MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{I}1$

IME I PREZIME: MARIO MILOŠ **BROJ INDEKSA:** 17-1-0170-2013

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 + \overline{1+i} = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 15+3

2. Gaussovom metodom riješi sustav linearnih jednadžbi, a zatim provjeri uvrštavanjem:

$$\begin{aligned} x + 2y - z + u &= -1 \\ 2x + 5y - z + 2u &= -2 \\ 3x - y - 2z + u &= 5 \\ x - y + 3z - 5u &= 6 \end{aligned}$$

Provjeri uvrštavanjem! 16+3

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 + x} - x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{x^2-4}{x^2+2}$. 20(graf)

5. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\sin(4x - 2))$. 15

6. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. 8

Ukupno: 59

MARIO MIKOŠ

②

$$\left|\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 6 \end{array}\right| \begin{array}{l} \cdot(-2) \\ + \\ \cdot(-3) \\ + \\ \cdot(-1) \\ + \end{array}$$

$$\sim \left|\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -7 & 1 & -2 & 8 \\ 0 & -3 & 4 & -6 & 7 \end{array}\right| \begin{array}{l} + \\ \cdot(-2) \\ + \\ \cdot 7 \\ + \\ \cdot 3 \\ + \end{array} \sim \left|\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -2 & 8 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right| :8$$

$$\sim \left|\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -4 & 1 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right| \begin{array}{l} + \\ + \\ \cdot 3 \\ + \\ \cdot(-1) \\ + \\ \cdot(-7) \\ + \end{array} \sim \left|\begin{array}{cccc|c} 1 & 0 & 0 & -11 & 2 \\ 0 & 1 & 0 & 4 & -1 \\ 0 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 22 & 0 \end{array}\right| :22$$

$$\sim \left|\begin{array}{cccc|c} 1 & 0 & 0 & -11 & 2 \\ 0 & 1 & 0 & 4 & -1 \\ 0 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right| \begin{array}{l} + \\ + \\ + \\ \cdot 11 \\ + \\ \cdot(-4) \\ + \\ \cdot 4 \\ + \end{array} \sim \left|\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right| \checkmark$$

PROVERA: ~~2 = 1 10~~

$2+2\cdot(-1)-1+0=-1$

$2\cdot 2+5\cdot(-1)-1+2\cdot 0=-2$ ✓

$3\cdot 2+1-2\cdot 1+0=5$

$2+1+3\cdot 1-5\cdot 0=6$

(5)

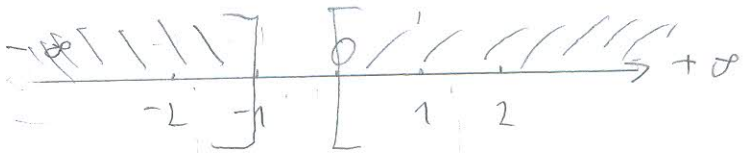
$f(x)=\ln(\sin(4x-2))$

$f'(x)=\frac{1}{\sin 4x-2}\cdot\left[\sin(4x-2)\right]'$

$f'(x)=\frac{4\cos(4x-2)}{\sin(4x-2)}=4\operatorname{ctg}(4x-2)$ ✓

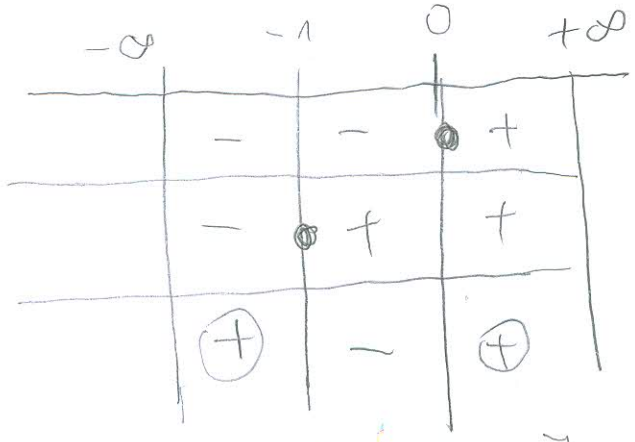
MARKO MILOŠ

③ $g(x)=\sqrt{x^2+x}-x$



$x^2+x-x\geq 0$

$x^2\geq 0$



$x=0$ $\quad x+1=0$

$x=-1$

$D_f: \langle -\infty, -1] \cup [0, +\infty\rangle$ ✓

ASIMPTOTE

A.V - NEMA JE

$\lim\limits_{x\to+\infty} \sqrt{x^2+x}-x \cdot \dfrac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x} = \lim\limits_{x\to\infty} \dfrac{x^2+x-x}{\sqrt{x^2+x}+x} \;/:x \;/:x = \dfrac{1}{2}$

$\boxed{y=\frac{1}{2}}$ DESNA H.A ✗

$\lim\limits_{x\to-\infty} \sqrt{x^2+x}-x \cdot \dfrac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}} = \lim\limits_{x\to-\infty} \dfrac{x^2+x-x^2}{\sqrt{x^2+x}+x} \;/:x \;/:x = -\dfrac{1}{0} = -\infty$

NEMA LIJEVE H.A

① $z^3 + \overline{1+i} = 0$

MARKO MILOŠ

④ $h(x) = \frac{x^2-4}{x^2+2}$

$$f''(x) = \frac{12(x^2+2)^2 - 12x(x^2-2)\cdot 4x}{(x^2+2)^4}$$

$$= \frac{(x^2+2)\left(12(x^2+2) - 48x^2\right)}{(x^2+2)^4} = \frac{12x^2+24-48x^2}{(x^2+2)^3} = \frac{-36x^2+24}{(x^2+2)^3}$$

$f(x) = 0$

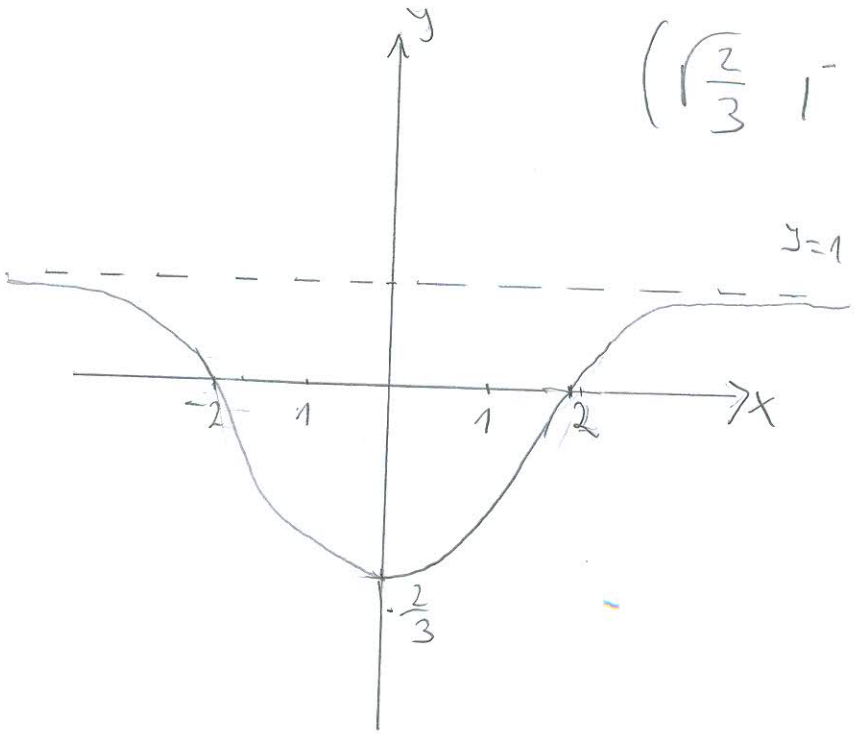
$-36x^2+24=0$

$x^2 = \frac{24}{36} = \frac{2}{3}$ $\quad x = \pm\sqrt{\frac{2}{3}}$

	$-\infty$	$-\sqrt{\frac{2}{3}}$	$\sqrt{\frac{2}{3}}$	$+\infty$
$f'(x)$	$-$	$+$	$-$
$f(x)$	$\cap$	$\cup$	$\cap$

$\left(-\sqrt{\frac{2}{3}}, -\frac{2}{3}\right)$ TOČKE

$\left(\sqrt{\frac{2}{3}}, -\frac{5}{4}\right)$ INFLEKSIJE



✓

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ I1

IME I PREZIME: MIRO KOVAČEVIĆ **BROJ INDEKSA:** 17-2-0080-11

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 + \overline{1+i} = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 15+3

2. Gaussovom metodom riješi sustav linearnih jednadžbi, a zatim provjeri uvrštavanjem:

$$\begin{aligned} x + 2y - z + u &= -1 \\ 2x + 5y - z + 2u &= -2 \\ 3x - y - 2z + u &= 5 \\ x - y + 3z - 5u &= 6 \end{aligned}$$

Provjeri uvrštavanjem! 16+3

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6. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. 8

Ukupno: 34

MIRO KOVAČEVIĆ

(2.)

$$x + 2y - z + u = -1$$
$$2x + 5y - z + 2u = -2$$
$$3x - y - 2z + u = 5$$
$$x - y + 3z - 5u = 6$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 6 \end{array}\right] \quad / \cdot(-2) \quad / \cdot(-3) \quad / \cdot(-1)$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -7 & 1 & -2 & 8 \\ 0 & -3 & 4 & -6 & 7 \end{array}\right] \quad / \cdot(-2) \quad / \cdot 7 \quad / \cdot 3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -1 & 8 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right] \quad / :8 \qquad = \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{8} & 1 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right] \quad / \cdot(-7) \quad / \cdot(-1) \quad / \cdot 3$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & 2 \\ 0 & 1 & 0 & \frac{1}{4} & -1 \\ 0 & 0 & 1 & -\frac{1}{8} & 1 \\ 0 & 0 & 0 & -\frac{17}{4} & 0 \end{array}\right] \quad / \cdot\left(-\frac{5}{12}\right) \qquad = \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & 2 \\ 0 & 1 & 0 & \frac{1}{4} & -1 \\ 0 & 0 & 1 & -\frac{1}{8} & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \quad / \cdot\frac{1}{4} \quad / \cdot\left(-\frac{1}{8}\right)$$

Rješenje →

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} \leftarrow x \\ \leftarrow y \\ \leftarrow z \\ \leftarrow u \end{array}$$

Provjera:

① $x + 2y - z + u = -1$

$2 + 2\cdot(-1) - 1 + 0 = -1$

$2 - 2 - 1 = -1$

② $2x + 5y - z + 2u = -2$

$4 \quad (-5) \quad (-1) + 0 = -2$

$-6 + 4 = -2$

-2

③ $3x - y - 2z + u = 5$

$6 + 1 - 2 + 0 = 5$

④ $x - y + 3z - 5u = 6$

$2 + 1 + 3 - 0 = 6$ ✓

---

⑤ $g(x) = \sqrt{x^2 + x} - x$

$\sqrt{x^2 + x} - x > 0$

$x^2 + x - x \neq 0$

$\sqrt{x^2 + x} = x \quad /^2$

$x^2 + x = x^2$?

$k = \frac{g(x)}{x} \qquad g = g(x) \cdot k$

$k = \frac{\sqrt{x^2 + x} - x}{x}$

$k = \frac{\sqrt{x^2}}{x}$?

---

⑤ $f(x) = \ln(\sin(4x - 2))$

$f'(x) = \frac{1}{\sin(4x-2)} \cdot (-\cos(4x-2)) \cdot 4$ ✓

$f'(x) = \frac{-\cos(4x-2)}{\sin(4x-2)} \cdot 4$

$f'(x) = -\operatorname{ctg}(4x-2) \cdot 4$

MIRO KOVAČIĆ

4. $h(x) = \frac{x^2-4}{x^2+2}$

1. NULTOČKE

$x^2 - 4 = 0$

$x^2 = 4 / \sqrt{\ }$

$x = \pm 2$

2. DOMENA

$x^2 + 2 \neq 0$

$x^2 = -2 / \sqrt{\ }$

$x = \pm\sqrt{2}$

$D_f \in \langle -\infty; \sqrt{2} \rangle \cup \langle \sqrt{2}, +\infty \rangle$

3. STACIONARNE TOČKE

$$h(x)' = \frac{(x^2-4)' \cdot (x^2+2) - (x^2-4) \cdot (x^2+2)'}{(x^2+2)^2}$$

$$h(x)' = \frac{2x(x^2+2) - (x^2-4)(2x)}{(x^2+2)^2}$$

$$= \frac{2x^3 + 4x - 2x^3 + 8x}{(x^2+2)^2}$$

$$= \frac{12x}{(x^2+2)^2}$$

$12x = 0$

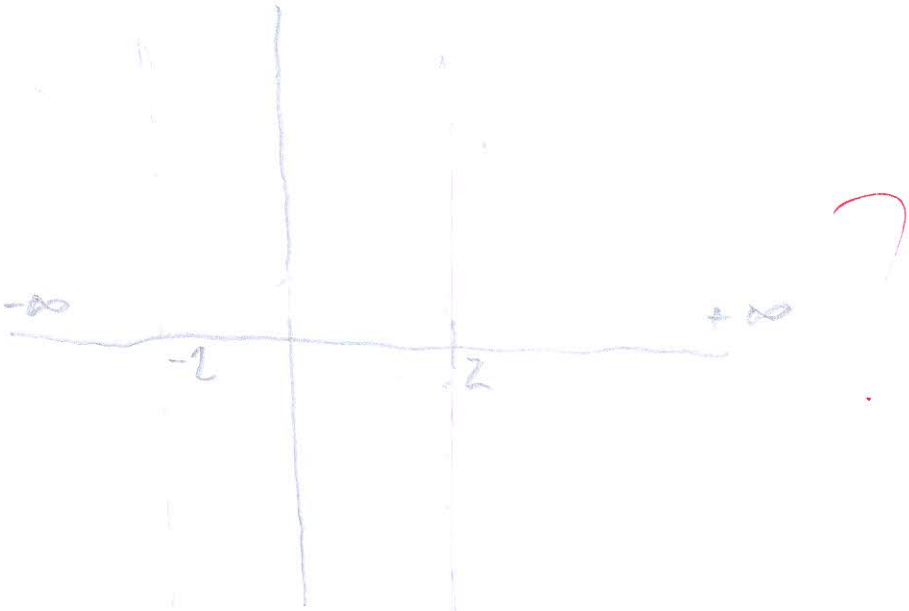
$x_{1,2} = \frac{-12 \pm \sqrt{144 - 0}}{0}$

$x_{1,2} = \pm\infty$

	$-\infty$		$-\sqrt{2}$		$\sqrt{2}$		$+\infty$
$12x$		$-$		$+$		$+$	
$(x^2+2)^2$		$+$		$+$		$+$	
$\frac{12x}{(x^2+2)^2}$		$-$		$-$		$+$	

MIN-

MAX-



7

(6)

$$\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ I1

IME I PREZIME: Ivan Stefančić **BROJ INDEKSA:** 17-2-0268-2013

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 + \overline{1+i} = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 15+3

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Ukupno: 23

6. $\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$

3 $g(x) = \sqrt{x^2+x} - x$

$x^2 + x \geq 0$

$x(x+1) \geq 0$

$x \geq 0 \quad x+1 \geq 0 \quad x \geq -1$

domena

$Df = [0, +\infty\rangle$ ✗

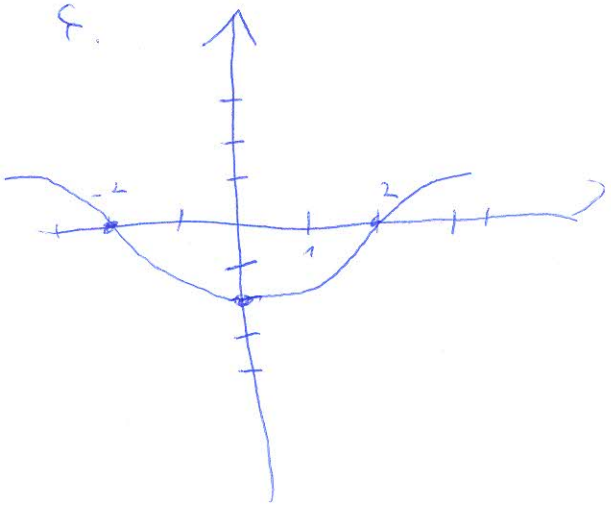
5. $\ln(\sin(4x-2)) = f(x)$

$f'(x) = \frac{1}{\sin(4x-2)} \cdot \cos(4x-2) \cdot 4$ ✓

$f'(x) = \frac{\cos(4x-2)}{\sin(4x-2)} \cdot 4$

$f'(x) = \frac{4\cos(4x-2)}{\sin(4x-2)}$

4.



? ODAKLE?

od $(-\infty, 2]$ pada
od $[2, \infty)$ raste

6.

$$\begin{bmatrix} 2 & 3 & 6 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 0 & 2 & 4 \\ 0 & 1 & 4 & -2 & 1 \end{bmatrix} \quad k_2 - 2k_1$$

$$\begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & -10 & 4 \\ 0 & 1 & 0 & 2 & 4 \\ 0 & 1 & 4 & -2 & 1 \end{bmatrix} \quad \begin{matrix} k_3 - k_2 \\ k_4 - k_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & -10 & 4 \\ 0 & 0 & 0 & 12 & 0 \\ 0 & 0 & 4 & 8 & -3 \end{bmatrix} \sim$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & -10 & 4 \\ 0 & 0 & 4 & 8 & -3 \\ 0 & 0 & 0 & 12 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & -10 & 4 \\ 0 & 0 & 4 & 8 & -3 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 4 & -3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 4 & 0 & -3 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

rang matrice je 4 ✓

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IME I PREZIME: Ante Šapić **BROJ INDEKSA:** 17-2-0211-2012

POPUNJAVA NASTAVNIK Broj ↓ bodova

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Ukupno: 16

(4) $\frac{x^2-4}{x^2+2}$

D: $x + 2 \neq 0$

$x^2 \neq -2$

$x \neq \sqrt{-2}$

$x \neq \pm i$

N, T $\frac{x^2-4}{x^2+2} = 0 \;/\cdot (x^2+2)$

$x^2 - 4 = 0$

$x^2 = 4$

$x_1 = 2$

$x_2 = -2$ //

ASIMPTOTE:

VERTIKALNA: $\pm i$

HORIZONTALNA: $\lim_{x \to \infty} f(x)$

$= \lim_{x \to \infty} \frac{x^2-4}{x^2+2} \quad \begin{matrix} 1 : x^2 \\ 1 : x^2 \end{matrix}$

$= \lim_{x \to \infty} \frac{1 - \frac{4}{x^2}}{1 + \frac{2}{x^2}} = \frac{1}{1} = 1$

KOSE: NEMA

EKSTREMI: $f(x) = f(x)'$

$$f(x)' \frac{x^2-4}{x^2+2} = \frac{2x\cdot(x^2+2)-(x^2-4)\cdot 2x}{(x^2+2)^2}$$

$$= \frac{2x^3+4x-2x^3+8x}{2x^4-4x^2+4} = \frac{12x}{2x^4+4x^2+4}$$

STACIONARNE TOČKE:

$12x = 0$

$x = 0$

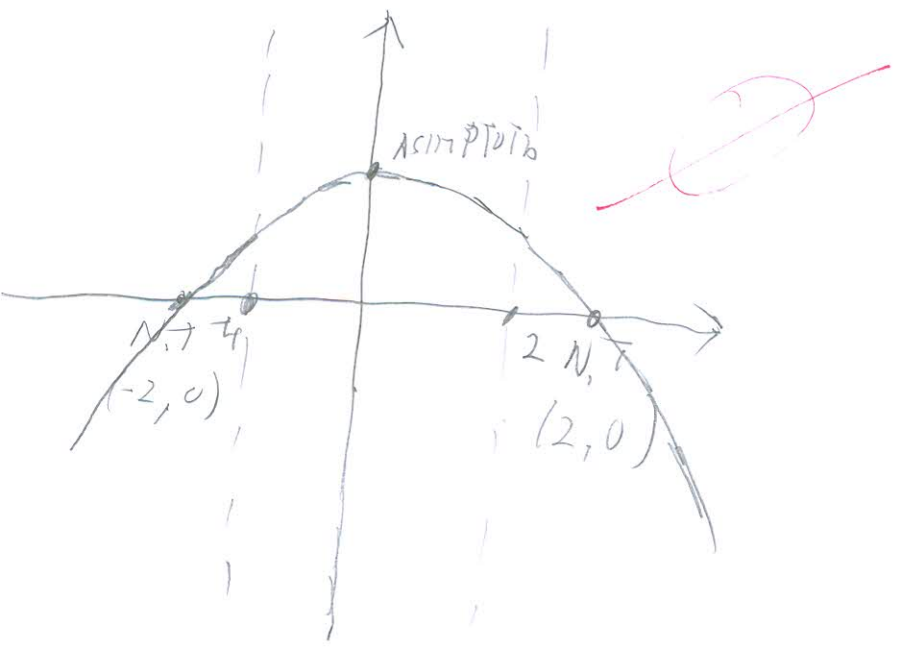
$$f(x)\ \frac{x^2-4}{x^2+2} = \frac{0^2-4}{0^2+2} = \frac{-4}{2} = -2$$

$(0, -2)$

↓

TOČKA MINIMUMA

x	$-\infty$		0		$+\infty$
$f'(x)$		$-$	0	$+$	
$f(x)$		↘	-2	↗	



(5) $f(x) = \ln(\tan(4x-2))$

Aste papir

$\ln(u) = \frac{1}{u} \cdot u'$

$f'(x) = \ln(\cos 4x-2)$

$f'(x) = \frac{1}{\cos 4x-2} \cdot \cos 4x-2$

$f'(x) = \frac{-\tan 4x-2}{\cos^2 4x-2} \cdot (-\tan 4x-2)$

$f'(x) = \frac{\tan^2 4x-2}{\cos^2 4x-2}$ ✗

(3) $g(x) = \sqrt{x^2+x}-x \quad /:x$

$= \frac{\sqrt{x^2-x}}{x} \cdot \frac{x}{x} = \frac{\sqrt{x^2-x}-x}{x}$

$D: x \neq 0 \quad D(f): R \setminus \{0\}$ ✗

ASIMPTOTE: VERTIKALNA $x=0$

KOSA: $y = kx + l$

$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\frac{\sqrt{x^2-x}-x}{x}}{x}$

$= \lim_{x\to\infty} \frac{\sqrt{x^2-x}-x}{x^2} \quad /:x^2 = \lim_{x\to\infty} \frac{\sqrt{1-\frac{x}{x^2}}-\frac{x^2}{x^2}}{\frac{x}{x^2}}$

$k = \frac{1}{\infty} = 1 \quad l = 0 \quad y = kx + l \quad \boxed{y = x}$ ✗

$l = \lim_{x\to\infty} (f(x) - k \cdot x)$

$= \lim_{x\to\infty} \left(\frac{\sqrt{x^2-x}-x}{x} - 1\right)$

$l = \lim_{x\to\infty} \frac{\sqrt{x^2-x}-x}{1} - x$

$= \lim_{x\to\infty} \frac{\sqrt{x^2-x}-x-x^2}{x}$

$= \lim_{x\to\infty} \frac{\sqrt{x^2-x}-x^3}{x} \quad /:x^3$

$= \lim_{x\to\infty} \frac{\sqrt{\frac{x^2}{x^3}-\frac{x}{x^3}}-\frac{x^3}{x^3}}{\frac{x}{x^3}} \to -1 = \frac{1}{0}$

2)
$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 6 \end{array}\right] \begin{array}{l} \\ II - 2I \\ III - 3I \\ IV - I \end{array} \approx \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -7 & 1 & -2 & 8 \\ 0 & -3 & 2 & -6 & 5 \end{array}\right] \begin{array}{l} I - 2II \\ \\ III + 7II \\ IV + 3II \end{array}$$

$$\approx \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -2 & 8 \\ 0 & 0 & 5 & -6 & 5 \end{array}\right] \begin{array}{l} \\ \\ :8 \\ \\ \end{array} \approx \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & 1 \\ 0 & 0 & 5 & -6 & 5 \end{array}\right] \begin{array}{l} I - III \\ II - III \\ \\ IV - 5III \end{array}$$

$$\approx \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{5}{4} & -2 \\ 0 & 1 & 0 & -\frac{1}{4} & 1 \\ 0 & 0 & 1 & -\frac{1}{4} & 1 \\ 0 & 0 & 0 & -\frac{19}{4} & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ \cdot(-\frac{4}{19}) \end{array} \approx \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{5}{4} & -2 \\ 0 & 1 & 0 & -\frac{1}{4} & 1 \\ 0 & 0 & 1 & -\frac{1}{4} & 1 \\ 0 & 0 & 0 & \frac{1}{4} & 0 \end{array}\right] \begin{array}{l} I - 5IV \\ II + IV \\ III + IV \\ \\ \end{array}$$

$$\approx \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & \frac{1}{4} & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ \cdot 4 \end{array} \approx \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$x = -2$
$y = 1$
$z = 1$ ✓
$v = 0$

$x + 2y - z + v = -1$

$-2 + 2 \cdot 1 - 1 + 0 = -1$

$-2 + 2 - 1 + 0 = -1$

$-1 = -1$ //

PROVJERA - DALJE -

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod ⇊ $\mathcal{I}1$

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: Marina Beštar

BROJ INDEKSA: 17-2-0193-2012

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 + \overline{1+i} = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 15+3

2. Gaussovom metodom riješi sustav linearnih jednadžbi, a zatim provjeri uvrštavanjem:

$$\begin{aligned} x + 2y - z + u &= -1 \\ 2x + 5y - z + 2u &= -2 \\ 3x - y - 2z + u &= 5 \\ x - y + 3z - 5u &= 6 \end{aligned}$$

Provjeri uvrštavanjem! 16+3

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 + x} - x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{x^2-4}{x^2+2}$. 20(graf)

5. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\sin(4x - 2))$. 15

6. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. 8

Ukupno: 35

Marina Bogez

3. $g(x) = \sqrt{x^2+x} - x$

1. DOMENA

$x^2+x \geq 0$

Df... $x \in \mathbb{R}$ ✗

2. N.T nema

4. ASIMPTOTE

V.A nema

$[\infty - \infty] \Rightarrow$ TREBA RACIONALIZACIJA

H.A

$$\lim_{x\to\pm\infty} \frac{\sqrt{x^2+x} - x}{1} \begin{matrix} :x \\ :x \end{matrix} = \lim_{x\to\pm\infty} \frac{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}} - 1}{\frac{1}{x}} \quad (1 \to 0)$$

$$\lim_{x\to\pm\infty} \frac{\sqrt{1} - 1}{0} = 1-1 = \frac{0}{0} = *$$

nema H.A

K.A $y = ax + b$

$$\lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{\sqrt{x^2+x} - x}{\frac{x}{1}} = \lim_{x\to\pm\infty} \frac{\sqrt{x^2+x} - x}{x} \begin{matrix} :x \\ :x \end{matrix} = \lim_{x\to\pm\infty} \frac{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}} - 1}{1}$$

$$\lim_{x\to\pm\infty} \frac{\sqrt{1} - 1}{1} = \frac{1-1}{1} = 0$$

$a = 0$

$b = \lim_{x\to\pm\infty} (f(x) - ax)$

$$\lim_{x\to\pm\infty} \frac{\sqrt{x^2+x} - x}{1} \begin{matrix} :x \\ :x \end{matrix} = \lim_{x\to\pm\infty} \frac{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}} - 1}{\frac{1}{x}} \quad (1 \to 0) \qquad \lim_{x\to\pm\infty} \frac{\sqrt{1} - 1}{0} = 0$$

$b = 0$

K.A $y = x$

$$f'(x) = \frac{1}{\sin(4x-2)} - \sin(4x-2)'$$

(5)

$$f'(x) = \frac{1}{\sin(4x-2)} - \sin(4x-2)'$$

$$f'(x) = \frac{1}{\sin(4x-2)} \cdot \cos(4x-2) \cdot 4$$

$$f'(x) = \frac{4 \cdot \cos(4x-2)}{\sin(4x-2)} \quad //$$ ✓

Marina Bešker

(5)

$$f'(x) = \frac{1}{\sin(4x-2)} \cdot \sin(4x-2)'$$

$$f'(x) = \frac{1}{\sin(4x-2)} \cdot \sin(4x-2)'$$

$$f'(x) \quad \frac{1}{\sin(4x-2)} \cdot \cos(4x-2) \cdot 4$$

$$f'(x) = \frac{4 \cdot \cos(4x-2)}{\sin(4x-2)} \quad //$$

(1.)

$$z^3 + \overline{1+i} = 0$$

$$z^3 = \overline{1+i}$$

$$z^3 = \overline{1+i}$$

$r = \sqrt{x^2+y^2}$

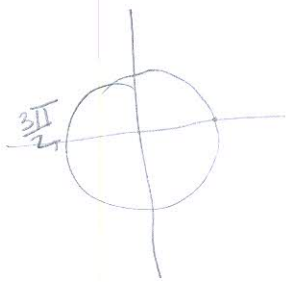
$r = \sqrt{1^2+1^2}$

$r = \sqrt{1}$

$r = 1$

$\operatorname{tg} \varphi = \frac{y}{x} = \frac{1}{1} = 1$

$\varphi = 45° = \frac{3\pi}{2}$



$k = 0, 1, 2$

$$z_{k+1} = \sqrt[n]{r}\left(\cos\frac{\varphi + k\pi}{n} + i\sin\frac{\varphi + k\pi}{n}\right)$$

$$z_{k+1} = \left(\cos\frac{45 \cdot 0 \cdot \pi}{3} + i\sin\frac{45 \cdot 0 \cdot \pi}{3}\right)$$

$z_1 = 30 + + i$

Marina Beša

(2.) $x + 2y - z + u = -1$

$2x + 5y - z + 2u = -2$

$3x - y - 2z + u = 5$

$x - y + 3z - 5u = 6$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 6 \end{array}\right] \begin{array}{l} \\ \cdot(-2) \\ + \\ \\ \sim \end{array} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 6 \end{array}\right] \begin{array}{l} \\ \cdot 3 \\ + \\ \sim \end{array} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -7 & 1 & -2 & 2 \\ 1 & -1 & 3 & -5 & 6 \end{array}\right] \begin{array}{l} \\ \cdot(-1) \\ + \\ \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -7 & 1 & -2 & 2 \\ 0 & -3 & 4 & -6 & 5 \end{array}\right] \begin{array}{l} \\ \cdot 7 \\ \sim \\ + \end{array} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -2 & 2 \\ 0 & -3 & 4 & -6 & 5 \end{array}\right] \begin{array}{l} \\ \\ \cdot(3) \\ + \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -2 & 2 \\ 0 & 0 & -7 & -6 & 5 \end{array}\right] \begin{array}{l} \\ \\ \\ \sim \end{array} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 0 & 7 & -6 & 5 \\ 0 & 0 & 8 & -2 & 2 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ \cdot 8 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 0 & 7 & -6 & 5 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 8 & -2 & 2 \end{array}\right] \begin{array}{l} \\ \\ \\ \cdot 8 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 1 \\ 0 & 0 & 7 & -6 & 5 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 2 \end{array}\right] //$$

$$1x + 2y - 1z + 1 = 1$$

$$-7z - 6u = 5$$

$$-1z = 0$$

$$-3z - 2u = 2$$

Marina Bešker

(4)

$$h(x) = \frac{x^2-4}{x^2+2}$$

1. DOMENA

Df ... $x \in \mathbb{R}$

$x^2+2 \neq 0$

$x^2 \neq -2 \mid \sqrt{\ }$

$x \neq \sqrt{-2}$

2. N.T

N.T $(-2, 2)$

$x^2-4=0$

$x^2=4 \mid \sqrt{\ }$

$x = \pm 2$

$(2x+1)'$

$2 \cdot 1 + 0$

2

3. PARNOST / NEPARNOST

$h(-x) = \frac{(-x)^2-4}{(-x)^2+2} = \frac{x^2-4}{x^2+2}$ funkcija je parna

$x^2 - 2x$

4. ASIMPTOTE

V.A nema

H.A

$$\lim_{x \to \pm\infty} \frac{x^2-4}{x^2+2} : x^2 : x^2 = \lim_{x \to \pm\infty} \frac{1 - \frac{4}{x^2}}{1 + \frac{2}{x^2}} = \frac{1}{1} = 1$$

$\boxed{y = 1}$ H.A

s lijeve i s desne strane

K.A nema

5. DERIVACIJA I STACIONARNE TOČKE

$$h'(x) = \frac{(x^2-4)' \cdot (x^2+2) - (x^2-4) \cdot (x^2+2)'}{(x^2+2)^2}$$

$$h'(x) = \frac{2x(x^2+2) - (x^2-4) \cdot 2x}{(x^2+2)^2}$$

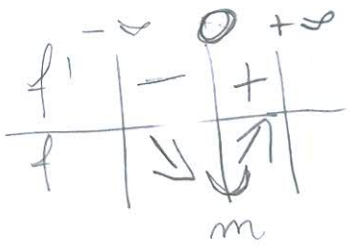
$$h'(x) = \frac{2x(x^2+2) - 2x(x^2-4)}{(x^2+2)^2}$$

(4)

Manna Bosz

$$h'(x) = \frac{2x^3 + 4x - 2x^3 + 8x}{(x^2+2)^2}$$

$$h'(x) = \frac{12x}{(x^2+2)^2} //$$



$-m(-1.33, 0)$

$12x = 0 \;/:12$

$x = \frac{0}{12}$

$x = 0$

DRUGA DERIVACIJA i TOČKE INFLEKSIJE

$$h''(x) = \frac{(12x)' \cdot (x^2+2)^2 - (12x) \cdot ((x^2+2)^2)'}{(x^2+2)^4}$$

$$h''(x) = \frac{12(x^2+2)^2 - (12x) \cdot 2(x^2+2) \cdot (x^2+2)'}{(x^2+2)^4}$$

$$h''(x) = \frac{12(x^2+2)^2 - (12x) \cdot 2(x^2+2) \cdot 2x}{(x^2+2)^4}$$

$$h''(x) = \frac{12(x^2+2)^2 - (12x) \cdot 4x(x^2+2)}{(x^2+2)^4}$$

(x^2+2)

$(x^2)^2 + 2 \cdot x^2 \cdot 2 + 2^2$

$x^4 + 4x^2 + 4$

$$h''(x) = \frac{12(x^2+2)^2 - 12x \cdot (4x^3 + 8x)}{(x^2+2)^4}$$

$$h''(x) = \frac{12(x^4 + 4x^2 + 4) - 48x^4 - 96x^2}{(x^2+2)^4}$$

$-48x^2$

$$h''(x) = \frac{12x^4 + 48x^2 + 48 - 48x^4 - 96x^2}{(x^2+2)^4}$$

$$h''(x) = \frac{-36x^4 - 48x^2 + 48}{(x^2+2)^4}$$

$$h''(x) = \frac{24 - 36x^2}{(x^2+2)^3} //$$

(4) Marine Berlu

$f'(x) = 0$

$-36x^2 - 24 = 0$

$-36x^2 = 24$

$x^2 = \frac{24}{36}$

$x^2 = \frac{8}{12}$

$x = \pm\sqrt{\frac{8}{12}} = \pm\frac{\sqrt{6}}{3}$ //

$x_1 = -\frac{\sqrt{6}}{3} = -0.81$

$x_2 = \frac{\sqrt{6}}{3} = 0.81$

	$-\infty$		-0.81		0.81		$+\infty$
$f'(x)$		+		+		+	
f		↗		↗		↗	

$y = \frac{x^2-4}{x^2+2}$

$I_1(-0.80, -0.81)$

$I_2(0.80, 0.81)$

(4) Maina Besker