MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO

IME I PREZIME: Antonijo Knežević **BROJ INDEKSA:** 57672

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Nađi partikularno rješenje koje zadovoljava: $xy' + y - e^x = 0$ i $y(1) = 1$. — 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = e^x - x + y^2 + 2y$. — 15
3. Skicirati razinske krivulje za funkciju $f(x,y) = x^2 - y$. — 15
4. $\int_0^\pi \frac{\sin x}{\cos x + 5} dx$? — 20
5. Izračunaj površinu lika omeđenog krivuljama $f(x) = x^2 - 3x - 4$ i $g(x) = -x^2 + 3x + 4$. — 15
6. Izračunaj $\int_1^4 \frac{1+\sqrt{x}}{x^2} dx$ i napravi provjeru izračunom aproksimacije integrala Simsponovom metodom. — 10+10

Ukupno: 45

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha\, dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x\, dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\, dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

2.) $f(x,y) = e^x - x + y^2 + 2y$

$z = e^x - x + y^2 + 2y$

$\frac{\partial z}{\partial x} = e^x - 1$ $\qquad \frac{\partial z}{\partial y} = 2y + 2$

$\left.\begin{array}{l} e^x - 1 = 0 \\ 2y + 2 = 0 \end{array}\right\}$ $\quad e^x = 1 \Rightarrow x = 0$, $\ y = -1$

$T(0,-1)$ stacionalna točka

$\frac{\partial^2 z}{\partial x^2} = e^x$

$\frac{\partial^2 z}{\partial x \partial y} = 0$

$\frac{\partial^2 z}{\partial y^2} = 2$

→

$$d^2z = e^x(dx)^2 - 0 + 2(dy)^2$$

$$\begin{vmatrix} e^x & 0 \\ 0 & 2 \end{vmatrix} = 2e^x > 0$$

MINIMUM U $T(0,-1)$ ✓

4) $$\int_0^{\pi} \frac{\sin x}{\cos x + 5}\,dx = \left[\begin{array}{l} t = \cos x + 5 \\ dt = -\sin x\,dx \\ dx = \frac{dt}{-\sin x} \end{array}\right]$$

$$= \int_0^{\pi} \frac{\cancel{\sin x}}{t} \cdot \frac{dt}{-\cancel{\sin x}} = \int_0^{\pi} -\frac{1}{t}\,dt =$$

$$= -\ln|t| \Big|_0^{\pi} = -\ln|\cos x + 5| \Big|_0^{\pi}$$

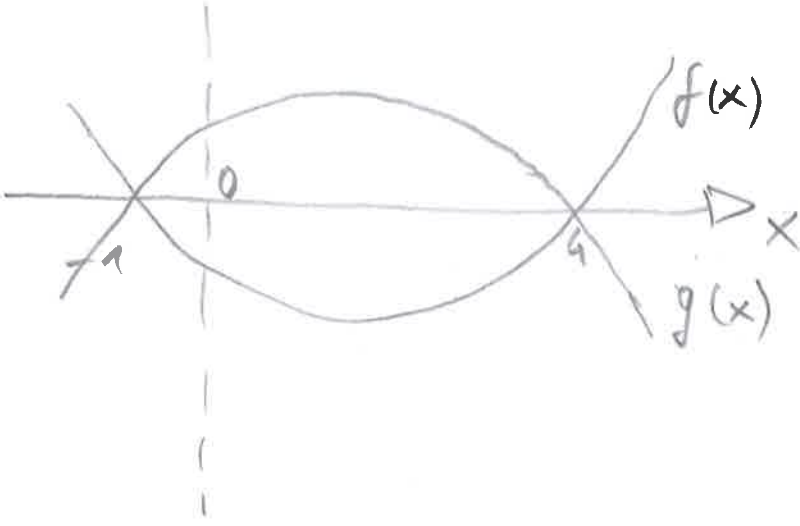
$$= -\ln|\cos\pi + 5| + \ln|\cos 0 + 5|$$

$$= -\ln|4| + \ln|6| = \ln 6 - \ln 4$$

$$= \ln\frac{6}{4}$$ ✓

5) $f(x) = x^2 - 3x - 4$

$$x_{1,2} = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2}$$



$$P = \int_{-1}^{4} \left[(-x^2 + 3x + 14) - (x^2 - 3x - 4) \right] dx$$

$$= \int_{-1}^{4} (-2x^2 + 6x + 8)\, dx$$ ✗

$$= -2 \frac{x^3}{3} \Big|_{-1}^{4} + 6 \frac{x^2}{2} \Big|_{-1}^{4} + 8x \Big|_{-1}^{4}$$

$$= \left(-2 \cdot \left(\frac{64}{3} - \frac{1}{3}\right) + 6 \cdot \left(8 - \frac{1}{2}\right) + 32 - 8\right.$$

$$= -2 \cdot 21 + 48 - 3 + 32 - 8 = 27$$

6) $\int_1^4 \frac{1+\sqrt{x}}{x^2}\,dx$

CANTONIJO KNEŽEVIĆ

$= \underbrace{\int_1^4 \frac{1}{x^2}\,dx}_{I_1} + \underbrace{\int_1^4 \frac{\sqrt{x}}{x^2}\,dx}_{II_2}$

$\frac{x^{\frac{1}{2}}}{x^2} = x^{\frac{1}{2}-2} = x^{\frac{1-4}{2}} = x^{-\frac{3}{2}}$

$= -\frac{1}{x}\Big|_1^4 + \int_1^4 x^{-\frac{3}{2}}\,dx$

$= -\frac{1}{4} + 1 + \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}\Big|_1^4$

$= \frac{3}{4} - \frac{2}{\sqrt{x}}\Big|_1^4 = \frac{3}{4} - 1 + 2 = -\frac{3}{4} + 1 = \frac{1}{4}$ ✓

1. HOMOGENA.

ANTONIJO KNEZEVIĆ

$$xy' + y = 0$$

$$x\frac{dy}{dx} + y = 0$$

$$x\,dy = -y\,dx$$

$$\frac{dy}{y} = -\frac{dx}{x}$$

$$\ln|y| = -\ln|x| + \ln|c|$$

$$y = \frac{c}{x}$$

$$y' = \frac{c'x - c}{x^2}$$

$$x \cdot \frac{c'x - c}{x^2} + \frac{c}{x} = 0$$

$$c'x = 0$$

$$c' = 0 \quad / \int$$

$$c = x$$

$$y = 1 \qquad x + 1 = e^x$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO

IME I PREZIME: MARKO ČULINA **BROJ INDEKSA:** A-1-0008-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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6. Izračunaj $\displaystyle\int_1^4 \frac{1+\sqrt{x}}{x^2}dx$ i napravi provjeru izračunom aproksimacije integrala Simsponovom metodom. ~~10+10~~

Ukupno: 30

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
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2) $f(x,y) = e^x - x + y^2 + 2y$

$\frac{\partial f}{\partial x} = e^x - 1 = 0 \rightarrow e^x = 1 \rightarrow x = 0$

$\frac{\partial f}{\partial y} = 2y + 2 = 0 \rightarrow 2y = -2$

$y = -1$

$\frac{\partial^2 f}{\partial x^2} = e^x \rightarrow A = 1$

$\frac{\partial^2 f}{\partial x \partial y} = 0 \rightarrow B = 0$

$\frac{\partial^2 f}{\partial y^2} = 2 \rightarrow C = 2$

$\Delta = AC - B^2$

$\Delta = 1 \cdot 2 - 0^2$

$\Delta = 2$

$A > 0$ - minimum

$f(0,-1) = e^0 - 0 + 1^2 + 2 \cdot 1 = 0$

$T(0,-1,0)$

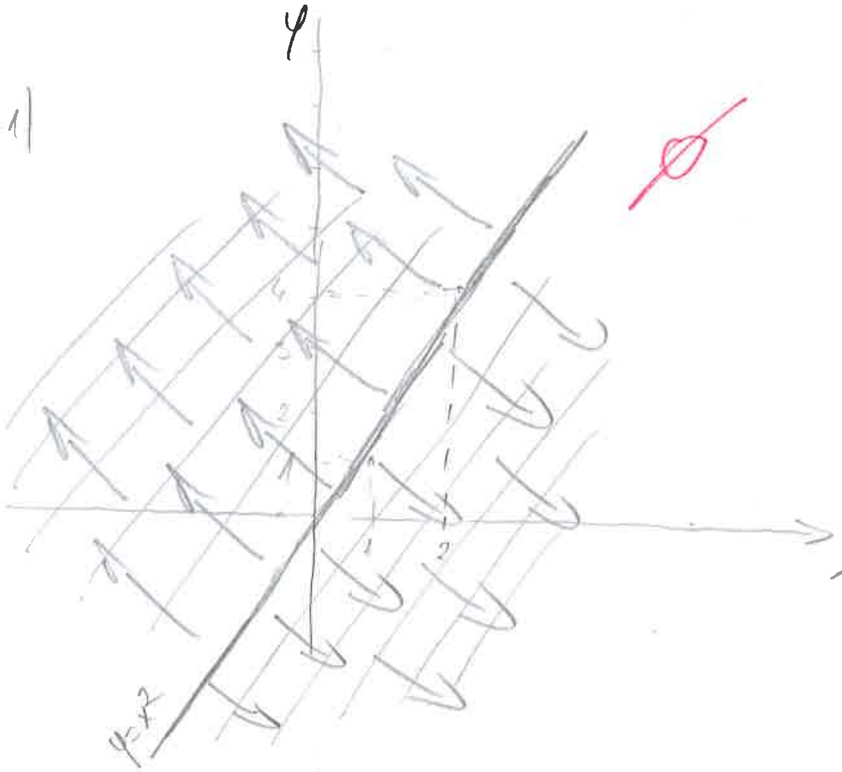
3) $f(x,y) = x^2 - y$

$x^2 - y = 0$

$-y = -x^2 \quad /(-1)$

$y = x^2$

x	0	1	2	3
y	0	1	4	9



5)

$f(x)\ x^2 - 3x - 4 \qquad g(x) = -x^2 + 3x + 4$

$x^2 - 3x - 4 = -x^2 + 3x + 4$

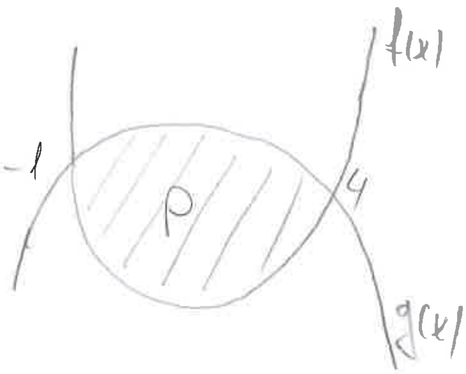
$x^2 - 3x - 4 + x^2 - 3x - 4 = 0$

$2x^2 - 6x - 8 = 0$

$x_{1,2} = \dfrac{6 \pm \sqrt{(-6)^2 - 4\cdot 2\cdot(-8)}}{4}$

$x_{1,2} = \dfrac{6 \pm 10}{4}$

$x_1 = 4 \qquad x_2 = -1$



$P = \int_{-1}^{4} (-x^2 + 3x + 4) - (x^2 - 3x - 4) = \int_{-1}^{4} (-x^2 + 3x + 4 - x^2 + 3x + 4)$

$P = \int_{-1}^{4} (-2x^2 + 6x + 8) = \left(-2\dfrac{x^3}{3} + 6\dfrac{x^2}{2} + 8x\right)\Big|_{-1}^{4}$

$P = \left(-2\dfrac{4^3}{3} + 6\cdot\dfrac{4^2}{2} + 8\cdot 4\right) - \left(2\dfrac{(-1)^3}{3} + 6\cdot\dfrac{(-1)^2}{2} + 8\cdot(-1)\right)$ ✓

$P = \dfrac{112}{3} + \dfrac{17}{3} = 43$

MARKO ČULINA – 17-1-0008-2010

$x^2 \neq 0$

6) $\int_1^4 \frac{1+\sqrt{x}}{x^2}$

$$\int_1^4 \frac{1+\sqrt{x}}{x^2}dx \left| \begin{matrix} x = t \\ dx = dt \end{matrix} \right| = \int_1^4 \frac{1+\sqrt{t}}{t^2}dt \int_1^4 \frac{1}{t^2} + \frac{t^{\frac{1}{2}}}{t^2}dt \int_1^4 \frac{1}{t^2} - t^{-\frac{3}{2}}dt$$

$$= \frac{t^{-1}}{-1} - \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}}\Bigg|_1^4 = \left(\frac{4^{-1}}{-1} - \frac{4^{-\frac{1}{2}}}{-\frac{1}{2}}\right) - \left(\frac{1^{-1}}{-1} - \frac{1^{-\frac{1}{2}}}{-\frac{1}{2}}\right) = \frac{3}{4} - 1 = -\frac{1}{4}$$

4) $$\int_0^{\pi} \frac{\sin x}{\cos x + 5}dx = \left| \begin{matrix} \sin x = t & x = \pi \ldots t = 0 \\ \cos x\, dx = dt & x = 0 \ldots t = 0 \end{matrix} \right| = \int_0^0 \frac{t}{5}dt = N/P$$

$\emptyset$

1) $xy' + y - e^x = 0$ $\qquad y(1) = 1$ MARKO ČULINA

$xy' + y = e^x \quad / \cdot \frac{1}{\cancel{x}}$

$y' + \frac{1}{x} y = \left(\frac{1}{x} \cdot e^x\right)$

$P(x) = \frac{1}{\cancel{x}} \qquad g(x) = \frac{e^x}{x}$

$\int P(x) = \frac{1}{x} = \ln|x| \qquad g(x) = \int e^{\ln|x|} \cdot \frac{e^x}{x}$

$g(x) = x \frac{x^2}{2} \cdot \frac{e^x}{x}$

$y = e^{-\ln|x|} \cdot \left(e^{\ln|x|} \cdot \frac{x^2}{2} \cdot \frac{e^x}{x}\right) + C$

$y = x^{-1}\left(x \cdot \frac{x^2}{2} \cdot \frac{e^x}{x}\right) + C$

$y(1) = 1 \qquad \cancel{x}^{-1}\left(1 \cdot \frac{x^2}{2} \cdot \frac{e^x}{x}\right) + C = 1$

$1\left(1 \cdot \frac{1}{2} \cdot 2{,}71\right) + C = 1$

PROVJERA . . .

$1{,}35 + C = 1$

$C = 1 - 1{,}35$

$C = -0{,}35$

$y_p = x^{-1}\left(x \cdot \frac{x^2}{2} \cdot \frac{e^x}{x}\right) - 0{,}35$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO

IME I PREZIME: MARIO PAVIW

BROJ INDEKSA: 57656-2009

POPUNJAVA NASTAVNIK

Broj ↓ bodova

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Ukupno: 25

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
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$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
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4. $\int_0^\pi \frac{\sin x}{\cos x + 5} dx = \left\{ \sin x = \frac{2t}{1+t^2} \quad t = tg\frac{x}{2} \quad \cos x = \frac{1-t^2}{1+t^2} \quad 0 \mapsto 0 \quad \pi \mapsto \infty \quad dx = \frac{2\,dt}{1+t^2} \right.$

$$\int_0^\infty \frac{\frac{2t}{1+t^2}}{\frac{1-t^2}{1+t^2}+5} \frac{2\,dt}{1+t^2} = \int_0^\infty \frac{\frac{2t}{1+t^2}}{\frac{1-t^2+5+5t^2}{1+t^2}}$$

6. $\int_1^4 \frac{1+\sqrt{x}}{x^2}dx = \int_1^4 \frac{1}{x^2}dx + \int_1^4 \frac{\sqrt{x}}{x^2}dx = -\frac{1}{x}\Big|_1^4 + \int_1^4 x^{-\frac{3}{2}}dx = \frac{1}{4}+1-\frac{2}{\sqrt{x}}\Big|_1^4$

$= \frac{3}{4} - \frac{2}{2} + \frac{2}{1} = \frac{3}{4} - 1 + 2 = \frac{7}{4}$ ✓

4. $\int_0^{\pi} \frac{\sin x}{\cos x + 5}dx = \left[\begin{array}{ll} t = \cos x & 0 \to 1 \\ dt = -\sin x\, dx & \pi \to 0 \\ dx = \frac{dt}{-\sin x} & \end{array}\right]$

$= \int_1^0 \frac{\sin x}{t+5}\cdot\frac{dt}{-\sin x} = -\int_1^0 \frac{dt}{t+5} = -\ln|t+5|\Big|_1^0 = -\ln 5 + \ln 6$

MARIO PAVIN 57656 - 2009

2. $f(x,y) = e^x - x + y^2 + 2y$

$$\frac{\partial f}{\partial x}(x,y) = e^x - 1$$

$$\frac{\partial f}{\partial y}(x,y) = 2y + 2 \quad \left.\right\} \quad \begin{array}{ll} e^x - 1 = 0 & 2y + 2 = 0 \\ e^x = 1 & y = -1 \\ x = 0 & \end{array}$$

STACIONARNA TOČKA $S(0,-1)$

$$\frac{\partial^2 f}{\partial x^2}(0,-1) = 1 \qquad \frac{\partial^2 f}{\partial y^2} = 2 \qquad \frac{\partial^2 f}{\partial x \partial y} = 0$$

$$\Delta_2(0,-1) = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2$$

$$\left.\begin{array}{l} \Delta_2 > 0 \\ \Delta_1 = \frac{\partial f}{\partial x^2}(0,-1) > 0 \end{array}\right\}$$ FUNKCIJA DOSTIŽE LOKALNI MINIMUM U TOČKI $(0,-1)$

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IME I PREZIME: NIKOLA CAREVIĆ **BROJ INDEKSA:** 17-1-0199-13
0165631010

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Broj ↓ bodova

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$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

2. $f(x,y) = e^x - x + y^2 + 2y$

$dx = e^x - 1 \Rightarrow x = 0$
$e^0 - 1$

$dy = 2y + 2 = 2y = -2 \;/:2$
$y = -1$

$dxx = e^x$

$dxy = 1$

$dyy = 2$

$T(0,-1)$ kritična točka

$$\begin{vmatrix} e^x & 1 \\ 1 & 2 \end{vmatrix} = 2e^x - 1 = 2e^0 - 1 = 2 - 1 = 1 \quad \boxed{\Delta > 0}$$

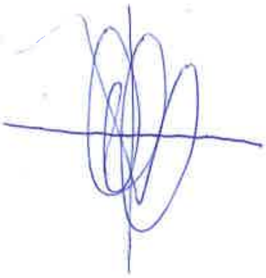
$1 > 0$

lokalni minimum.

⑥ $\int_1^4 \frac{1+\sqrt{x}}{x^2} = \int_1^4 \frac{1}{x^2}dx + \int_1^4 \frac{\sqrt{x}}{x^2}dx = \int_1^4 \frac{dx}{x^2} + \int_1^4 \frac{x^{\frac{1}{2}}}{x^2}dx = \int_1^4 \frac{dx}{x \cdot x} + \int_1^4 \frac{x^{\frac{1}{2}}}{x^2}dx = \ln|x| \cdot x \Big|_1^4 +$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Ⓑ. ⑤. $x^2 - 3x - 4$ $x^2 + 3x + 4$ $x_{1/2} =$



$x_{3,4}$

$\frac{-3 \pm \sqrt{9-4\cdot 4}}{2} = \frac{3 \pm \sqrt{-7}}{2}$

$\frac{3 + \sqrt{9 - 4\cdot(-4)}}{2}$

$= \frac{3 \pm 5}{2} = \frac{8}{2} = \boxed{x_1 = 4}$

$\frac{2}{2} = \boxed{x_2 = 1}$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: DOMAGOJ GROZAJ **BROJ INDEKSA:** 19-1-0056-2011	

1. Nađi partikularno rješenje koje zadovoljava: $xy' + y - e^x = 0$ i $y(1) = 1$. — 15
2. Odrediti lokalne ekstreme funkcije $f(x, y) = e^x - x + y^2 + 2y$. — 15
3. Skicirati razinske krivulje za funkciju $f(x, y) = x^2 - y$. — 15
4. $\int_0^\pi \frac{\sin x}{\cos x + 5} dx$? — 20
5. Izračunaj površinu lika omeđenog krivuljama $f(x) = x^2 - 3x - 4$ i $g(x) = -x^2 + 3x + 4$. — ~~15~~
6. Izračunaj $\int_1^4 \frac{1+\sqrt{x}}{x^2} dx$ i napravi provjeru izračunom aproksimacije integrala Simsponovom metodom. — 10+10

$t = \cos x + 5$
$dt = -\sin x \cdot dx$
$dx = \frac{dt}{-\sin x}$

$\int_0^\pi \frac{dt}{t}$ $\quad \ln|t| \quad \ln|\cos x + 5|$

Ukupno: ~~Ø~~

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
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$\ln|\cos \pi + 5|$
$- \ln|\cos 0 + 5|$

(5) $f(x) = x^2 - 3x - 4 \quad g(x) = -x^2 + 3x + 4$

$x^2 - 3x - 4 = -x^2 + 3x + 4$
$x^2 - 3x - 4 + x^2 - 3x - 4 = 0$
$2x^2 - 6x - 8 = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{6 \pm \sqrt{36 - 4 \cdot 2 \cdot (-8)}}{4}$

$\frac{6 \pm 10}{\cancel{2}\ 4}$

$x_1 = 8$

$x_2 = -2$

$\int_{-2}^{8} (x^2 - 3x - 4) - (-x^2 + 3x + 4)\, dx$

$\int_{-2}^{8} x^2 - 3x - 4 + x^2 - 3x - 4\, dx$

$\int_{-2}^{8} 2x^2 - 6x - 8\, dx$

$\int_{-2}^{8} 2x^2\, dx - \int_{-2}^{8} 6x\, dx - \int_{-2}^{8} 8\, dx$

$2\frac{x^3}{3}\Big|_{-2}^{8} - 6\frac{x^2}{2}\Big|_{-2}^{8} - 8x\Big|_{-2}^{8} + C$

$\left(2\frac{8^3}{3} - 2\frac{-2^3}{3}\right) - \left(6\frac{8^2}{2} - 6\frac{-2^2}{2}\right) - \left((8 \cdot 8) - (-8 \cdot (-2))\right)$

$(341{,}3 + 5{,}3) - (192 - 12) - (64 + 16) = \boxed{86{,}6}$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO

IME I PREZIME: ROKO DUŠEVIĆ **BROJ INDEKSA:** [redacted]

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Nađi partikularno rješenje koje zadovoljava: $xy' + y - e^x = 0$ i $y(1) = 1$. 15
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3. Skicirati razinske krivulje za funkciju $f(x, y) = x^2 - y$. 15
4. $\int_0^{\pi} \frac{\sin x}{\cos x+5} dx$? ~~20~~
5. Izračunaj površinu lika omeđenog krivuljama $f(x) = x^2 - 3x - 4$ i $g(x) = -x^2 + 3x + 4$. 15
6. Izračunaj $\int_1^4 \frac{1+\sqrt{x}}{x^2} dx$ i napravi provjeru izračunom aproksimacije integrala Simsponovom metodom. 10+10

Ukupno: ∅

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
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Tablica nekih integrala

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$\int x^\alpha \, dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
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$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4.)

$$\int_0^{\pi} \frac{\sin x}{\cos x+5} dx = \left[\begin{matrix} \sin x = \frac{2t}{1+t^2} & dx = \frac{2dt}{1+t^2} \\ \cos x = \frac{1-t^2}{1+t^2} & \end{matrix} \right] = \int_0^{\pi} \frac{\frac{2t}{1+t^2}}{\frac{1-t^2}{1+t^2}+5} \cdot \frac{2dt}{1+t^2} = \int_0^{\pi} \frac{\frac{2t}{1+t^2}}{\frac{1-t^2+5+5t^2}{1+t^2}} \cdot \frac{2dt}{1+t^2} =$$

$$= \int_0^{\pi} \frac{\frac{2t}{1+t^2}}{\frac{4t^2+6}{1+t^2}} \cdot \frac{2dt}{1+t^2} = \int_0^{\pi} \frac{1+t^2}{(2t+6)(1+t^2)} \cdot \frac{2dt}{1+t^2} = \int_0^{\pi} \frac{1+t^2}{2t+2t^3+6t^2+6} \cdot \frac{2dt}{1+t^2}$$

= ... ?

2.

$f(x,y) = e^x - x + y^2 + 2y$

$\frac{\partial f}{\partial x} = e^x - 1$ $\frac{\partial^2 f}{\partial x^2} = e^x$

$\frac{\partial f}{\partial y} = 2y + 2$ $\frac{\partial^2 f}{\partial y^2} = 2$

$\delta = \begin{vmatrix} e^x & 0 \\ 0 & 2 \end{vmatrix}$

$\delta = 2e^x$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO

IME I PREZIME: Mateja Pečarić **BROJ INDEKSA:** 17-0032-2010

POPUNJAVA NASTAVNIK

	Broj ↓ bodova
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5. Izračunaj površinu lika omeđenog krivuljama $f(x) = x^2 - 3x - 4$ i $g(x) = -x^2 + 3x + 4$.	~~15~~
6. Izračunaj $\int_1^4 \frac{1+\sqrt{x}}{x^2} dx$ i napravi provjeru izračunom aproksimacije integrala Simsponovom metodom.	10+10
	Ukupno:

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
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5. $f(x) = x^2 - 3x - 4$ $\quad g(x) = -x^2 + 3x + 4$

$x^2 - 3x - 4 = -x^2 + 3x + 4$

$2x^2 - 6x - 8 = 0$

$x^2 + 3x - 4 = 0$

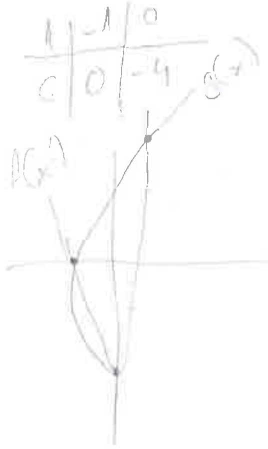
$x_{1,2} = \frac{-3 \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-4)}}{2}$

$= \frac{-3 \pm \sqrt{9+16}}{2}$

$= \frac{-3 \pm 5}{2} =$

$x_1 = -4 \quad x_2 = 1$



$P = \int_{-4}^{1} (x^2+3x+4) - (x^2-3x-4) dx$

$= \int_{-4}^{1} x^2+3x+4-x^2+3x+4\, dx$

$= \frac{x^3}{3} + 3\frac{x^2}{2} - \frac{x^3}{3} + 3\frac{x^2}{2} \Big|_{-4}^{1}$

$= \frac{3x^2}{2} + \frac{3x^2}{2}$

$= \frac{6x^2}{2} = 3x^2$

$= 3(1)^2 + 3\cdot(-4)^2$

$= 3 + 48$

$= 51$

1. $xy' + y - e^x = 0 \qquad y(1) = 1$

$xy + y = 0$

$y(x+1) = 0$

$x = 1$

$y = 0$

4. $\int_0^{\pi} \frac{\sin x}{\cos x + 5} dx = \begin{Bmatrix} \cos x = t \\ \sin x\, dx = dt \end{Bmatrix} = \int \frac{dt}{t+5} = \frac{1}{5} \int \frac{dt}{t}$

$= \frac{1}{5} \ln|t| = \left(\frac{1}{5} \ln|\pi|\right) - \left(\frac{1}{5} \ln|0|\right)$

$= \frac{1}{5} \ln|\pi|$

$= 0.2289$

2. $f(x,y) = e^x - x + y^2 + 2y$