MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: Ivan Colić **BROJ INDEKSA:** 17-2-0152-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1,\ x'(0) = 0.$$

2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,0)$, a $\overset{\frown}{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int_{\overset{\frown}{\partial K}} (2x+3)\,\mathrm{d}y$? 20

3. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint_K (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$? 20

4. Zadan je P paraboloid $x^2 + y^2 = 4z$, $z \le 4$. Izračunati $\iint_P 3\mathrm{d}S$? 20

5. Izračunati $\int_{(3,2)}^{(5,5)} x\mathrm{d}y + y\mathrm{d}x$. 20

Ukupno: 80

① $x'''(t) + x'(t) = 0 \qquad x(0) = x''(0) = 1 \qquad x'(0) = 0$

$s^3F(s) - s^2f(0) - sf'(0) - f''(0) + sF(s) - f(0) = 0$

$f(0) = 1,\ f'(0) = 0,\ f''(0) = 1$

$s^3F(s) - s^2 - 1 + sF(s) - 1 = 0$

$F(s)(s^3 + s) = s^2 + 1 + 1$

$F(s)\,s(s^2+1) = s^2 + 2$

$F(s) = \frac{s^2+2}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} \quad / \cdot s(s^2+1)$

$s^2 + 2 = A(s^2+1) + (Bs+C)\cdot s$

$s^2 + 2 = As^2 + A + Bs^2 + Cs$

$A + B = 1 \Rightarrow 2 + B = 1$

$C = 0 \qquad B = 1 - 2$

$A = 2 \qquad B = -1$

$F(s) = \frac{2}{s} + \frac{-1s}{s^2+1}$

$F(s) = 2\cdot\frac{1}{s} - \frac{s}{s^2+1}$

$F(s) = 2 - \cos t$

$X(s) = 2 - \cos t$ ✓

$X(0) = 2 - \cos 0 = 2 - 1 = 1$ ✓

$X'(0) = 2' - (\cos t)' = 0 + \sin 0 = 0 + 0 = 0$ ✓

$X''(0) = 0 - (-\cos 0) = 0 + 1$

$X''(0) = 1$ ✓ ✓

2

$P(x,y)=2x \qquad Q(x,y)=3$

Krug.

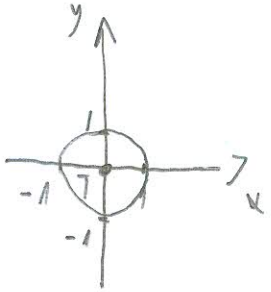
$r=1$

$T(0,0)$

$\int (2x+3)dy$

∂k

$\frac{\partial P}{\partial x}-\frac{\partial Q}{\partial y}=\frac{\partial 2x}{\partial x}-\frac{\partial 3}{\partial y}=2-0=2$



$r\in[0,1]$

$\varphi\in[0,2\pi]$

$x=r\cos\varphi$

$y=r\sin\varphi$

$dxdy=rdrd\varphi$

$$\int_0^{2\pi}\int_0^1 2\,r\,dr\,d\varphi=\int_0^{2\pi}\left.2\cdot\frac{r^2}{2}\right|_0^1 d\varphi=\int_0^{2\pi}1^2\,d\varphi$$

$$=\int_0^{2\pi}1\,d\varphi=\left.\varphi\right|_0^{2\pi}=2\pi-0=2\pi$$ ✓

3.

Kugla

$r=2$

$T(0,0)$

$\iiint_k (2x+3)\,dxdydz$

$r\in[0,2]$

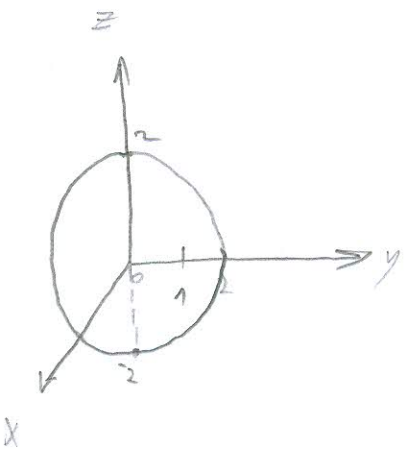
$\varphi\in[0,2\pi]$

$x=r\cos\varphi$

$y=r\sin\varphi$

$dxdydz=rdrd\varphi dz$

$z\in[-2,2]$



$$\int_0^{2\pi}\int_{-2}^{2}\int_0^{2}(2r\cos\varphi+3)\,r\,dr\,dz\,d\varphi=\int_0^{2\pi}\int_{-2}^{2}\int_0^{2}2r^2\cos\varphi+3r\,dr\,dz\,d\varphi=$$

$$\int_0^{2\pi}\int_{-2}^{2}\left.2\cdot\frac{r^3}{3}\cos\varphi+3\cdot\frac{r^2}{2}\right|_0^2 dz\,d\varphi=\int_0^{2\pi}\int_{-2}^{2}\left(2\cdot\frac{2^3}{3}\cos\varphi+3\cdot\frac{2^2}{2}\right)dz\,d\varphi=\int_0^{2\pi}\int_{-2}^{2}\frac{16}{3}\cos\varphi+6\,dz\,d\varphi=$$

$$=\int_{-2}^{2}\int_0^{2\pi}\frac{16}{3}\cos\varphi+6\,d\varphi\,dz=\int_{-2}^{2}\left.\frac{16}{3}\sin\varphi+6\varphi\right|_0^{2\pi}dz=\int_{-2}^{2}\left(\frac{16}{3}\sin 2\pi+6\cdot 2\pi\right)-\left(\frac{16}{3}\cdot\sin 0+0\right)$$

$$=\int_{-2}^{2}\left(\frac{16}{3}\cdot 0+12\pi-0\right)dz=\int_{-2}^{2}12\pi\,dz=\left.12\pi z\right|_{-2}^{2}=12\pi\cdot(2)-(12\pi\cdot(-2))=24\pi+24\pi=$$

$$=48\pi$$

$x^2+y^2=4z$, $z \leq 4$,

zračunati $\iint_P 3\,dS$

$$\sqrt{1+\left(\frac{\partial x}{\partial z}\right)^2+\left(\frac{\partial y}{\partial z}\right)^2}=\sqrt{1+\left(\frac{x}{2}\right)^2+\left(\frac{y}{2}\right)^2}=\sqrt{1+\frac{x^2}{4}+\frac{y^2}{4}}=$$

$$=\sqrt{\frac{4+x^2+y^2}{4}}$$

$4z=x^2+y^2$

$4z\,\partial z=x^2\,\partial x$

$\frac{\partial z}{\partial x}=\frac{2x}{4}=\frac{x}{2}$

$4z=x^2+y^2$

$4z\,\partial z=y^2\,\partial y$

$\frac{\partial z}{\partial y}=\frac{2y}{4}=\frac{y}{2}$

$$=\sqrt{\frac{4+r^2}{4}}$$

$$=\frac{\sqrt{4+r^2}}{2}$$

$$=\frac{1}{2}\sqrt{4+r^2}$$

$x^2+y^2=4z$

$x^2+y^2=r^2$

$r^2=4\cdot 4$

$r^2=16$

$r=\sqrt{16}$

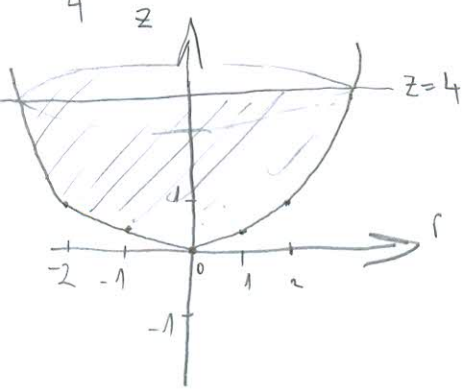
$r=4$

$r\in[0,4]$

$r^2=4z$

$z=\frac{r^2}{4}$

r	-1	0	1	2	-2
$z=\frac{r^2}{4}$	$\frac{1}{4}$	0	$\frac{1}{4}$	1	1



$$\iint_P 3\,dS=\int_0^{2\pi}\int_0^4 3\cdot\frac{1}{2}\sqrt{4+r^2}\,r\,dr\,d\varphi=$$

$$=\int_0^{2\pi}\left(\frac{1}{2}\cdot\sqrt{(4+r^2)^3}\right)\Big|_0^4 d\varphi=\int_0^{2\pi}\left(\left(\frac{1}{2}\cdot\sqrt{(4+4^2)^3}\right)-\left(\frac{1}{2}\cdot\sqrt{(4+0^2)^3}\right)\right)d\varphi=$$

$$\int_0^{2\pi}(20\sqrt{5}-4)\,d\varphi=\int_0^{2\pi}16\sqrt{5}\,d\varphi=16\sqrt{5}\,\varphi\Big|_0^{2\pi}=16\sqrt{5}\cdot 2\pi-16\sqrt{5}\cdot 0$$

$$=32\sqrt{5}\pi$$

$$\int\frac{3}{2}\cdot\sqrt{4+r^2}\,r\,dr=\left[\begin{matrix}4+r^2=t\\ 2r\,dr=dt\\ r\,dr=\frac{dt}{2}\end{matrix}\right]=\frac{3}{2}\cdot\int\sqrt{t}\,\frac{dt}{2}=\frac{3}{2}\cdot\frac{1}{2}\int\sqrt{t}\,dt=\frac{3}{4}\int_1^2 t^{\frac{3}{2}}\,dt=\frac{3}{2}\cdot\frac{\sqrt{t^3}}{3}=\frac{3}{2}\cdot\frac{1}{3}\sqrt{t^3}=$$

$$=\frac{1}{2}\cdot\sqrt{(4+r^2)^3}$$

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5. Izračunati

$$\int_{(3,2)}^{(5,5)} x\,dy + y\,dx = \int_{(3,2)}^{(5,5)} y\,dx + x\,dy =$$

$$\begin{pmatrix} y \\ x \end{pmatrix} = \bar{g}rad\,f = \begin{pmatrix} -\partial_x f \\ -\partial_y f \end{pmatrix}$$

$\partial x = -y \Rightarrow \partial f = \int -y\,dx = -xy + c(y)$ $\qquad f(x,y) = -xy + c_y$

$\partial y = -x \Rightarrow -x + c(y) = -x$

$c(y) = 0$ $\qquad f(x,y) = -xy + 0 = -xy$

$f(x,y) = (-3 \cdot 2) - (-5 \cdot 5) = -6 - (-25) = -6 + 25 = 19$ ✓

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POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: Luka Bikavac

BROJ INDEKSA: A-2-0022-2010

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Ukupno: 40

(4.) $P = ?$ $\quad z \leq 4$ $\quad \iint_P 3\,dS$

$x^2+y^2 = 4z$

$r^2 = 4z$

$r = \sqrt{4z}$

$\varphi \; [0, 2\pi]$

$r \; [0, \sqrt{4z}]$ ×

$x = r\sin\varphi$

$y = r\cos\varphi$

$z = z$

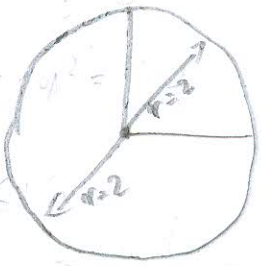
$z \; [0, 4]$

$$3\int_0^{2\pi} d\varphi \int_0^4 dz \int_0^{\sqrt{4z}} r\,dr = 3\int_0^{2\pi} d\varphi \int_0^4 \frac{r^2}{2}\Big|_0^{\sqrt{4z}} dz$$

$$3\int_0^{2\pi} d\varphi \int_0^4 \left(\frac{4z}{2}\right) dz = 3\int_0^{2\pi} \frac{4z^2}{4}\Big|_0^4 d\varphi = 3\int_0^{2\pi} (16)\,d\varphi$$

$$48\varphi\Big|_0^{2\pi} = 96\pi$$

3. KULA



$r = 2$

$$\iiint_K (2x+3)\,dx\,dy\,dz$$

$\varphi \in [0, 2\pi]$

$\vartheta \in [0, \pi]$

?