MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxx

IME I PREZIME: MATEO BAČIĆ **BROJ INDEKSA:** 17-2-0225-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Nađi koliko iznosi $f(2.5)$ ako f zadovoljava $\sin x\, dy = y \ln y\, dx$ i $y(1) = 2$. 15
2. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' + 4y = 0$, uz $y(0) = 0$ i $y'(0) = 2$. Na kraju provjeri rješenje. 15
3. Skicirati razinske krivulje za $f(x, y) = x^2 + y^2$. Ima li ekstrema? Pronađi tangencijalnu ravninu u točki koju možeš sam odabrati. 15
4. $\int_0^1 3x\, e^{x+1}\, dx = ?$ 20
5. $\int_1^3 \frac{dx}{x^2-2x+4} = ?$ 15
6. Neka je $f(x) = \tan x$. Skicirati graf funkcije f i površinu određenu integralom. Odrediti $\int_0^{\pi/2} f(x)\, dx$. Kolika je skicirana površina ispod grafa funkcije f? 20

Ukupno: 85

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4) $\int_0^1 3x\, e^{x+1}\, dx = \left[\begin{matrix} 3x = u \\ 3dx = du \end{matrix}\right.$ $\quad dv = e^{x+1} dx$

$v = \int e^{x+1} dx \quad \left\{\begin{matrix} x+1 = t \\ dx = dt \end{matrix}\right\}$

$= \left[3x \cdot e^{x+1} - \int e^{x+1}\, 3dx\right]_0^1 \quad v = \int e^t dt$

$v = e^t = e^{x+1}$

$= \left[3xe^{x+1} - 3e^{x+1}\right]_0^1$

$= \left[3 \cdot 1 \cdot e^2 - 3 \cdot e^2 - 0 + 3e^1\right]$

$= 8{,}1548$ ✓

5) $\int_1^3 \frac{dx}{x^2-2x+4} = \int_1^3 \frac{dx}{x^2-2x+1+3}$

$= \int_1^3 \frac{dx}{(x-1)^2+\sqrt{3}^2} = \left[\frac{1}{\sqrt{3}} \operatorname{arctan} \frac{x-1}{\sqrt{3}}\right]_1^3$

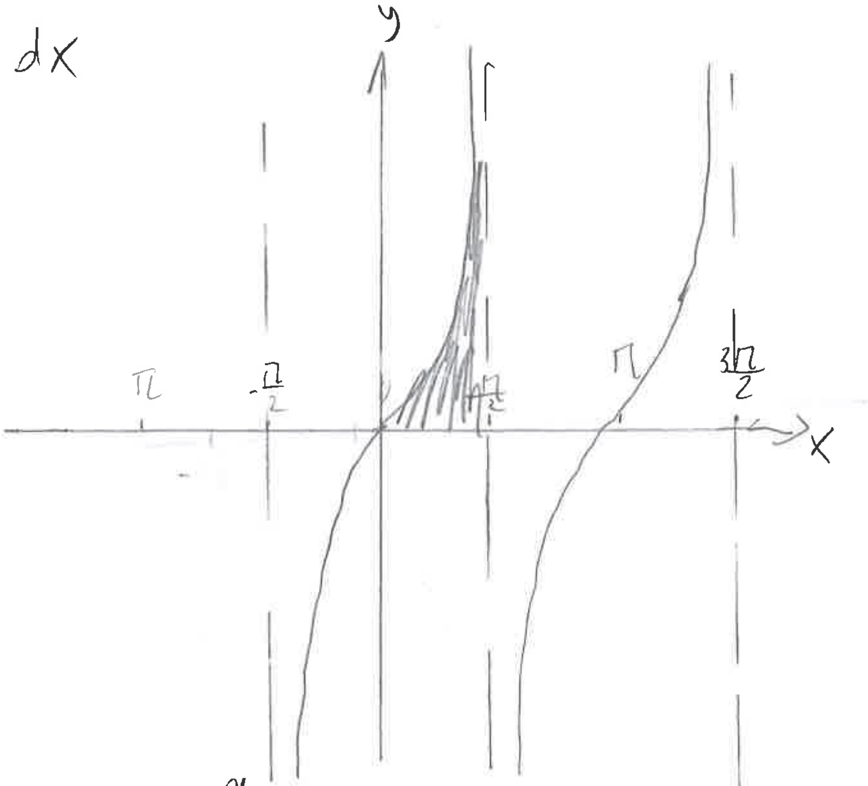
$\begin{cases} x-1=t \\ dx=dt \end{cases}$

$= \frac{1}{\sqrt{3}} \operatorname{arctan} \frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}} \operatorname{arctan} 0$

$= 0,4948$

6) $f(x,y) = \tan x$

$\int_0^{\frac{\pi}{2}} f(x)\, dx$



GRAF NE SMIJE PRELAZITI ASIMPTOTE

$\int_0^{\frac{\pi}{2}} \tan x\, dx = \lim_{a \to \frac{\pi}{2}} \int_0^a \tan x\, dx$

$= \lim_{a \to \frac{\pi}{2}} \left[-\ln|\cos x|\right]_0^a$

$= \lim_{a \to \frac{\pi}{2}} (-\ln|\cos a| + \ln|\cos 0|)$

$= -\ln|\cos \frac{\pi}{2}| + 0$

$= -\ln 0 = -\infty$

F

~~3) $f(x,y) = x^2 + y^2$~~

1) $\sin x \, dy = y \ln y \, dx$

$$\frac{dy}{y \ln y} = \frac{dx}{\sin x}$$

$$\int \frac{dy}{y\ln y} = \left[\begin{array}{l} \ln y = t \\ \frac{1}{y} dy = dt \end{array} \right] = \int \frac{dt}{t} =$$

$$\int \frac{dx}{\sin x} = \int \frac{\frac{2dt}{1+t^2}}{\frac{2t}{1+t^2}} = \int \frac{dt}{t}$$

$$= \ln |t| = \ln \left| \tan \frac{x}{2} \right| \checkmark$$

$$\ln |\ln y| = \ln \left| \tan \frac{x}{2} \right| + c$$

$$\ln |\ln 2| = \ln \left| \tan \frac{1}{2} \right| + c$$

$$c = 0{,}238$$

$$\ln |\ln y| = \ln \left| \tan \frac{x}{2} \right| + 0{,}238$$

$$f(2{,}5)$$

$$\ln |\ln y| = \ln \left| \tan \frac{2{,}5}{2} \right| + 0{,}238$$

$$\ln |\ln y| = 1{,}339797$$

$$\ln y = e^{1{,}339797}$$

$$\ln y = 3{,}818$$

$$y = 45{,}51 \checkmark$$

$3) \; f(x,y) = x^2 + y^2$

$f(x,y) = 0$

$c = 0, 1, 2, 4, 9$

$x^2 + y^2 = 0$

$x^2 + y^2 = 1$

$x^2 + y^2 = 2$

2)

$(y_p)' = c_1(-\sin(2x)) \cdot 2 + c_2 \cos(2x) \cdot 2$

$y_p' = -2c_1 \sin(2x) + 2c_2 \cos(2x)$

$0 = c_1 \cos(0) + c_2 \sin(0)$

$\boxed{0 = c_1}$

5

$2 = -2c_1 \sin(0) + 2c_2 \cos(0)$

$2 = 2c_2 \Rightarrow c_2 = 1$

$y_p = \sin(2x)$ ✓

PROVJERA

$y_p = \sin 2x$

$0 = \sin(2 \cdot 0)$

$0 = 0$ ✓

$y_p = \cos(2x) \cdot 2$

$2 = \cos(0) \cdot 2$

$2 = 2$

✓

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxx

POPUNJAVA NASTAVNIK

IME I PREZIME: Roberto Jurko **BROJ INDEKSA:** 17-2-0195-2012

Broj ↓ bodova

1. Nađi koliko iznosi $f(2.5)$ ako f zadovoljava $\sin x\,dy = y\ln y\,dx$ i $y(1) = 2$. 15
2. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' + 4y = 0$, uz $y(0) = 0$ i $y'(0) = 2$. Na kraju provjeri rješenje. 15
3. Skicirati razinske krivulje za $f(x,y) = x^2 + y^2$. Ima li ekstrema? Pronađi tangencijalnu ravninu u točki koju možeš sam odabrati. 15
4. $\int_0^1 3x\,e^{x+1}\,dx = ?$ 20
5. $\int_1^3 \frac{dx}{x^2-2x+4} = ?$ 15
6. Neka je $f(x) = \tan x$. Skicirati graf funkcije f i površinu određenu integralom. Odrediti $\int_0^{\pi/2} f(x)\,dx$. Kolika je skicirana površina ispod grafa funkcije f? 20

Ukupno: 70

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

4.

$$\int_0^1 3x\,e^{x+1}\,dx = \left|\begin{array}{l} u = 3x \\ du = 3dx \end{array}\right. \quad \begin{array}{l} dv = e^{x+1}dx \\ v = e^{x+1} \end{array}$$

$$= \left(3x\,e^{x+1}\right)\Big|_0^1 - \int_0^1 e^{x+1} \cdot 3dx$$

$$= 3e^2 - \left(3e^{x+1}\right)\Big|_0^1 =$$

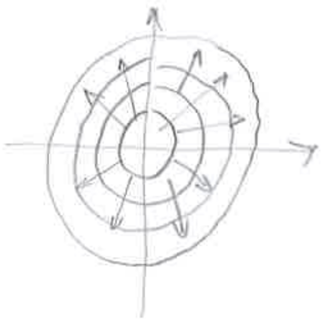
$$= 3e^2 - 3e^2 + 3e = 3e$$

(5) $\int_1^3 \frac{dx}{x^2-2x+4} = \int_1^3 \frac{dx}{(x-1)^2+3} = \left(\frac{1}{\sqrt{3}} \operatorname{arctg} \frac{x-1}{\sqrt{3}}\right)\Big|_1^3 =$

$= \frac{1}{\sqrt{3}}\left(\operatorname{arctg}\frac{2}{\sqrt{3}} - \operatorname{arctg} 0\right)$

$= \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{2}{\sqrt{3}}$

(3) $f(x,y) = x^2 + y^2$ $\qquad D(f) = \mathbb{R}^2$



$\frac{\partial f}{\partial x} = 2x \qquad \frac{\partial f}{\partial y} = 2y$

$x = 0 \qquad y = 0$

$\frac{\partial^2 f}{\partial x^2} = 2 \qquad \frac{\partial^2 f}{\partial x \partial y} = 0 \qquad \frac{\partial^2 f}{\partial y^2} = 2$

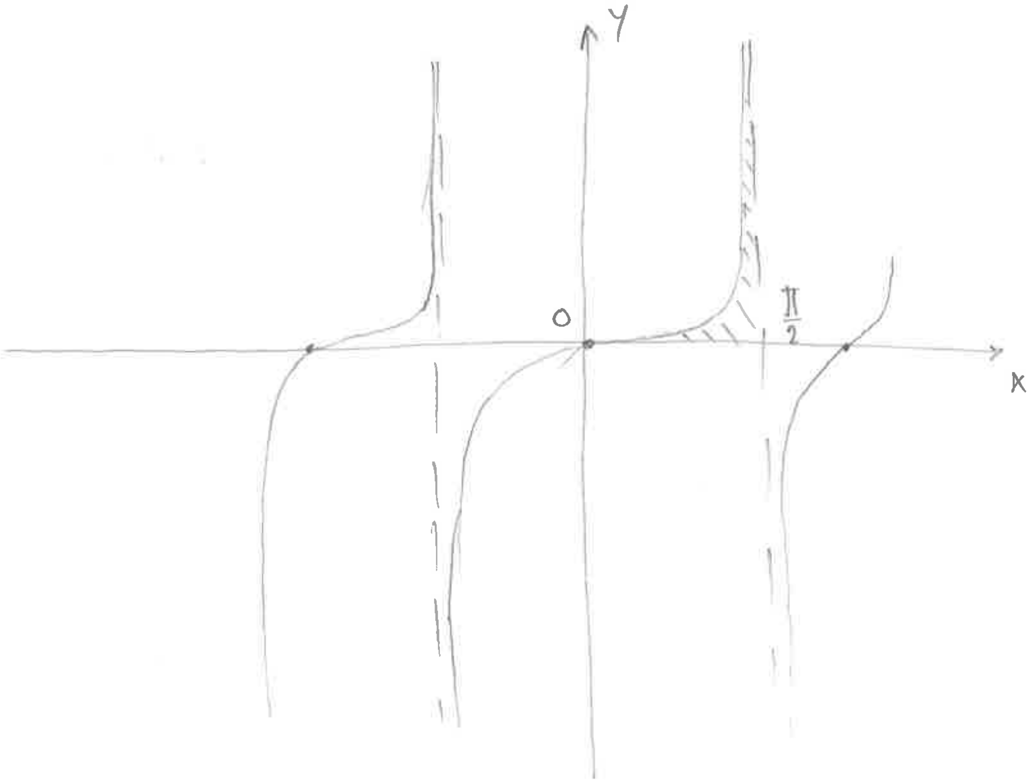
TOČKA $(0,0,\cancel{0})$ JE MINIMUM

$x = 1 \qquad y = 1 \qquad z = 2 \qquad \frac{\partial f}{\partial x} = 2 \qquad \frac{\partial f}{\partial y} = 2$

T.... $z - 2 = 2(x-1) + 2(y-1)$

$2x + 2y - z - 2 = 0$

(6.)



$$\int_0^{\frac{\pi}{2}} tg\,x\,dx = \lim_{b \to \frac{\pi}{2}} \int_0^b tg\,x\,dx =$$

$$= \lim_{b \to \frac{\pi}{2}} \left(-\ln|\cos x| \right) \Big|_0^b =$$

$$= \lim_{b \to \frac{\pi}{2}} \left(-\ln|\cos b| + \ln|\cos 0| \right)$$

$$= -\ln\left|\cos \frac{\pi}{2}\right| + \ln 1 = -\ln 0 = +\infty$$

DIVERGIRA. ✓

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxx

IME I PREZIME: VESNA MILOŠ **BROJ INDEKSA:** 17-2-0310-2013

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Nađi koliko iznosi $f(2.5)$ ako f zadovoljava $\sin x\, dy = y \ln y\, dx$ i $y(1) = 2$. 15
2. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' + 4y = 0$, uz $y(0) = 0$ i $y'(0) = 2$. Na kraju provjeri rješenje. 15
3. Skicirati razinske krivulje za $f(x, y) = x^2 + y^2$. Ima li ekstrema? Pronađi tangencijalnu ravninu u točki koju možeš sam odabrati. 15
4. $\displaystyle\int_0^1 3x\, e^{x+1}\, dx = ?$ 20
5. $\int_1^3 \frac{dx}{x^2-2x+4} = ?$ 15
6. Neka je $f(x) = \tan x$. Skicirati graf funkcije f i površinu određenu integralom. Odrediti $\int_0^{\pi/2} f(x)\, dx$. Kolika je skicirana površina ispod grafa funkcije f? 20

Ukupno: 35

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

TANGENCIJALNA RAVNINA

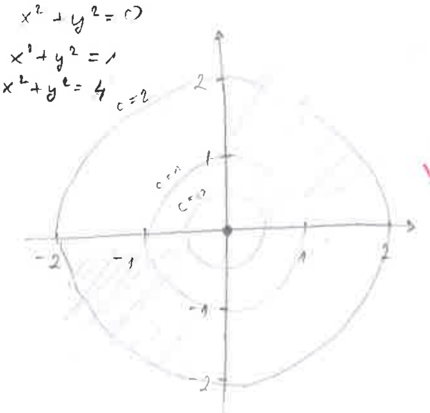
$z - z_0 = f_x(x-x_0) + f_y(y-y_0)$

3) $f(x,y) = x^2 + y^2$

$c=0 \quad x^2+y^2=0$

$c=1 \quad x^2+y^2=1$

$c=2 \quad x^2+y^2=4$



$T(1,1)$

$z_0 = 1^2 + 1^2$

$z_0 = 1 + 1$

$\boxed{z_0 = 2}$

$R(T) \ldots z - 2 = 2(x-1) + 2(y-2)$

$z = 2x - 2 + 2y - 4 + 5$

$z = 2x + 2y - 1$

normala... $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{-1}$

$\partial f_x = 2x = 2$

$\partial f_y = 2y = 2$

$\partial f_{xx} = 2$

$\partial f_{xy} = 0 = \partial f_{yx}$

$\partial f_{yy} = 2$

$\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 > 0$

$2x = 0$
$x = 0$

$2y = 0$
$y = 0$

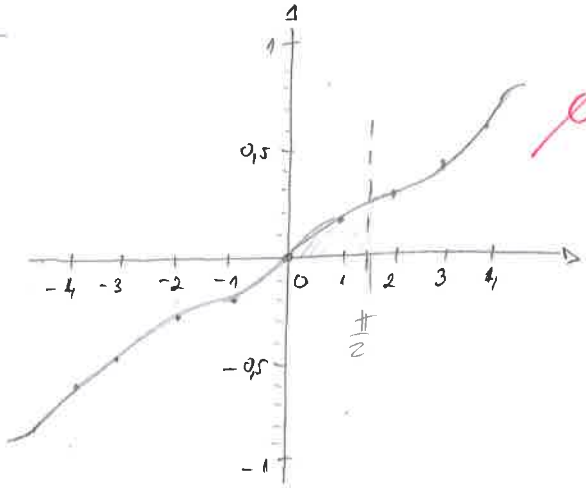
$T(0,0)$

FUNKCIJA IMA MINIMUM u $T(0,0)$

6. $f(x) = \tan x$

$\int_0^{\frac{\pi}{2}} \tan x \, dx$

x	-1	-2	0	1	2	3	4	-3	-4
y	-0,02	-0,03	0	0,02	0,03	0,05	0,07	-0,05	-0,07



$$\int_0^{\frac{\pi}{2}} \tan x \, dx = -\ln|\cos x| \Big|_0^{\frac{\pi}{2}} = -\ln\left|\cos\frac{\pi}{2} - \cos 0\right|$$

$$= -\ln\left|\cos\frac{\pi}{2} - 1\right| + C$$

$\frac{\pi}{4} \approx 0{,}785$

$\frac{\pi}{2} \approx 1{,}57$

k	0	1	2
x_k	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$y(x_k)$	0	0,0137	0,0274
$H = d \cdot y(x_k)$	0	0,0215	0,043018

$P \approx 0{,}064527$

$$\frac{1{,}57}{6}(0 + 4 \cdot 0{,}0137 + 0{,}0274)$$

$$= \frac{1{,}57}{6}(0{,}0822)$$

$$= 0{,}021509$$

5. $$\int_1^3 \frac{dx}{x^2-2x+4} = \left\{ \begin{aligned} x^2 - 2x + 4 &= (x-a)^2 + b^2 \\ &= x^2 - 2ax + a^2 + b^2 \end{aligned} \quad (x-1)^2 + 1^2 \right\}$$

TIP B

$$= \int_1^3 \frac{dx}{(x-1)^2+1^2} = \left\{ \begin{aligned} t &= x-1 \\ dt &= dx \end{aligned} \right\} = \int_1^3 \frac{dt}{t^2+1^2} = \int_1^3 \frac{dt}{1^2+t^2}$$

$$= \left[\arctan x\right]_1^3 = (\arctan 3 - \arctan 1) + C$$

$-2x = -2ax$

$-2x = a(-2x) \,/ : (-2x)$

$\boxed{a = 1}$

$a^2 + b^2 = 4 \,/\sqrt{\ }$

$a + b = 2$

$b = 2 - 1$

$\boxed{b = 1}$

4. $$\int_0^1 3x \cdot e^{x+1} dx = 3\int_0^1 x e^{x+1} dx = \left\{ \begin{aligned} u &= x \,/' & du &= dx \\ dv &= e^{x+1} dx & v &= \int dv = e^{x+1} \end{aligned} \right\}$$

$$\int e^{x+1} dx = \left\{ \begin{aligned} t &= x+1 \\ dt &= dx \end{aligned} \right\} = \int e^t dt = e^{x+1} + C$$

$$= 3\left([uv]_0^1 - \int_0^1 v\,du\right) = 3\left(\left[x e^{x+1}\right]_0^1 - \int_0^1 e^{x+1} dx\right)$$

$$= 3\left(e^2 - \left[e^{x+1}\right]_0^1\right) = 3\left(e^2 - \left(e^{1+1} - e^{0+1}\right)\right)$$

$$= 3\left(e^2 - (e^2 - e^1)\right) = 3(7{,}389056 - 7{,}389056 + 2{,}71828)$$

$$= 3(2{,}71828) = 8{,}1548$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXX

IME I PREZIME: IVAN VELEMIR

BROJ INDEKSA: 17-2-0067-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Nađi koliko iznosi $f(2.5)$ ako f zadovoljava $\sin x\,dy = y\ln y\,dx$ i $y(1)=2$. — 15
2. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y''+4y=0$, uz $y(0)=0$ i $y'(0)=2$. Na kraju provjeri rješenje. — ~~15~~
3. Skicirati razinske krivulje za $f(x,y)=x^2+y^2$. Ima li ekstrema? Pronađi tangencijalnu ravninu u točki koju možeš sam odabrati. — ~~15~~ 15
4. $\int_0^1 3x\,e^{x+1}\,dx = ?$ — 20
5. $\int_1^3 \frac{dx}{x^2-2x+4} = ?$ — 15
6. Neka je $f(x)=\tan x$. Skicirati graf funkcije f i površinu određenu integralom. Odrediti $\int_0^{\pi/2} f(x)\,dx$. Kolika je skicirana površina ispod grafa funkcije f? — 20

Ukupno: 35

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha>0)$	$\alpha^x\ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a}+C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

(3) TANG. RAVNINA

$\frac{\partial f}{\partial x} = 2x$ $\qquad \frac{\partial f}{\partial y} = 2y$

$z - z_0 = \frac{\partial f}{\partial x}(T)(x-x_0) + \frac{\partial f}{\partial y}(T)(y-y_0)$

$T = (1,1,1)$

$z - z_0 = 2(x-1) + 2(y-1)$

$z - 1 = 2x - 2 + 2y - 2$

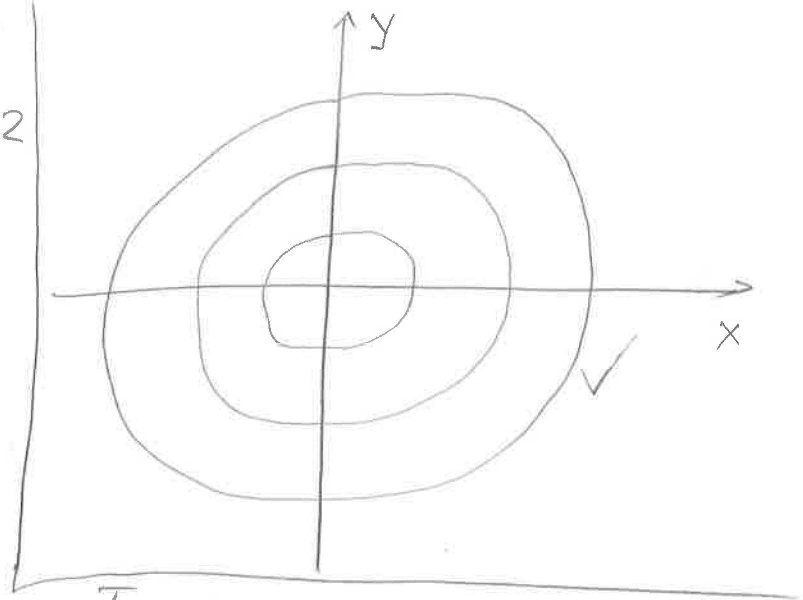
$z - 1 = 2x + 2y - 4$

$z - 2x - 2y + 2 = 0$ .... tang. ravnina

→

$$\frac{\partial f}{\partial x \partial x} = 2 \qquad \frac{\partial f}{\partial y \partial y} = 2$$

SKICA



$2x = 0$

$x = 0$

$2y = 0$

$y = 0$

$\Rightarrow T(0,0)$ ✓

T ≠ lokalni ekstrem, sedlo

$$\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 - 0 = 4 > 0$$

$$\boxed{\frac{\partial f}{\partial x \partial x} = 2 > 0 \quad \text{min.}}$$

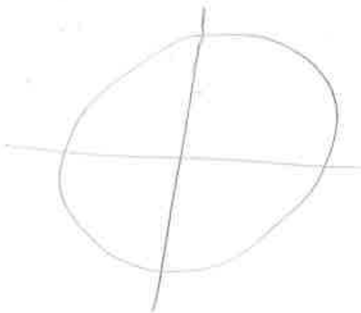
(6.) $\int_0^{\frac{\pi}{2}} \tan x \, dx =$

$\int_0^{\frac{\pi}{2}} \frac{\tan x}{\cos x} dx = \left\{ \begin{matrix} \cos x = t / \\ -\sin x \, dx = dt \end{matrix} \right\}$

$= \int_0^{\frac{\pi}{2}} \frac{-dt}{t} = - \int_0^{\frac{\pi}{2}} \frac{dt}{t} =$

$= -\ln t \Big|_0^{\frac{\pi}{2}} = -\ln(\cos x) \Big|_0^{\frac{\pi}{2}}$

$= -[\ln(\underbrace{\cos \frac{\pi}{2}}_{0}) - \ln(\underbrace{\cos 0}_{1})]$



ln 0 ne postoji

NEPAVI INTEGRAL!

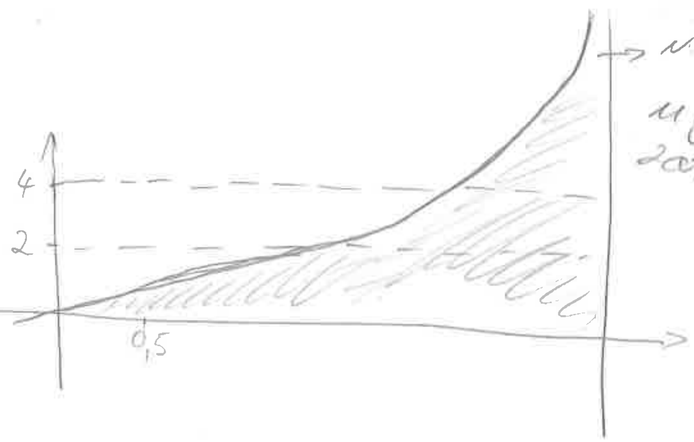
$= \lim_{b \to \frac{\pi}{2}} \int_0^b \frac{\sin x}{\cos x} dx = \lim_{b \to \frac{\pi}{2}} \left(-\ln[\cos x]\right)\Big|_a^b$

$= \lim_{b \to \frac{\pi}{2}} (-[\ln[\underbrace{\cos b}_{0}] - [\ln(\cos 0)]]$

$= \lim_{b \to \frac{\pi}{2}} (-\ln(\cos b) - \underbrace{\ln(1)}_{0}]$

$= -\ln(\lim_{b \to \frac{\pi}{2}} \cos b) - 0 = \infty$ ✓

(divergira)



→ NEMA POV.

nije zatvoren

(2)

$r^2 + 4 = 0$

$r^2 = -4$

$r_1 = \sqrt{-4}$, $r_2 = -\sqrt{-4}$

$r_1 = \sqrt{4}\, i$ $r_2 = -2i$

$r_1 = 2i$

$y(x) = e^{-8x}(c_1 \cos 2x + c_2 \sin(2x)$

$y(x) = (c_1 \cos 2x + c_2 \sin 2x)$

$y(x) = c_1 \cos 2x + c_2 \sin 2x \; //$

$y'(x) = -c_1 \sin 2x \cdot 2 + 2 c_2 \cos 2x$

$= c_1 \cos(2 0) + c_2 \sin(2 0) = 0$

$= -c_{12} \sin(2 0) + 2 c_2 (\cos 2 0) = 2$

$\cos 0 = 1$

$\sin 0 = 0$

$\rightarrow c_1 \cdot 1 + c_2 \cdot 0 = 0 \Rightarrow c_1 = 0$

$-c_1^2 \cdot 0 + 2 c_2 \cdot 1 = 2 \Rightarrow c_2 = 1$

$y(x) = \sin 2x \; /'$

$y'(x) = 2 \cos(2x)$

$y''(x) = 4(\sin 2x)$

$-4 \sin 2x + 4 \cdot \sin 2x = 0$

$0 = 0$ ✓

Ø

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxx

IME I PREZIME: NIKOLA TOMASOV

BROJ INDEKSA: 17-2-0161-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Nađi koliko iznosi $f(2.5)$ ako f zadovoljava $\sin x\, dy = y \ln y\, dx$ i $y(1) = 2$. 15
2. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' + 4y = 0$, uz $y(0) = 0$ i $y'(0) = 2$. Na kraju provjeri rješenje. ~~15~~
3. Skicirati razinske krivulje za $f(x, y) = x^2 + y^2$. Ima li ekstrema? Pronađi tangencijalnu ravninu u točki koju možeš sam odabrati. ~~15~~ 10
4. $\displaystyle\int_0^1 3x\, e^{x+1}\, dx = ?$ ~~20~~
5. $\int_1^3 \frac{dx}{x^2-2x+4} = ?$ ~~15~~
6. Neka je $f(x) = \tan x$. Skicirati graf funkcije f i površinu određenu integralom. Odrediti $\int_0^{\pi/2} f(x)\, dx$. Kolika je skicirana površina ispod grafa funkcije f? ~~20~~

Ukupno: 10

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

2. $y'' + 4y = 0$ $\quad y(0) = 0 \quad y'(0) = 2$

$r^2 + 4 = 0$

$r^2 = -4$

$r_{1,2} = \pm\sqrt{-4}$

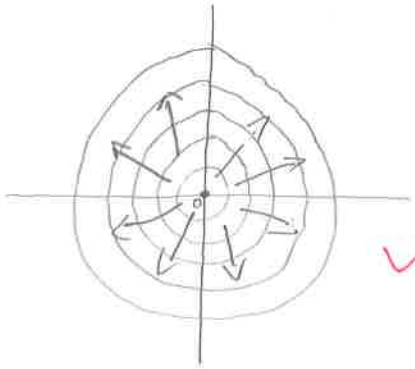
$r_1 = \pm 2i$

0

(3) $f(x,y) = x^2+y^2$

$x^2+y^2=0$

$\mathcal{D}(f) = \mathbb{R} \setminus \{0,0\}$



$T(1,1,z_0)$

$z_0 = 1^2 + 1^2 = \underline{\underline{1}}$

$\frac{df}{dx} = (x^2+y^2)' = 2x$

$f_x(T) = 2 \cdot 1 = \underline{\underline{2}}$

$\frac{df}{dy} = (x^2+y^2)' = 2y$

$f_y(T) = 2 \cdot 1 = \underline{\underline{2}}$

$z - z_0 = f_x(T)(x-x_0) + f_y(T)(y-y_0)$

$z - 1 = 2(x-1) + 2(y-1)$

$z - 1 = 2x - 2 + 2y - 2$

$z - 1 - 2x + 2 - 2y + 2 = 0$

$-2x - 2y + 3 + z = 0 \; / \cdot (-1)$

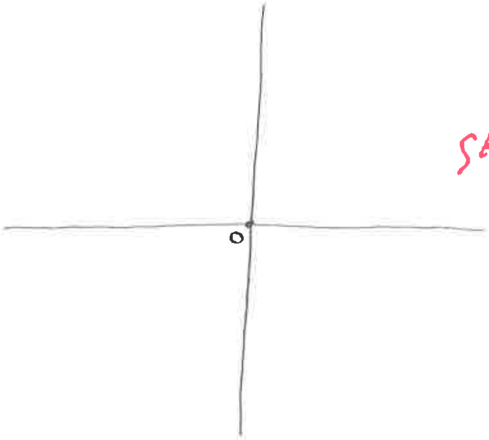
$2x + 2y - 3 - z = 0 \; \ldots \quad Rt$

NIKOLA TOMASOV

6. $f(x) = \tan x$

$$P=\int_0^{\pi/2} \tan x \, dx = \left(-\ln|\cos x| + c\right)\Big/_0^{\pi/2} = \left(-\ln\left|\cos\frac{\pi}{2}\right| + \ln|\cos 0|\right) = 0+0=0$$

? NEMA POVRŠINE



SKICA?

4. $$\int_0^1 \underbrace{3x}_{I_1}\,\underbrace{e^{x+1}}_{I_2}\,dx =$$

$$I_1 = \int_0^1 3x\,dx = 3\cdot\frac{x^2}{2}\Big/_0^1 = \left(3\cdot\frac{1}{2}\right) - \left(3\cdot\frac{0}{2}\right) = \frac{3}{2} - 0 = \frac{3}{2}$$

$$I_2 = \int_0^1 e^{x+1}\,dx = \left[\begin{array}{c|cc} t = x+1 & x=0 & x=1 \\ dt = dx & t=1 & t=2 \end{array}\right] = \int_1^2 e^t\,dt = e^t\Big/_1^2 = \left(e^2 - e^1\right) = e$$

$$I = I_1 + I_2 = \frac{3}{2} + e$$

1. $\sin x\,dy = y \ln y\,dx$ $\quad y(\overset{x}{1}) = \overset{y}{2}$ $\quad f(2,5)$

5. $\int_{1}^{3} \frac{dx}{x^2-2x+4}$

$x^2-2x+4 \neq 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_{1,2} = \frac{2 \pm \sqrt{4-16}}{2}$

$x_{1,2} = \frac{2 \pm \sqrt{-12}}{2}$

NEMA NULTOČAKA

$= \int_{1}^{3} \frac{1}{x^2} dx - \int_{1}^{3} \frac{1}{2x} dx + \frac{1}{4} \int_{1}^{3} dx = \int_{1}^{3} x^{-2} dx - \frac{1}{2} \int_{1}^{3} \frac{1}{x} dx + \frac{1}{4} \int_{1}^{3} dx$ X

$= \left(\frac{x^{-1}}{-1}\right) \Big/_{1}^{3} - \left(\underbrace{\frac{1}{2} \ln|x|}_{0}\right) \Big/_{1}^{3} + \frac{1}{4} \cdot x \Big/_{1}^{3} = \frac{2}{3} - 0 + \frac{1}{2} = \frac{2}{3} + \frac{1}{2} = \frac{7}{6}$