MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OXX

IME I PREZIME: TOMISLAV BULIĆ **BROJ INDEKSA:** 17-2-0271-2013

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Riješi diferencijalnu jednadžbu $(1+e^x)\,yy' = e^x$ uz početni uvjet $y(0)=1$. — 20
2. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x\sin x$. — 15
3. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x,y) = x + 2y + 1$. — 5+5+5
4. Numeričkom integracijom odrediti vrijednost $\int_{-\pi/2}^{\pi/2} \cos x \, dx$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lesssim 3\%$, 8 za rel. grešku $\lesssim 6\%$) — 20
5. $\displaystyle\int_0^2 \frac{2x^2+x+2}{x^2-1}\,dx = ?$ — 15
6. Integriranjem izračunati površinu između krivulja $x=0$ i $y^2 = x+4$. — 15

Ukupno:

MOBITEL U RUCI NEKOLIKO PUTA ZA REDOM U KRATKOM VREMENSKOM INTERVALU I GLEDA ĆAS U MOBITEL ĆAS U ZADAĆU I PIŠE.

STUDENT SE OPRAVDAVA DA JE GLEDAO KOLIKO JE SATI!

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x\,dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

5. $\displaystyle\int_0^2 \frac{2x^2+x+2}{x^2-1}\,dx = \int_0^2 \frac{2x^2}{x^2-1}\,dx + \int_0^2 \frac{x}{x^2-1}\,dx + \int_0^2 \frac{2}{x^2-1}\,dx$

$= 2\int_0^2 \frac{x^2}{x^2-1}\,dx + \int_0^2 \frac{x}{x^2-1}\,dx + 2\int_0^2 \frac{dx}{x^2-1}$

$x^2 - 1 = t$

$dt = 2x\,dx$

$dx = \frac{dt}{2x}$

$= 2\int_0^{1} \frac{x^2}{t+1}\frac{dt}{2x}$

→

~~4~~ $x=0$

$y^2 = x+4$

$y = 0$

$x^2 = -y+4 \Rightarrow y = -x^2 - 4$

$-x+4=0$

$x = \sqrt{4}$

$x_1 = 2$

$x_2 = -2$

x	0
y	0

$y = -x^2+4$

x	1		2
y	3	4	0

$P = \int_{-2}^{2} -x^2 + 4 - 0$

$P = \int_{-2}^{2} \frac{-x^3}{3} + 4x$

$P = -\frac{2^3}{3} + 4\cdot 2 - \frac{(-2)^3}{3} - 4(-2)$

$P = -\frac{8}{3} + 8 + \frac{8}{3} + 8$ DRUGI PAPIR

$P = 16$

(5) $\int_0^2 \frac{2x^2+x+2}{x^2-1}\,dx = \quad 2x^2+x+2 : (x^2-1) = 2 + \frac{x+4}{x^2-1}$

$\quad -2x^2+2$

$\quad x+4$

$= \int_0^2 \frac{2x^2+x+2}{x^2-1}\,dx = \int_0^2 \frac{x+4}{x^2-1} + 2$

$\frac{x+4}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1} \Big/ \cdot (x-1)(x+1) \quad x+4 = Ax + Bx + A - B$

$A+B=1$
$A-B=4$ $\Rightarrow A = \frac{5}{2}$, $B = -\frac{3}{2}$

$\int_0^2 2 + \int_0^2 \frac{\frac{5}{2}}{x-1} - \frac{\frac{3}{2}}{x+1} = \int_0^2 2 + \frac{5}{2}\int_0^2 \frac{1}{x-1} - \frac{3}{2}\int_0^2 \frac{1}{x+1} =$

$\Big|_0^2 2x - \frac{3}{2}\Big|_0^2 \ln|x+1| + \frac{5}{2}\Big|_0^{1-} \ln|x-1| + \frac{5}{2}\Big|_{1+}^2 \ln|x-1| =$

$4 - \frac{3}{2}\ln|3| + (-\infty) - (-\infty)$

$= 4 - \frac{3}{2}\ln|3| + \infty - \infty \quad N/P$

TOMISLAV BUDIC

(2) $4y'' - y = 2x \sin x$

y_H

$4r^2 - r = 0$

$r(4r-1) = 0$

$r_1 = 0 \quad 4r-1=0$

$r_2 = \frac{1}{4}$

$y_H = C_1 \cdot e^{0x} + C_2 \cdot e^{\frac{1}{4}x}$

y_P

$f(x) = 2x \sin x$

$f(x) = e^{0x}[0 \cdot \cos x + 2x \sin x]$

$a = 0 \quad P_k(x) = 0 \quad Q_l(x) = 2x$

$b = 1 \quad k = 0 \quad l = 1$

$t = 1$

$a \pm bi = 0 \pm 1i \rightarrow$ NIJE RJEŠENJE KARAKTERISTIČNE JEDNADŽBE

$y_P = e^{0x}[(Ax+B)\cos x + (Cx+D)\cos x]$

$y = y_H + y_P$

$y = C_1 + C_2 \cdot e^{\frac{1}{4}x} + [(Ax+B)\cos x + (Cx+D)\cos x]$

(3) $f(x,y) = x+2y+1$

$D_f = \mathbb{R}^2 \qquad K_f = \mathbb{R}^2$

$D_f = \{(x,y) \mid x \in R ; y \in R\} \qquad K_f = \{(x,y$

→

6. $x=0$
$y^2=x+4$ $\Rightarrow$ ZAMJENA x sa y

$\Rightarrow y=0$

$x^2=y+4$

$y=0$

$y=x^2-4$

$\Rightarrow x^2-4=0 \Rightarrow x^2=4$

$x_1=2$

$x_2=-2$

$$\int_{-2}^{2} -x^2+4 = -\int_{-2}^{2} x^2 + \int_{-2}^{2} 4 = \frac{1}{3}\Big|_{-2}^{2} -x^3 + 4\Big|_{-2}^{2} x$$

$$= -\frac{1}{3}\cdot(8+8)+4\cdot(2+2) = -\frac{16}{3}+16 = \frac{32}{3}$$

1. $(1+e^x)yy' = e^x$

$y\frac{dy}{dx} = \frac{e^x}{1+e^x} \Big| dx$

$\int y dy = \int \frac{e^x}{1+e^x} dx =$

$\frac{1}{2}y^2 = \int \frac{e^x}{1+e^x} dx = \Big| \begin{matrix} 1+e^x=t \\ e^x dx = dt \end{matrix}$

$\frac{1}{2}y^2 = \int \frac{dt}{t}$

$\frac{1}{2}y^2 = \ln|1+e^x| + \ln c$

$\frac{1}{2}y^2 = \ln|1+e^x| \cdot c$

$y(0)=1$

$y(0)=1$

$\frac{1}{2}\cdot 1^2 = \ln|1+e^0| \cdot c$

$\frac{1}{2} = \ln 2 \cdot c \,|\cdot 2$

$1 = 2\ln 2 \cdot c$

$c = \frac{1}{2}\ln 2$

$\frac{1}{2}y^2 = \ln(1+e^x)\frac{1}{2}\ln 2$

$\rightarrow$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OXX

IME I PREZIME: DOMAGOJ LOTOVAC **BROJ INDEKSA:** 0269082198

POPUNJAVA NASTAVNIK | Broj ↓ bodova

1. Riješi diferencijalnu jednadžbu $(1+e^x)\,yy' = e^x$ uz početni uvjet $y(0)=1$. — 20
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6. Integriranjem izračunati površinu između krivulja $x=0$ i $y^2 = x+4$. — 15

Ukupno: 35

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha>0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
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Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x\,dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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1. $(1+e^x)\,yy' = e^x \qquad y(0)=1$

$$yy' = \frac{e^x}{1+e^x}$$

$$y\frac{dy}{dx} = \frac{e^x}{1+e^x}$$

$$y\,dy = \frac{e^x}{1+e^x}\,dx \quad \Big/ \int$$

$$\frac{1}{2}y^2 = \int \frac{e^x}{1+e^x}\,dx$$

$$\int \frac{e^x}{1+e^x}\,dx = \left| \begin{matrix} 1+e^x = t \ /' \\ e^x\,dx = dt \end{matrix} \right| = \int \frac{1}{t}\,dt = \ln\lvert t\rvert + C = \ln\lvert 1+e^x\rvert + C$$

$$\frac{1}{2}y^2 = \ln\lvert 1+e^x\rvert + C$$

$$\frac{1}{2} = \ln\lvert 2\rvert + C$$

$$C = \frac{1}{2} - \ln\lvert 2\rvert$$

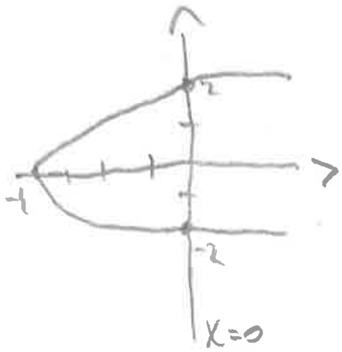
$$\boxed{\frac{1}{2}y^2 = \ln\lvert 1+e^x\rvert + \frac{1}{2} - \ln\lvert 2\rvert}$$

6.
$x = 0$

$y^2 = x+4$

$y = \pm\sqrt{x+4}$

x	0	-4
y	± 2	0



$$P = 2 \cdot \int_{-4}^{0} \sqrt{x-4}\, dx$$

$$\int \sqrt{x+4}\, dx = \left| \begin{matrix} x+4 = t \;/' \\ dx = dt \end{matrix} \right| =$$

$$= \int \sqrt{t}\, dt = \int t^{1/2}\, dt = \frac{2}{3} t^{3/2} + C$$

$$= \frac{2}{3}\sqrt{(x+4)^2} + C$$

$$P = 2 \cdot \int_{-4}^{0} \sqrt{x+4}\, dx = 2 \cdot \left(\frac{2}{3}\sqrt{(x+4)^2} \right) \Big|_{-4}^{0}$$

$$= 2 \cdot \left(\frac{2}{3}\sqrt{(0+4)^3} - \frac{2}{3}\sqrt{(-4+4)^3} \right) = 2 \cdot \left(\frac{2}{3}\sqrt{64} - \frac{2}{3}\sqrt{0^3} \right) = \frac{4}{3} \cdot (8-0) = \frac{32}{3}$$

⑤ $\int_0^2 \frac{2x^2+x+2}{x^2-1}\,dx = \int_0^2 \left(2+\frac{x+4}{x^2-1}\right)dx = \int_0^2 2\,dx + \int_0^2 \frac{x+4}{x^2-1}\,dx =$

$= 2x\Big|_0^2 + \int_0^2 \left[-\frac{3}{2(x+1)} + \frac{3}{2(x-1)}\right]dx = 2\cdot 2 - 2\cdot 0 + \int_0^2 -\frac{3}{2(x+1)}\,dx + \int_0^2 \frac{5}{2(x-1)}\,dx$

$= 4 - \frac{3}{2}\int_0^2 \frac{1}{x+1}\,dx + \frac{5}{2}\int_0^2 \frac{dx}{x-1} =$

1 ES V.A. OCULTAR [0,2]

$= 4 - \frac{3}{2}\ln|x+1|\Big|_0^2 + \frac{3}{2}\ln|x-1|\Big|_0^2 =$

$= 4 - \frac{3}{2}\left(\ln|2+1| - \ln|0+1|\right) + \frac{5}{2}\left(\ln|2-1| - \ln|0-1|\right) =$

$= 4 - \frac{3}{2}(\ln 3 - 0) + \frac{5}{2}(\ln|1| - \ln 1) =$

$= 4 - \frac{3}{2}\ln 3$

$2x^2+x+2 \,|\, (x^2-1) = 2$

$-2x^2+2$

$x+2+2$

$x+4$

$\frac{x+4}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1} \quad | (x+1)(x-1)$

$x+4 = Ax - A + Bx + B$

$x+4 = x(A+B) + B - A$

$\left.\begin{array}{l} A+B=1 \\ -A+B=4 \end{array}\right\} +$

$2B = 5$

$B = \frac{5}{2}$

$A = 1 - B$

$A = 1 - \frac{5}{2} = -\frac{3}{2}$

$\frac{x+4}{x^2-1} = -\frac{3}{2(x+1)} + \frac{5}{2(x-1)}$

DOMAGOJ KOTOVAC

(2) $4y'' - y = 2x \sin x$

$$\underbrace{2x \sin x}_{y_1}$$

$4\lambda^2 - 1 = 0$

$\lambda^2 = \frac{1}{4}$

$\lambda = \pm \frac{1}{2}$

$y_0 = C_1 e^{\frac{1}{2}x} + C_2 e^{-\frac{1}{2}x}$

$y = y_0 + y_1$

$y = C_1 e^{\frac{1}{2}x} + C_2 e^{-\frac{1}{2}x} + Ax + B$ X

$f(x) = 2x \sin x = 2x \sin x \; e^{0}x$

$a = 0 \; , \; f(a) \neq 0$

$P_n = 2x \, , \; n = 1$

$Q_{1n} = Ax + B$

$Q_n' = A$

$Q_n'' = 0$

$4Q_{1n}'' - Q_n = 2x \sin x$

$-Ax - B = 2x \sin x$

$B = 0$

$A = -2 \sin x$

$y = C_1 e^{\frac{1}{2}x} + C_2 e^{-\frac{1}{2}x} - 2x \sin x$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OXX

IME I PREZIME: HATE RADAŠ **BROJ INDEKSA:** 17-2-0183-2012

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješi diferencijalnu jednadžbu $(1+e^x)\,yy' = e^x$ uz početni uvjet $y(0)=1$. 20
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$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
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$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

6.) $x = 0$ $\quad f(y) = x$

$y^2 = x + 4$ $\quad g(y) = y^2$

$y^2 = 4 \;/\sqrt{\ }$

$y = \pm 2$

$y_1 = 2 \Rightarrow a = 2$

$y_2 = -2 \Rightarrow b = -2$

$$\int_a^b f(y) - g(y)\,dx = \int_2^{-2} x - y^2\,dy = \int_2^{-2} 0 - (x+4)\,dx =$$

$$= -\frac{x^2}{2} - 4x \Big|_{2}^{-2} = \left(-\frac{(-2)^2}{2} - 4\cdot(-2)\right) - \left(-\frac{(2)^2}{2} - 4\cdot 2\right) = -2 + 8 + 2 + 8 =$$

$$= 16$$

4.) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx = \sin x \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \sin\frac{\pi}{2} - \sin(-\frac{\pi}{2}) = 1-(-1) = 2$

TRAŽI SE NUMERIČKA INTEGRACIJA. -

5.) $\int_0^2 \frac{2x^2+x+1}{x^2-1} dx = \left| \begin{matrix} x^2-1=u \\ du = 2x \\ dv \ \ 2x^2+x+1 \\ v = \frac{2x^3}{3}+1 \end{matrix} \right| = uv - \int v \, du \, dx$

$\varnothing$

$= (x^2-1) \cdot \frac{2x^3}{3} + 1 - \int \left(\frac{2x^3}{3}+1\right) \cdot 2x \, dx = \frac{2x^5}{3} + x^2 - \frac{2x^3}{3} - 1 -$

$\int \frac{4x^4}{3} + 2x \, dx = \frac{2x^5}{3} + x^2 - \frac{2x^3}{3} - 1 - \left(\left(\frac{4}{3} \cdot \frac{x^5}{5}\right) + \left(2 \cdot \frac{x^2}{2}\right)\right)\Big|_0^2 =$

$= \frac{2x^5}{3} + \cancel{x^2} - \frac{2x^3}{3} - 1 - \frac{4x^5}{15} - \cancel{x^2} \Big|_0^2 =$

$= \left(\frac{2 \cdot 2^5}{3} - \frac{2 \cdot 2^3}{3} - 1 - \frac{4 \cdot 2^5}{15}\right) - \left(\frac{2 \cdot 0^5}{3} - \frac{2 \cdot 0^3}{3} + 1 - \frac{4 \cdot 0^5}{15}\right) =$

$= \left(\frac{64}{3} - \frac{16}{3} - 1 - \frac{128}{15}\right) + 1 =$

$= \left(\frac{320 - 80 - 15 - 128}{15}\right) + 1 = \frac{97}{15} + 1 = \frac{97+15}{15} = \frac{112}{15}$

1.) $(1+e^x)yy' = e^x$ uz početni uvjet $y(0)=1 \Rightarrow$ diferencijalna jednadžba

$$y\,dy = \frac{e^x}{1+e^x}dx \Big/ \int$$

$$\int y\,dy = \int \frac{e^x}{1+e^x}dx$$

$$\frac{y^2}{2} = \int \frac{e^x}{1+e^x} = \begin{vmatrix} 1+e^x = t \\ e^x = dt \end{vmatrix} = \int \frac{dt}{t} = \ln|t| + c = \ln|1+e^x| + c$$

$$\frac{y^2}{2} = \ln|1+e^x| + c$$

$$y = \sqrt{2\ln|1+e^x| + 2c}$$

$$y = \sqrt{2\ln|1+e^x| + 0.39}$$

$y(0)=1$

$$1 = \sqrt{1.39 + 2c} \Big/ ^2$$

$$1 = 1.39 + 2c$$

$$0.39 = 2c \Big/ :2$$

$$c = 0.19$$

2.) $4y'' - y = 2x\sin x \Rightarrow$ diferencijalna jednadžba

MATE RADAŠ

3. $f(x,y) = x + 2y + 1$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** oxx	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: Goran Kovaček BROJ INDEKSA: 17-1-0219-2013	

1. Riješi diferencijalnu jednadžbu $(1+e^x)\,yy' = e^x$ uz početni uvjet $y(0)=1$. 20
2. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x\sin x$. 15
3. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x,y) = x + 2y + 1$. 5+5+5
4. Numeričkom integracijom odrediti vrijednost $\int_{-\pi/2}^{\pi/2} \cos x \, dx$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lesssim 3\%$, 8 za rel. grešku $\lesssim 6\%$) 20
5. $\int\limits_0^2 \frac{2x^2+x+2}{x^2-1} dx = ?$ 15
6. Integriranjem izračunati površinu između krivulja $x=0$ i $y^2 = x+4$. 15

Ukupno: $\emptyset$

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

2. $4y'' - y = 2x\sin x$

$4r^2 - r = 0$

$r(4r-1) = 0$

$\boxed{r_1 = 0}$ $\quad 4r - 1 = 0$

$4r = 1$

$\boxed{r_2 = \frac{1}{4}}$

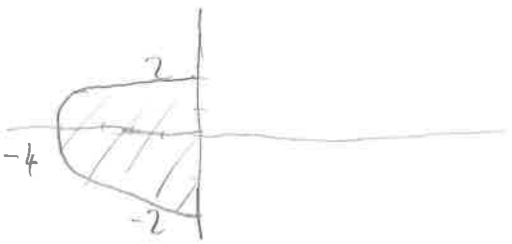
$y_{H(x)} = C_1 \cdot e^{0 \cdot x} + C_2 \cdot e^{\frac{1}{4} \cdot x}$

$2x\sin x = e^{\alpha x}(P_n(x) \cdot \cos(\beta x) + Q_n(x) \cdot \sin(\beta x))$

$\alpha = 0$; $\beta = 1$; $n = 1$

?

6. $p = \int_{-2}^{2} y^2 - 4 \, dy = \frac{1}{3} 4^2 - 4 \cdot 4 \Big|_{-2}^{2} = -\frac{32}{3}$ X



1. $(1+e^x)yy' = e^x \quad / : (1+e^x)$

$$yy' = \frac{e^x}{1+e^x}$$

4. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx = \left[\sin x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} =$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** oxx

IME I PREZIME: SANDRO GRDOVIĆ **BROJ INDEKSA:** 17-2-0213-2012

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Riješi diferencijalnu jednadžbu $(1+e^x)\,yy' = e^x$ uz početni uvjet $y(0)=1$. ~~20~~
2. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x\sin x$. 15
3. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x,y) = x + 2y + 1$. ~~5+5+5~~
4. Numeričkom integracijom odrediti vrijednost $\int_{-\pi/2}^{\pi/2}\cos x\,dx$. (bodovanje: 20 za rel. grešku $\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$) 20
5. $\displaystyle\int_0^2 \frac{2x^2+x+2}{x^2-1}\,dx = ?$ ~~15~~
6. Integriranjem izračunati površinu između krivulja $x=0$ i $y^2 = x+4$. ~~15~~

Ukupno: $\emptyset$

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha>0)$	$\alpha^x\ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq -1$	$\int \tan x\,dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\,dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\|x+\sqrt{x^2\pm a^2}\right\| + C$
$\int e^x\,dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

1. $(1-e^x)yy' = e^x, \quad y(0)=1$

$yy' = \frac{e^x}{1+e^x} \qquad y' = \frac{dx}{dy}$

$y\frac{dy}{dx} = \frac{e^x}{1+e^x} \quad / \, dx$

$y\,dy = \frac{e^x}{1+e^x}\,dx \quad / \int$

$\int y\,dy = \int \frac{e^x}{1+e^x}\,dx$

$\int y\,dy = \frac{y^2}{2} \quad ||$

$\int \frac{e^x}{1+e^x}\,dx \quad \left\{ \begin{matrix} 1+e^x = t \\ e^x dx = dt \end{matrix} \right\} = \int dt$

$= \ln|t| + C$

$= \ln|1+e^x|$

$\frac{y^2}{2} = \ln|1+e^x| + C$

$y(0) = 1$

$(0,1)$

$\frac{1}{2} = \ln|1+1| + C$

$\frac{1}{2} = \ln 0.69 + C$

$C = -0.19$

3. $f(x,y) = x + 2y + 1$

Domena $\quad f(x,3) = c$

$x - 2y + 1 = c$

$c = 0$

$x + 2y + 1 = 0$

$2y = -x - 1 \;/2$

$y = -\frac{x}{2} - \frac{1}{2}$

x	2	3	-1
y	$-\frac{3}{2}$	-1	0

$c = 1$

$x + 2y + 1 = 1$

$2y = -x$

$y = -\frac{x}{2}$

x	1	2	-1
y	$-\frac{1}{2}$	-1	$\frac{1}{2}$

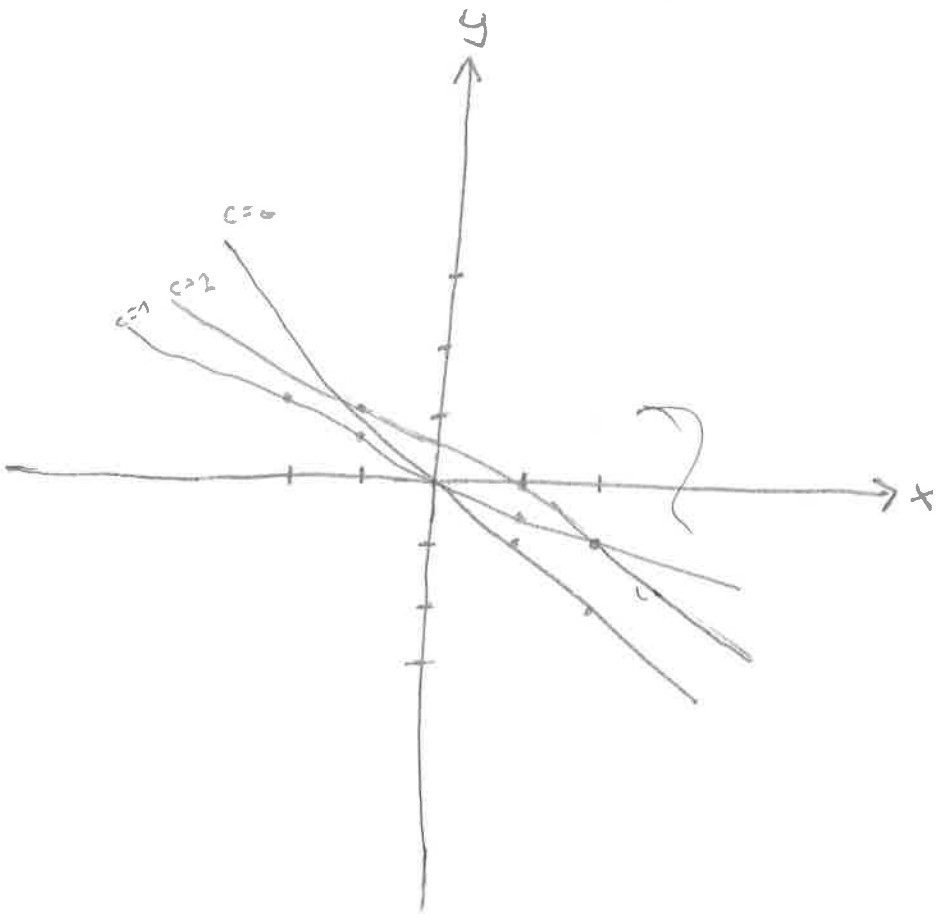
$c = 2$

$x + 2y - 1 = 2$

$2y = -x - 1 \;/2$

$y = -\frac{x}{2} + \frac{1}{2}$

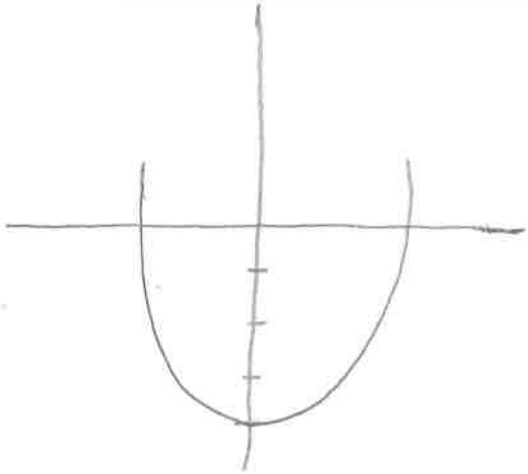
x	1	-1	2
y	0	1	$-\frac{1}{2}$



SANDRO GRDOVIĆ

6. Zamjena varijabli

$y = 0 \qquad x^2 = y + 4$



$$\int_{-2}^{2} (x^2 - 4)\, dx$$

$$= \frac{-32}{3} \times$$

$x^2 = y + 4$

$= dy = x^2 - 4$

5 $$\int_0^2 \frac{2x^2 + x + 2}{x^2 - 1}$$

$2x^2 + x : x^2 - 1 =$

$2x^2 - 2$

$x + 4$

$$\int_0^2 \left[\left(\frac{x+4}{x^2-1}\right) + 2\right] dx$$

$$\int_0^2 \left(\frac{x}{x^2-1} + \frac{4}{x^2-1} + 2 \right) dx$$

$$\int_0^2 \frac{x}{x^2-1}\, dx = \left\{ \begin{array}{l} x^2 - 1 = t \\ 2x\, dx = dt \\ x\,dx = \frac{dt}{2} \end{array} \right\} = \int \frac{\frac{dt}{2}}{t} = \frac{1}{2} \int \frac{dt}{t}$$

$$= \frac{1}{2} \ln |x^2 - 1|$$

$$\int \frac{4}{x^2-1}\, dx = 4 \int \frac{dx}{x^2-1} = \cancel{4}\, \frac{1}{\cancel{2}} \ln \left| \frac{x-1}{x+1} \right| + C$$

$$= \frac{1}{2} \ln |x^2 - 1| \Big|_0^2 + 2 \ln \left| \frac{x-1}{x+1} \right| \Big| + 2$$

$$= \left(\frac{1}{2} \ln |3| - \frac{1}{2} \ln \underbrace{|-1|}_{=1} + \left(2 \ln \left|\tfrac{1}{3}\right| + 2 \ln \left| \tfrac{-1}{1} \right| + 2 \right)\right.$$

NEPRAVI

NEPRAVI

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	OXX
IME I PREZIME: STJEPAN KOŠTA	**BROJ INDEKSA:** 17-2-0187-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi diferencijalnu jednadžbu $(1+e^x)\,yy' = e^x$ uz početni uvjet $y(0) = 1$. — 20
2. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x\sin x$. — 15
3. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x, y) = x + 2y + 1$. — 0 5+5+5
4. Numeričkom integracijom odrediti vrijednost $\int_{-\pi/2}^{\pi/2}\cos x\,dx$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$) — 20
5. $\displaystyle\int_0^2 \frac{2x^2+x+2}{x^2-1}\,dx = ?$ — 15
6. Integriranjem izračunati površinu između krivulja $x = 0$ i $y^2 = x + 4$. — 15

Ukupno: 5

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
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Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\,dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\|x+\sqrt{x^2\pm a^2}\right\| + C$
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$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

1.) $(1+e^x)\,yy' = e^x$

$yy' = \frac{e^x}{1+e^x}$

$Y_0 = C_1 e^{-x} + C_2 e^{0x}$

$Y_0 = C_1 e^{-x} + C_2$

$1+k=0$

$\boxed{k=-1}$

$Y = \frac{e^x(Cx^2+D^2+1)}{(1+e^x)(A+AD-CB)}$

$Y' = \frac{A\cdot(1+Cx+D)-(Ax+B)\cdot C}{(Cx+D+1)^2} = \frac{A+ACx+AD-ACx-CB}{Cx^2+D^2+1} = \frac{A+AD-CB}{Cx^2+D^2+1}$

$Y = Y_0 + Y$
$= C_1e^{-x} + C_2 + \frac{e^x(Cx^2+D^2+1)}{(1+e^x)(A+AD-CB)}$

2.) $4y'' - y = 2x \sin x$

$4k^2 - 1 = 0$

$k^2 = \frac{1}{4}$

$k_1 = \frac{1}{2} \quad k_2 = -\frac{1}{2}$

$Y_0 = C_1 e^{\frac{1}{2}x} + C_2 e^{-\frac{1}{2}x}$

$Y = \cancel{(Ax+B)} + C\cos x + D\sin x$ ×

$Y' = A - C\sin x + D\cos x$

$Y'' = -C\cos x - D\sin x$

$-4C\cos x - 4D\sin x - (Ax+B) - C\cos x - D\sin x = 2x\sin x$

$-5C\cos x - 5D\sin x - Ax - B = 2x\sin x$

$A = -2$

$B = 0$

$-5C = 0 \Rightarrow \boxed{0}$

$-5D = 1$

$\boxed{D = \frac{1}{5}}$

$Y = Y_0 + Y$

$\boxed{= C_1 e^{\frac{1}{2}x} + C_2 e^{-\frac{1}{2}x} + \frac{1}{5}\sin x - 2}$ ×

3.) $f(x,y) = x + 2y + 1$

$D_f \in \mathbb{R}$ ✓

$f'_x = 2y \qquad f'_x = 0 \Rightarrow y = 0 \qquad T(-2,0)$?

$f'_y = x + 2 \qquad f'_y = 0 \Rightarrow x = -2$

$f''_{xx} = 0 \qquad A = f''_{xx}(-2,0) = 0 \qquad \Delta = AC - B^2$

$f''_{xy} = 1 \qquad B = f''_{xy}(-2,0) = 1 \qquad = 0 - 1$

$f''_{yy} = 0 \qquad C = f''_{yy}(-2,0) = 0 \qquad = -1$

$A; C < 0$

MAXIMUM

$MAX(-2, 0, -1)$ ×

STJEPAN KOŠTA

5.) $\int_0^2 \frac{2x^2+x+2}{x^2-1}\,dx = \rightarrow \frac{1}{2}\ln|0|+C$

$\int F(1)-F(0) = \int \frac{2\cdot 2^2+2+2}{\frac{1}{2}\ln|0|} - \frac{2\cdot 0^2+0+2}{\frac{1}{2}\ln|0|}$

$= \int \frac{12}{\frac{1}{2}\ln|0|} - \frac{2}{\frac{1}{2}\ln|0|}$

$= \int \frac{10}{\frac{1}{2}\ln|0|}$

$= 10$