MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: JOSIP ŠIMIČEV

BROJ INDEKSA:

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) - y(t) = t, \quad y(0) = 1,\ y''(0) = 2,\ y'(0) = 1.$$

2. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \frac{t^3}{3}\mathbf{k}$, $t \in [0, 2]$. Zadano je skalarno polje $f(x, y, z) = \frac{1}{x+1}$. Izračunaj $\int_C f\,ds$. 20

3. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{2}{2}, 0)$, $B(2, \frac{3}{2})$ i $C(\frac{2}{2}, \frac{4}{2})$. Izračunati dvostruki integral 20

$$\iint_X x^3 \mathrm{d}x\mathrm{d}y$$

4. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2 + y^2 + z^2 = 4$ za koji vrijedi $z \geq 1$. 20

5. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\partial K} (2x + 3)\,dxdy$? 20

Ukupno: 80

5. $a = 2$

$\iint_{\partial K} (2x+3)\,dxdy \qquad W = \begin{pmatrix} 0 \\ 0 \\ 2x+3 \end{pmatrix} \quad \operatorname{div} W = 0$

$x \in [0,2] \quad z \in [0,2)$

$y \in [0,2]$

poz

$$\iint_{\partial K} (2x+3)\,dxdy = \iiint_K \operatorname{div} W\,dxdydz = \iiint_K 0\,dxdydz = 0$$

1) $y'''(t) - y(t) = t \qquad y(0)=1 \quad y''(0)=2 \quad y'(0)=1$

$$s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) - Y(s) = \frac{1}{s^2}$$

$$s^3 Y(s) - s^2 - s - 2 - Y(s) = \frac{1}{s^2}$$

$$Y(s)(s^3-1) = \frac{1}{s^2} + s^2 + s + 2 = \frac{s^4+s^3+2s^2+1}{s^2}$$

$$Y(s) = \frac{s^4+s^3+2s^2+1}{s^2(s^3-1)} = \frac{s^4+s^3+2s^2+1}{(s^5-s^2)}$$

$s(s^2-1) \quad (s+1)(s-1)$

$(s^2-s) \quad (s^2-s+s-1)$

$s(s+1)(s-1) \quad (s-1)\cdot s \quad s^3-s \quad (s^2+1)(s^3-s)$

$(s^3-s)(s^2-1)\cdot s \qquad s^5-s^3+s^3-s$

$(s^3-s)(s^2+1)\cdot s \quad (s^3-s)(s^2-1) \quad (s^2-s)(s^3-s)$

$s^5+s^3-s^3-s\cdot s \quad (s^5-s) \quad s^5-s^2-s^3-s^2$

$$= \frac{s^4+s^3+2s^2+1}{s^2(s+1)(s-1)(s^2-1)\cdot s}$$

$s^2-s \cdot s^3-s = s^4-s^2-s^2+s$

s^3-s

$$\frac{A}{s} + \frac{B}{s+1} + \frac{C}{s-1} + \frac{Ds+E}{s^2+1} \Big/ \cdot s(s+1)(s-1)(s^2-1)\cdot s \Rightarrow As(s^5-s) + Bs^5 - 2Bs^2 - Bs^3 + Cs^5 - Cs + Ds^4 - Ds^2 - Es^3 - Es$$

$$As^6 - As^2 + Bs^5 - 2Bs^2 - Bs^3 + Cs^5 - Cs + Ds^4 - Ds^2 - Es^3 - Es$$

$A=0$

$B+C=0$

$D=1$

$-B-E=1$

$-A-2B-D=2$

$A=0$

$D=1$

$B+C=0$

$-B-E=1$

$-2B-D=2$

$-2B-D=2$

$-2B=2+D=3$

$\boxed{B=-\frac{3}{2}}$

$-B-E=1$

$-E=1+B=1-\frac{3}{2}=-\frac{1}{2} \quad -\frac{1}{2}=E$

$B+C=0$

$C=-B=\frac{3}{2}$

$-A=2+2B+D$

$=2-3+1 \quad =-1+1=0$

$$Y(s) = -\frac{3}{2}\cdot\frac{1}{s+1} + \frac{3}{2}\cdot\frac{1}{s-1} + 1\,\frac{s}{s^2+1} + \frac{1}{2}\,\frac{1}{s^2+1}$$

$y(t) = -\frac{3}{2}\cdot e^{-t} + \frac{3}{2}e^{t} + \cos t + \frac{1}{2}\sin t \qquad y(0) = -\frac{3}{2} + \frac{3}{2} + 1 = 1$

$y'(t) = \frac{3}{2}e^{-t} + \frac{3}{2}e^{t} - \sin t + \frac{1}{2}\cos t \quad y'(0) = \frac{7}{2} \Rightarrow \times$

$y''(t) = -\frac{3}{2}e^{-t} + \frac{3}{2}e^{t} - \cos t + \frac{1}{2}\sin(t) = -1$

2. $r(t) = t i + t^2 j + \frac{t^3}{3} k \quad t \in [0,2] \quad f(x,y,z) = \frac{1}{x+1}$

$$\int_C f ds = ?$$

$$r(t) = \begin{pmatrix} t \\ t^2 \\ \frac{1}{3}t^3 \end{pmatrix} \Rightarrow r'(t) = \begin{pmatrix} 1 \\ 2t \\ t^2 \end{pmatrix} \checkmark$$

$$\|r'(t)\| = \sqrt{1^2 + (2t)^2 + (t^2)^2} = \sqrt{1 + 4t^2 + t^4}$$

$$\int f ds = \int_0^2 \frac{1}{t+1} \cdot \sqrt{1+4t^2+t^4}\, dt \checkmark \quad \underline{20}$$

DALJE JE NETOČNO, ALI ZADAN JE DOSTA TEŽAK INTEGRAL! RIJEŠAVATI NUMERIČKOM INTEGRACIJOM!

$1+4t^2+t^4 = u$

$8t+4t^3 dt = du$

$dt = \frac{1}{8t+4t^3} du \quad = \int \sqrt{u} \cdot du = \int u^{\frac{1}{2}} \cdot du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \cdot \quad = \frac{2}{3} \cdot u^{\frac{3}{2}} \cdot \frac{1}{8t+4t^3}$

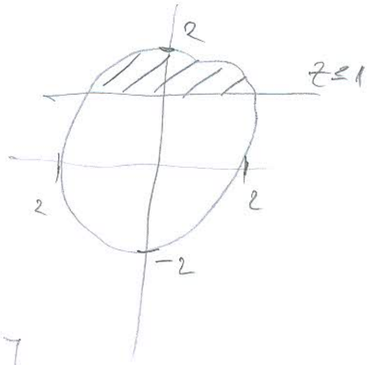
$$= \frac{2}{3} \cdot \sqrt{u^3} \cdot \frac{1}{8t+4t^3} = \frac{2}{24t+12t^3} \cdot \sqrt{(1+4t^2+4t)^3} \Big|_0^2 *$$

$$\overset{*}{=} \frac{2}{24 \cdot 2 + 12 \cdot 2^3} \cdot \left(\sqrt{1+(4 \cdot 2^2)+4 \cdot 2)^3}\right) - \left(0 \cdot \left(\sqrt{1^3}\right)\right)$$

$$= \frac{1}{72} \cdot 125 - 1 = \frac{125}{72} - 1 = \frac{125-72}{72} = \frac{53}{72}$$

JOSIP ŠIMIČEV

4. $x^2+y^2+z^2=4 \qquad z \geq 1$



$x^2+y^2+z^2=4$

$r^2+z^2=4$

$r^2=4-z^2$

$r=\sqrt{4-z^2}$

$z\geq 1$

$r=2$

$z\in[1,2]$

$r\in[0,\sqrt{4-z^2}]$

$\rho\in[0,2\pi]$

$$V=\int_0^{2\pi}\int_1^2\int_0^{\sqrt{4-z^2}} r\,\cancel{dr}\,dr\,dz\,d\rho$$

$$=\int_0^{2\pi}\int_1^2 \frac{1}{2}r^2\Big|_0^{\sqrt{4-z^2}}dz\,d\rho=\int_0^{2\pi}\int_1^2\frac{1}{2}\cdot\left(\sqrt{4-z^2}\right)^2dz\,d\rho=\int_0^{2\pi}\int_1^2\frac{1}{2}\cdot(4-z^2)\,dz\,d\rho=\int_0^{2\pi}\int_1^2 2-\frac{z^2}{2}\,dz\,d\rho$$

$$=\int_0^{2\pi}2z-\frac{1}{2}\frac{z^3}{3}\Big|_1^2d\rho=\int_0^{2\pi}\left(2\cdot2-\frac{1}{2}\cdot\frac{8}{3}\right)-\left(2\cdot1-\frac{1}{2}\cdot\frac{1}{3}\right)d\rho$$

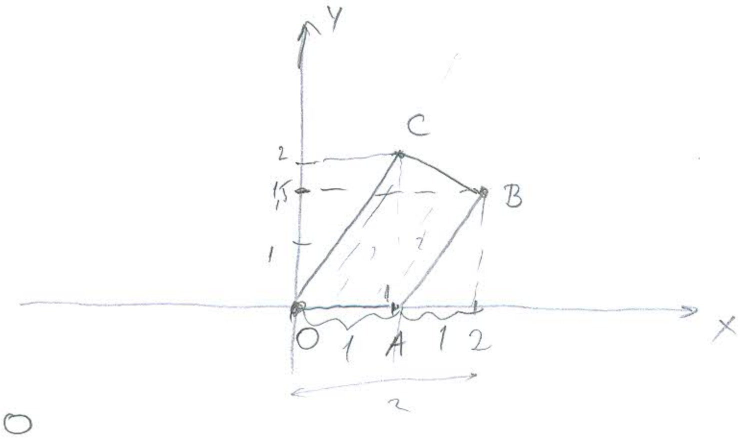
$$=\int_0^{2\pi}\frac{8}{3}-\frac{11}{6}\,d\rho=\int_0^{2\pi}\frac{5}{6}\,d\rho=\frac{5}{6}\rho\Big|_0^{2\pi}=\frac{5}{3}\pi$$

JOSIP ŠIMIČEV

3. četverokut $= O(0,0)\ A\left(\frac{2}{2},0\right)\ B\left(2,\frac{3}{2}\right)$ i $C\left(\frac{2}{2},\frac{4}{2}\right)$

$$\iint\limits_X x^3\,dx\,dy \Rightarrow ?$$

$O(0,0)\ A(1,0)\ B\left(2,\frac{3}{2}\right)\ C(1,2)$



$\overline{OA} = y = 0$

$\overline{AB}\quad A(1,0)\ B\left(2,\frac{3}{2}\right)$

$(2-1)(y-0) = \left(\frac{3}{2}-0\right)(x-1)$

$y = \frac{3}{2}x - \frac{3}{2}$

$\overline{BC}\quad B\left(2,\frac{3}{2}\right)\ C(1,2)$

$(1-2)\left(y-\frac{3}{2}\right) = \left(2-\frac{3}{2}\right)(x-2)$

$-1y + \frac{3}{2} = \frac{1}{2}x - 1$

$-y = \frac{1}{2}x - 1 - \frac{3}{2} = \frac{1}{2}x - \frac{5}{2}\ /:(-1)$

$y = -\frac{1}{2}x + \frac{5}{2}$

$\overline{OC}\ O(0,0)\ C(1,2)$

$(1-0)(y-0) = (2-0)(x-0)$

$y = 2x$

$$\int_0^1\int_0^{2x} x^3\,dy\,dx + \int_1^2\int_{\frac{3}{2}x-\frac{3}{2}}^{-\frac{1}{2}x+\frac{5}{2}} x^3\,dy\,dx = \int_0^1 yx^3\Big|_0^{2x}dx + \int_1^2 yx^3\Big|_{\frac{3}{2}x-\frac{3}{2}}^{-\frac{1}{2}x+\frac{5}{2}}dx$$

$$= \int_0^1 2x^4\,dx + \int_1^2\left(-\frac{1}{2}x+\frac{5}{2}\right)-\left(\frac{3}{2}x-\frac{3}{2}\right)x^3\,dx = \int_0^1 2x^4\,dx + \int_1^2(-2x+4)x^3\,dx$$

$$= \int_0^1 2x^4\,dx + \int_1^2 -2x^4+4x^3\,dx = \frac{2}{5}x^5\Big|_0^1 + \left(-\frac{2}{5}x^5+\frac{4}{4}x^4\right)\Big|_1^2 \Rightarrow$$

NASTAVAK

$$= \frac{2}{5}x^5\Big|_0^1 + \left(-\frac{2}{5}x^5 + \frac{4}{4}x^4\right)\Big|_1^2$$

$$= \frac{2}{5}\cdot 1^5 + \left(-\frac{2}{5}\cdot 2^5 + 2^4\right) - \left(-\frac{2}{5}\cdot 1^5 + 1^4\right)$$

$$\frac{2}{5}$$

$$= \frac{2}{5} + \left(-\frac{64}{5} + 16\right) - \left(-\frac{2}{5} + 1\right)$$

$$= \frac{2}{5} + \left(\frac{16}{5} - \left(\frac{3}{5}\right)\right) = \frac{2}{5} + \frac{13}{5} = \frac{15}{5} = 3$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: Tomislav Kraljev

BROJ INDEKSA: 17-01-0052-2011

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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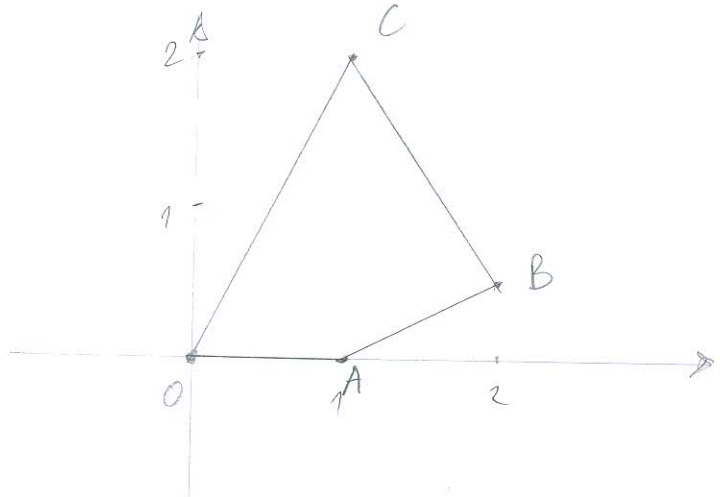
Ukupno: 35

Tomislav Kraljev 17-01-0052-2011

19.09.2014.

$\boxed{3}$ $O(0,0)$ $A(1,0)$ $B(2,\frac{3}{2})$ $C(1,2)$

$$\iint_X x^3\,dx\,dy$$



$O\begin{pmatrix} x_1 & y_1 \\ 0, & 0 \end{pmatrix}$ $C\begin{pmatrix} x_2 & y_2 \\ 1, & 2 \end{pmatrix}$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(1-0)(y-0)=(2-0)(x-0)$

$1\cdot y = 2\cdot x$

$\overline{OC}\ldots\ y=2x$

$A\begin{pmatrix} x_1 & y_1 \\ 1, & 0 \end{pmatrix}$ $B\begin{pmatrix} x_2 & y_2 \\ 2, & \frac{3}{2} \end{pmatrix}$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(2-1)(y-0)=(\frac{3}{2}-0)(x-1)$

$1\cdot y = \frac{3}{2}x-\frac{3}{2}$

$\overline{AB}\ldots\ y=\frac{3}{2}x-\frac{3}{2}$

$B\begin{pmatrix} x_1 & y_1 \\ 2, & \frac{3}{2} \end{pmatrix}$ $C\begin{pmatrix} x_2 & y_2 \\ 1, & 2 \end{pmatrix}$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(1-2)(y-\frac{3}{2})=(2-\frac{3}{2})(x-2)$

$-y+\frac{3}{2}=\frac{1}{2}x-1$

$-y=\frac{1}{2}x-1-\frac{3}{2} \quad / \cdot(-1)$

$y=-\frac{1}{2}x+1+\frac{3}{2} \Rightarrow y=-\frac{1}{2}x+\frac{5}{2}\ \ldots\ \overline{BC}$

$$\int_0^1\int_0^{2x} x^3\,dy\,dx + \int_1^2\int_{\frac{3}{2}x-\frac{3}{2}}^{-\frac{1}{2}x+\frac{5}{2}} x^3\,dy\,dx =$$

<u>1,5</u>

$$= \int_0^1 x^3\cdot y\Big|_0^{2x} dx + \int_1^2 x^3\cdot y\Big|_{\frac{3}{2}x-\frac{3}{2}}^{-\frac{1}{2}x+\frac{5}{2}} dx =$$

$$= \int_0^1 x^3\cdot(2x-0)\,dx + \int_1^2 x^3\cdot\left(\left(-\frac{1}{2}x+\frac{5}{2}\right)-\left(\frac{3}{2}x-\frac{3}{2}\right)\right)dx =$$

$$= \int_0^1 2x^4\,dx + \int_1^2 x^3\cdot\left(-\frac{1}{2}x+\frac{3}{2}x+\frac{5}{2}+\frac{3}{2}\right)dx = \int_0^1 2x^4 dx + \int_1^2 x^3\cdot(x+4)\,dx =$$

$$= \int_0^1 2\cdot x^4 dx + \int_1^2 x^4+4x^3\,dx = 2\cdot\frac{1}{5}x^5\Big|_0^1 + \left[\left(\frac{1}{5}\cdot x^5+\frac{4}{4}x^4\right)\Big|_1^2\right] =$$

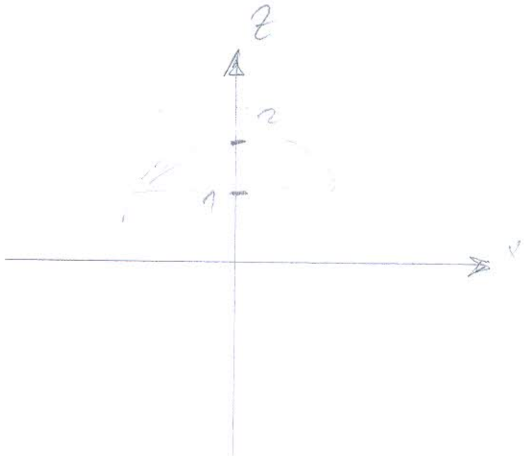
$$= \frac{2}{5}\cdot 1 + \left[\frac{1}{5}\cdot(2^5)+2^4-\left(\frac{1}{5}\cdot 1^5+1^4\right)\right] = \frac{2}{5}+\frac{32}{5}+16-\frac{6}{5} =$$

$$= \frac{2+32+80-6}{5} = \frac{108}{5} \;//$$

Tomislav Kraljev   17-01-0052-2011

$\boxed{19.09.2014}$

$\boxed{4}$ $x^2+y^2+z^2=4 \qquad z \geq 1$



$r^2+z^2=4$

$r^2=4-z^2$

$r=\sqrt{4-z^2}$

$r \in \left[0, \sqrt{4-z^2}\right]$

$z \in [1,2]$

$\varphi \in [0,2\pi]$

$dx\,dy\,dz = r\,dr\,d\varphi\,dz$

$$\int_0^{2\pi}\int_1^{2}\int_0^{\sqrt{4-z^2}} r\,dr\,dz\,d\varphi = \int_0^{2\pi}\int_1^{2} \frac{r^2}{2}\Big|_0^{\sqrt{4-z^2}} dz\,d\varphi =$$

$$= \int_0^{2\pi}\int_1^{2} \frac{1}{2}\cdot\left(\sqrt{4-z^2}\right)^2 dz\,d\varphi = \int_0^{2\pi}\int_1^{2} \frac{1}{2}\cdot\left(4-z^2\right) =$$

$$= \int_0^{2\pi}\int_1^{2} 2-\frac{1}{2}\cdot z^2\,dz\,d\varphi = \int_0^{2\pi} 2-\frac{1}{2}\cdot\frac{z^3}{3}\Big|_1^2\,d\varphi =$$

$$= \int_0^{2\pi} 2-\frac{1}{6}\cdot\left(2^3-1^3\right) = \int_0^{2\pi} 2-\frac{1}{6}\cdot(8-1) = \int_0^{2\pi} 2-\frac{7}{6}\,d\varphi = \frac{5}{6}\Big|_0^{2\pi}$$

$$= \frac{5}{6}\cdot 2\pi = \frac{5}{3}\pi$$

✓

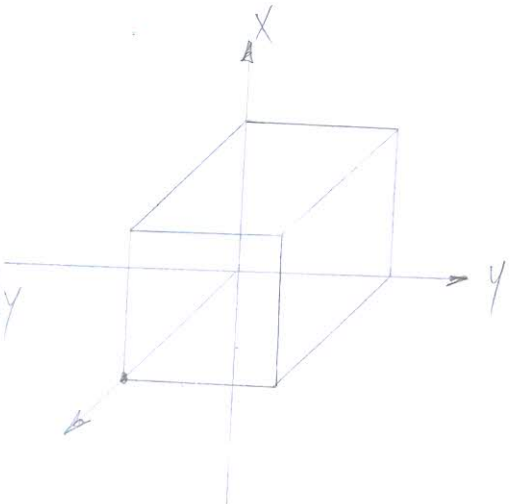
Tomislav Kraljev

17-01-0052-2011

19.09.2014

$\boxed{5}$ $a=2$ = Kocka $\iint\limits_{\partial K} (2x+3)\,dx\,dy = ?$

$T(0,0,0)$



$\iint\limits_{\partial K} w_x\,dy\,dz + w_y\,dx\,dz + w_z\,dx\,dy$

$w = \begin{pmatrix} w_x \\ w_y \\ w_z \end{pmatrix} \quad w = \begin{pmatrix} 0 \\ 0 \\ 2x+3 \end{pmatrix}$

$\operatorname{div} w = \frac{\partial 0}{\partial x} + \frac{\partial 0}{\partial y} + \frac{\partial(2x+3)}{\partial z} = 0+0+0 = 0$

$\iint\limits_{\partial K} (2x+3)\,dx\,dy = \iint\limits_{\partial K} (w|ds) = \iiint\limits_{K} \operatorname{div} w\,dx\,dy =$

$= \iiint 0\,dx\,dy\,dz = 0$ //

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: BORIS KREŠIĆ BROJ INDEKSA: 17-1-0022-2010	Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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3. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{2}{2},0)$, $B(2,\frac{3}{2})$ i $C(\frac{2}{2},\frac{4}{2})$. Izračunati dvostruki integral 20

$$\iint_X x^3 \mathrm{d}x\mathrm{d}y$$

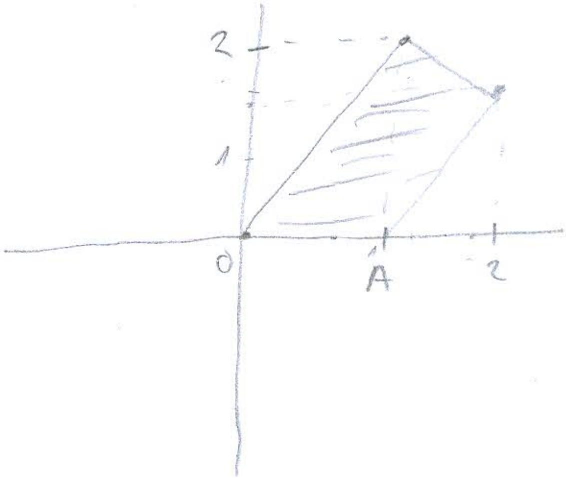
4. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2 = 4$ za koji vrijedi $z \geq 1$. 20

5. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\partial K} (2x+3)\,\mathrm{d}x\mathrm{d}y$? ~~20~~

Ukupno: 40

(3.) X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{3}{2},0)$, $B(2,\frac{3}{2})$ i $C(\frac{3}{2},2)$. Izračunati dvostruki integral $\iint_X x^3 dx dy$

$$\iint_X x^3 dx dy$$



$$I = \iint_X x^3 dx dy$$

$$I_1 = \int_0^1 \int_0^{2x} x^3 dy dx = \int_0^1 x^3 \cdot y \Big|_0^{2x} dx = 2\int_0^1 x^4 dx$$

$$= 2 \cdot \frac{x^5}{5} \Big|_0^1 = \frac{2}{5}$$

$$I_2 = \int_1^2 \int_{-\frac{3}{2}+\frac{3}{2}x}^{\frac{5}{2}-\frac{1}{2}x} x^3 dy dx = \int_1^2 x^3 \cdot \left(\frac{5}{2} - \frac{1}{2}x + \frac{3}{2} + \frac{3}{2}x\right) dx$$

$$= \int_1^2 x^3 \cdot (4-2x) dx = 4 \cdot \frac{x^4}{4} \Big|_1^2 - 2\frac{x^5}{5} \Big|_1^2$$

$$= (16-1) - \frac{2}{5}(32-1) = \frac{13}{5}$$

$$I_1 + I_2 = \frac{15}{3} = 3$$

OKRENITE =>

BORIS KREŠIĆ

5) Neka je K kocka stranice duljine $a=2$ centrirana u ishodištu. Izračunati $\iint_{\partial K} (2x+3)\,dx\,dy$.

$a=2$

$S(0,0,0)$

$x \in (-1,1)$

$y \in (-1,1)$

$z \in (-1,1)$

$$I = \iint_{\partial K} (2x+3)\,dx\,dy \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \operatorname{div}(2x+3)\,dz\,dy\,dx$$

$$\operatorname{div} = (2x+3) = 2 \quad \times$$

$$I = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} 2\,dz\,dy\,dx = 2\cdot 2\cdot 2\cdot 2 = 16$$

BORIS KREŠIĆ

(4) $x^2+y^2+z^2=4 \qquad z \geq 1$

$r^2=4 \,/\sqrt{\ }$

$r=\sqrt{4}=2$

$\varphi \in [0, 2\pi]$

$z \in [1, 2]$

$r \in [0, \sqrt{4-z^2}]$

$$V=\int_0^{2\pi}\int_1^{2}\int_0^{\sqrt{4-z^2}} r\,dz\,dr\,d\varphi = 2\pi \int_1^2 \frac{r^2}{2}\Big|_0^{\sqrt{4-z^2}} dz$$

$$= \cancel{2}\pi \int_1^2 \frac{4-z^2}{\cancel{2}}\,dz = \pi\left(4z\Big|_1^2 - \frac{z^3}{3}\Big|_1^2\right)$$

$$= \pi \cdot \left(4 - \left(\frac{8}{3}-\frac{1}{3}\right)\right)$$

$$= \pi\left(4-\frac{7}{3}\right) = \frac{5\pi}{3}$$

BORIS KRETIĆ

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: Kristian Jozić **BROJ INDEKSA:** 17-1-0012-2010.	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) - y(t) = t, \quad y(0) = 1, \, y''(0) = 2, \, y'(0) = 1.$$

2. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \frac{t^3}{3}\mathbf{k}$, $t \in [0, 2]$. Zadano je skalarno polje $f(x, y, z) = \frac{1}{x+1}$. Izračunaj $\int\limits_C f ds$. 20

3. X je zadan kao četverokut s vrhovima $O(0, 0)$, $A(\frac{2}{2}, 0)$, $B(2, \frac{3}{2})$ i $C(\frac{2}{2}, \frac{4}{2})$. Izračunati dvostruki integral 20

$$\iint\limits_X x^3 dxdy$$

4. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2 + y^2 + z^2 = 4$ za koji vrijedi $z \geq 1$. 20

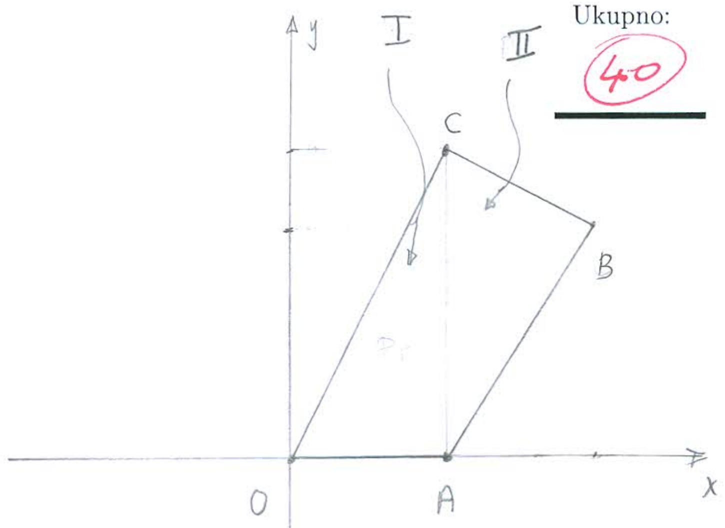
5. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (2x + 3) \, dxdy$? 20

Ukupno: 40

(3) $O(0,0)$ $B(2, \frac{3}{2})$ (x_2, y_2)

$A(\frac{2}{2}, 0)$ $C(\frac{2}{2}, \frac{4}{2})$ (x_1, y_1)

$$\iint\limits_X x^3 dx dy$$



$x_1 = 0 \quad y_1 = 0$

$x_2 = 1 \quad y_2 = 2$

$\overline{OC} \ldots \; y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y = \frac{2 - 0}{1 - 0}(x)$

$\overline{OC} \ldots \; \underline{y = 2x}$

$\overline{OA} \ldots \; \underline{y = 0}$

$\overline{AB} \ldots \; y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y = \frac{\frac{3}{2} - 0}{2 - 1}(x - 1)$

$\underline{y = \frac{3}{2}x - \frac{3}{2}}$

$x_1 = 2 \quad y_1 = \frac{3}{2}$

$x_2 = 1 \quad y_2 = \frac{4}{2}$

$\overline{BC} \ldots \; y - \frac{3}{2} = \frac{2 - \frac{3}{2}}{1 - 2}(x - 2)$ $\underline{y = -\frac{1}{2}x + \frac{5}{2}}$

$y - \frac{3}{2} = \frac{\frac{1}{2}}{-1}(x - 2)$

$y - \frac{3}{2} = -\frac{1}{2}x + 1$

nastavak iza ↘

$$I = \iint x^3\,dx\,dy = \int_0^1 x^3\,dx \int_0^{2x} dy = \int_0^1 x^3\,dx\,(2x-0) = 2\int_0^1 x^4\,dx$$

$$= 2\cdot\left(\frac{x^5}{5}\right)\Bigg|_0^1 = \frac{2}{5}\cdot\left(1^5-0^5\right) = \frac{2}{5}\cdot(1-0) = \frac{2}{5}$$

$$II = \iint x^3\,dx\,dy = \int_1^2 x^3\,dx \int_{\frac{3}{2}x-\frac{3}{2}}^{-\frac{1}{2}x+\frac{5}{2}} dy = \int_1^2 x^3 \left(\left(-\frac{1}{2}x+\frac{5}{2}\right)-\left(\frac{3}{2}x-\frac{3}{2}\right)\right) dx$$

$$= \int_1^2 x^3\left(-\frac{1}{2}x+\frac{5}{2}-\frac{3}{2}x+\frac{3}{2}\right)dx = \int_1^2 x^3(-2x+4)\,dx$$

$$= -2\int_1^2 x^4\,dx + 4\int_1^2 x^3\,dx = -2\cdot\left(\frac{x^5}{5}\right)\Bigg|_1^2 + 4\cdot\left(\frac{x^4}{4}\right)\Bigg|_1^2$$

$$= -2\cdot\left(\frac{2^5}{5}-\frac{1^5}{5}\right) + 4\cdot\left(\frac{2^4}{4}-\frac{1^4}{4}\right)$$

$$= -2\cdot\left(\frac{31}{5}\right) + 4\cdot\left(\frac{15}{4}\right) = -\frac{62}{5} + 15 = \frac{15}{5}$$

$$\iint_X x^3\,dx\,dy = I + II = \frac{2}{5} + \frac{13}{5} = 3$$

KRISTIAN JOZIĆ

17-1-0012-2010.

4. $V = ?$ kugla: $x^2 + y^2 + z^2 = 4$ vrijedi: $z \geq 1$

Prijelaz na cilindrične koordinate:

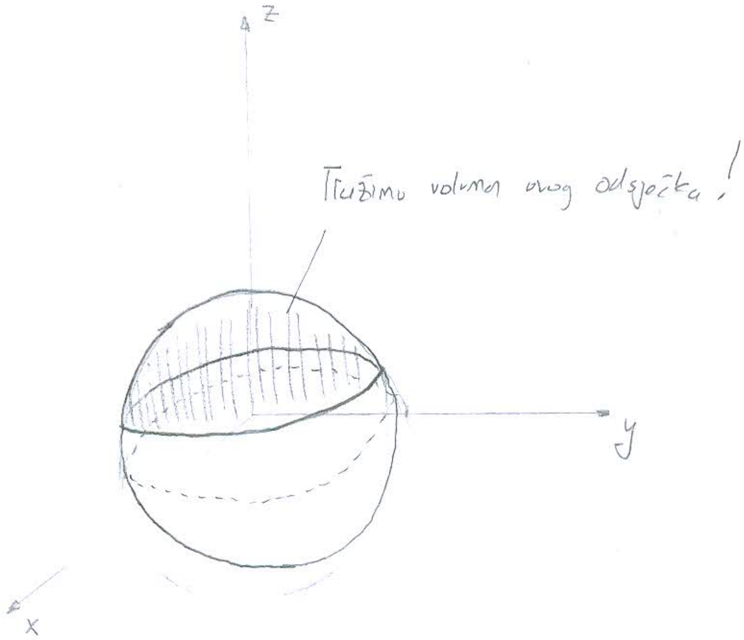
$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

Još vrijedi:

$dx\,dy\,dz = r\,dr\,d\varphi\,dz$



$x^2 + y^2 + z^2 = 4$

$(r\cos\varphi)^2 + (r\sin\varphi)^2 + z^2 = 4$

$r^2 \underbrace{(\cos^2\varphi + \sin^2\varphi)}_{1} + z^2 = 4$

$r^2 + z^2 = 4$

$r^2 = 4 - z^2$

$r = \sqrt{4 - z^2}$

$\varphi \in [0, 2\pi]$

$r \in [0, \sqrt{4 - z^2}]$

$z \in [1, 2]$

nastavak iza ↘

$$V = \int_0^{2\pi}\int_1^{2}\int_0^{\sqrt{4-z^2}} r\,dr\,d\varphi\,dz = \int_0^{2\pi} d\varphi \int_1^{2} dz \int_0^{\sqrt{4-z^2}} r\,dr$$

$$V = \frac{1}{2}\int_0^{2\pi} d\varphi \int_1^{2} dz \cdot \left(r^2\right)\Big|_0^{\sqrt{4-z^2}} = \frac{1}{2}\int_0^{2\pi} d\varphi \int_1^{2} \cdot \left(\sqrt{4-z^2}\right)^2 dz$$

$$V = \frac{1}{2}\int_0^{2\pi} d\varphi \int_1^{2} 4 - z^2 \;dz = \frac{1}{2}\int_0^{2\pi} d\varphi \left(4z - \frac{z^3}{3}\right)\Big|_1^{2}$$

$$V = \frac{1}{2}\int 4\cdot 2 - \frac{2^3}{3} - 4\cdot 1 - \frac{1^3}{3} \;d\varphi$$

$$V = \frac{1}{2}\int_0^{2\pi} \frac{16}{3} - \frac{11}{3} = \frac{1}{2}\int_0^{2\pi} \frac{5}{3}\,d\varphi = \frac{5}{6}\int_0^{2\pi} d\varphi = \frac{5}{6}\cdot 2\pi = \frac{5}{3}\pi$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: KREŠIMIR KALCINA BROJ INDEKSA: 57/181-2009	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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Ukupno: 40

(2) $r(t) = ti + t^2 j + \frac{t^3}{3}k \qquad t \in [0,2]$

$f' \cdot g + f \cdot g'$

$t^{3'} \cdot 3 + t^3 \cdot 3'$

$\frac{3t^2 \cdot 3 \cancel{t}}{3^2} = \frac{9t^2}{} = t^2$

$r = \begin{bmatrix} x = t \\ y = t^2 \\ z = \frac{t^3}{3} \end{bmatrix} \qquad r' = \begin{bmatrix} x = 1 \\ y = 2t \\ z = t^2 \end{bmatrix} \quad \|r'\| = \sqrt{1^2 + (2t)^2 + (t^2)^2}$

$\boxed{= \sqrt{1^2 + (2t + t^2)^2}} = 1 + 2t + t^2 \Longrightarrow \boxed{\sqrt{2^2 + 2^2} \neq 4}$

$\int f ds = \frac{1}{x+1} = \int_0^2 \frac{1}{1 + 2t + t^2 + 1} dt = \int_0^2 \frac{1}{t^2 + 2t + 2} dt = \int_0^2 \frac{dt}{(t+1)^2 + 1}$

$(t+1)(t+1) = t^2 + t + t + 1$
$= t^2 + 2t + 1 -$
$(t+1)^2 + 1$

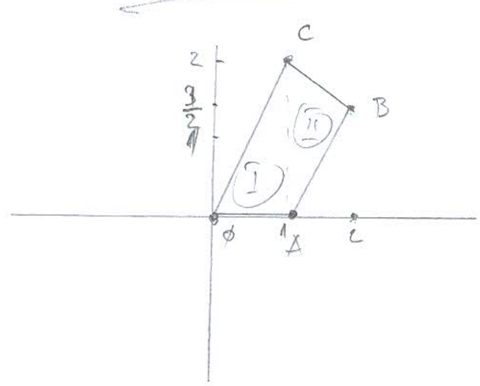
$= \frac{1}{1} \arctan \frac{(t+1)}{1} \Big|_0^2 = \Big[\underbrace{(\arctan(2+1))}_{1.25} - \underbrace{(\arctan(0+1))}_{0.79}\Big]$

$= 0.463$

VIDI ŠIMIČEV.

③ $O(0,0)$, $A(\overset{x_1\ y_1}{1,0})$, $B(\overset{x_2\ y_2}{2,\tfrac{3}{2}})$ $C(1,2)$

$$\iint_X (x^3)\,dx\,dy$$



$\overline{OC}$ $(x_2-x_1)(y-y_1) = (y_2-y_1)(x-x_1)$

$(1-0)(y-0) = (2-0)(x-0)$

$\overline{OC} \Rightarrow y = 2x$

$\overline{CB}$ $(1-2)\left(y-\frac{3}{2}\right) = \left(2-\frac{3}{2}\right)(x-2)$

$-y + \frac{3}{2} = \frac{1}{2}x - 1$

$-y = \frac{1}{2}x - 1 - \frac{3}{2}$

$-y = \frac{1}{2}x - \frac{5}{2}$

$\overline{CB} \Rightarrow y = -\frac{1}{2}x + \frac{5}{2}$

$-y + \frac{3}{2} = \frac{1}{2}x - 1$

$-y = \frac{1}{2}x - 1 - \frac{3}{2}$

$-y = \frac{1}{2}x - 2.5$

$-1 - 1.5 = 2{,}5$

$-\frac{2}{2} - \frac{3}{2} = \frac{5}{2}$

$\overline{AB} \Rightarrow (2-1)(y-0) = \left(\frac{3}{2}-0\right)(x-1)$

$\overline{AB} \Rightarrow y = \frac{3}{2}x - \frac{3}{2}$

$$P_{I,II} = \int_0^1 \int_0^{2x} x^3\,dy\,dx + \int_1^2 \int_{\frac{3x}{2}-\frac{3}{2}}^{-\frac{1}{2}x+\frac{5}{2}} x^3\,dy\,dx = \int_0^1 x^3(2x-0)\,dx + \int_1^2 x^3\Big(\underbrace{-\frac{1}{2}x + \frac{5}{2} - \frac{3x}{2} + \frac{3}{2}}_{-\frac{4}{2}x + \frac{8}{2}}\Big)dx \Rightarrow (-2x+4)$$

$$= \int_0^1 2x^4\,dx + \int_1^2 -2x^4 + 4x^3$$

$$= \frac{2}{5}x^5\Big|_0^1 + \left(-\frac{2}{5}x^5\right) + \frac{4}{4}x^4\Big|_1^2$$

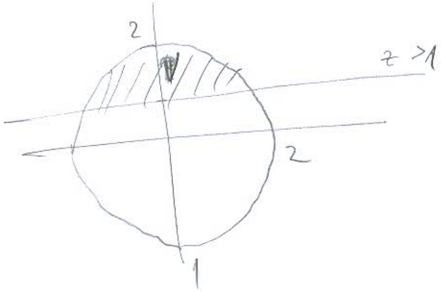
$$= \frac{2}{5} + \left[\left(-\frac{2}{5}\cdot 32\right) + 16\right] - \left[-\frac{2}{5} + 1\right]$$

$$= \frac{2}{5} + \left[\left(-\frac{64}{5}\right) + \frac{80}{5}\right] - \Big[\underbrace{-\frac{2}{5} + \frac{5}{5}}_{\frac{3}{5}}\Big]$$

$$= \frac{2}{5} + \frac{16}{5} - \frac{3}{5}$$

$$P_D = \frac{15}{5} = 3$$ ✓

(4) $\underbrace{x^2+y^2}_{r^2}+z^2=4$ $\quad r=2$



$r^2+z=4$

$r=\sqrt{4-z^2}$

$\varphi \in [0, 2\pi]$

$r \in [0, \sqrt{4-z^2}]$

$z \in [1, 2]$

$$\int_0^{2\pi}\int_1^2\int_0^{\sqrt{4-z^2}} r\,dr\,dz\,d\varphi = \int_0^{2\pi}\int_1^2 \frac{r^2}{2}\Big|_0^{\sqrt{4-z^2}} = \int_0^{2\pi}\int_1^2 \frac{4-z^2}{2}\,dz\,d\varphi = \int_0^{2\pi}\int_1^2 2-\frac{z^2}{2}\,dz\,d\varphi$$

$$= \int_0^{2\pi} 2z - \frac{z^3}{6}\Big|_1^2 d\varphi = \int_0^{2\pi}\left(2\cdot 2 - \frac{2^3}{6}\right) - \left(2\cdot 1 - \frac{1^3}{6}\right)$$

$$\underbrace{4-\frac{8}{6}} - \underbrace{2-\frac{1}{6}}$$

$$\left(\frac{24}{6}-\frac{8}{6}\right) - \left(\frac{12}{6}-\frac{1}{6}\right)$$

$$= \int_0^{2\pi} \frac{16}{6} - \frac{11}{6}\,d\varphi = \frac{16}{6}\varphi - \frac{11}{6}\varphi\Big|_0^{2\pi} = \frac{16}{3}\pi - \frac{11}{3}\pi$$

$$V = \frac{5}{3}\pi$$ //

✓

KREŠIMIR KALCINA

$$y'''(t) - y(t) = t \qquad y(0)=1 \quad y''(0)=2 \quad y'(0)=1$$

$$\underbrace{s^3F(s) - s^2f(0) - sf' - f''(0)} - y(t) = \frac{1}{s^2}$$

$$\underbrace{s^3F_s - s^2 - s - 2} - y(t) = \frac{1}{s^2}$$

$$F(s)\,(s^3-1) - s^2 - s - 2 = \frac{1}{s^2}$$

$$F(s)\,(s^3-1) = \frac{1}{s^2} - s^2 - s - 2$$

$$F_s\,(s^3-1) = \frac{1 - s^4 - s^3 - 2s^2}{s^2}$$

$$F(s) = \frac{1 - s^4 - s^3 - 2s^2}{s^2(s^3-1)} \qquad (s^3-1) = (s-1)\,(s^2+s+1)$$

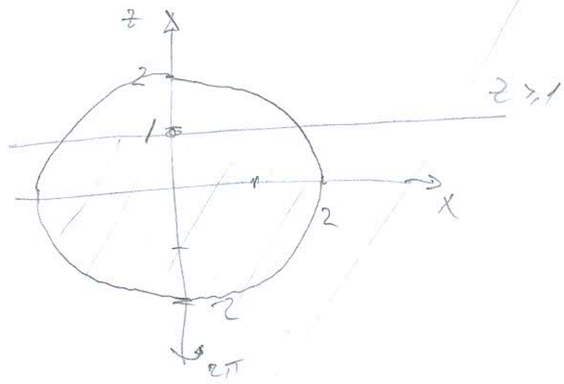
$$F_s = \frac{1 - s^4 - s^3 - 2s^2}{s^2(s-1)\underbrace{(s^2+s+1)}_{s(s+1)+1}} \qquad (s^2+s+1) = s(s+1)+1$$

$$F_s = \frac{1 - s^4 - s^3 - 2s^2}{s^2(s-1)[s(s+1)+1]} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D}{s-1} \quad \Big/ \cdot s^2(s-1)s(s+1)$$

0

(4) $x^2+y^2+z^2=4$ | KREŠIMIR KALCINA |

$z \geq 1$ $\quad r=4$



$r^2+z^2=4$

$r^2=4-z^2$

$r=\sqrt{4-z^2}$

$\varphi \in [0, 2\pi]$

$r \in [0, \sqrt{4-z^2}]$

$z \in [-2, 1]$

$$\int_0^{2\pi}\int_{-2}^{1}\int_0^{\sqrt{4-z^2}} r\,dr\,dz\,d\varphi = \int_0^{2\pi}\int_{-2}^{1} \frac{r^2}{2}\Big|_0^{\sqrt{4-z^2}} dz\,d\varphi = \int_0^{2\pi}\int_{-2}^{1} \frac{(\sqrt{4-z^2})^2}{2}\,dz\,d\varphi$$

$\varphi \quad z \quad r$

$$= \int_0^{2\pi}\int_{-2}^{1} \frac{4-z^2}{2}\,dz\,d\varphi = \int_0^{2\pi}\int_{-2}^{1} \frac{4}{2} - \frac{z^2}{2}\,dz\,d\varphi$$

$$= \int_0^{2\pi} \frac{4}{2}z - \frac{z^3}{6}\Big|_{-2}^{1} d\varphi = \int_0^{2\pi} \left(2\cdot 1 - \frac{1^3}{6}\right) - \left(2\cdot(-2) - \frac{(-2)^3}{6}\right) d\varphi$$

$$\left[\left(\frac{12}{6} - \frac{1}{6}\right) - \left(-4 - \frac{-8}{6}\right)\right] d\varphi$$

$$= \int_0^{2\pi} \frac{11}{6} - \left(\underbrace{-\frac{24}{6} - \left(\frac{-8}{6}\right)}_{\underbrace{-\frac{26}{6} + \frac{8}{6}}_{-\frac{18}{6}}}\right)$$

$$\int_0^{2\pi} \frac{11}{6} + \frac{18}{6}\,d\varphi$$

KRIVO UZET [r]

$$= \frac{11}{6}\varphi + \frac{18}{6}\varphi = \frac{11\pi}{3} + \frac{18\pi}{3} = \frac{29\pi}{3} \,//$$

~~ZADATAK~~ NA DRUGOM PAPIRU!!

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IME I PREZIME: DAMIR DVORNIK

BROJ INDEKSA: 17-1-0041-2010

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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Ukupno:

20

DAMIR DVORNIK

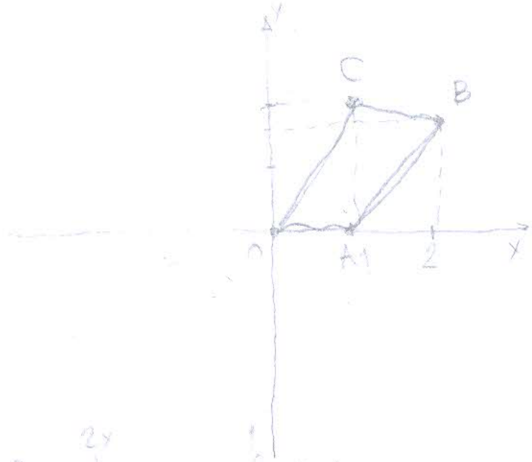
MATEMATIKA 3.

3

$A\left(\frac{2}{2}, 0\right)$

$B\left(2, \frac{3}{2}\right)$

$C\left(\frac{2}{2}, \frac{4}{2}\right)$



$$I_1 = \int_0^1 \int_0^{2x} x^3 \, dy \, dx = \int_0^1 x^3 \cdot y \Big|_0^{2x} dx = 2\int_0^1 x^4 \, dx$$

$$I_1 = 2 \cdot \frac{x^5}{5} \Big|_0^1 = \frac{2}{5}$$

$$I_2 = \int_1^2 \int_{-\frac{3}{2}+\frac{3}{2}x}^{\frac{5}{2}-\frac{1}{2}x} x^3 \, dy \, dx = \int_1^2 x^3 \left(\frac{5}{2} - \frac{1}{2}x + \frac{3}{2} - \frac{3}{2}x\right) dx$$

$$= \int_1^2 x^3 \cdot (4 - 2x) \, dx = 4 \cdot \frac{x^4}{4} \Big|_1^2 - 2 \frac{x^5}{5} \Big|_1^2$$

$$= (16-1) - \frac{2}{5}(32-1) = \frac{13}{5}$$

$$I_1 + I_2 = \frac{15}{5} = 3$$

2) $r(t) = t\vec{i} + t^2\vec{j} + \frac{t^3}{3}\vec{k}$

$t \in [0, 2]$

$f(x, y, z) = \frac{1}{x+1}$

$\emptyset$

$$\int_C f \, ds$$

DAMIR DVORNIK MATEMATIKA 3.

1. $Y'''(t) - y(t) = t$ $\quad y(0)=1$

$y''(0)=2$

$y'(0)=1$

$$S^3Y(s) - S^2y(0) - Sy'(0) - y''(0) - Y(t) = \frac{1}{S^2}$$

$\varnothing$

5. $a=2$

$S(0,0,0)$

$x \in [-1,1]$

$y \in [-1,1]$

$z \in [-1,1]$

$\varnothing$

$$I = \iint_{\partial K} (2x+3)\,dx\,dy = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \operatorname{div}(2x+3)\,dz\,dy\,dx$$

$$\operatorname{div}(2x+3) = 2 \quad \times$$

$$I = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} 2\,dz\,dy\,dx = 2\cdot 2\cdot 2\cdot 2 = 16$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: MARKO PARANCIN BROJ INDEKSA: 17-1-0062-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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4. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2=4$ za koji vrijedi $z \geq 1$. 20

5. Neka je K kocka stranice duljine $a=2$ centrirana u ishodištu. Izračunati $\iint_{\partial K} (2x+3)\,dx\,dy$? 20

Ukupno: 20

$Y(0)=1,\ y''(0)=2,\ y'(0)=1$

① $s^3F(s) - s^2f(0) - sf'(0) - f''(0) - sF(s) = \frac{1}{s}$

$s^3F(s) - s^2(1) - s\cdot(1) - 2 - sF(s) = \frac{1}{s}$

$s^3F(s) - s^2 - s - 2 - sF(s) = \frac{1}{s}$

$s^3F_s - s^3 - 2 - sF(s) = \frac{1}{s}$

$F_s(s^3 - s) = \frac{1}{s} + s^3 + 2 \quad / : s^3 - s$

$F_s = \frac{1+s^3+2}{s(s^3-s)} = \frac{1+s^3+2}{s(s^2-1)(s^2-1)}$

$F_s = \frac{A}{s} + \frac{Bs}{s^2-1} + \frac{C}{s^2-1} = A(s) + Bs(s^2-1) + C(s^2-1)$

$F_s = As + Bs^3 - Bs + Cs^2 - C$

$s^3 = A = Bs \Rightarrow A = 1$

$s^2 = B = Cs \quad 3 = 2$

$s^0 - C = A - B \quad C = A - B = -1$

$F_s = \frac{1}{s} + \frac{2s}{s^2-1} + \frac{(-1)}{s^2-1}$

$F_s = -1 + \cos(t) - \sin(t)$ //

X

→

(2) $r(t) = ti + t^2 j + \frac{t^3}{3} k \quad , \quad t \in [0,2]$

$\int_C t \, ds \, ?$

$r_t \begin{pmatrix} x = t \\ y = t^2 \\ z = \frac{t^3}{3} \end{pmatrix} = r'(t) \begin{pmatrix} x = 1 \\ y = 1 \\ z = \frac{1}{3} \end{pmatrix}$ ✗

$\|r t\| = \int_0^2 \frac{1}{x+1} t \, ds$

$\|r' t\| = \sqrt{\frac{1}{x+1}} = \sqrt{\frac{1}{2}} \, t \, ds = \int_0^2 \frac{\sqrt{2}}{2} \Big/_0^2$ ✗

$= \frac{2\sqrt{2}}{4 \, 2} = \frac{\sqrt{2}}{2} \,//$

(5) $a = 2$

$\iint_{\partial K} 2x + 3 \, dx \, dy$

$W = \begin{pmatrix} x = 0 \\ y = 0 \\ z = 2x + 3 \end{pmatrix} \begin{matrix} = 0 \\ = 0 \\ = 0 \end{matrix} \quad dx \, dy = r \, dr$

$W = \cancel{\partial_x i} \cdot \cancel{\partial_y j} \cdot \cancel{\partial_z k} = 0 \qquad \int_K r \, dr \, W = \emptyset$ ✓

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: RIKARDO PEROVIĆ

BROJ INDEKSA:

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$y'''(t) - y(t) = t, \quad y(0) = 1,\ y''(0) = 2,\ y'(0) = 1.$$

2. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \frac{t^3}{3}\mathbf{k}$, $t \in [0, 2]$. Zadano je skalarno polje $f(x, y, z) = \frac{1}{x+1}$. Izračunaj $\int_C f \mathrm{d}s$. ~~20~~

3. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{2}{2}, 0)$, $B(2, \frac{3}{2})$ i $C(\frac{2}{2}, \frac{4}{2})$. Izračunati dvostruki integral ~~20~~

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4. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2 + y^2 + z^2 = 4$ za koji vrijedi $z \geq 1$. ~~20~~ 15

5. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\partial K} (2x + 3)\, \mathrm{d}x\mathrm{d}y$? ~~20~~

Ukupno: 15

4.) $x^2+y^2+z^2=4 \qquad z \geq 1$

$r^2+z^2=4 \qquad z^2=4$

$r^2=4-z^2 \qquad z=\sqrt{4}$

$r=\sqrt{4-z^2} \qquad z=2$

15

$\varphi \in [0, 2\pi]$

$r \in [0, \sqrt{4-z^2}]$

$z \in [1, 2]$

$$V = \int_0^{2\pi}\int_1^{2}\int_0^{\sqrt{4-z^2}} r\,\mathrm{d}r\,\mathrm{d}z\,\mathrm{d}\varphi$$

RIKARDO PEROVIĆ

1.) $y'''(t) - y(t) = t$ $\qquad y(0)=1 \quad y''(0)=2 \quad y'(0)=1$

$$s^3Y(s) - s^2y(0) - sy'(0) - y''(0) - Y(s) = \frac{1}{s^2}$$

$$s^3Y(s) - s^2 - s - 2 - Y(s) = \frac{1}{s^2}$$

$$Y(s)\left(s^3-1\right) = \frac{1}{s^2} + s^2 + s$$

$$Y(s)\left(s^3-1\right) = \frac{1+s^3+s^2}{s^2} \quad / \cdot \frac{1}{(s^3-1)}$$

$$Y(s) = \frac{s^3+s^2+1}{s^2(s^3-1)}$$

2.) $r(t) = ti + t^2 j + \frac{t^3}{3} k \qquad t \in [0,2] \qquad f(x,y,z) = \frac{1}{x+1}$

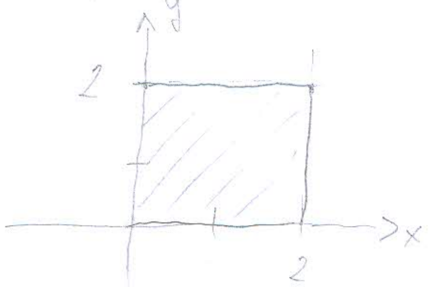
$$= \frac{1}{t+1}$$

$$r(t) = \begin{bmatrix} t \\ t^2 \\ \frac{t^3}{3} \end{bmatrix} \qquad r'(t) = \begin{bmatrix} 1 \\ 2t \\ 3t^2 \end{bmatrix} \times$$

$$\|r'\| = \sqrt{1^2 + (2t)^2 + (3t^2)^2} = \sqrt{1 + 4t^2 + 9t^4} = \sqrt{1 + t^2(4 + 9t^2)}$$

$$\int_0^2 \frac{1}{t+1} \cdot \sqrt{1 + t^2(4+9t^2)}\, dt$$

3.) $a = 2\,cm$



OVO BI BIO RAČUN ZA PLOŠNI INTEGRAL 1. VRSTE

U ZADATKU SE RIJEČ O PLOŠNOM INTEGRALU VEKTORSKE FUNKCIJE

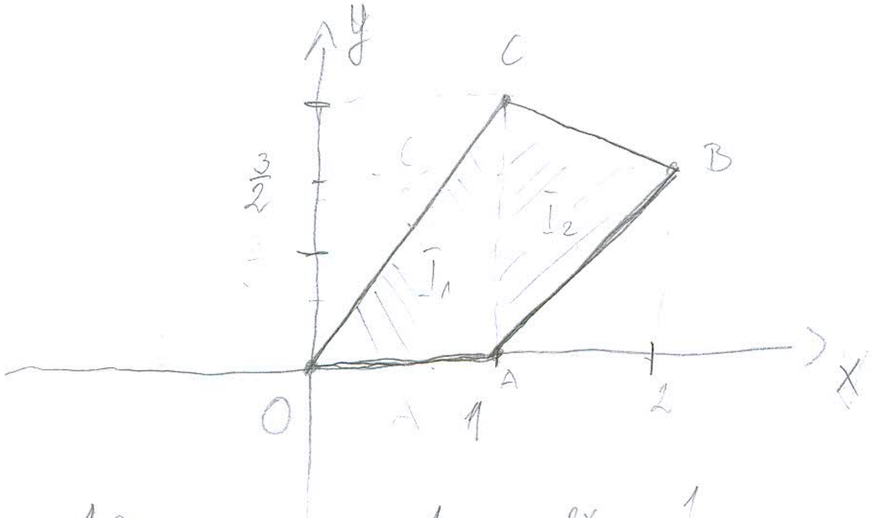
$$\int_0^2\int_0^2 (2x+3)\,dx\,dy = \underbrace{\int_0^2 2\cdot\frac{x^2}{2} + 3x \Big|_0^2 dy}_{\times} = \int_0^2 2\cdot\frac{2^2}{2} + 3\cdot 2 = \int_0^2 10\,dy$$

$$= 10y \Big|_0^2 = 20$$

RIKARDO PEROVIC

3.) $O(0,0)$ $A(\frac{2}{2},0)$, $B(2,\frac{3}{2})$ i $C(\frac{2}{2},\frac{4}{2})$

$$\iint\limits_X x^3\,dx\,dy$$



$\overline{OA}$ ... $= 0$

$\overline{OC}$ ... $(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$O(0,0)$

$C(\frac{2}{2},\frac{4}{2})$

$$(1-0)(y-0)=(2-0)(x-0)$$

$$\boxed{\overline{OC}\ \ y = 2x}$$

$\overline{AB}$ $A(\frac{2}{2},0)$ $B(2,\frac{3}{2})$

$$(2-1)(y-0)=(\frac{3}{2}-0)(x-1)$$

$$\overline{AB} \ldots \boxed{y=\frac{3}{2}x-\frac{3}{2}}$$

$\overline{BC}$ $B(2,\frac{3}{2})$ $C(\frac{2}{2},\frac{4}{2})$

$$(1-2)(y-\frac{3}{2})=(2-\frac{3}{2})(x-2)$$

$$-y+\frac{3}{2}=\frac{1}{2}x-1 \quad /\cdot(-1)$$

$$y-\frac{3}{2}=\frac{1}{2}x+1$$

$$y=\frac{1}{2}x+1+\frac{3}{2}$$

$$\overline{BC} \ldots \boxed{y=\frac{1}{2}x+\frac{5}{2}}$$

$$I_1=\int_0^1\int_0^{2x} x^3\,dy\,dx = \int_0^1 yx^3 \Big|_0^{2x} dx = \int_0^1 2x\cdot x^3\,dx$$

$$=\int_0^1 2x^4\,dx = 2\cdot\frac{x^5}{5}\Big|_0^1 \; 2\cdot\frac{1}{5} = \frac{2}{5}$$

$$I_2=\int_1^2\int_{\frac{3}{2}x-\frac{3}{2}}^{\frac{1}{2}x+\frac{5}{2}} x^3\,dy\,dx = \int_1^2 yx^3\Big|_{\frac{3}{2}x-\frac{3}{2}}^{\frac{1}{2}x+\frac{5}{2}} dx =$$

$$=\int_1^2 x^3\left[\frac{1}{2}x+\frac{5}{2}-\left(\frac{3}{2}x-\frac{3}{2}\right)\right]dx=\int_1^2 x^3\left(\frac{1}{2}x+\frac{5}{2}-\frac{3}{2}x+\frac{3}{2}\right)dx$$

$$=\int_1^2 x^3\left(-x+\frac{8}{2}\right)dx=\int_1^2\left(-x^4+4x^3\right)dx$$

$$=-\frac{x^5}{5}+4\cdot\frac{x^4}{4}\Big|_1^2 = -\frac{32}{5}+16-\left(-\frac{1}{5}+1\right)$$

$$=-\frac{32}{5}+16+\frac{1}{5}-1 = \frac{44}{5}$$

$$I=I_1+I_2=\frac{2}{5}+\frac{44}{5}=\frac{46}{5}$$

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IME I PREZIME: ANTE RAŽOV **BROJ INDEKSA:** 0269082684

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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Ukupno: 20

1 $\quad y'''(t) - y(t) = t \qquad y(0) = 1 \quad y''(0) = 2, \quad y'(0) = 1$

$$s^3 Y(s) - s^2 \overset{=1}{y(0)} - s\overset{=1}{y'(0)} - \overset{=2}{y''(0)} - Y(s) = \frac{1}{s^2}$$

$$Y(s)(s^3 - 1) - s^2 - s - 2 = \frac{1}{s^2}$$

$$Y(s)(s^3 - 1) = s^2 + s + 2 + \frac{1}{s^2} = \frac{s^4 + s^3 + 2s^2 + 1}{s^2} \quad \Big/ : (s^3 - 1)$$

$$Y(s) = \frac{s^4 + s^3 + 2s^2 + 1}{s^2(s^3 - 1)} = \frac{s^4 + s^3 + 2s^2 + 1}{s^2 \cdot s(s^2 - \frac{1}{s})} = \frac{s^4 + s^3 + 2s^2 + 1}{s^3(s^2 - \frac{1}{s})} =$$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds + E}{s^2 - \frac{1}{s}} \quad \Big| \cdot s^3(s^2 - \frac{1}{s})$$

$$\boxed{\begin{aligned} A &= 0 \\ B &= -1 \\ C &= 2 \\ D &= 1 \\ E &= 2 \end{aligned}}$$

$$s^4 + s^3 + 2s^2 + 1 = As^2(s^2 - \frac{1}{s}) + Bs(s^2 - \frac{1}{s}) + Cs^2 - C\frac{1}{s} + Ds^4 + Es^3$$

$$s^4 + s^3 + 2s^2 + 1 = \underline{As^4} - \underline{As} + \underline{Bs^3} - B + \underline{Cs^2} - \underline{\frac{C}{s}} + \underline{Ds^4} + \underline{Es^3}$$

$A + D = 1 \qquad B + E = 1 \qquad \boxed{C = 2} \quad \boxed{A = 0} \qquad -\frac{C}{s} = 0 \qquad -B = 1 \quad \boxed{B = -1} \quad \longrightarrow$

$-1 + E = 1 \qquad E = 1 + 1 = 2$

$$Y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2-\frac{1}{s}} = 0 - \frac{1}{s^2} + 2\frac{1}{s^3} + \frac{s+2}{s^2-\frac{1}{s}} =$$

$$Y(s) = -\frac{1}{s^2} + 2\frac{1}{s^3} + \frac{s}{s^2-\frac{1}{s}} + 2\frac{1}{s^2-\frac{1}{s}} =$$

$$Y(t) = -t + \frac{2}{3}t^2 + \cosh\left(\frac{1}{\sqrt{\pi t}}\right)^2 + 2\sinh\left(\frac{1}{\sqrt{\pi t}}\right)^2$$

$$\frac{1}{s} = \left(\frac{1}{\sqrt{s}}\right)^2$$

0

PROVJERA

$$f(0) = -0 + \frac{2}{3}\cdot 0^2 + \cosh\left(\frac{1}{\sqrt{\pi 0}}\right)^2 + 2\sinh\left(\frac{1}{\sqrt{\pi 0}}\right)$$

DIJELJENJE NULOM?

(4) $x^2+y^2+z^2=4$ $\quad z \geq 1$ $\quad$ Volumen dijela kugle

$$r^2+z^2=4$$
$$r^2 = 4-z^2$$
$$r = \sqrt{4-z^2}$$

$$V = \int_0^{2\pi}\int_1^2\int_0^{\sqrt{4-z^2}} r\,dr\,dz\,d\varphi =$$

$$V = \int_0^{2\pi}\int_1^2 \frac{1}{2}r^2\Big|_0^{\sqrt{4-z^2}} dz\,d\varphi = \int_0^{2\pi}\int_1^2 \frac{1}{2}\cdot(4-z^2)\,dz\,d\varphi = \int_0^{2\pi}\int_1^2 2-\frac{1}{2}z^2\,dz\,d\varphi = \int_0^{2\pi} 2z - \frac{1}{2}\frac{z^3}{3}\Big|_1^2 d\varphi =$$

$$V = \int_0^{2\pi} 2z - \frac{z^3}{6}\Big|_1^2 d\varphi = \int_0^{2\pi}\left(2\cdot 2 - \frac{2^3}{6}\right) - \left(2\cdot 1 - \frac{1^3}{6}\right)d\varphi = \int_0^{2\pi}\left(4-\frac{8}{6}\right)-\left(2-\frac{1}{6}\right)d\varphi =$$

$$V = \int_0^{2\pi}\left(4-\frac{8}{6}-2+\frac{1}{6}\right)d\varphi = \int_0^{2\pi}\left(\frac{24-8-12+1}{6}\right)d\varphi = \int_0^{2\pi}\frac{5}{6}\,d\varphi = \frac{5}{6}\cdot 2\pi = \frac{5}{3}\pi$$

ANTE RAŽON BR IND. 0269082684

(2)

$$r = \begin{pmatrix} t \\ t^2 \\ \frac{t^3}{3} \end{pmatrix} \to r' = \begin{pmatrix} \frac{t^2}{2} \\ \frac{t^3}{3} \\ \frac{t^5}{12} \end{pmatrix}$$

$t \in (0,2)$

$f(x,y,z) = \frac{1}{x+1}$

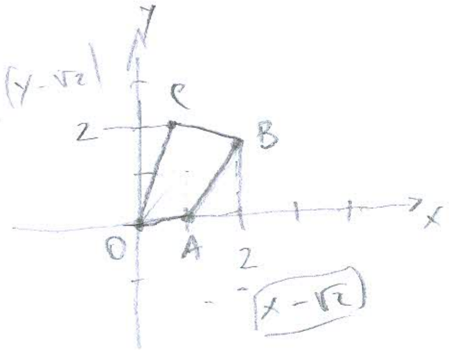
$$\|r'\| = \sqrt{\left(\frac{t^2}{2}\right)^2 + \left(\frac{t^3}{3}\right)^2 + \left(\frac{t^5}{12}\right)^2} =$$

$$\int_C f\,ds = \int_0^2 \frac{1}{x+1} \cdot \|r'\|\,ds = \int_0^2 1 \cdot \|r'\|\,ds = \int_0^2 \left(\frac{1}{2}t^2 + \frac{1}{3}t^3 + \frac{1}{12}t^5\right) ds =$$

$$= \left(\frac{1}{2}\cdot\frac{t^3}{3} + \frac{1}{3}\cdot\frac{t^4}{4} + \frac{1}{12}\cdot\frac{t^5}{5}\right)\Bigg|_0^2 = \frac{t^3}{6} + \frac{t^4}{12} + \frac{t^5}{60}\Bigg|_0^2 = \frac{2^3}{6} + \frac{2^4}{12} + \frac{2^5}{60} = \frac{8}{6} + \frac{16}{12} + \frac{32}{60}$$

$$= \frac{4}{3} + \frac{4}{3} + \frac{8}{15} = \frac{8}{3} + \frac{8}{15} = \frac{40+8}{15} = \frac{48}{15} = \frac{16}{5} = 3\frac{1}{5}$$

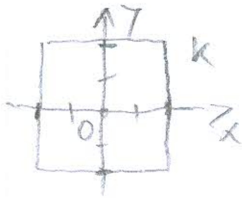
(3)



$$= \int_0^2\int_0^2 x^3\,dx\,dy = \int_0^2 (x^3 y)\Big|_0^2 dx = \int_0^2 \left(x^3\cdot 2 - (x^3\cdot 0)\right) dx =$$

$$= \int_0^2 2x^3\,dx = \frac{2x^4}{4}\Bigg|_0^2 = \frac{2\cdot 2^4}{4} = \frac{2\cdot 16}{4} = 8$$

(5) $= \int_{-2}^{2}\int_{-2}^{2}(2x+3)\,dx\,dy = \int_{-2}^{2}(2xy+3y)\Big|_{-2}^{2}dx =$



$= \int_{-2}^{2}\Big((2x\cdot 2+3\cdot 2)-(2x\cdot(-2)+3\cdot(-2))\Big)dx = \int_{-2}^{2}\big(4x+6-(-4x-6)\big)dx = \int_{-2}^{2}(4x+4x+12)dx =$

$= \int_{-2}^{2}(8x+12)dx = \left(\frac{8x^2}{2}+12x\right)\Big|_{-2}^{2} = (4x^2+12x)\Big|_{-2}^{2} = (4\cdot 2^2+12\cdot 2)-(4\cdot(-2)^2+12\cdot(-2)) =$

$= 16+24-(16-24) = \underline{\underline{48}}$

VIDI NAPOMENU

KOD PEROVIĆ