MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: Ivan Colić

BROJ INDEKSA: 17-2-0152-2011

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1,\ x'(0) = 0.$$

2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,0)$, a $\partial\widehat{K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int_{\partial\widehat{K}} (2x + 3)\,dy$? 20

3. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint_K (2x + 3)\,dxdydz$? 20

4. Zadan je P paraboloid $x^2 + y^2 = 4z$, $z \le 4$. Izračunati $\iint_P 3\,dS$? 20

5. Izračunati $\int_{(3,2)}^{(5,5)} x dy + y dx$. 20

Ukupno: 80

1) $x'''(t) + x'(t) = 0 \qquad x(0) = x''(0) = 1 \qquad x'(0) = 0$

$s^3F(s) - s^2 f(0) - s f'(0) - f''(0) + sF(s) - f(0) = 0$

$f(0) = 1, \quad f'(0) = 0, \quad f''(0) = 1, \quad f(0) = 1$

$s^3F(s) - s^2 - 1 + sF(s) - 1 = 0$

$F(s)(s^3 + s) = s^2 + 1 + 1$

$F(s)\, s(s^2+1) = s^2 + 2$

$F(s) = \frac{s^2+2}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} \quad / \cdot s(s^2+1)$

$s^2 + 2 = A(s^2+1) + (Bs+C)\cdot s$

$s^2 + 2 = As^2 + A + Bs^2 + Cs$

$A + B = 1 \Rightarrow 2 + B = 1$

$C = 0 \qquad B = 1 - 2$

$A = 2 \qquad B = -1$

$F(s) = \frac{2}{s} + \frac{-1s}{s^2+1}$

$F(s) = 2\cdot\frac{1}{s} - \frac{s}{s^2+1}$

$F(s) = 2 - \cos t$

$X(s) = 2 - \cos t$

$X(0) = 2 - \cos 0$
$= 2 - 1$
$= 1$

$X'(0) = 2' - (\cos t)'$
$= 0 + \sin 0$
$= 0 + 0$
$= 0$

$X''(0) = 0 - (-\cos 0)$
$= 0 + 1$
$X''(0) = 1$

2.

$P(x,y) = 2x \qquad Q(x,y) = 3$

Krug.

$r = 1$

$T(0,0)$

$\oint\limits_{\partial K} (2x+3)\,dy$

$r \in [0,1]$

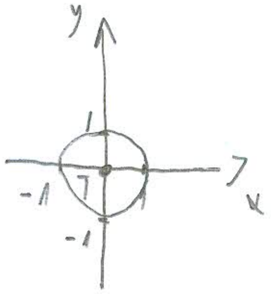
$\varphi \in [0, 2\pi]$

$x = r\cos\varphi$

$y = r\sin\varphi$

$dx\,dy = r\,dr\,d\varphi$

$$\frac{\partial P}{\partial x} - \frac{\partial Q}{\partial y} = \frac{\partial 2x}{\partial x} - \frac{\partial 3}{\partial y} = 2 - 0 = 2$$



$$\int_0^{2\pi}\int_0^1 2r\,dr\,d\varphi = \int_0^{2\pi} 2\cdot\frac{r^2}{2}\Big|_0^1 d\varphi = \int_0^{2\pi} 1^2\,d\varphi$$

$$= \int_0^{2\pi} 1\,d\varphi = \varphi\Big|_0^{2\pi} = 2\pi - 0 = \underline{\underline{2\pi}}$$

3.

Kugla

$r = 2$

$T(0,0)$

$\iiint\limits_K (2x+3)\,dx\,dy\,dz$

$r \in [0,2]$

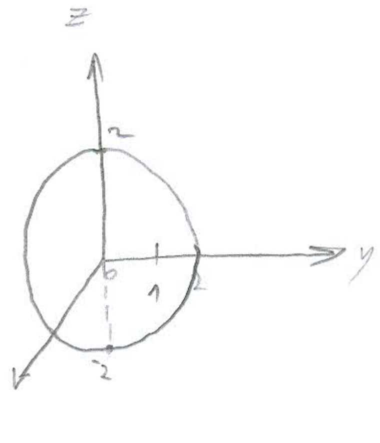
$\varphi \in [0, 2\pi]$

$x = r\cos\varphi$

$y = r\sin\varphi$

$dx\,dy\,dz = r\,dr\,d\varphi\,dz$

$z \in [-2, 2]$



$$\int_0^{2\pi}\int_{-2}^{2}\int_0^{2} [(2r\cos\varphi)+3]\,r\,dr\,dz\,d\varphi = \int_0^{2\pi}\int_{-2}^{2}\int_0^{2} 2r^2\cos\varphi + 3r\,dr\,dz\,d\varphi =$$

$$\int_0^{2\pi}\int_{-2}^{2}\left(2\cdot\frac{r^3}{3}\cos\varphi + 3\cdot\frac{r^2}{2}\right)\Big|_0^2 dz\,d\varphi = \int_0^{2\pi}\int_{-2}^{2}\left(2\cdot\frac{2^3}{3}\cos\varphi + 3\cdot\frac{2^2}{2}\right) dz\,d\varphi = \int_0^{2\pi}\int_{-2}^{2}\frac{16}{3}\cos\varphi + 6\,dz\,d\varphi =$$

$$= \int_{-2}^{2}\int_0^{2\pi}\frac{16}{3}\cos\varphi + 6\,d\varphi\,dz = \int_{-2}^{2}\frac{16}{3}\sin\varphi + 6\varphi\Big|_0^{2\pi} dz = \int_{-2}^{2}\left(\frac{16}{3}\sin 2\pi + 6\cdot 2\pi\right) - \left(\frac{16}{3}\cdot\sin 0 + 0\right)$$

$$= \int_{-2}^{2}\left(\frac{16}{3}\cdot 0 + 12\pi - 0\right) dz = \int_{-2}^{2} 12\pi\,dz = 12\pi z\Big|_{-2}^{2} = 12\pi\cdot(2) - (12\pi\cdot(-2)) = 24\pi + 24\pi =$$

$$= \underline{\underline{48\pi}}$$

$x^2+y^2=4z$, $z \leq 4$,

zračunati $\iint_P 3\,dS$

$$\sqrt{1+\left(\frac{\partial x}{\partial z}\right)^2+\left(\frac{\partial y}{\partial z}\right)^2}=\sqrt{1+\left(\frac{x}{2}\right)^2+\left(\frac{y}{2}\right)^2}=\sqrt{1+\frac{x^2}{4}+\frac{y^2}{4}}=$$

$$=\sqrt{\frac{4+x^2+y^2}{4}}$$

$$=\sqrt{\frac{4+r^2}{4}}$$

$$=\frac{\sqrt{4+r^2}}{2}$$

$$=\frac{1}{2}\sqrt{4+r^2}$$

$4z=x^2+y^2$

$4z\,\partial z=x^2\,\partial x$

$\frac{\partial z}{\partial x}=\frac{\cancel{2}x}{\cancel{4}_2}=\frac{x}{2}$

$4z=x^2+y^2$

$4z\,\partial z=y^2\,\partial y$

$\frac{\partial z}{\partial y}=\frac{\cancel{2}y}{\cancel{4}}=\frac{y}{2}$

$x^2+y^2=4z$

$x^2+y^2=r^2$

$r^2=4\cdot 4$

$r^2=16$

$r=\sqrt{16}$

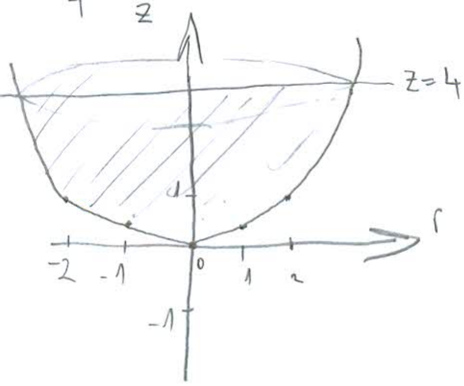
$r=4$

$r\in[0,4]$

$r^2=4z$

r	-1	0	1	2	-2
$z=\frac{r^2}{4}$	1/4	0	1/4	1	1

$z=\frac{r^2}{4}$



$$\iint_P 3\,dS=\int_0^{2\pi}\int_0^4 3\cdot\frac{1}{2}\sqrt{4+r^2}\;r\,dr\,d\varphi=$$

$$=\int_0^{2\pi}\left(\frac{1}{2}\cdot\sqrt{(4+r^2)^3}\right)\Big|_0^4 d\varphi=\int_0^{2\pi}\left(\left(\frac{1}{2}\cdot\sqrt{(4+4^2)^3}\right)-\left(\frac{1}{2}\cdot\sqrt{(4+0^2)^3}\right)\right)d\varphi=$$

$$\int_0^{2\pi}(20\sqrt5-4)\,d\varphi=\int_0^{2\pi}16\sqrt5\,d\varphi=16\sqrt5\,\varphi\Big|_0^{2\pi}=16\sqrt5\cdot 2\pi-16\sqrt5\cdot 0$$

$$=32\sqrt5\,\pi$$

$$\int\frac{3}{2}\cdot\sqrt{4+r^2}\;r\,dr=\begin{bmatrix}4+r^2=t\\ 2r\,dr=dt\\ r\,dr=\frac{dt}{2}\end{bmatrix}=\frac{3}{2}\cdot\int\sqrt{t}\,\frac{dt}{2}=\frac{3}{2}\cdot\frac{1}{2}\int\sqrt{t}\,dt=\frac{3}{\cancel{4}_2}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{\cancel{2}_1}}=\frac{3}{2}\cdot\frac{\sqrt{t^3}}{3}=\frac{\cancel{3}}{2}\cdot\frac{1}{\cancel{3}}\sqrt{t^3}=$$

$$=\frac{1}{2}\cdot\sqrt{(4+r^2)^3}$$

IVAN COLIĆ

5. Izračunati

IVAN COHĆ

$$\int_{(3,2)}^{(5,5)} x\,dy + y\,dx = \int_{(3,2)}^{(5,5)} y\,dx + x\,dy =$$

$$\begin{pmatrix} y \\ x \end{pmatrix} = \overline{\text{grad}}\, f = \begin{pmatrix} -\partial_x f \\ -\partial_y f \end{pmatrix}$$

$\partial x = -y \Rightarrow \partial f = \int -y\,dx = -xy + C(y)$

$f(x,y) = -xy + C_y$

$\partial y = -x \Rightarrow \cancel{-x} + C(y)' = \cancel{-x}$

$C(y) = 0$

$f(x,y) = -xy + 0 = -xy$

$f(x,y) = (-3\cdot 2) - (-5\cdot 5) = -6 - (-25) = -6 + 25 = \underline{\underline{19}}$ ✓

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

POPUNJAVA NASTAVNIK

Broj ↓ bodova

IME I PREZIME: Luka Bihavac

BROJ INDEKSA: A-2-0022-2010

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,0)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int\limits_{\widehat{\partial K}} (2x+3)\,\mathrm{d}y$? 20

3. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$? ~~20~~

4. Zadan je P paraboloid $x^2 + y^2 = 4z$, $z \le 4$. Izračunati $\iint\limits_P 3\mathrm{d}S$? ~~20~~

5. Izračunati $\int\limits_{(3,2)}^{(5,5)} x\mathrm{d}y + y\mathrm{d}x$. 20

Ukupno: 40

(4.) $P = P_a$

$z \le 4$

$\iint\limits_P 3ds$

$x^2+y^2=4z$

$r^2 = 4z$

$r = \sqrt{4z}$

$\varphi\ [0, 2\pi]$

$x = r\sin\varphi$

$y = r\cos\varphi$

$r\ [0, \sqrt{4z}]$ ✗

$z = z$

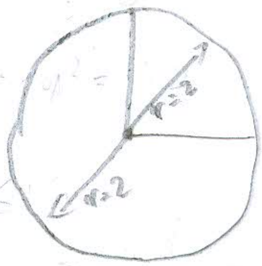
$z\ [0, 4]$

$$3\int_0^{2\pi} d\varphi \int_0^4 dz \int_0^{\sqrt{4z}} r\,dr = 3\int_0^{2\pi} d\varphi \int_0^4 \frac{r^2}{2}\Big|_0^{\sqrt{4z}} dz$$

$$3\int_0^{2\pi} d\varphi \int_0^4 \left(\frac{4z}{2}\right) dz = 3\int_0^{2\pi} \frac{4z^2}{4}\Big|_0^4 d\varphi = 3\int_0^{2\pi} (16)\, d\varphi$$

$$48\varphi\Big|_0^{2\pi} = 96\pi$$

3. KUGLA



r = 2

$$\iiint_K (2x+3)\,dx\,dy\,dz$$

$\varphi [0, 2\pi]$

$r [0, 2]$

?

① $x'''(t) + x'(t) = 0$

$x(0) = 1$

$x''(0) = 1$

$x'(0) = 0$

Luka Bekavac

17-2-0022-2010

$$s^3 y(s) \underbrace{- s^2 y(0)}_{1} \underbrace{- s y'(0)}_{0} \underbrace{- y''(0)}_{1} + s y(s) \underbrace{- y(0)}_{1} = 0$$

$$s^3 y(s) - s^2 - 1 + s y(s) - 1 = 0$$

$$y(s)(s^3 + s) = s^2 + 2$$

$$y(s) = \frac{s^2+2}{(s^3+s)} = \frac{s^2+2}{s(s^2+1)} =$$

$$y(s) = \frac{A}{s} + \frac{Bs + C}{(s^2+1)} \quad / \; s(s^2+1)$$

$$s^2+2 = A(s^2+1) + s(Bs+c)$$

$s=0$

$\boxed{A=2}$

$\boxed{C=0}$

$$s^2+2 = 2s^2 + 2 + Bs^2 + cs$$

$2 + B = 1$

$\boxed{B = -1}$

$\boxed{A=2}$ $\boxed{B=-1}$ $\boxed{C=0}$

$$y(t)=\frac{2}{s}-\frac{s}{(s^2+1)}$$

$$\boxed{y(t)=2-\cos t}$$

Provjera:

$$f'(t)=\sin t$$

$$f''(t)=\cos t$$

$$f'''(t)=-\sin t$$

$$f(t)=-\cancel{\sin t}+\cancel{\sin t}=0$$

$$\boxed{f(t)=0=0}$$

Luka Bilanac

17-2-0022-2010

(2) $r=1$ $\quad T(0,0)$

$$\int_{\partial K} (2x+3)\,dy$$

$x = r\cos t$

$y = r\sin t$

$$r(t) = \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix} \qquad r' = \begin{bmatrix} -\sin(t) \\ \cos(t) \end{bmatrix}$$

$$\|r'(t)\| = \sqrt{\sin^2(t) + \cos^2(t)}$$

$$= \sqrt{1}$$

$$= 1$$

$$\int_0^{2\pi} (2\cos^2 t + 3\cos t)\,dt = 2\int_0^{2\pi} \cos^2 t\,dt + 3\int_0^{2\pi} \cos t\,dt$$

$$= 2\int_0^{2\pi} \frac{1+\cos 2t}{2}\,dt = 2\pi$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: Ravana Naval BROJ INDEKSA: 17-2-0097-2011.

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1,\ x'(0) = 0.$$

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3. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint_K (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$? ~~20~~

4. Zadan je P paraboloid $x^2 + y^2 = 4z$, $z \le 4$. Izračunati $\iint_P 3\mathrm{d}S$? ~~20~~

5. Izračunati $\int_{(3,2)}^{(5,5)} x\mathrm{d}y + y\mathrm{d}x$. ~~20~~ 10

Ukupno: 40

(2) $r = 1$ $\quad$ $T(0,0)$ $\quad$ $\int (2x+3)\,dy$ $\quad$ $x = r\cos\varphi$, $y = r\sin\varphi$ $\quad$ $x = -\sin\varphi$, $y = \cos\varphi$

$$\int_0^{2\pi} (2\cos\varphi + 3\cos\varphi)\,d\varphi = \int_0^{-2\pi} 2\cos^2\varphi + 3\cos\varphi\,d\varphi$$

$$= 2\cdot\frac{1}{2}\left|\cos\varphi\cos\varphi + \varphi\right|_0^{2\pi} + 3\,\cancel{\sin\varphi}\Big|_0^{-2\pi}$$ 10

$$= -2\pi$$

ORIJENTACIJA U SMJERU KAZALJKE NA SATU

(5) $$\int_{(3,2)}^{(5,5)} x\,dy + y\,dx = \varphi_1(3,2) - \varphi_2(5,5)$$

$$= -6 - 25 = -31$$

$$\frac{\partial\varphi}{\partial x} = y \qquad \varphi_1 = xy + C \checkmark$$

$$\frac{\partial\varphi}{\partial y} = -y \qquad \varphi_2 = -xy \checkmark$$ 10

(4.) $x^2+y^2=4z$

$z \leq 4$

Izračunaj $\iint_P 3\,ds$?

$z=\frac{x^2}{4}+\frac{y^2}{4}$

$x=r\cos\varphi$

$y=r\sin\varphi$

$z=\frac{r^2}{4}$

$\frac{r^2}{4}\leq 4$

$r^2 \leq 4$

$r \geq$

$3\int_{40}^{2\pi}\int_0^4 r\,d\varphi\,dr$

$\iint_P 3\,ds$

$= 3\int_0^{2\pi} \frac{r^2}{2}\Big|_0^{40} = 6\pi \cdot \frac{4^2}{2} = 48\pi$ 0

$\iint_P 3\,ds$

$r\in[0,4]$

(1.)

$y(s)[s^3+s]-1-s^2-1=0$

$y(s)[s^3-s]=s^2+2$

$y(s)=\frac{s^2+2}{s(s^2+1)}=\frac{A}{s}+\frac{3}{s^2+1}=\frac{2}{s}-\frac{5}{s^2-1}$

$\frac{A}{s}+\frac{3s+C}{s^2-1}=\frac{s^2+2}{s(s^2+1)}$

$\boxed{x(t)=2-\cos(t)}$

$A(s^2+1)+B\cdot s^2-1=s+2$

$A+B=1 \qquad B=-1$

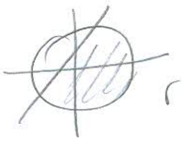
$C=0$

$A=2$

(3) $r = 2$

$T(6,4)$

$\iiint_Z (2x+3)\,dx\,dy\,dz$



$r = 2$

$x = r\cos\varphi = 2\cos\varphi$

$y = r\sin\varphi = 2\sin\varphi$

$z = 2r\cos\varphi + 3$

$x^2 + y^2 = 4$

$2x + 3 = 0$

$x = \pm \frac{3}{2}$

$y = \pm \frac{\sqrt{7}}{2}$

$$\int_{-\frac{3}{2}}^{\frac{3}{2}} \int_{-\frac{\sqrt{7}}{2}}^{\frac{\sqrt{7}}{2}} \int_{-2}^{2} (2x+3)\,dx\,dy\,dz =$$

$$= 4\sqrt{7} \cdot \left| \left(\frac{2x^2}{2} + 3x \right) \right|_{-\frac{3}{2}}^{\frac{3}{2}}$$

$$= 4\sqrt{7} \left(\cancel{\frac{9}{4}} + \frac{9}{2} - \cancel{\frac{9}{4}} + \frac{9}{2} \right)$$

$$= 36\sqrt{7}$$

$$\int_{-2}^{2}\int_{-2}^{2}\int_{-2}^{2} (2x+3)\,dx\,dy\,dz$$

$$= 16 \cdot (x^2 + 3x) \Big|_{-2}^{2} = 16 \cdot (\cancel{4} + 6 - \cancel{4} + 6) = 16 \cdot 12 = 192$$

$16 \cdot 12$
$+32$
192

Romana Novosel

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: MARIN SMOLIĆ **BROJ INDEKSA:** 55376-2007

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1,\ x'(0) = 0.$$

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3. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$? ~~20~~

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5. Izračunati $\int\limits_{(3,2)}^{(5,5)} x\mathrm{d}y + y\mathrm{d}x.$ 20

Ukupno: ~~Ø~~

2. $r = 1$

$T = (0,0)$

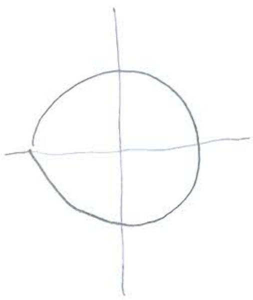
$$\int\limits_{\partial K} (2x+3)\,dy \qquad x = r\cos\varphi$$

$r \in [0,1]$

$\varphi \in [0, 2\pi]$

$$\int_0^{2\pi} (2r\cos\varphi + 3)\, r\, d\varphi$$

X



$$\int_0^{2\pi} (2\cos\varphi + 3)\, d\varphi$$

$$= 2\sin\varphi + 3\varphi \Big|_0^{2\pi}$$

$$= 2\sin 2\pi + 6\pi$$

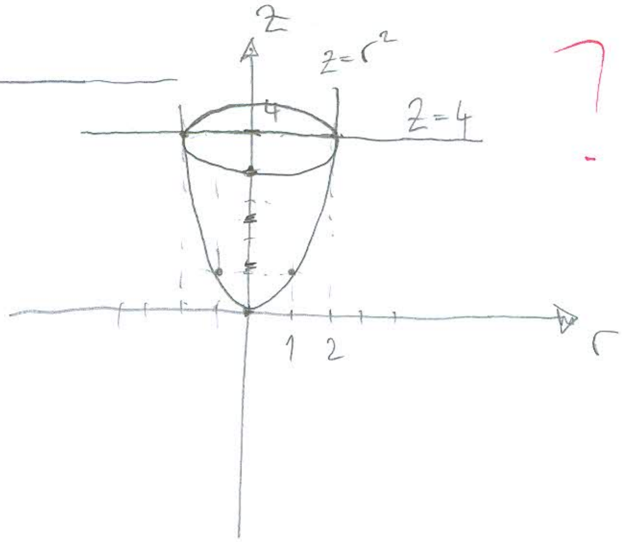
$$= \underline{6\pi} \,//$$

4. $x^2+y^2=4z \qquad z \leqslant 4 \qquad \iint_P 3\,ds = ?$

$z=4$

$x^2+y^2=r^2$

r	0	1	-1	2	-2
$z=r^2$	0	1	1	4	4



?

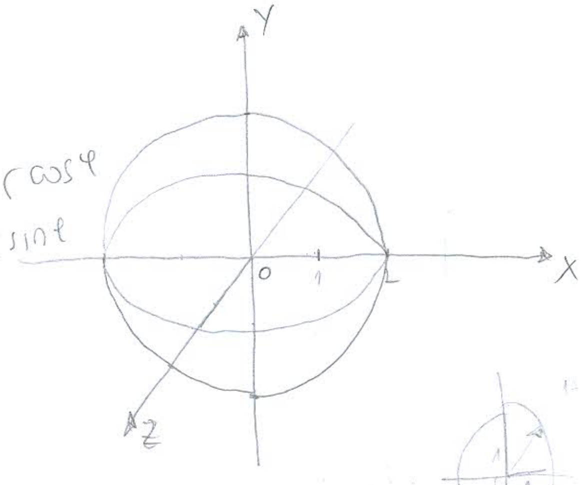
3. $r=2$ $\quad r\in[0,2]$ ✓

$T=(\overset{x_1}{0},\overset{y_1}{0},\overset{z_1}{0})$ $\quad \varphi\in[0,2\pi]$ ✓

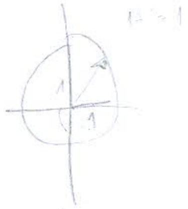
$z\in[0,2\ ]$ ✗

$x=r\cos\varphi$

$y=r\sin\varphi$



$$\iiint_K (2x+3)\,dx\,dy\,dz$$



$$\int_0^{2\pi}\int_0^{2}\int_0^{2}(2r\cos\varphi+3)\,dr\,dz\,d\varphi=$$ ✗

$$\int_0^{2\pi}\int_0^{2}\cancel{2}\cdot\frac{r^2}{\cancel{2}}\cos\varphi+3r\Big|_0^2\,dz\,d\varphi=\int_0^{2\pi}\int_0^{2}4\cos\varphi+6\;dz\,d\varphi$$

$$\int_0^{2\pi}4z\cos\varphi+6z\Big|_0^2\,d\varphi=\int_0^{2\pi}8\cos\varphi+12\;d\varphi=8\sin\varphi+12\varphi\Big|_0^{2\pi}$$

$$=\cancel{8\sin 2\pi}+12\cdot 2\pi=\underline{24\pi}\,\|$$

MARIN SMOLIČ

1. $x'''(t)+x'(t)=0 \quad , \quad x(0)=x''(0)=1 \quad x'(0)=0$

$$s^3X(s)-s^2x(0)-sx'(0)-x''(0)+sX(s)-x(0)=0$$

$$s^3X(s)-s^2-1+sX(s)-1=0$$

$$s^3X(s)+sX(s)=2-s^2$$

$$X(s)(s^3+s)=2-s^2 \quad /:(s^3+s)$$

$$X(s)=\frac{2-s^2}{s^3+s}=\frac{2-s^2}{s(s^2+1)}=\frac{A}{s}+\frac{Bs+C}{s^2+1}$$

$$\frac{2-s^2}{s(s^2+1)}=\frac{A}{s}+\frac{Bs+C}{s^2+1} \quad \Big| \cdot s(s^2+1)$$

$$2-s^2=A(s^2+1)+s(Bs+C)$$

$$2-s^2=As^2+A+Bs^2+Cs$$

$$-s^2+2=s^2(A+B)+s(C)+A$$

$A+B=-1 \qquad A+B=-1$

$\boxed{C=0} \qquad 2+B=-1$

$\boxed{A=2} \qquad \boxed{B=-3}$

$$X(s)=2\cdot\frac{1}{s}+\frac{-3s+0}{s^2+1}$$

$$X(s)=2\cdot\frac{1}{s}-\frac{3}{1}\cdot\frac{s}{s^2+1^2}$$

$$X(s)=2\cdot 1-\frac{3}{1}\cdot\cos t$$

$$X(s)=\underline{2-3\cos t}\,\|$$

X

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: VEDRAN ČIZMIN BROJ INDEKSA: 17-2-0089-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1,\ x'(0) = 0.$$

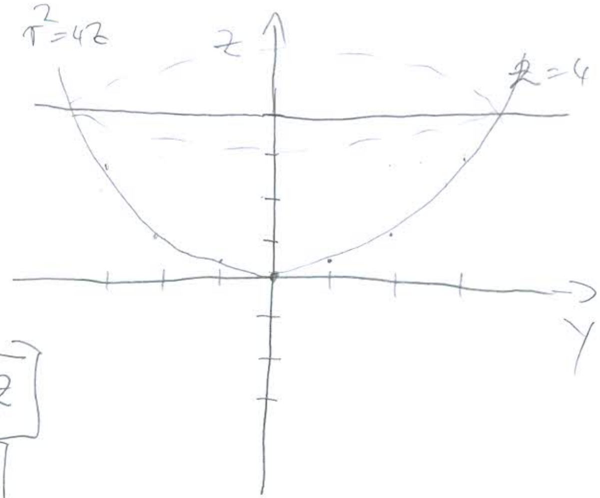
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,0)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int_{\widehat{\partial K}} (2x+3)\,dy$? ~~20~~

3. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint_K (2x+3)\,dxdydz$? ~~20~~

4. Zadan je P paraboloid $x^2 + y^2 = 4z$, $z \leq 4$. Izračunati $\iint_P 3\,dS$? ~~20~~

5. Izračunati $\int_{(3,2)}^{(5,5)} x\,dy + y\,dx$. 20

Ukupno: Ø



(4.) $x^2 + y^2 = 4z \qquad z \leq 4$

$x^2 + y^2 = r^2$

$r^2 = 4z \qquad r \in [0, 2\sqrt{z}]$

$r = \sqrt{4z} \qquad z \in [0, 4]$

$r = 2\sqrt{z} \qquad \varphi \in [0, 2\pi]$

r	0	1	-1	2	-2	3	-3
$z = \frac{r^2}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}$	1	1	$\frac{9}{4}$	$\frac{9}{4}$

$$\int_0^{2\pi}\int_0^{2\sqrt{z}}\int_0^{4} 3r\,dr\,dz\,d\varphi \quad \times$$

$$= \int_0^{2\pi}\int_0^{2\sqrt{z}} 3rz \Big|_0^4 dr\,d\varphi = \int_0^{2\pi}\int_0^{2\sqrt{z}} 3r\big((4-0)\big) = \int_0^{2\pi}\int_0^{2\sqrt{z}} 12r\,dr\,d\varphi$$

$$= \int_0^{2\pi} 12\frac{r^2}{2}\Big|_0^{2\sqrt{z}} d\varphi = \int_0^{2\pi} 12\left(\frac{(2\sqrt{z})^2}{2} - 0\right)d\varphi = \int_0^{2\pi} 12\left(\frac{4z}{2}\right)d\varphi = \int_0^{2\pi} 24z\,d\varphi$$

$$= 24z\varphi\Big|_0^{2\pi} = 24z \cdot 2\pi = 48\pi z$$

3.) kugla $r=2$ $\quad T(0,0)$ $\quad \iiint\limits_K (2x+3)\,dx\,dy\,dz$

$x^2+y^2+z^2=r^2$

$x^2+y^2=r^2$

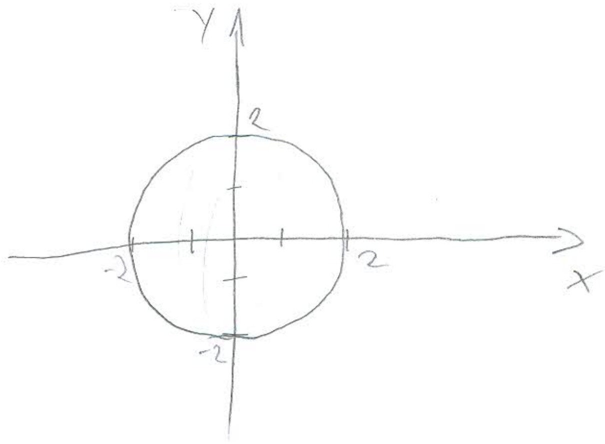
$x=r\cos\varphi$

$y=r\sin\varphi$

$\varphi\in[0,2\pi]$

$r\in[0,2]$

$z\in[0,2-r]$



$r^2+z^2=4$

$z^2=4-r^2$

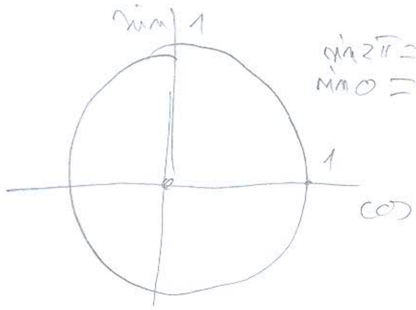
$z=\sqrt{4-r^2}$ ✓

$z=2-r$ ✗

$$\int_0^{2\pi}\int_0^{2}\int_0^{2-r}(2r\cos\varphi+3)\,r\,dr\,d\varphi\,dz=\int_0^{2\pi}\int_0^{2}\int_0^{2-r}2r^2\cos\varphi+3r\,dr\,d\varphi\,dz$$

$$=\int_0^{2\pi}\int_0^{2}2r^2z\cos\varphi+3rz\Big|_0^{2-r}dr\,d\varphi=\int_0^{2\pi}\int_0^{2}\left(2r^2(2-r)\cos\varphi+3r(2-r)\right)-0\,dr\,d\varphi$$

$$=\int_0^{2\pi}\int_0^{2}\left(4r^2\cos\varphi-2r^3\cos\varphi+6r-3r^2\right)dr\,d\varphi$$



$\sin 2\pi=0$
$\sin 0=0$

$$=\int_0^{2\pi}\left(4\frac{r^3}{3}\cos\varphi-2\frac{r^4}{4}\cos\varphi+6\frac{r^2}{2}-3\frac{r^3}{3}\right)\Big|_0^2 d\varphi$$

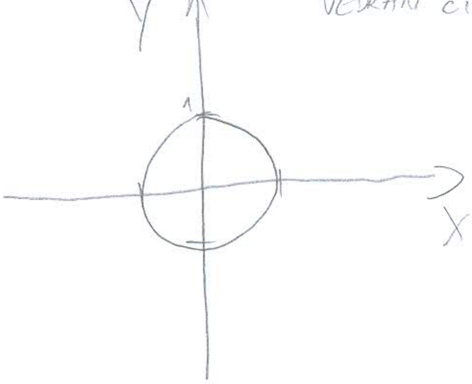
$$=\int_0^{2\pi}\left(4\cdot\frac{2^3}{3}\cos\varphi-2\frac{2^4}{4}\cos\varphi+6\frac{2^2}{2}-3\frac{2^3}{3}\right)-0\,d\varphi=\int_0^{2\pi}\left(\frac{32}{3}\cos\varphi-\frac{32}{4}\cos\varphi+\frac{24}{2}-\frac{24}{3}\right)d\varphi$$

$$=\int_0^{2\pi}\left(\frac{32}{3}\cos\varphi-8\cos\varphi+\frac{12}{3}\right)d\varphi=\frac{32}{3}\sin\varphi-8\sin\varphi+\frac{12}{3}\varphi\Big|_0^{2\pi}=$$

$$=\Big(\underbrace{\frac{32}{3}\sin 2\pi-8\sin 2\pi}_{0}+\frac{12}{3}2\pi\Big)-\underbrace{\left(\frac{32}{3}\sin 0-8\sin 0+\frac{12}{3}\cdot 0\right)}_{0}=\frac{24}{3}\pi=8\pi$$

VEDRAN CIZMIN

(2.) $r = 1 \quad T(0,0) \qquad \oint_{\partial K} (2x+3)\,dy$



$x = r\cos\varphi \qquad \varphi \in [0, -2\pi]$

$y = r\sin\varphi \qquad r \in [0,1]$

$$\int_0^{-2\pi}\int_0^1 (2x+3)\,r\,dr\,d\varphi = \int_0^{-2\pi}\int_0^1 (2r^2\cos\varphi + 3r)\,dr\,d\varphi$$

✗

$$= \int_0^{-2\pi}\left(2\frac{r^3}{3}\cos\varphi + 3\frac{r^2}{2}\right)\Big|_0^1 d\varphi = \int_0^{-2\pi}\left(\frac{2}{3}\cos\varphi + \frac{3}{2}\right)d\varphi = \left(\frac{2}{3}\sin\varphi + \frac{3}{2}\varphi\right)\Big|_0^{-2\pi}$$

$$= \left(\underbrace{\frac{2}{3}\sin(-2\pi)}_{0} + \frac{3}{2}\cdot(-2\pi)\right) - \underbrace{\left(\frac{2}{3}\sin 0 + 0\right)}_{0} = -\frac{6\pi}{2} = -3\pi$$

✗