dx} = x \sin x$$

$$y\,dy = x \sin x \, dx$$

$$\frac{y^2}{2} = \int x \sin x \, dx = \left| \begin{array}{ll} u = x & dv = \sin x \, dx \\ du = dx & v = -\cos x \end{array} \right.$$

$$\frac{y^2}{2} = -x\cos x + \int \cos x \, dx$$

$$\frac{y^2}{2} = -x\cos x + \sin x + C$$

$$y^2 = -2x\cos x + 2\sin x + 2C$$

1)
$$y'' + 2y' = 1$$

$$r^2 + 2r = 0$$

$$r_1 = 0 \qquad r_2 = -2$$

$$y_H = C_1 + C_2 e^{-2x}$$

$$g(x) = 1 \longrightarrow y_p = Ax \qquad y_p' = A \qquad y_p'' = 0$$

$$2A = 1 \qquad A = \frac{1}{2}$$

$$y = C_1 + C_2 e^{-2x} + \frac{1}{2}x$$

$$y' = -2C_2 e^{-2x} + \frac{1}{2}$$

$$y(0) = 0 \longrightarrow C_1 + C_2 = 0$$

$$y'(0) = 0 \longrightarrow -2C_2 + \frac{1}{2} = 0 \longrightarrow C_2 = \frac{1}{4}$$

$$C_1 = -\frac{1}{4}$$

$$y = -\frac{1}{4} + \frac{1}{4}e^{-2x} + \frac{1}{2}x$$

PROVJERA:

$$y' = -\frac{1}{2}e^{-2x} + \frac{1}{2}$$

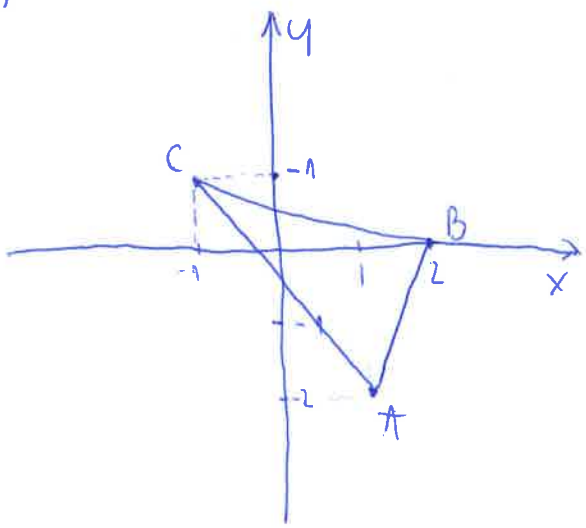
$$y'' = e^{-2x}$$

$$y'' + 2y' = 1 \longrightarrow e^{-2x} - e^{-2x} + 1 = 1$$

$$\boxed{1 = 1}$$

ŠIMURINA FRANKO

6)



$p_{AB} \ldots \quad y = \frac{-2}{1-2}(x-2)$

$y = 2x-4$

$p_{AC} \ldots \quad y-1 = \frac{-2-1}{1+1}(x+1)$

$y = -\frac{3}{2}x - \frac{1}{2}$

$p_{BC} \ldots \quad y = \frac{1}{-1-2}(x-2)$

$y = -\frac{1}{3}x + \frac{2}{3}$

$$P = \int_{-1}^{1}\left(-\frac{1}{3}x + \frac{2}{3} - 2x + 4\right)dx + \int_{1}^{2}\left(-\frac{1}{3}x + \frac{2}{3} + \frac{3}{2}x + \frac{1}{2}\right)dx$$

$$= \left(-\frac{7x^2}{6} + \frac{14}{3}x\right)\Big|_{-1}^{1} + \left(\frac{7x^2}{12} + \frac{7}{6}x\right)\Big|_{1}^{2}$$

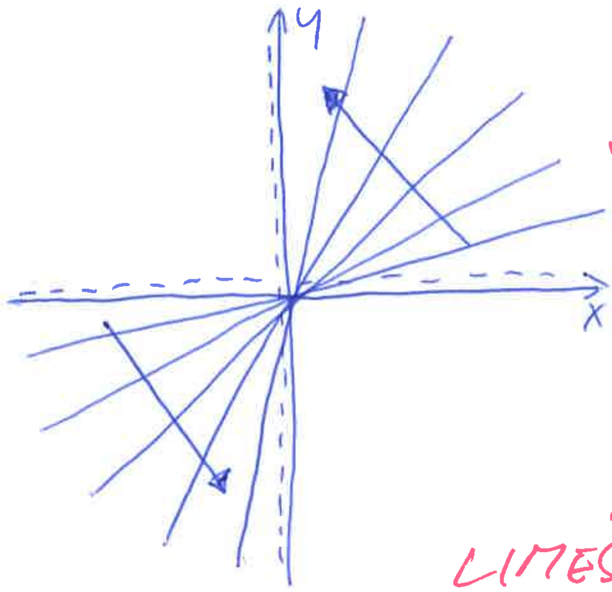
$$= -\cancel{\frac{7}{6}} + \frac{14}{3} + \cancel{\frac{7}{6}} + \frac{14}{3} + \frac{7}{3} + \frac{7}{3} - \frac{7}{12} - \frac{7}{6} = \frac{49}{4}$$

ŠIMURINA FRANKO

3) $f(x,y) = \ln\left(\frac{y}{x}\right)$

$\frac{y}{x} > 0$

$x > 0$, $y > 0$ $\qquad$ $x < 0$, $y < 0$



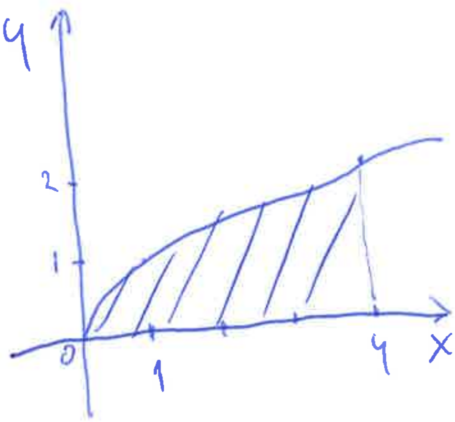
✓

$K(f) = \mathbb{R}$ ✓

$D(f) = \{x, y \in \mathbb{R}^2 \mid \frac{y}{x} > 0\}$ ✓ 16

LIMES?

5) $f(x) = \sqrt{x}$



$$P = \int_0^4 \sqrt{x}\,dx = \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)\Bigg|_0^4 = \left(\frac{2}{3}\sqrt{x^3}\right)\Bigg|_0^4 =$$

$$= \frac{2}{3}\sqrt{4^3} - \frac{2}{3}\sqrt{0^3} = \frac{16}{3}$$

$\cancel{0}$

TRAPEZNA FORMULA ?

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OOX

IME I PREZIME: DINO DOKOZA **BROJ INDEKSA:** 1219036348

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y''+2y'=1$, uz $y(0)=0$ i $y'(0)=0$. Na kraju provjeri rješenje. 15 15

2. Nađi implicitno rješenje jednadžbe $\frac{y'}{x}=\frac{\sin x}{y}$. 15 15

3. Za funkciju $f(x,y)=\ln\left(\frac{y}{x}\right)$ odrediti domenu, kodomenu, razinske krivulje i limes u ishodištu (ako postoji). ~~20~~ 16

4. $\int\limits_0^2 \frac{x-1}{x^2+3x+2}\,dx=?$ 15 15

5. Zadana je funkcija $f(x)=\sqrt{x}$. Traži se površina ispod grafa funkcije (do osi apcise) na segmentu $[0,4]$. Podijeliti segment na nekoliko dijelova i preko trapezne formule procijeniti traženu površinu. Skicirati graf funkcije, površinu koja je dobivena procjenom i vizualno ocijeniti grešku numeričkog postupka. 20

6. Integriranjem izračunati površinu trokuta zadanog točkama $A(1,-2)$, $B(2,0)$, $C(-1,1)$. ~~15~~

Ukupno: 61

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha>0)$	$\alpha^x\ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int\frac{dx}{a^2+x^2}=\frac{1}{a}\arctan\frac{x}{a}+C$	$\int\frac{dx}{a^2-x^2}=\frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\|+C$
$\int x^\alpha\,dx=\frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq-1$	$\int\tan x\,dx=-\ln\|\cos x\|+C$	$\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\|+C$
$\int\frac{dx}{x}=\ln\|x\|+C$	$\int\cot x\,dx=\ln\|\sin x\|+C$	$\int\frac{dx}{\sqrt{x^2\pm a^2}}=\ln\left\|x+\sqrt{x^2\pm a^2}\right\|+C$
$\int e^x\,dx=e^x+C$	$\int\frac{dx}{\cos^2 x}=\tan x+C$	$\int\frac{dx}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$
$\int a^x\,dx=\frac{a^x}{\ln a}+C$	$\int\frac{dx}{\sin^2 x}=-\cot x+C$	$\int\frac{dx}{\sqrt{2ax-x^2}}=\arccos\left(1-\frac{x}{a}\right)+C$
$\int\sin x\,dx=-\cos x+C$	$\int\sqrt{x^2\pm a^2}\,dx=\frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]+C$	
$\int\cos x\,dx=\sin x+C$	$\int\sqrt{a^2-x^2}\,dx=\frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$	

1. $y''+2y'=1$

$y''+2y'=0$

1° HOMOGENO RJEŠENJE

$\lambda^2+2\lambda=0$

$\lambda(\lambda+2)=0$

$\lambda=0 \quad \lambda=-2$

$y_H=C_1+C_2e^{-2x}, \quad C_1, C_2\in\mathbb{R}$

2° PARTIKULARNO RJEŠENJE

$y_p=Ax \qquad y_p''=0$

$y_p'=A$

$0+2A=1 \Rightarrow A=\frac{1}{2}$

$y_p=\frac{1}{2}x$

$$y = y_H + y_P = C_1 + C_2 e^{-2x} + \frac{1}{2}x \qquad y' = -2C_2 e^{-2x} + \frac{1}{2}$$

$y(0) = 0 \qquad y'(0) = 0$

$0 = C_1 + C_2$

$0 = -2C_2 + \frac{1}{2} \Rightarrow C_2 = \frac{1}{4} \quad \Rightarrow C_1 = -\frac{1}{4}$

$$y = -\frac{1}{4} + \frac{1}{4}e^{-2x} + \frac{1}{2}x \quad ✓$$

PROVJERA:

$y' = -\frac{1}{2}e^{-2x} + \frac{1}{2}$

$y'' = e^{-2x}$

$e^{-2x} + 2\left(-\frac{1}{2}e^{-2x} + \frac{1}{2}\right) = 1$

$\cancel{e^{-2x}} - \cancel{e^{-2x}} + 1 = 1 \quad ✓✓$

② $\frac{y'}{x} = \frac{\sin x}{y} \quad / \cdot xy$

$y'y = \sin x \cdot x \quad / \int$

$\int y\,dy = \int x \sin x\,dx$

$\frac{y^2}{2} = -x\cos x + \sin x + C \quad ✓$

$y^2 = -2x\cos x + 2\sin x + C$

$y = \pm\sqrt{-2x\cos x + 2\sin x + C}$

$\int x \sin x\,dx = \left[\begin{matrix} u = x & dv = \sin x\,dx \\ du = dx & v = -\cos x \end{matrix}\right]$

$= -x\cos x + \int \cos x\,dx$

$= -x\cos x + \sin x + C$

4 $\int_0^2 \frac{x-1}{x^2+3x+2}\,dx = ?$

DINO DOKOZA

$x^2+3x+2=0$

$x_{1,2} = \frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm 1}{2} \Rightarrow x^2+3x+2 = (x+1)(x+2)$

$x_1 = -2 \quad x_2 = -1$

$\frac{x-1}{x^2+3x+2} = \frac{A}{x+1} + \frac{B}{x+2} \quad / \cdot (x+1)(x+2)$

$x-1 = A(x+2) + B(x+1)$

$x-1 = x(A+B) + (2A+B)$

$\Rightarrow A+B=1 \quad \Rightarrow B = 1-A$

$2A+B = -1 \quad \rightarrow \quad 2A+1-A = -1$

$\underline{\underline{A=-2}} \qquad \underline{\underline{B=3}}$

$\frac{x-1}{x^2+3x+2} = \frac{-2}{x+1} + \frac{3}{x+2}$

$\int_0^2 \frac{x-1}{x^2+3x+2}\,dx = -2\int_0^2 \frac{dx}{x+1} + 3\int_0^2 \frac{dx}{x+2}$

$= -2\ln|x+1|\Big|_0^2 + 3\ln|x+2|\Big|_0^2$

$= -2(\ln 3 - \ln 1) + \cancel{\left(3\ln|x+2|\Big|_0^2\right)}\ 3(\ln 4 - \ln 2)$

$= \underline{-2\ln 3 + 3\ln 4 - 3\ln 2}$

$= -0.1177830357$ ✓

DINO DOKOZA

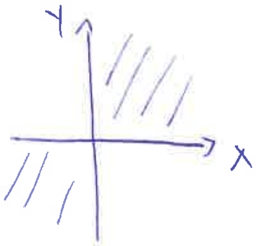
③ $f(x,y) = \ln\left(\frac{y}{x}\right)$

UVJETI NA $\ln$: $\frac{y}{x} > 0 \Rightarrow y > 0,\ x > 0$

ili $y < 0,\ x < 0$

✓

DOMENA:



$(x,y) \in \mathbb{R}^2$ t.d. $x > 0,\ y > 0$ ili $x < 0,\ y < 0$

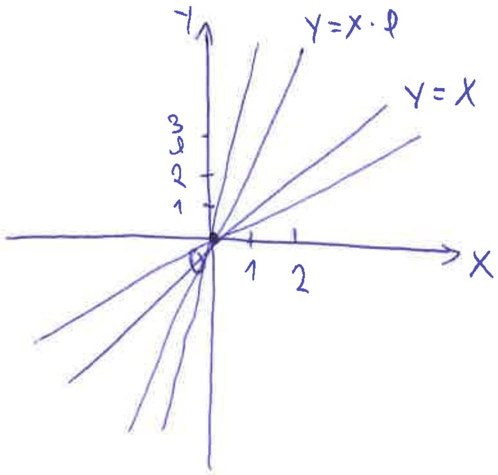
KODOMENA: $\mathbb{R}$

$\ln\left|\frac{y}{x}\right| = c \quad / e$ ✓

$\frac{y}{x} = e^c$

$y = x \cdot e^c$

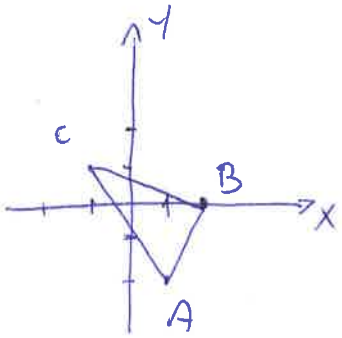
RAZINSKE KRIVULJE (PRAVCI) ✓



UMRES U ISHODIŠTU?

6. $A(1,-2)$ $B(2,0)$ $C(-1,1)$

DINO DOKOZA



pravac AB
$$y-0=\frac{-2}{-1}(x-2)$$
$$y=2(x-2)$$
$$y=2x-4$$

pravac BC
$$y-0=\frac{1}{-1-2}(x-2)$$
$$y=-\frac{1}{3}(x-2)$$
$$y=-\frac{1}{3}x+\frac{2}{3}$$

pravac AC
$$y-1=\frac{-2-1}{1+1}(x+1)$$
$$y=-\frac{3}{2}(x+1)+1$$
$$y=-\frac{3}{2}x-\frac{1}{2}$$

$$P=\int_{-1}^{1}\left(-\frac{1}{3}x+\frac{2}{3}-\left(-\frac{3}{2}x-\frac{1}{2}\right)\right)dx+\int_{1}^{2}\left(-\frac{1}{3}x+\frac{2}{3}-(2x-4)\right)dx$$
$$=\int_{-1}^{1}\left(\frac{7}{6}x+\frac{7}{6}\right)dx+\int_{1}^{2}\left(-\frac{7}{8}x+\frac{14}{3}\right)dx$$
$$=\left(\frac{7}{6}\frac{x^2}{2}+\frac{7}{6}x\right)\Big|_{-1}^{1}+\left(-\frac{7}{3}\frac{x^2}{2}+\frac{14}{3}x\right)\Big|_{1}^{2}$$

DINO DOKOZA

$$= \cancel{\frac{7}{12}} + \frac{7}{6} - \cancel{\frac{7}{12}} + \frac{7}{6} + \left(-\cancel{\frac{14}{3}} + \cancel{\frac{28}{3}} + \frac{7}{6} - \cancel{\frac{14}{3}} \right)$$

$$= \frac{7}{6} + \frac{7}{3} = \frac{21}{6}$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

oox

IME I PREZIME: Goran Marjanović **BROJ INDEKSA:** 17-2-0170-2012

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y''+2y'=1$, uz $y(0)=0$ i $y'(0)=0$. Na kraju provjeri rješenje. ~~15~~ 13
2. Nađi implicitno rješenje jednadžbe $\frac{y'}{x}=\frac{\sin x}{y}$. 15
3. Za funkciju $f(x,y)=\ln\left(\frac{y}{x}\right)$ odrediti domenu, kodomenu, razinske krivulje i limes u ishodištu (ako postoji). ~~20~~
4. $\int_0^2 \frac{x-1}{x^2+3x+2}\,dx=?$ 15
5. Zadana je funkcija $f(x)=\sqrt{x}$. Traži se površina ispod grafa funkcije (do osi apcise) na segmentu $[0,4]$. Podijeliti segment na nekoliko dijelova i preko trapezne formule procijeniti traženu površinu. Skicirati graf funkcije, površinu koja je dobivena procjenom i vizualno ocijeniti grešku numeričkog postupka. 20
6. Integriranjem izračunati površinu trokuta zadanog točkama $A(1,-2)$, $B(2,0)$, $C(-1,1)$. 15

Ukupno: 43

f	$\frac{df}{dx}$
x^α $(\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
α^x $(\alpha>0)$	$\alpha^x\ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha\neq-1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x\,dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

1.

$y''+2y'=1$

$y(0)=y'(0)=0$

$s^2+2s=0$

$s(s+2)=0$

$s_1=0$

$s_2=-2$

$Y_h = C_1 e^{0x} + C_2 e^{-2x} = C_1 + C_2 e^{-2x}$

$Y_p = Ax + B$

$Y_p'' + 2Y_p' = 1$

$2A = 1$

$A = \frac{1}{2}$

$\boxed{Y_p = \frac{1}{2}x}$ partikularno rješenje

$y = Y_h + Y_p = C_1 + C_2 e^{-2x} + \frac{1}{2}x$

$y'(0) = -2C_2 + \frac{1}{2} = 0$

$2C_2 = \frac{1}{2}$ $\quad C_2 = \frac{1}{4}$

PROVJERA?

$y(0) = 0 = C_1 + C_2 \Rightarrow C_1 = -C_2$ $\quad C_1 = -\frac{1}{4}$

$\boxed{y = -\frac{1}{4} + \frac{1}{4}e^{-2x} + \frac{1}{2}x}$ → Totalno rješenje

2. $\frac{y'}{x} = \frac{\sin x}{y}$ $\quad$ $y\,dy = x\sin x\,dx \;/\!\int$

$\frac{dy}{dx} = \frac{x\sin x}{y}$ $\quad$ $\frac{y^2}{2} = \int x\sin x\,dx =$ $\Big|$ $u = x$ $\quad dv = \sin x\,dx$; $du = dx$ $\quad v = -\cos x$

$= -x\cos x + \int \cos x\,dx = -x\cos x + \sin x + C$

$\frac{y^2}{2} = -x\cos x + \sin x + C$ ✓

$\frac{y^2}{2} + x\cos x - \sin x - C = 0$ //

3. $f(x,y) = \ln\left(\frac{y}{x}\right)$

~~f(x,y)=~~ $\quad \frac{y}{x} = 0 \quad , \quad x \neq 0$

$C = \ln\left(\frac{y}{x}\right)$ $\quad$ $\lim_{x\to 0} \ln\left(\frac{0}{x}\right) = \lim_{x\to 0} \ln\left(\frac{0}{0}\right) = -\infty$

$\frac{y}{x} = C$

$y = Cx$ – nivo krivulje ZA KAKAV c?

$T(x, 0)$

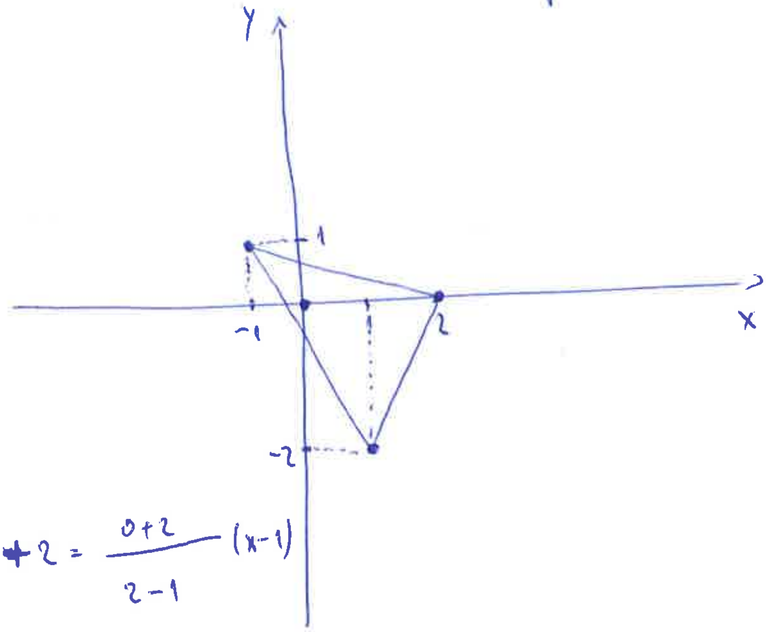
?

6. A (1, -2)

B (2, 0)

C (-1, 1)

Goran Marjanović



$$Y_1 - 1 = \frac{-2-1}{1+1}(x+1)$$

$$Y_1 = -\frac{3}{2}x - \frac{1}{2}$$

$$Y_3 + 2 = \frac{0+2}{2-1}(x-1)$$

$$Y_3 = 2x - 4$$

$$Y_2 - 1 = \frac{0-1}{2+1}(x+1)$$

$$Y_2 = -\frac{1}{3}x + \frac{2}{3}$$

$$P = \int_{-1}^{0}\left(-\frac{1}{3}x+\frac{2}{3}\right)dx - \int_{-1}^{0}\left(-\frac{3}{2}x-\frac{1}{2}\right)dx + \int_{0}^{1}\left(-\frac{1}{3}x+\frac{2}{3}\right)dx - \int_{0}^{1}\left(-\frac{3}{2}x-\frac{1}{2}\right)dx +$$

$$+ \int_{1}^{2}\left(-\frac{1}{3}x+\frac{2}{3}\right)dx - \int_{1}^{2}(2x-4)\,dx =$$

$$= \frac{7}{12} + \frac{7}{4} + \frac{7}{6} = \frac{7}{2}$$

4. $\int_0^2 \frac{x-1}{x^2+3x+2} = \int_0^2 \frac{x-1}{(x+\frac{3}{2})^2-\frac{1}{4}}\,dx =$

$= \int_0^2 \frac{x}{(x+\frac{3}{2})^2-\frac{1}{4}}\,dx - \int_0^2 \frac{dx}{(x+\frac{3}{2})^2-\frac{1}{4}} =$ $\left| \begin{array}{l} t = x+\frac{3}{2} \\ dt = dx \\ x = t-\frac{3}{2} \\ x=0 \to t=\frac{3}{2} \\ x=2 \to t=\frac{3}{2} \end{array} \right.$

$= \int_{\frac{3}{2}}^{\frac{7}{2}} \frac{t-\frac{3}{2}}{t^2-\frac{1}{4}}\,dt - \int_{\frac{3}{2}}^{\frac{7}{2}} \frac{dt}{t^2-\frac{1}{4}} = \int_{\frac{3}{2}}^{\frac{7}{2}} \frac{t}{t^2-\frac{1}{4}}\,dt - \frac{5}{2}\int_{\frac{3}{2}}^{\frac{7}{2}} \frac{dt}{t_2-\frac{1}{4}}$

$= \int_{\frac{3}{2}}^{\frac{7}{2}} \frac{t}{t^2-\frac{1}{4}}\,dt - \frac{3}{2}\cdot\frac{1}{2\cdot\frac{1}{2}}\cdot \ln\left|\frac{x-\frac{1}{2}}{x+\frac{1}{2}}\right|\Bigg|_{\frac{3}{2}}^{\frac{7}{2}} =$ $\left| \begin{array}{l} k = t^2-\frac{1}{4} \\ dk = 2t\,dt \\ dt = \frac{dk}{2t} \end{array} \right.$

$= \frac{1}{2}\int \frac{dk}{k} - \frac{5}{2}\cdot\left[\ln\left|\frac{3}{4}\right| - \ln\left|\frac{1}{2}\right|\right] =$

$= \frac{1}{2}\ln\left|t^2-\frac{1}{4}\right|\Bigg|_{\frac{3}{2}}^{\frac{7}{2}} - \frac{5}{2}\ln\frac{3}{4} + \frac{5}{2}\ln\frac{1}{2} =$

$= \frac{1}{2}\ln 12 - \frac{1}{2}\ln 2 - \frac{5}{2}\ln\frac{3}{4} + \frac{5}{2}\ln\frac{1}{2} = ?$

KOLIKO IZNOSI RJEŠENJE U DECIMALNOM ZAPISU.

SAMI IZRAČUNAJTE PA JAVITE AKO SE ISPAVAN REZULTAT.

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** oox

IME I PREZIME: NEMANJA KORDA **BROJ INDEKSA:** 17-2-0297-2013 0269076510

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y''+2y'=1$, uz $y(0)=0$ i $y'(0)=0$. Na kraju provjeri rješenje. 15
2. Nađi implicitno rješenje jednadžbe $\frac{y'}{x}=\frac{\sin x}{y}$. 15
3. Za funkciju $f(x,y)=\ln\left(\frac{y}{x}\right)$ odrediti domenu, kodomenu, razinske krivulje i limes u ishodištu (ako postoji). ~~20~~
4. $\int\limits_0^2 \frac{x-1}{x^2+3x+2}\,dx=?$ ~~15~~
5. Zadana je funkcija $f(x)=\sqrt{x}$. Traži se površina ispod grafa funkcije (do osi apcise) na segmentu $[0,4]$. Podijeliti segment na nekoliko dijelova i preko trapezne formule procijeniti traženu površinu. Skicirati graf funkcije, površinu koja je dobivena procjenom i vizualno ocijeniti grešku numeričkog postupka. ~~20~~
6. Integriranjem izračunati površinu trokuta zadanog točkama $A(1,-2)$, $B(2,0)$, $C(-1,1)$. 15

Ukupno: 15

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha>0)$	$\alpha^x\ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x+C$	$\int\frac{dx}{a^2+x^2}=\frac{1}{a}\arctan\frac{x}{a}+C$	$\int\frac{dx}{a^2-x^2}=\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert+C$
$\int x^\alpha dx=\frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq-1$	$\int\tan x\,dx=-\ln\lvert\cos x\rvert+C$	$\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert+C$
$\int\frac{dx}{x}=\ln\lvert x\rvert+C$	$\int\cot x\,dx=\ln\lvert\sin x\rvert+C$	$\int\frac{dx}{\sqrt{x^2\pm a^2}}=\ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert+C$
$\int e^x dx=e^x+C$	$\int\frac{dx}{\cos^2 x}=\tan x+C$	$\int\frac{dx}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$
$\int a^x dx=\frac{a^x}{\ln a}+C$	$\int\frac{dx}{\sin^2 x}=-\cot x+C$	$\int\frac{dx}{\sqrt{2ax-x^2}}=\arccos\left(1-\frac{x}{a}\right)+C$
$\int\sin x\,dx=-\cos x+C$	$\int\sqrt{x^2\pm a^2}\,dx=\frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]+C$	
$\int\cos x\,dx=\sin x+C$	$\int\sqrt{a^2-x^2}\,dx=\frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$	

4. $\int\limits_0^2 \frac{x-1}{x^2+3x+2}dx$

$= \int\limits_0^2 x\,dx + 2\int\limits_0^2 dx + \int\limits_0^2 \frac{4\,dx}{\underbrace{x^2+3x+2}_{\square}}$

$= \frac{x^2}{2}\Big|_0^2 + 2x\Big|_0^2 + \square$

$\int\frac{4dx}{x^2+3x+2} = 4\int$

$x^2+3x+2 : x-1 = \boxed{x}$

$-(x^2-x)$

$\cancel{4x+2} : x-1 = \boxed{2}$

$-(2x-2)$

$\boxed{4}$

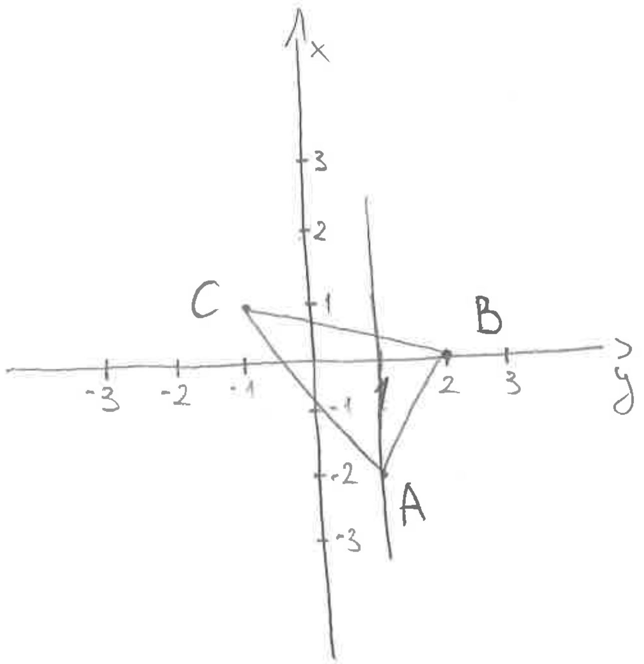
$x^2+3x+2 =$

$\left(x+\frac{3}{2}\right)^2 +$

$x^2+2\cdot(x\cdot\frac{3}{2}) + \left(\frac{3}{2}\right)^2$

$\left(x^2+3x+\frac{9}{4}\right)-\frac{1}{4}$

⑥ $A(1,-2)\quad B(2,0)\quad C(-1,1)$



$\boxed{\overline{AB}}\quad A\begin{pmatrix}x_1 & y_1\\ 1, & -2\end{pmatrix}\quad B\begin{pmatrix}x_2 & y_2\\ 2, & 0\end{pmatrix}$

$(y-y_1)(x_2-x_1)=(x-x_1)(y_2-y_1)$

$(y+2)(2-1)=(x-1)(0+2)$

$y+2=2x-2$

$\boxed{y=2x-4}$

$\boxed{\overline{BC}}\quad B\begin{pmatrix}x_1 & y_1\\ 2, & 0\end{pmatrix}\quad C\begin{pmatrix}x_2 & y_2\\ -1, & 1\end{pmatrix}$

$(y-0)(-1-2)=(x-2)(1-0)$

$-3y=x-2\ |:(-3)$

$\boxed{y=\frac{-x}{3}+\frac{2}{3}}$

$\overline{AC}\quad A\begin{pmatrix}x_1 & y_1\\ 1, & -2\end{pmatrix}\quad C\begin{pmatrix}x_2 & y_2\\ -1, & 1\end{pmatrix}$

$(y+2)(-1-1)=(x-1)(1+2)$

$-2y-4=3x-3$

$-2y=3x+1\ |:(-2)$

$\boxed{y=\frac{-3x}{2}-\frac{1}{2}}$

$P_1=\int_{-1}^{1} BC-AC$

$=\int_{-1}^{1}\frac{-x}{3}+\frac{2}{3}-\left(\frac{-3}{2}x-\frac{1}{2}\right)$

$\int_{-1}^{1}\left(\frac{-x}{3}+\frac{2}{3}+\frac{3}{2}x+\frac{1}{2}\right)dx$

$-\frac{1}{3}\int_{-1}^{1}x+\frac{2}{3}\int_{-1}^{1}dx+\frac{3}{2}\int_{-1}^{1}x\,dx+\frac{1}{2}\int_{-1}^{1}dx$

$\left[\frac{-x^2}{6}+\frac{2}{3}x+\frac{3x^2}{4}+\frac{1}{2}x\right]_{-1}^{1}=$

$P_1=\left(\frac{-1}{6}+\frac{2}{3}+\frac{3}{4}+\frac{1}{2}\right)-\left(\frac{-1}{6}-\frac{2}{3}+\frac{3}{4}-\frac{1}{2}\right)$

$P_1=\frac{7}{4}-\left(-\frac{7}{12}\right)=\frac{7}{4}+\frac{7}{12}=\frac{7}{3}$

$P_2=\int_{1}^{2} BC-AB$

$P_2=\int_{1}^{2}\frac{-x}{3}+\frac{2}{3}-(2x-4)=\int_{1}^{2}\left(\frac{-x}{3}+\frac{2}{3}-2x+4\right)dx=-\frac{1}{3}\int_{1}^{2}x\,dx+\frac{2}{3}\int_{1}^{2}dx-2\int_{1}^{2}x\,dx+4\int_{1}^{2}dx$

$\left[\frac{-x^2}{6}+\frac{2}{3}x-\cancel{\frac{2x^2}{2}}+4x\right]_{1}^{2}=\left(\frac{-\cancel{4}^{2}}{\cancel{6}_{3}}+\frac{4}{3}-4+8\right)-\left(\frac{-1}{6}+\frac{2}{3}-1+4\right)=\frac{14}{3}-\left(\frac{7}{2}\right)=\boxed{\frac{7}{6}}$

$P_{\Delta ABC}=P_1+P_2=\frac{7}{3}+\frac{7}{6}=\frac{7}{2}=\boxed{3.5}$

4.

$$\int_0^2 \frac{x-1}{x^2+3x+2}\,dx =$$

$$\cancel{x^2}+3x+2 : x-1 = \boxed{x}$$
$$-(\cancel{x^2}-x)$$
$$4x+2 : x-1 = \boxed{4}$$
$$-(4x-4)$$
$$\boxed{6}$$

$$\int_0^2 x\,dx + 4\int_0^2 dx + 6\underbrace{\int_0^2 \frac{dx}{x^2+3x+2}}_{\square}$$

REZULTAT!

$$* \left[\frac{x^2}{2} + 4x + 6\cdot\blacksquare\right]_0^2 = \frac{4^2}{2} + 8 - (0) + 6\cdot\blacksquare = 10 + (6\cdot 0.41) = 10 + 2.4 = \boxed{12.4}$$

TABLIČNI INT.

$$\blacksquare \int_0^2 \frac{dx}{x^2+3x+2} = \int_0^2 \frac{dx}{\left(x+\frac{3}{2}\right)^2-\frac{1}{4}} = \frac{dt}{t^2-\left(\frac{\sqrt{1}}{2}\right)^2}$$

$$\left| \begin{array}{l} x+\frac{3}{2}=t \\ dx=dt \end{array} \right|$$

$$= \left[\frac{1}{\underbrace{2\cdot\frac{\sqrt{1}}{2}}_{2\cdot\frac{1}{2}=\frac{2}{2}=1}} \ln\left|\frac{t-\frac{\sqrt{1}}{2}}{t+\frac{\sqrt{1}}{2}}\right|\right]_0^2 = \ln\left[\left|\frac{\left(x+\frac{3}{2}\right)-\frac{1}{2}}{\left(x+\frac{3}{2}\right)+\frac{1}{2}}\right|\right]_0^2 =$$

$$= \ln\left|\frac{2+\frac{3}{2}-\frac{1}{2}}{2+\frac{3}{2}+\frac{1}{2}}\right| - \left(\ln\left|\frac{0+\frac{3}{2}-\frac{1}{2}}{0+\frac{3}{2}+\frac{1}{2}}\right|\right) = \ln\frac{3}{4} - \left(\ln\frac{1}{2}\right)$$

$$\boxed{\approx 0.4}$$

NEMANJA KORDA

BROJ INDEKSA: 0269076510

③ $f(x,y) = \ln\left(\frac{y}{x}\right)$

UVJET NAZIVNIKA $x \neq 0$

UVJET LOGARITMA $x > 0$

a) $\frac{y}{x} > 0 \Rightarrow \boxed{\begin{matrix} x > 0 \\ y > 0 \end{matrix}}$ X

b) $x \neq 0$

$y \geq 0$

RAZINSKE KRIVULJE:

$c_0 = \ln \frac{x}{y} = 0$

$c_1 = \ln \frac{x}{y} = 0$

$c_2 \ln \frac{x}{y} = 0$

?

⑤

$$\int_0^4 \sqrt{x}$$

k	0	1	2
x_k	0	2	4
f_{x_k}	0	1.41	2

$d = \frac{4-0}{2}$

$d = 2$

$S = \frac{d}{6} \Big($ ___ = ?

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** oox	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: TONI STOŠIĆ **BROJ INDEKSA:** 57817-2009	

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' + 2y' = 1$, uz $y(0) = 0$ i $y'(0) = 0$. Na kraju provjeri rješenje. ~~15~~

2. Nađi implicitno rješenje jednadžbe $\frac{y'}{x} = \frac{\sin x}{y}$. 15

3. Za funkciju $f(x,y) = \ln\left(\frac{y}{x}\right)$ odrediti domenu, kodomenu, razinske krivulje i limes u ishodištu (ako postoji). ~~20~~

4. $\displaystyle\int_0^2 \frac{x-1}{x^2+3x+2}\,dx = ?$ 15

5. Zadana je funkcija $f(x) = \sqrt{x}$. Traži se površina ispod grafa funkcije (do osi apcise) na segmentu $[0,4]$. Podijeliti segment na nekoliko dijelova i preko trapezne formule procijeniti traženu površinu. Skicirati graf funkcije, površinu koja je dobivena procjenom i vizualno ocijeniti grešku numeričkog postupka. 20

6. Integriranjem izračunati površinu trokuta zadanog točkama $A(1,-2)$, $B(2,0)$, $C(-1,1)$. ~~15~~

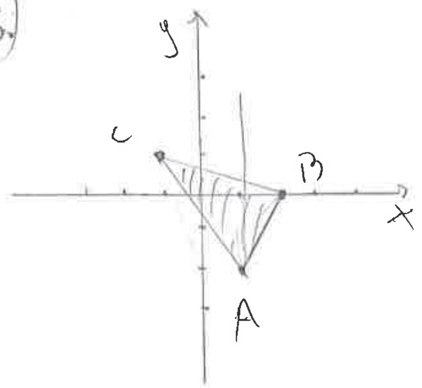
Ukupno: 15

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_a x$ $(a > 0)$	$\frac{1}{x \ln a}$
e^x	e^x
a^x $(a > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

6.



$A(1,-2)$ $B(2,0)$
$x_1\ y_1$ $x_2\ y_2$

$(x_2-x_1)(y-y_1) = (y_2-y_1)(x-x_1)$

$(2-1)(y+2) = (0+2)(x-1)$

$y+2 = 2x-2$

$y = 2x-2-2$

$\overline{AB} \ldots\ y = 2x-4$

$B(2,0)$ $C(-1,1)$
$x_1\ y_1$ $x_2\ y_2$

$(x_2-x_1)(y-y_1) = (y_2-y_1)(x-x_1)$

$(-1-2)(y-0) = (1-0)(x-2)$

$-3y = x-2 \ /:-3$

$\overline{BC} \ldots\ y = -\frac{1}{3}x + \frac{2}{3}$

$P = \int ($

$C(-1,1)$ $A(1,-2)$
$x_1\ y_1$ $x_2\ y_2$

$(x_2-x_1)(y-y_1) = (y_2-y_1)(x-x_1)$

$(1+1)(y-1) = (-2-1)(x+1)$

$2y-2 = -3x-3$

$2y = -3x-3+2$

$2y = -3x-1 \ /:2$

$\overline{CA} \ldots\ y = -\frac{3}{2}x - \frac{1}{2}$

—7

$$P=\int_{-1}^{1}\left[-\frac{1}{3}x+\frac{2}{3}-\left(-\frac{3}{2}x-\frac{1}{2}\right)\right]dx+\int_{1}^{2}\left[-\frac{1}{3}x+\frac{2}{3}-(2x-4)\right]dx$$

$$P=\int_{-1}^{1}\left(-\frac{1}{3}x+\frac{2}{3}+\frac{3}{2}x+\frac{1}{2}\right)dx+\int_{1}^{2}\left(-\frac{1}{3}x+\frac{2}{3}-2x+4\right)dx$$

$$P=\int_{-1}^{1}\left(-\frac{1}{3}x+\frac{3}{2}x+\frac{7}{6}\right)dx+\int_{1}^{2}\left(-\frac{1}{3}x-2x+\frac{14}{3}\right)dx$$

$$\frac{14}{12}+\frac{-5}{6}=\frac{14-10}{12}=\frac{4}{12}=\frac{1}{3}$$

$$P=\left(-\frac{1}{3}\cdot\frac{x^2}{2}+\frac{3}{2}\cdot\frac{x^2}{2}+\frac{7}{6}x\right)\Big|_{-1}^{1}+\left(-\frac{1}{3}\cdot\frac{x^2}{2}-\cancel{2}\cdot\frac{x^2}{\cancel{2}}+\frac{14}{3}x\right)\Big|_{1}^{2}=\left(-\frac{1}{3}\cdot\frac{1^2}{2}+\frac{3}{2}\cdot\frac{1^2}{2}+\frac{7}{6}\cdot1-\left(-\frac{1}{3}\cdot\frac{(-1)^2}{2}+\frac{3}{2}\cdot\frac{(-1)^2}{2}+\frac{7}{6}\cdot(-1)\right)\right)+$$

$$\left(-\frac{1}{3}\cdot\frac{2^2}{2}-2^2+\frac{14}{3}\cdot2-\left(-\frac{1}{3}\cdot\frac{1^2}{2}-1^2+\frac{14}{3}\cdot1\right)\right)=$$

$$=\left(-\frac{1}{6}+\frac{3}{4}+\frac{7}{6}+\frac{1}{6}-\frac{3}{4}+\frac{7}{6}\right)+\left(-\frac{2}{3}-4+\frac{28}{3}+\frac{1}{6}-1-\frac{14}{3}\right)=\frac{-2+9+14+2-9+14}{12}+\frac{-4-24+56+1-6-28}{6}\quad \boxed{P=\frac{1}{3}}$$

4. $$\int_0^2\frac{x-1}{x^2+3x+2}\,dx=$$

$D(f): \mathbb{R}\setminus\{-2,-1\}$

$[0,2]\in D(f)$

INTEGRAL NIJE NEPRAVI

$$=3\cdot\ln|x+2|-2\ln|x+1|\Big|_0^2$$

$$=3\ln|2+2|-2\ln|2+1|-(3\ln|0+2|-2\ln|0+1|)$$

$$=3\ln4-2\ln3-3\ln2+2\ln1=4.28$$

$x^2+3x+2=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-3\pm\sqrt{3^2-4\cdot1\cdot2}}{2}$$

$$x=\frac{-3\pm\sqrt{9-8}}{2}$$

$$x=\frac{-3\pm\sqrt{1}}{2}$$

$$x_1=\frac{-3-1}{2}=\frac{-4}{2}=-2$$

$$x_2=\frac{-3+1}{2}=\frac{-2}{2}=-1$$

$a(x-x_1)(x-x_2)$

$(x+2)(x+1)$

$$\int\frac{x-1}{x^2+3x+2}=\int\frac{x-1}{(x+2)(x+1)}=\int\frac{3}{x+2}dx+\int\frac{-2}{x+1}dx=$$

$$3\int\frac{dx}{x+2}-2\int\frac{dx}{x+1}=3\cdot\ln|x+2|-2\ln|x+1|$$

$$\frac{x-1}{(x+2)(x+1)}=\frac{A}{(x+2)}+\frac{B}{(x+1)}\ /\cdot(x+2)(x+1)$$

$$x-1=A(x+1)+B(x+2)$$

$$x-1=Ax+A+Bx+2B$$

$$x-1=(A+B)x+A+2B$$

$$A+B=1\ /\cdot(-1)$$
$$A+2B=-1$$

$$-A-B=-1$$
$$A+2B=-1$$

$$B=-2$$

$$A+2\cdot(-2)=-1$$
$$A-4=-1$$
$$A=-1+4$$
$$A=3$$

TONI STOŠIĆ

1. $y'' + 2y' = 1$

$r^2 + 2r = 0$

$r(r+2) = 0$

$r_1 = 0$

$r + 2 = 0$

$r_2 = -2$

$y_H = C_1 \cdot e^{r_1 x} + C_2 \cdot e^{r_2 x}$

$y_H = C_1 \cdot e^{0} + C_2 \cdot e^{-2x}$

$y_H = C_1 + C_2 \cdot e^{-2x}$

$y_p = x^k \cdot e^{\alpha x}\left(S_N(x) \cdot \cos(\beta x) + T_N(x) \cdot \sin(\beta x)\right)$

$y_p = x \cdot S_N(x)$

$y_p = x \cdot (Ax + B)$

$y_p = Ax^2 + Bx$

$y_p' = 2Ax + B$

$y_p'' = 2A$

$y^2 + 2y' = 1$

$2A + 2Ax + B = 1$

$2A + B = 1$

$2A = 0 \quad / \cdot (-2)$

$2A + B = 1$

$-2A = 0$

$B = 1$

$2A + 1 = 1$

$2A = 1 - 1$

$A = 0$

$y_p = Ax^2 + Bx$

$y_p = 0x^2 + 1x$

$y_p = x$

UVJETI: $0 = C_1 + C_2 \cdot e^{-2\cdot 0} + 0$

$0 = C_1 + C_2$

$0 = -2C_2 \cdot e^{-2\cdot 0} + 1$

$0 = -2C_2 + 1$

$-1 = -2C_2$

$C_2 = \frac{1}{2}$

$C_1 + C_2 = 0$

$C_2 = \frac{1}{2} \quad / \cdot (-1)$

$C_1 + C_2 = 0$

$-C_2 = -\frac{1}{2}$

$C_1 = -\frac{1}{2}$

$1 = e^{\alpha x}\left(P_m(x) \cdot \cos(\beta x) + Q_n(x) \cdot \sin(\beta x)\right)$

$\alpha = 0$

$\beta = 0$

$\alpha + \beta i = 0 + 0i = 0 = r_1 \neq r_2 \Rightarrow \boxed{k = 1}$

$1 = e^{0}\left(P_m(x) \cdot \cos 0 + Q_n(x) \cdot \sin 0\right)$

$1 = P_m(x) \qquad m = 0$

$Q_n(x) = 0 \qquad n = N/P$

$N = \max\{0, NP\} = 0$

PROVJERA: $y = -\frac{1}{2} + \frac{1}{2}e^{-2x} + x$

$y' = -e^{-2x} + 1$

$y'' = 2e^{-2x}$

$y'' + 2y' = 2e^{-2x} + 2(-1e^{-2x} + 1) = 2 \neq 1$

$y = y_H + y_p$

$y = C_1 + C_2 \cdot e^{-2x} + x$

$y' = C_2 \cdot e^{-2x} \cdot (-2) + 1$

$y' = -2C_2 \cdot e^{-2x} + 1$

$y' = -2 \cdot \frac{1}{2} \cdot e^{-2x} + 1$

$y = C_1 + C_2 \cdot e^{-2x} + x$

$y = -\frac{1}{2} + \frac{1}{2} \cdot e^{-2x} + x$ ✗

PROVJERA:

Uz $y(0) = 0$

$-\frac{1}{2} + \frac{1}{2} \cdot e^{-2\cdot 0} + 0 = 0$

$0 = 0$

Uz $y'(0) = 0$

$-2 \cdot \frac{1}{2} \cdot e^{-2\cdot 0} + 1 = 0$

$-1 + 1 = 0$

$0 = 0$

3.

$f(x,y) = \ln \frac{y}{x}$

$\ln > 0$

$\frac{y}{x} > 0$

$D(f) \in \left\{ \frac{y}{x} \right\}$?

$y =$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** oox

IME I PREZIME: Ante Podlišić **BROJ INDEKSA:** 17-1-0114-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' + 2y' = 1$, uz $y(0) = 0$ i $y'(0) = 0$. Na kraju provjeri rješenje. 15
2. Nađi implicitno rješenje jednadžbe $\frac{y'}{x} = \frac{\sin x}{y}$. 15
3. Za funkciju $f(x, y) = \ln\left(\frac{y}{x}\right)$ odrediti domenu, kodomenu, razinske krivulje i limes u ishodištu (ako postoji). 20
4. $\int\limits_0^2 \frac{x-1}{x^2+3x+2}\,dx = ?$ ~~15~~
5. Zadana je funkcija $f(x) = \sqrt{x}$. Traži se površina ispod grafa funkcije (do osi apcise) na segmentu $[0, 4]$. Podijeliti segment na nekoliko dijelova i preko trapezne formule procijeniti traženu površinu. Skicirati graf funkcije, površinu koja je dobivena procjenom i vizualno ocijeniti grešku numeričkog postupka. ~~20~~ 5
6. Integriranjem izračunati površinu trokuta zadanog točkama $A(1, -2)$, $B(2, 0)$, $C(-1, 1)$. ~~15~~

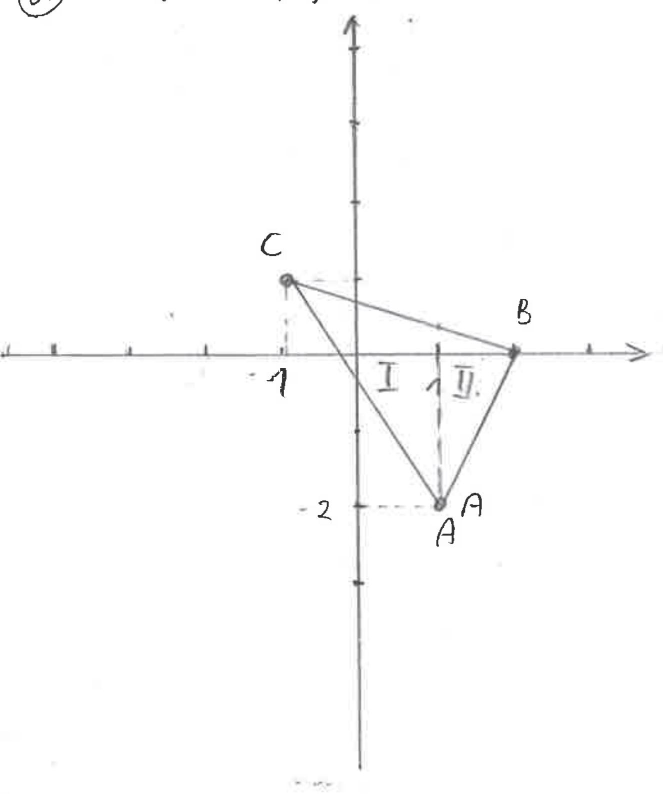
Ukupno: 5

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

6. $A(1, -2)$, $B(2, 0)$, $C(-1, 1)$



$(y-1)(x_2-x_1) = (y_2-y_1)(x-x_1)$

$\boxed{CB}$

$(y-1)\overbrace{(2+1)}^{3} = \overbrace{(0-1)}^{-1}(x-2)$

$3y - 3 = -x + 2 \Rightarrow 3y = -x + 5$

$\boxed{CB = y = \frac{-x+5}{3}}$

$\boxed{CA}$

$(y-1)\overbrace{(1+1)}^{2} = \overbrace{(-2-1)}^{-3}\cdot(x+1)$

$2y - 2 = -3x - 3 \quad 2y = -3x - 1$

$\boxed{CA = \frac{-3x-1}{2}}$

$\boxed{AB}$

$(y-1)\cdot\overbrace{(2-1)}^{1} = \overbrace{(0+2)}^{2}\cdot(x-1)$

$y - 1 = 2x - 2 \Rightarrow y = 2x - 1$

AB

$\overline{AB} \Rightarrow y = 2x - 1$ $\quad \overline{CA} = \frac{-3x+1}{2}$

$\overline{CB} \Rightarrow y = \frac{-x+5}{3}$

$$P_1 = \int_{-1}^{1} (\overline{CB} - \overline{CA})\,dx = \int_{-1}^{1} \left(\frac{-x+5}{3} - \left(\frac{-3x+1}{2}\right)\right) dx$$

$$= \int_{-1}^{1} \left(\frac{-x+5}{3} - \frac{3x+1}{2}\right) dx = \int_{-1}^{1} \left(\frac{-2x+10-9x+3}{6} \right) dx$$

$$= \int_{-1}^{1} \left(\frac{-11x + 7}{6} \right) dx = \int_{-1}^{1} -\frac{11}{6} x\,dx - \int_{-1}^{1} \frac{7}{6}\,dx$$

$$= -\frac{11}{6} \cdot \frac{x^2}{2} - \frac{7}{6} x \Big|_{-1}^{1} = -2{,}06 - 0{,}25 = -1{,}83 \Rightarrow \boxed{P_1 = 1{,}83}$$

$$\overline{P_2} = \int_1^2 (\overline{CB} - \overline{AB})\,dx = \int_1^2 \left(\frac{-x+5}{3} - (2x-1)\right) dx = \int_1^2 \left(\frac{-x+5-6x+3}{3} \right) dx =$$

$$= \int_1^2 \left(\frac{-7x+8}{3} \right) dx = \int_1^2 -\frac{7}{3} x\,dx + \int_1^2 \frac{8}{3}\,dx = -\frac{7}{3} \cdot \frac{x^2}{2} + \frac{8}{3} x \Big|_1^2 =$$

$$= \frac{2}{3} - \frac{3}{2} = -\frac{5}{6} = -0{,}83 \Rightarrow P_{II} = 0{,}834$$

$$\boxed{P = P_I + P_{II}} = 1{,}83 + 0{,}834 = \boxed{2{,}664}$$

×

(4.) $\int_0^2 \frac{x-1}{x^2+3x+2}\,dx =$

$x^2+3x+2 = 0$

$x_{1,2} = \frac{-3 \pm \sqrt{9-8}}{2}$

$x_{1,2} = \frac{-3 \pm 1}{2}$

$x_1 = -1, \; x_2 = -2$

$x^2+3x+2 = (x-1) \cdot (x+2)$

$$\int_0^2 \frac{x-1}{(x-1)(x+2)}\,dx = \int_0^2 \frac{dx}{x+2} = \ln(x+2)\Big|^2$$ ×

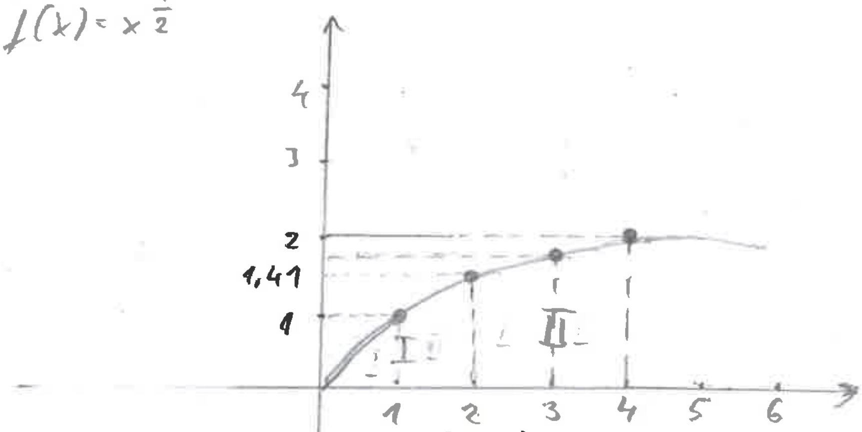
$$= \ln 4 - \ln 2 = \ln \frac{4}{2} = \boxed{\ln 2} = 0{,}693147$$

Ante Pedišić 17-1-0114-2012

(51) $f(x)=\sqrt{x}$

$f(x)=x^{\frac{1}{2}}$

$P[0,4]$



x	f(x)
0	0
1	1
2	1,41
3	1,73
4	2

$$\int_0^4 x^{\frac{1}{2}}dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2x^{\frac{3}{2}}}{3}\Big|_0^4 = \frac{16}{3}+0 \Rightarrow \boxed{P=\frac{16}{3}=5,33}$$

P na segmentu od $[0,4]$

2

$P_I = m \cdot h = 1,41 \Rightarrow \boxed{P_I = 1,41}$

(m = 1, h = 1,41)

Površina trapeza: $\boxed{P=m \cdot h}$

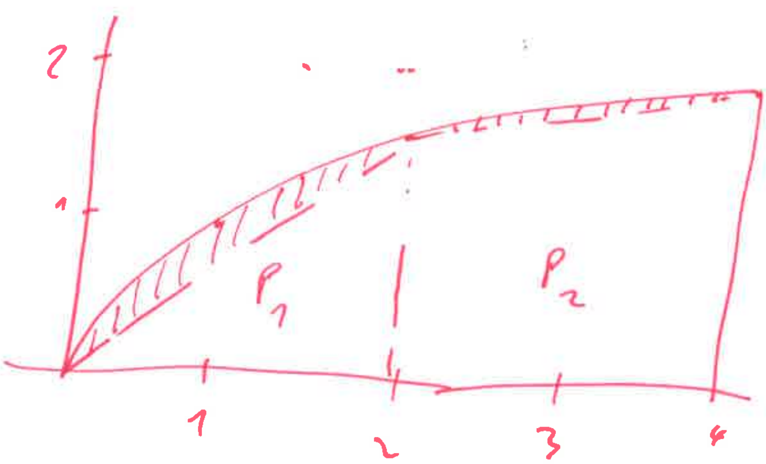
$P_{II} = m \cdot h = 1,73 \cdot 2 = 3,46 \Rightarrow \boxed{P_{II} \approx 3,46}$

(m = 1,73, h = 2)

$\boxed{4,87 \neq 5,46}$

$P_I + P_{II} = 1,41 + 3,46 = 4,87 \Rightarrow \boxed{P \approx 4,87}$

RADILI STE METODOM SREDNJE TOČKE, TE NAPRAVILI ZNAČAJNU POGREŠKU



$P_1 = \frac{\sqrt{0}+\sqrt{2}}{2}\cdot 2$

$P_2 = \frac{\sqrt{2}+\sqrt{4}}{2}\cdot 2$

GREŠKA (CRTKANA POVRŠINA) UGRUBO OKO 10%.

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OOX

IME I PREZIME: JOSIP PREDOVAN **BROJ INDEKSA:** 17-1-0126-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y''+2y'=1$, uz $y(0)=0$ i $y'(0)=0$. Na kraju provjeri rješenje. 15
2. Nađi implicitno rješenje jednadžbe $\frac{y'}{x}=\frac{\sin x}{y}$. ~~15~~
3. Za funkciju $f(x,y)=\ln\left(\frac{y}{x}\right)$ odrediti domenu, kodomenu, razinske krivulje i limes u ishodištu (ako postoji). ~~20~~
4. $\int\limits_0^2 \frac{x-1}{x^2+3x+2}\,dx=?$ ~~15~~
5. Zadana je funkcija $f(x)=\sqrt{x}$. Traži se površina ispod grafa funkcije (do osi apcise) na segmentu $[0,4]$. Podijeliti segment na nekoliko dijelova i preko trapezne formule procijeniti traženu površinu. Skicirati graf funkcije, površinu koja je dobivena procjenom i vizualno ocijeniti grešku numeričkog postupka. 20
6. Integriranjem izračunati površinu trokuta zadanog točkama $A(1,-2)$, $B(2,0)$, $C(-1,1)$. ~~15~~

Ukupno: $\varnothing$

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha>0)$	$\alpha^x\ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int\frac{dx}{a^2+x^2}=\frac{1}{a}\arctan\frac{x}{a}+C$	$\int\frac{dx}{a^2-x^2}=\frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\|+C$
$\int x^\alpha dx=\frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq-1$	$\int\tan x\,dx=-\ln\|\cos x\|+C$	$\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\|+C$
$\int\frac{dx}{x}=\ln\|x\|+C$	$\int\cot x\,dx=\ln\|\sin x\|+C$	$\int\frac{dx}{\sqrt{x^2\pm a^2}}=\ln\left\|x+\sqrt{x^2\pm a^2}\right\|+C$
$\int e^x dx=e^x+C$	$\int\frac{dx}{\cos^2 x}=\tan x+C$	$\int\frac{dx}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$
$\int a^x dx=\frac{a^x}{\ln a}+C$	$\int\frac{dx}{\sin^2 x}=-\cot x+C$	$\int\frac{dx}{\sqrt{2ax-x^2}}=\arccos\left(1-\frac{x}{a}\right)+C$
$\int\sin x\,dx=-\cos x+C$	$\int\sqrt{x^2\pm a^2}\,dx=\frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]+C$	
$\int\cos x\,dx=\sin x+C$	$\int\sqrt{a^2-x^2}\,dx=\frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$	

(4) $\int\limits_0^2 \frac{x-1}{x^2+3x+2}\,dx = \frac{A}{(x+1)}+\frac{B}{(x+2)}\ \Big/\cdot(x+1)\cdot(x+2)$

$x_{1,2}=\frac{-3\pm\sqrt{3^2-4\cdot1\cdot2}}{2\cdot1}=\frac{-3\pm1}{2}$ $x_1=-1$, $x_2=-2$

$x-1=A\cdot(x+2)+B\cdot(x+1)$

$x-1=Ax+2A+Bx+B$

$x-1=x\cdot(A+B)+2A+B$

$A+B=1$

$2A+B=-1\ /\cdot(-1)$

$A+B=1$

$-2A-B=1$

$-A=2\ /\cdot(-1)$

$A=-2$

$2A+B=-1$

$2\cdot(-2)+B=-1$

$-4+B=-1$

$B=-1+4$

$B=3$

$$\int_0^2 \frac{-2}{(x+1)}dx + \frac{3}{(x+2)}dx = \underbrace{\int_0^2 \frac{-2}{x+1}dx}_{I_1} + \underbrace{\int_0^2 \frac{3}{x+2}dx}_{I_2}$$

$$\int_0^2 \frac{-2}{x+1}dx = -2\int_0^2 \frac{dx}{x+1} \qquad \left| \begin{matrix} x+1=t & t=3 \\ dx=dt & t=1 \end{matrix} \right|$$

$$-2\cdot\int_1^3 \frac{dt}{t} = \ln|t| \Big|_1^3 = -2\cdot\ln|3| + 2\cdot|\ln|1|| = -2\ln|3| = -2{,}197$$

$$\int_0^2 \frac{3}{x+2}dx = 3\int_0^2 \frac{dx}{x+2} \qquad \left| \begin{matrix} x+2=t & t=4 \\ dx=dt & t=2 \end{matrix} \right|$$

$$3\int_2^4 \frac{dt}{t} = \ln|t| \Big|_3^4 = 3\cdot\ln|4| - 3\ln|3| = 0{,}863$$

$$I_1+I_2 = -2{,}197 + 0{,}863 = -1{,}334$$

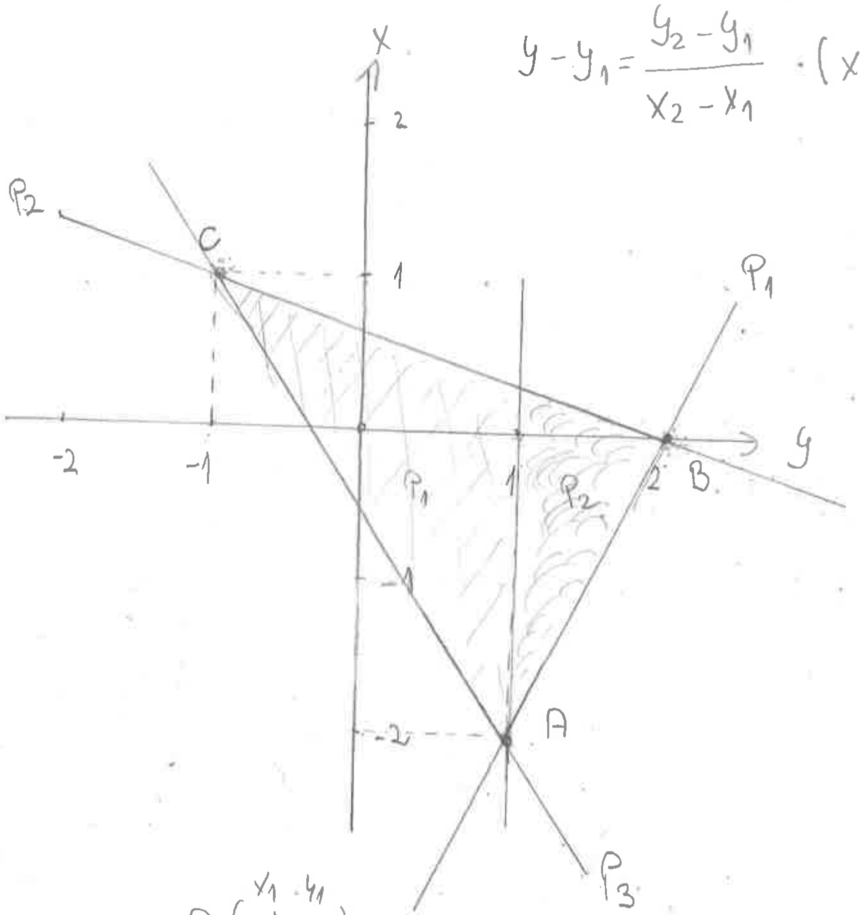
JOSIP PREDOVAN (17-1-0126-2012)

6. $A(1,-2)$

$B(2,0)$

$C(-1,1)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1)$$



$A\overset{x_1\ y_1}{(1,-2)}$

$B\overset{x_2\ y_2}{(2,0)}$

$$y+2 = \frac{0+2}{2-1} \cdot (x-1)$$

$$y+2 = 2\cdot(x-1)$$

$$y+2 = 2x-2$$

$$y = 2x - 2 - 2$$

$$\boxed{p_1 \ldots y = 2x - 4}$$

$B\overset{x_1\ y_1}{(2,0)}$

$C\overset{x_2\ y_2}{(-1,1)}$

$$y - 0 = \frac{1-0}{-1-2} \cdot (x-2)$$

$$y - 0 = -\frac{1}{3}\cdot(x-2)$$

$$y - 0 = -\frac{1}{3}x + \frac{2}{3}$$

$$\boxed{p_2 \ldots y = -\frac{1}{3}x + \frac{2}{3}}$$

$A\overset{x_1\ y_1}{(1,-2)}$

$C\overset{x_2\ y_2}{(-1,1)}$

$$y+2 = \frac{1+2}{-1-(1)} (x-1)$$

$$y+2 = -\frac{3}{2}\cdot(x-1)$$

$$y+2 = -\frac{3}{2}x + \frac{3}{2}$$

$$y = -\frac{3}{2}x + \frac{3}{2} - 2$$

$$\boxed{p_3 \ldots y = -\frac{3}{2}x - \frac{1}{2}}$$

$$P_1 = \int_{-1}^{1} -\frac{1}{3}x + \frac{2}{3} - \int_{-1}^{1} -\frac{3}{2}x - \frac{1}{2}$$

$$P_1 = -\frac{1}{3}\cdot\frac{x^2}{2} + \frac{2}{3}x + \frac{3}{2}\,\frac{x^2}{2} - \frac{1}{2}x$$

$$P_1 = \left(-\frac{1}{3}\cdot\frac{1^2}{2} + \frac{2}{3}\cdot 1 + \frac{3}{2}\cdot\frac{1^2}{2} - \frac{1}{2}\cdot 1\right) - \left(-\frac{1}{3}\cdot\frac{(-1)^2}{2} + \frac{2}{3}(-1) + \frac{3}{2}\cdot\frac{(-1)^2}{2} - \frac{1}{2}(-1)\right)$$

$$P_1 = \frac{3}{4} - \frac{5}{12} = \frac{1}{3}$$

$P_2 - P_1$

$$P_2 = \int_1^2 -\frac{1}{3}x + \frac{2}{3} - \int_1^2 2x - \frac{2}{3}4$$

$$P_2 = -\frac{1}{3} \cdot \frac{x^2}{2} + \frac{2}{3}x - 2 \cdot \frac{x^2}{2} - 4x \Big|_1^2$$

$$P_2 = -\frac{1}{3} \cdot \frac{2^2}{2} + \frac{2}{3} \cdot 2 - 2 \cdot \frac{2^2}{2} - 4 \cdot 2 - \left(-\frac{1}{3} \cdot \frac{1^2}{2} + \frac{2}{3} \cdot 1 - 2 \cdot \frac{1^2}{2} - 4 \cdot 1\right)$$

$$-\frac{34}{3} + \frac{19}{2} = -\frac{41}{6}$$

$$P_2 = \int_1^2 2x - 4 + \int_1^2 \frac{1}{3}x + \frac{2}{3}$$

$$P_2 = 2 \cdot \frac{x^2}{2} - 4x + \frac{1}{3} \cdot \frac{x^2}{2} + \frac{2}{3}x$$

$$P_2 = \left(2 \cdot \frac{2^2}{2} - 4 \cdot 2 + \frac{1}{3} \cdot \frac{2^2}{2} + \frac{2}{3} \cdot 2\right) - \left(2 \cdot \frac{1^2}{2} - 4 \cdot 1 + \frac{1}{3} \cdot \frac{1^2}{2} + \frac{2}{3} \cdot 1\right)$$

$$P_2 = -2 + \frac{16}{3} \boxed{= \frac{10}{3}}$$

$$\boxed{P = P_1 + P_2}$$

$$P = \frac{1}{3} + \frac{10}{3} = \frac{11}{3}$$

X

3

$$f(x,y) = \ln\left(\frac{y}{x}\right)$$

D_f $\langle 0, +\infty)$?

2.

$$\frac{y'}{x} = \frac{\sin x}{y} \quad / \cdot x \cdot y$$

$$\frac{y'}{y} = \frac{\sin x}{x}$$

X

$$\int \frac{dy}{y} = \int \frac{\sin x}{x}$$