

**MATEMATIKA 3:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: JOSIP ŠIMIČEV

BROJ INDEKSA:

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačbu:

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$$y'''(t) - y(t) = t, \quad y(0) = 1, \quad y''(0) = 2, \quad y'(0) = 1.$$

2. Neka je  $C$  krivulja sa parametrizacijom  $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \frac{t^3}{3}\mathbf{k}$ ,  $t \in [0, 2]$ . Zadano je skalarno polje  $f(x, y, z) = \frac{1}{x+1}$ . Izračunaj  $\int_C f ds$ .

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3.  $X$  je zadan kao četverokut s vrhovima  $O(0, 0)$ ,  $A(\frac{2}{2}, 0)$ ,  $B(2, \frac{3}{2})$  i  $C(\frac{2}{2}, \frac{4}{2})$ . Izračunati dvostruki integral

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$$\iint_X x^3 dx dy$$

4. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle  $x^2 + y^2 + z^2 = 4$  za koji vrijedi  $z \geq 1$ .

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5. Neka je  $K$  kocka stranice duljine  $a = 2$  centrirana u ishodištu. Izračunati  $\iint_{\partial K} (2x + 3) dx dy$ ?

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Ukupno:

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5.  $a = 2$

$$\iint_{\partial K} (2x + 3) dx dy$$

$$\mathbf{w} = \begin{pmatrix} 0 \\ 0 \\ 2x + 3 \end{pmatrix} \quad \text{div } \mathbf{w} = 0$$

$$x \in [0, 2] \quad z \in [0, 2] \\ y \in [0, 2]$$

$$\iint_{\partial K} (2x + 3) dx dy = \iiint_K \text{div } \mathbf{w} dx dy dz = \iiint_K 0 dx dy dz = 0 \quad \checkmark$$

$$1) \quad y'''(t) - y(t) = t \quad y(0) = 1 \quad y''(0) = 2 \quad y'(0) = 1$$

$$s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) - Y(s) = \frac{1}{s^2}$$

$$s^3 Y(s) - s^2 - s - 2 - Y(s) = \frac{1}{s^2}$$

$$Y(s)(s^3 - 1) = \frac{1}{s^2} + s^2 + s + 2 = \frac{s^4 + s^3 + 2s^2 + 1}{s^2}$$

$$Y(s) = \frac{s^4 + s^3 + 2s^2 + 1}{s^2(s^3 - 1)} = \frac{s^4 + s^3 + 2s^2 + 1}{s^2(s-1)(s^2+s+1)}$$

$$\frac{s(s^2-1)}{(s^2-s)} \quad \frac{(s+1)(s-1)}{(s^2+s+1)}$$

$$\frac{s(s+1)(s-1)}{(s^2-s)(s^2+1) \cdot s} \quad \frac{s^2-s}{(s^2-1) \cdot s} \quad \frac{(s^2-1)(s^2-s)}{s^5 - s^3 + s^3 - s}$$

$$= \frac{s^4 + s^3 + 2s^2 + 1}{s^2(s+1)(s-1)(s^2-1) \cdot s} \quad \frac{(s^3-s)(s^2+1) \cdot s}{s^5 + s^3 - s^2 - s} \quad \frac{(s^2-s)(s^2-1)}{(s^5-s)} \quad \frac{(s^2-s)(s^2-s)}{s^5 - s^2 - s^3 - s^2}$$

$$\frac{A}{s} + \frac{B}{s+1} + \frac{C}{s-1} + \frac{Ds+E}{s^2+1} \Rightarrow As(s^5-s) + Bs^5 - 2Bs^2 - Bs^3 + Cs^5 - Cs + Ds^4 - Ds^2 - Es^3 - Es$$

$$As^6 - As^2 + Bs^5 - 2Bs^2 - Bs^3 + Cs^5 - Cs + Ds^4 - Ds^2 - Es^3 - Es$$

$$A=0$$

$$A=0$$

$$-2B - D = 2$$

$$-A = 2 + 2B + D$$

$$B+C=0$$

$$D=1$$

$$-2B - 2 + D = 3$$

$$= 2 - 3 + 1 = -1 + 1 = 0$$

$$D=1$$

$$B+C=0$$

$$-B-E=1$$

$$\boxed{B = -\frac{3}{2}}$$

$$-B-E=1$$

$$-2B-D=2$$

$$-B-E=1$$

$$E = 1 + B = 1 - \frac{3}{2} = -\frac{1}{2} = \frac{1}{2} = E$$

$$-A - 2B - D = 2$$

$$B+C=0$$

$$Y(s) = -\frac{3}{2} \cdot \frac{1}{s+1} + \frac{3}{2} \cdot \frac{1}{s-1} + 1 \cdot \frac{s}{s^2+1} + \frac{1}{2} \cdot \frac{1}{s^2+1}$$

$$y(t) = -\frac{3}{2} e^{-t} + \frac{3}{2} e^t + \cos t + \frac{1}{2} \sin t \quad y(0) = -\frac{3}{2} + \frac{3}{2} + 1 = 1$$

$$y'(t) = \frac{3}{2} e^{-t} + \frac{3}{2} e^t - \sin t + \frac{1}{2} \cos t \quad y'(0) = \frac{7}{2} \Rightarrow \text{X}$$

$$y''(t) = -\frac{3}{2} e^{-t} + \frac{3}{2} e^t - \cos t + \frac{1}{2} \sin t = -1$$

$$2. \quad r(t) = ti + t^2j + \frac{t^3}{3}k \quad t \in [0, 2] \quad f(x, y, z) = \frac{1}{x+1}$$

$$\int_C f ds = ?$$

$$r(t) = \begin{pmatrix} t \\ t^2 \\ \frac{t^3}{3} \end{pmatrix} \Rightarrow r'(t) = \begin{pmatrix} 1 \\ 2t \\ t^2 \end{pmatrix} \checkmark$$

$$\|r'(t)\| = \sqrt{1^2 + (2t)^2 + (t^2)^2} = \sqrt{1 + 4t^2 + t^4}$$

$$\int_C f ds = \int_0^2 \frac{1}{t+1} \cdot \sqrt{1+4t^2+t^4} dt \checkmark \underline{20}$$

$$1+4t^2+t^4 = u$$

$$8t+4t^3 dt = du$$

$$dt = \frac{1}{8t+4t^3} du$$

DA JE JE NETOČNO, ALI ZADAN JE DOSTA TEŽAK INTEGRAL! RIJEŠAVATI NUMERIČKOM INTEGRACIJOM!

$$= \int \sqrt{u} \cdot \frac{1}{8t+4t^3} du = \int u^{\frac{1}{2}} du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3} \cdot u^{\frac{3}{2}} \cdot \frac{1}{8t+4t^3}$$

$$= \frac{2}{3} \cdot \sqrt{u^3} \cdot \frac{1}{8t+4t^3} = \frac{2}{24t+12t^3} \cdot \sqrt{(1+4t^2+t^4)^3} \Big|_0^2$$

$$= \frac{2}{24 \cdot 2 + 12 \cdot 2^3} \left( \sqrt{(1+(4 \cdot 2^2)+4 \cdot 2)^3} \right) - \left( 0 \cdot \sqrt{1^3} \right)$$

$$= \frac{1}{72} \cdot 125 - 0 = \frac{125}{72} - 0 = \frac{125-72}{72} = \frac{53}{72}$$

JOSIP ŠIMICĀEV

$$4. \quad x^2 + y^2 + z^2 = 4 \quad z \geq 1$$

$$x^2 + y^2 + z^2 = 4$$

$$r^2 + z^2 = 4$$

$$r^2 = 4 - z^2$$

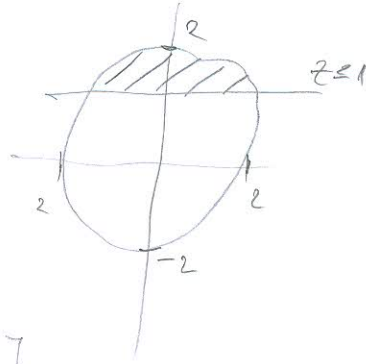
$$r = \sqrt{4 - z^2}$$

$$z \geq 1 \quad z \in [1, 2]$$

$$r = 2$$

$$r \in [0, \sqrt{4 - z^2}]$$

$$\rho \in [0, 2\pi]$$



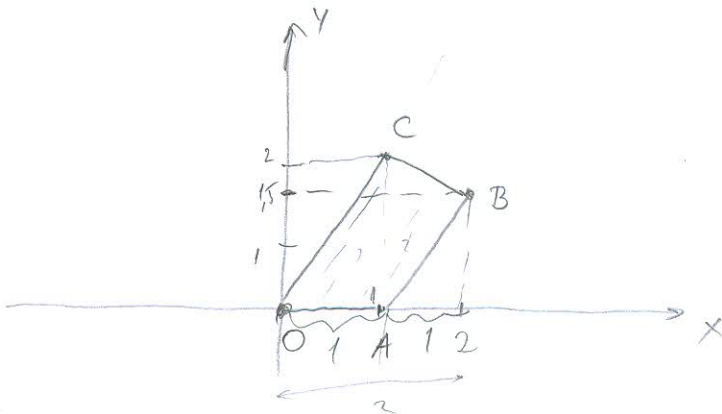
$$\begin{aligned} V &= \int_0^{2\pi} \int_0^2 \int_0^{\sqrt{4-z^2}} r \, dr \, dz \, d\rho \\ &= \int_0^{2\pi} \int_0^2 \frac{1}{2} r^2 \Big|_0^{\sqrt{4-z^2}} dz \, d\rho = \int_0^{2\pi} \int_0^2 \frac{1}{2} (4 - z^2) dz \, d\rho = \int_0^{2\pi} \int_0^2 2 - \frac{z^2}{2} dz \, d\rho \\ &= \int_0^{2\pi} \left( 2z - \frac{1}{2} \frac{z^3}{3} \Big|_0^2 \right) d\rho = \int_0^{2\pi} \left( 2 \cdot 2 - \frac{1}{2} \cdot \frac{8}{3} \right) - \left( 2 \cdot 0 - \frac{1}{2} \cdot \frac{0}{3} \right) d\rho \\ &= \int_0^{2\pi} \frac{8}{3} - \frac{4}{3} d\rho = \int_0^{2\pi} \frac{4}{3} d\rho = \frac{4}{3} \rho \Big|_0^{2\pi} = \frac{4}{3} 2\pi = \frac{8}{3} \pi \quad \checkmark \end{aligned}$$

JOŠIP ŠIMIČEV

3. četvorkut =  $O(0,0)$   $A(\frac{2}{2},0)$   $B(2, \frac{3}{2})$  i  $C(\frac{2}{2}, \frac{4}{2})$

$$\iint_x x^3 dx dy \Rightarrow$$

$O(0,0)$   $A(1,0)$   $B(2, \frac{3}{2})$   $C(1,2)$



$$\overline{OA} = y = 0$$

$$\overline{AB} \quad A(1,0) \quad B(2, \frac{3}{2})$$

$$\overline{BC} \quad B(2, \frac{3}{2}) \quad C(1,2)$$

$$(2-1)(y-0) = (\frac{3}{2}-0)(x-1) \quad (1-2)(y-\frac{3}{2}) = (2-\frac{3}{2})(x-2)$$

$$y = \frac{3}{2}x - \frac{3}{2}$$

$$-1y + \frac{3}{2} = \frac{1}{2}x - 1$$

$$-y = \frac{1}{2}x - 1 - \frac{3}{2} = \frac{1}{2}x - \frac{5}{2} \quad /: (-1)$$

$$y = -\frac{1}{2}x + \frac{5}{2}$$

$$\overline{OC} \quad O(0,0) \quad C(1,2)$$

$$(1-0)(y-0) = (2-0)(x-0)$$

$$y = 2x$$

$$\int_0^1 \int_0^{2x} x^3 dy dx + \int_1^2 \int_{\frac{3}{2}x - \frac{3}{2}}^{-\frac{1}{2}x + \frac{5}{2}} x^3 dy dx = \int_0^1 y x^3 \Big|_0^{2x} dx + \int_1^2 y x^3 \Big|_{\frac{3}{2}x - \frac{3}{2}}^{-\frac{1}{2}x + \frac{5}{2}} dx$$

$$= \int_0^1 2x^4 dx + \int_1^2 (-\frac{1}{2}x + \frac{5}{2}) - (\frac{3}{2}x - \frac{3}{2}) x^3 dx = \int_0^1 2x^4 dx + \int_1^2 (-2x + 4) x^3 dx$$

$$= \int_0^1 2x^4 dx + \int_1^2 -2x^4 + 4x^3 dx = \frac{2}{5}x^5 \Big|_0^1 + (-\frac{2}{5}x^5 + \frac{4}{4}x^4) \Big|_1^2 \Rightarrow$$

NASTAVAK

$$= \frac{2}{5} \times 5^1 + \left( -\frac{2}{5} \times 5 + \frac{4}{5} \times 4 \right)^2$$

$$= \frac{2}{5} \cdot 1^5 + \left( -\frac{2}{5} \cdot 2^5 + 2^4 \right) - \left( -\frac{2}{5} \cdot 1^5 + 1^4 \right)$$

$\left( \frac{2}{5} \right)$

$$= \frac{2}{5} + \left( -\frac{64}{5} + 16 \right) - \left( -\frac{2}{5} + 1 \right)$$

$$= \frac{2}{5} + \left( \frac{16}{5} - \left( \frac{3}{5} \right) \right) = \frac{2}{5} + \frac{13}{5} = \frac{15}{5} = 3 \checkmark$$



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IME I PREZIME:

Tomislav Kaljev

BROJ INDEKSA:

17-01-0052-2011

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