

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: *Poko Šimurina*

BROJ INDEKSA: *17-1-0029-2010*

85

- Riješi jednadžbu među kompleksnim brojevima: $z^3 - 5 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
- Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2 + 3}$ 10
- Ispitati asimptote funkcije: $h(x) = \sqrt{x^2 + 2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10 (asimptote)
20 (graf)
- Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat
(a) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) =$ 7+2
(b) $\lim_{n \rightarrow \infty} \left(\frac{x^2 + 5}{x^2} \right) =$ 4+2
- Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5
$$\begin{array}{rccccrcr} 4x & - & y & + & z & + & 2u & = & -1 \\ 2x & + & y & & & - & 3u & = & 4 \\ x & - & y & + & 2z & + & u & = & 2 \\ 2x & + & y & + & z & - & 4u & = & 7 \end{array}$$
- Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$. 10

Ukupno:

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$$6. f'(x) = \frac{1}{\cos(2x^2-1)} \cdot (-\sin(2x^2-1) \cdot 4x = -4x \cdot \frac{1}{\sin(2x^2-1)})$$

5.

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 4 & -1 & 1 & 2 & -1 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 0 & -3 & 3 & -9 \\ 0 & 0 & -1 & -1 & 3 \end{array} \right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -3 & 3 & -9 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Roko Simurina

2. $g(x) = \sqrt{x^2+3}$

$$x^2+3 \geq 0$$

$$x^2 \geq -3$$

$$D(g) = \mathbb{R}$$

$$g'(x) = \frac{x}{\sqrt{x^2+3}}$$

$$g'(x) = 0 \Rightarrow x = 0$$

$$g(0) = \sqrt{3}$$

$$\min(0, \sqrt{3}) \checkmark$$

	$-\infty$	0	$+\infty$
$g'(x)$		-	+

↓ ↑
min

4. a) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) = ?$

b) $\lim_{n \rightarrow \infty} \left[\frac{x^2+5}{x^2} \right]_{\substack{1: x^2 \\ 1: x^2}}^{\substack{1: x^2 \\ 1: x^2}} = \lim_{h \rightarrow \infty} \frac{1 + \frac{5}{h^2}}{1} = 1 \checkmark$

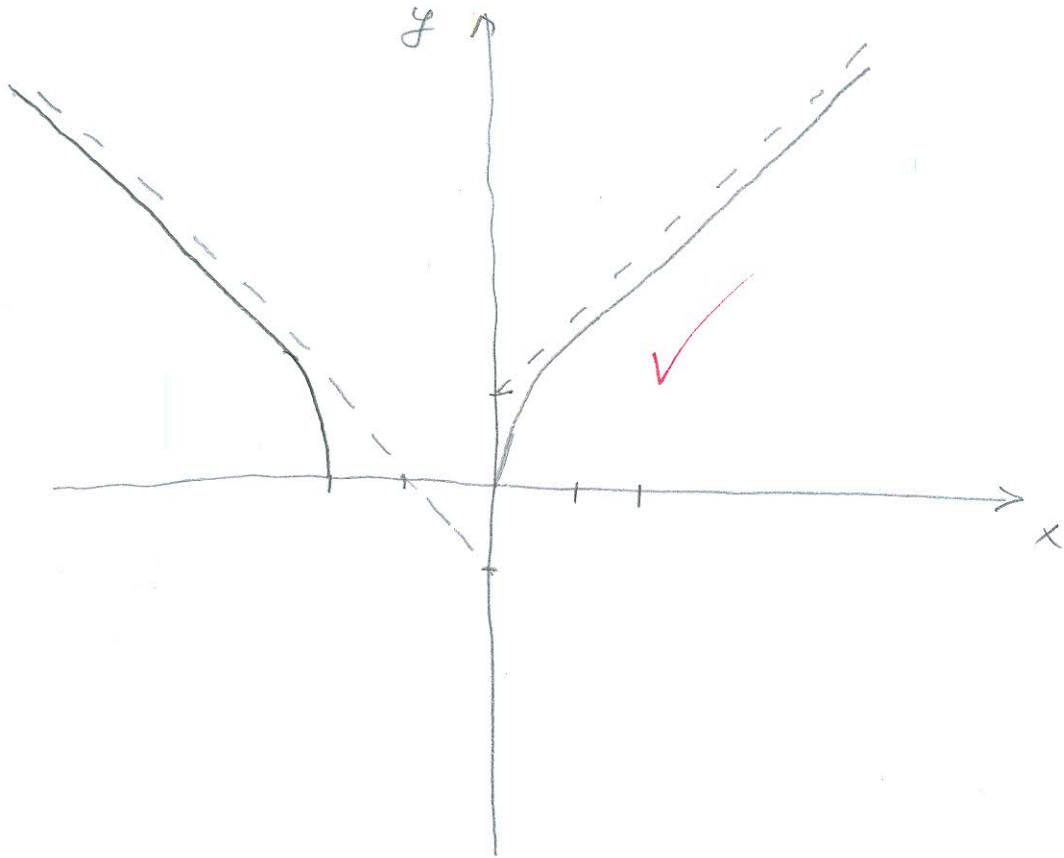
$$h(x) = 0 \Rightarrow \sqrt{x^2 + 2x} = 0$$

$$x = 0 \quad x = -2$$

$$h'(x) = \frac{2x+2}{2\sqrt{x^2+2x}} = \frac{x+1}{\sqrt{x^2+2x}}$$

	$-\infty$	-2	0	$+\infty$
$h'(x)$	-	/	+	

$$h'(x) = 0 \Rightarrow x+1=0$$
$$x = -1$$



Roko Simurina

3.

$$h(x) = \sqrt{x^2 + 2x}$$

$$x^2 + 2x \geq 0$$

$$x(x+2) \geq 0$$

$$x_1 = 0 \quad x_2 = -2$$

$-\infty$	-2	0	$+\infty$
x^2+2x	+	-	+

$$D(h) = \langle -\infty, -2 \rangle \cup [0, +\infty \rangle$$

$$\lim_{x \rightarrow -2^-} \sqrt{x^2 + 2x} = 0$$

$$\lim_{x \rightarrow 0^+} \sqrt{x^2 + 2x} = 0$$

Newa V. A

$$\lim_{x \rightarrow -\infty} \sqrt{x^2 + 2x} = \infty \quad \text{Newa H. A.}$$

$$LKA = k = \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 2x}}{x} = \lim_{x \rightarrow -\infty} -\sqrt{1 + \frac{2}{x}} = 1$$

$$l = \lim_{x \rightarrow -\infty} (\sqrt{x^2 + 2x} + x) \cdot \frac{\sqrt{x^2 + 2x} - x}{\sqrt{x^2 + 2x} - x} = \lim_{x \rightarrow -\infty} \frac{x^2 + 2x - x^2}{\sqrt{x^2 + 2x} - x} \cdot \frac{1}{x}$$

$$= \lim_{x \rightarrow -\infty} \frac{2}{-\sqrt{1 + \frac{2}{x}} - 1} = -1$$

$$\boxed{LKA \dots y = -x - 1} \quad \checkmark$$

$$DKA \quad k = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + 2x}}{x} = \lim_{x \rightarrow +\infty} \sqrt{1 + \frac{2}{x}} = 1$$

$$l = \lim_{x \rightarrow +\infty} (\sqrt{x^2 + 2x} - x) \cdot \frac{\sqrt{x^2 + 2x} + x}{\sqrt{x^2 + 2x} + x} = \lim_{x \rightarrow +\infty} \frac{x^2 + 2x - x^2}{\sqrt{x^2 + 2x} + x} \cdot \frac{1}{x^2}$$

$$= \lim_{x \rightarrow +\infty} \frac{2}{\sqrt{1 + \frac{2}{x}} + 1} = 1$$

$$\boxed{DKA \dots y = x + 1} \quad \checkmark$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

ε5

IME I PREZIME: ANTONIO ŠARIN

BROJ INDEKSA: 17-2-0301-13

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

1. Riješi jednačbu među kompleksnim brojevima: $z^3 - 5 + 3i = 0$. Prikaži rješenja u kompleksnoj ravnini! 12+3

2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2 + 3}$

10
5
10 (asimptote)
20 (graf) 13

3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2 + 2x}$. Zatim dovršiti ispitivanje toka i skicirati graf.

4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

(a) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) =$

7+2

(b) $\lim_{n \rightarrow \infty} \left(\frac{x^2 + 5}{x^2} \right) =$

4+2

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

15+5

$$\begin{aligned} 4x - y + z + 2u &= -1 \\ 2x + y - 3u &= 4 \\ x - y + 2z + u &= 2 \\ 2x + y + z - 4u &= 7 \end{aligned}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$.

10

Ukupno:

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2) $g(x) = \sqrt{x^2 + 3}$
 $g'(x) = \frac{1}{2\sqrt{x^2 + 3}} \cdot 2x = \frac{x}{\sqrt{x^2 + 3}}$
 $g'(x) = 0 \Rightarrow x = 0$
 $g''(x) = \frac{\sqrt{x^2 + 3} - x \cdot \left(\frac{1}{\sqrt{x^2 + 3}} \cdot 2x \right)}{(x^2 + 3)^{3/2}}$
 $= \frac{\sqrt{x^2 + 3} - \frac{2x^2}{\sqrt{x^2 + 3}}}{(x^2 + 3)^{3/2}}$
 $g''(0) = \frac{\sqrt{3} - 0}{3} > 0$ MINIMUM
(0, √3) minimum

6) $f(x) = \ln(\cos(2x^2 - 1))$
 $f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot (-\sin(2x^2 - 1)) \cdot 4x$
 $= -4x \tan(2x^2 - 1)$

~~Handwritten scribbles and crossed-out text at the top of the page.~~

4) a) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) \cdot \frac{\sqrt{3+x} + \sqrt{3}}{\sqrt{3+x} + \sqrt{3}}$

$= \lim_{x \rightarrow 0} \frac{3+x-3}{x(\sqrt{3+x} + \sqrt{3})} = \dots$

$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{3+x} + \sqrt{3}} = \frac{1}{2\sqrt{3}} // \checkmark$

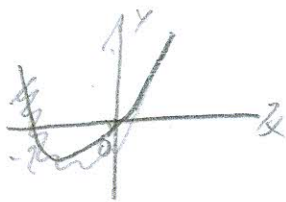
b) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5}{x^2} \right) = (L'HOSPITAL)$

$= \lim_{x \rightarrow \infty} \frac{2x}{2x} = 1 // \checkmark$

3) $h(x) = \sqrt{x^2 + 2x}$ $x_1 = 0$

$x^2 + 2x \geq 0$ $x_2 = -2$

$x(x+2) \geq 0$



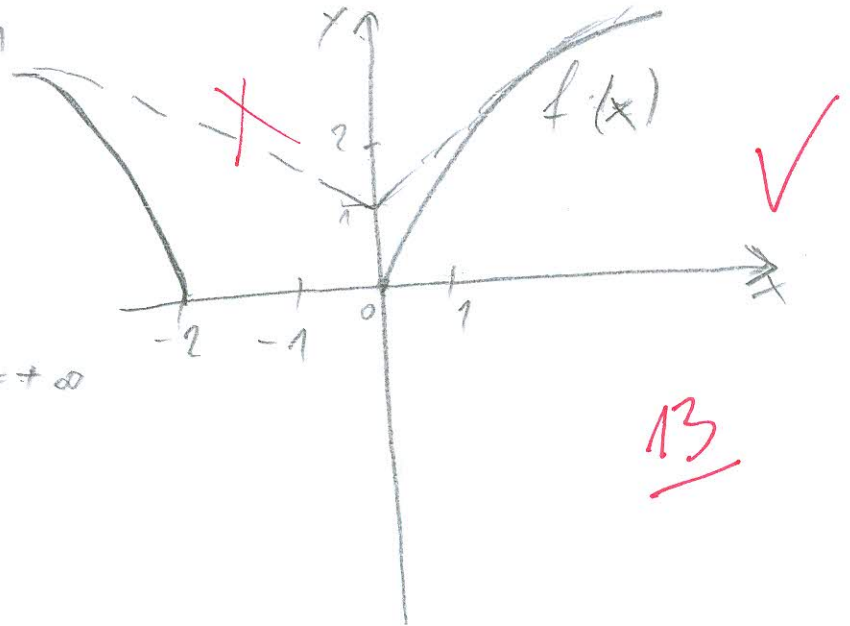
$OB(x) \in (-\infty, -2] \cup [0, +\infty)$

- 1) VERTIKALNE ASIMPTOTE NEMA
- 2) HORIZONTALNE ASIMPTOTE

$\lim_{x \rightarrow +\infty} \sqrt{x^2 + 2x} = +\infty$

$\lim_{x \rightarrow -\infty} \sqrt{x^2 + 2x} = \lim_{x \rightarrow +\infty} \sqrt{x^2 - 2x} = +\infty$

NEMA HORIZONTALNE



13

koje

$h = \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 2x}}{x} =$

$= \lim_{x \rightarrow +\infty} \frac{\sqrt{1 + \frac{2}{x}}}{1} = 1 //$

$$L = \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2 + 2x} - x}{\sqrt{x^2 + 2x} + x} \right) =$$

$$= \lim_{x \rightarrow \infty} \frac{x^2 + 2x - x^2}{\sqrt{x^2 + 2x} + x} \cdot x = 1$$

DESNA KOSA: $y = x + 1$

LIJEVA KOSA?

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

ε5

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: *Saša Pelc*

BROJ INDEKSA: *0269082855*

- Riješi jednačbu među kompleksnim brojevima: $z^3 - 5 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
- Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2 + 3}$ 10
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(b) $\lim_{n \rightarrow \infty} \left(\frac{x^2 + 5}{x^2} \right) =$ 4+2
- Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5
 - $4x - y + z + 2u = -1$
 - $2x + y - 3u = 4$
 - $x - y + 2z + u = 2$
 - $2x + y + z - 4u = 7$
- Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$. 10

Ukupno:

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$$\begin{aligned}
 5) \quad & 4x - y + z + 2u = 1 \\
 & 2x + y - 3u = 4 \\
 & x - y + 2z + u = 2 \\
 & 2x - y + z - 4u = 7
 \end{aligned}$$

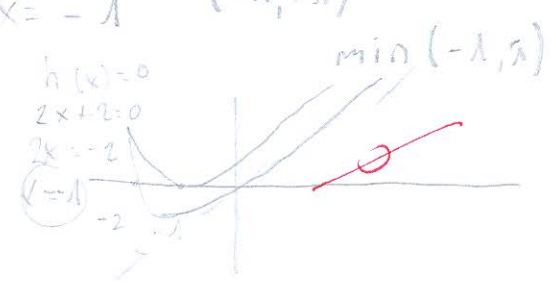
$$\begin{aligned}
 & \left[\begin{array}{cccc|c} 4 & -1 & -1 & 2 & 1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & -1 & 1 & -4 & 7 \end{array} \right] \cdot \frac{1}{4} \sim \left[\begin{array}{cccc|c} 1 & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & -1 & 1 & -4 & 7 \end{array} \right] \begin{array}{l} \cdot (-2) \cdot (-1) / (-2) \\ \checkmark \\ \checkmark \\ \checkmark \end{array} \\
 & \left[\begin{array}{cccc|c} 1 & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{2} & \frac{3}{2} & -4 & \frac{7}{2} \\ 0 & -\frac{3}{2} & \frac{5}{2} & \frac{3}{2} & \frac{7}{2} \\ 0 & \frac{3}{2} & \frac{3}{2} & -5 & \frac{15}{2} \end{array} \right] \cdot \frac{2}{3} \sim \left[\begin{array}{cccc|c} 1 & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & -\frac{1}{3} & -\frac{4}{3} & \frac{7}{3} \\ 0 & -\frac{3}{4} & \frac{7}{4} & \frac{1}{2} & \frac{7}{4} \\ 0 & \frac{3}{2} & \frac{1}{2} & -5 & \frac{15}{2} \end{array} \right] \begin{array}{l} \cdot \frac{1}{3} \cdot \frac{3}{4} \\ \cdot (-\frac{3}{2}) \\ \cdot (-\frac{1}{2}) \cdot (-1) / (-2) \end{array} \\
 & \left[\begin{array}{cccc|c} 1 & 0 & \frac{1}{6} & -\frac{1}{6} & \frac{1}{2} \\ 0 & 1 & -\frac{1}{3} & -\frac{2}{3} & 3 \\ 0 & 0 & \frac{3}{2} & -\frac{3}{2} & \frac{9}{2} \\ 0 & 0 & 1 & -1 & 3 \end{array} \right] \cdot \frac{2}{3} \sim \left[\begin{array}{cccc|c} 1 & 0 & \frac{1}{6} & -\frac{1}{6} & \frac{1}{2} \\ 0 & 1 & -\frac{1}{3} & -\frac{2}{3} & 3 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -1 & 3 \end{array} \right] \begin{array}{l} \cdot (-\frac{1}{6}) \\ \cdot (-\frac{1}{3}) \\ \cdot (-1) \\ \cdot (-1) \end{array}
 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -3 & 1 & 4 \\ 0 & 0 & 1 & -1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

PROVJETA

$$\begin{aligned}
 0 - 4 + 3 + 0 &= 1 \\
 0 + 4 - 0 &= 4 \\
 0 - 4 + 6 &= 2 \\
 0 + 4 + 3 - 0 &= 7
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{2} \sqrt{x^2 + 2x} - 2x + 2 \\
 & \frac{x+1}{\sqrt{x^2+2x}} \rightarrow x = -1 \\
 & (-1, 1)
 \end{aligned}$$



$x = 0$
 $y - 3u = 4$
 $z - u = 3$

NJE
KJESENJA...

$$\begin{aligned}
 x &= 0 \\
 y &= 4 \\
 z &= 3 \\
 u &= 0
 \end{aligned}$$

3.) $h(x) = \sqrt{x^2 + 2x}$

$$k = \lim_{x \rightarrow 0} \frac{f(x)}{x} = \lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 2x}}{x} = \lim_{x \rightarrow 0} \sqrt{\frac{x^2 + 2x}{x^2}} = \lim_{x \rightarrow 0} \sqrt{1 + \frac{2}{x}} \rightarrow 0$$

MULTOČKE

$$\begin{aligned}
 h(x) &= 0 \\
 \sqrt{x^2 + 2x} &= 0 / ^2 \\
 x^2 + 2x &= 0
 \end{aligned}$$

$$\begin{aligned}
 l &= \lim_{x \rightarrow 0} (f(x) - kx) = \lim_{x \rightarrow 0} (\sqrt{x^2 + 2x} - x) = L.H. \lim_{x \rightarrow 0} \left(\frac{1}{2} \cdot \frac{1}{\sqrt{x^2 + 2x}} \cdot (2x + 2) \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{(x+1)}{\sqrt{x^2 + 2x}} - 1 \right) = \lim_{x \rightarrow 0} \left(\frac{(x+1) - \sqrt{x^2 + 2x}}{\sqrt{x^2 + 2x}} \right) = \lim_{x \rightarrow 0} \frac{1 - \sqrt{1}}{1} = 0
 \end{aligned}$$

P. y=x (x=0) (x=-2)

$$2.) g(x) = \sqrt{x^2+3} = (x^2+3)^{\frac{1}{2}}$$

$$g'(x) = \frac{1}{2} (x^2+3)^{-\frac{1}{2}} \cdot 2x$$

$$g'(x) = \frac{3}{2} x^2 \cdot \frac{1}{\sqrt{x^2+3}}$$

$$g'(x) = 0$$

$$\frac{3}{2} x^2 \cdot \frac{1}{\sqrt{x^2+3}} = 0$$

$$x = 0 \rightarrow (0, \sqrt{3}) \quad \checkmark$$

$$g(0) = \sqrt{3}$$

$$4.) a) \lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) = \sqrt{3} - \sqrt{3} = 0$$

$$\lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) = LH = \lim_{x \rightarrow 0} \frac{\frac{1}{2} \cdot \frac{1}{\sqrt{3+x}} - 0}{1} = \frac{1}{2\sqrt{3}} \quad \checkmark$$

$$b) \lim_{n \rightarrow \infty} \left(\frac{x^2 + 5}{x^2} \right) = \lim_{n \rightarrow \infty} \frac{1 + \frac{5}{x^2}}{1} = 1 \quad \checkmark$$

$$c) f(x) = \ln(\cos(2x^2 - 1))$$

$$f'(x) = \frac{1}{\cos(2x^2-1)} \cdot (-\sin(2x^2-1)) \cdot 4x \quad \checkmark$$

$$= -\operatorname{tg}(2x^2-1) \cdot 4x$$

$$1) z^3 = 5 - 3i$$

$$z^3 = 5 (\cos(-30,96^\circ) + i \sin(-30,96^\circ))$$

$$z = 5^{\frac{1}{3}} \left(\cos \frac{-30,96 + 2k\pi}{3} + i \sin \left(\frac{-30,96 + 2k\pi}{3} \right) \right) \quad ? \quad \rho = \operatorname{arctg} \left(-\frac{3}{5} \right) = 30,96^\circ$$

$$z_1 = 5^{\frac{1}{3}} \left(\cos \frac{-30,96}{3} + i \sin \left(\frac{-30,96}{3} \right) \right)$$

$$z_2 = 5^{\frac{1}{3}} \left(\cos \frac{-30,96 + 2\pi}{3} + i \sin \left(\frac{-30,96 + 2\pi}{3} \right) \right)$$

$$z_3 = 5^{\frac{1}{3}} \left(\cos \frac{-30,96 + 4\pi}{3} + i \sin \left(\frac{-30,96 + 4\pi}{3} \right) \right)$$

$$r = \sqrt{2r-9}$$

$$r = 5$$

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POPUNJAVA
NASTAVNIK
Broj ↓
bodova

ε5

IME I PREZIME:

BROJ INDEKSA:

MIŠEL GOBIN

17-1-0034-2010

- Riješi jednadžbu među kompleksnim brojevima: $z^3 - 5 + 3i = 0$. Prikaži rješenja u kompleksnoj ravnini! 12+3
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- Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$.

10

Ukupno:

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~~17-1-0034-2010~~

6) (a) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right)$

$$= \frac{\sqrt{3+x} + \sqrt{3}}{\sqrt{3+x} + \sqrt{3}}$$

$$\lim_{x \rightarrow 0} \frac{x + x - 0}{x (\sqrt{3+x} + \sqrt{3})}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{3+x} + \sqrt{3}} = \frac{1}{2\sqrt{3}} \quad \checkmark$$

b) $\lim_{n \rightarrow \infty} \left(\frac{x^2 + 5}{x^2} \right) = (L'HOSPITAL)$

$$= \lim_{x \rightarrow +\infty} \frac{2x}{2x} = 1 = \checkmark$$

② $g(x) = \sqrt{x^2 + 3}$

$$g'(x) = \frac{1}{\sqrt{x^2 + 3}} \cdot 2x$$
$$= \frac{x}{\sqrt{x^2 + 3}}$$

$g'(x) = 0 \Rightarrow x = 0$

$$g''(x) = \frac{\sqrt{x^2 + 3} - x \cdot \left(\frac{1}{2\sqrt{x^2 + 3}} \cdot 2x \right)}{(x^2 + 3)}$$
$$= \frac{\sqrt{x^2 + 3} - \frac{x^2}{\sqrt{x^2 + 3}}}{(x^2 + 3)}$$

$g''(0) = \frac{\sqrt{3}}{3} > 0$ Minimum

$T(0, \sqrt{3})$ Minimum ✓

③ $f(x) = \ln(\cos(2x^2 - 1))$

$$f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot (-\sin(2x^2 - 1)) \cdot 4x$$
 ✓

$$= -4x \tan(2x^2 - 1)$$

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

ε5

IME I PREZIME: **KARLO FRANOV**

BROJ INDEKSA: **17-2-0311-2013**

1. Riješi jednačbu među kompleksnim brojevima: $z^3 - 5 + 3i = 0$. Prikaži rješenja u kompleksnoj ravnini! 12+3

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20(graf) 5

4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

(a) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) = \lim_{x \rightarrow 0} \frac{\sqrt{3+0} - \sqrt{3}}{0} = \frac{\sqrt{3} - \sqrt{3}}{0} = \frac{0}{0} = 0$ 7+2

(b) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5}{x^2} \right) = \lim_{x \rightarrow \infty} \frac{x^2 + 5}{x^2}$ 4+2

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{aligned} 4x - y + z + 2u &= -1 \\ 2x + y & - 3u &= 4 \\ x - y + 2z + u &= 2 \\ 2x + y + z - 4u &= 7 \end{aligned}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$. 10

Ukupno:

25

1. $z^3 - 5 + 3i = 0$
 $z^3 = 5 - 3i \quad | \cdot \sqrt[3]{1}$
 $z = \sqrt[3]{5 - 3i}$

$|z| = \sqrt{x^2 + y^2} = \sqrt{5^2 + (-3)^2} = \sqrt{25 + 9} = \sqrt{34}$
 $\arg z = \frac{y}{x} = \frac{-3}{5} = -0,6 \Rightarrow -0,5104 \text{ IV KVADRANT}$

$z = 0 \Rightarrow z = \sqrt[3]{\sqrt{34}} \cdot \left(\cos \frac{-0,5104 \pi}{3}, \sin \frac{-0,5104 \pi}{3} \right)$

$z = 1 \Rightarrow z = \sqrt[3]{\sqrt{34}} \cdot \left(\cos \frac{-5,7692}{3}, \sin \frac{-5,7692}{3} \right)$

$z = 2 \Rightarrow z = \sqrt[3]{\sqrt{34}} \cdot \left(\cos \frac{-6,4139}{3}, \sin \frac{-6,4139}{3} \right)$

6. $f(x) = \ln(\cos(2x^2 - 1))$

$f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot -\sin(2x^2 - 1) \cdot 4x$

$f'(x) = \frac{-\sin(2x^2 - 1) \cdot 4x}{\cos(2x^2 - 1)} = -4x \tan(2x^2 - 1)$ ✓

5. $4x - y + z + 2M = -1$
 $2x + y + 0 + 3M = 4$
 $x - y + 2z + M = 2$
 $2x + y + z - 4M = 7$

$\begin{bmatrix} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{bmatrix}$ $\cdot (-2) \quad (-4) \quad (-2)$

$\sim \begin{bmatrix} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{bmatrix}$ $1:3$

$\sim \begin{bmatrix} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{bmatrix}$ $1 \cdot (-3) \quad 1 \cdot (-3)$

$\sim \begin{bmatrix} 1 & 0 & \frac{2}{3} & -\frac{2}{3} & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 0 & 4 & \frac{5}{3} & -9 \\ 0 & 0 & 4 & -1 & 3 \end{bmatrix}$ $1:4$

$\sim \begin{bmatrix} 1 & 0 & \frac{2}{3} & -\frac{2}{3} & 1 & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 1 & 0 \\ 0 & 0 & 1 & \frac{4}{3} & 1 & -9 \\ 0 & 0 & 4 & -1 & 1 & 3 \end{bmatrix}$ $1 \cdot (-\frac{2}{3}) \quad 1 \cdot (\frac{4}{3}) \quad (-4)$

$\sim \begin{bmatrix} 1 & 0 & 0 & -\frac{16}{3} & 1 & \frac{7}{2} \\ 0 & 1 & 0 & -\frac{14}{3} & 1 & -3 \\ 0 & 0 & 1 & \frac{4}{3} & 1 & -9 \\ 0 & 0 & 0 & -\frac{20}{3} & 1 & 12 \end{bmatrix}$ $1: (-\frac{21}{5})$

$\sim \begin{bmatrix} 1 & 0 & 0 & -\frac{16}{3} & 1 & \frac{7}{2} \\ 0 & 1 & 0 & -\frac{14}{3} & 1 & -3 \\ 0 & 0 & 1 & \frac{4}{3} & 1 & -9 \\ 0 & 0 & 0 & -\frac{20}{3} & 1 & 12 \end{bmatrix}$ $1 \cdot (\frac{6}{5}) \quad 1 \cdot (\frac{2}{5}) \quad (-\frac{4}{5})$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & \frac{14}{3} \\ 0 & 1 & 0 & 0 & 1 & -\frac{3}{2} \\ 0 & 0 & 1 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & 1 & 1 & -\frac{20}{7} \end{bmatrix}$ $\begin{matrix} x \\ y \\ z \\ M \end{matrix}$

$$4 a) \lim_{x \rightarrow 0} \frac{\sqrt{3+x} - \sqrt{3}}{0} = \frac{\sqrt{3} - \sqrt{3}}{0} = \frac{0}{0} = 0 = \lim_{x \rightarrow 0} \frac{1}{2\sqrt{3+x}} = \frac{1}{2\sqrt{3}}$$

UVIŠTAVANJE 0 U IZRAZ DOBIJENO OBLIKE $\frac{0}{0}$ KPI
PRIMENJUJEMO L'HOSPITALOVO PRAVILO ZA OBAVUJNE RJEŠAVANJE

$\frac{1}{2\sqrt{3+x}} = \frac{1}{2\sqrt{3}}$ ✓
KARLO FRANOV
OKAZUJE NA BESKONAČNOST.
LINESA DERIVIRAMO KRAJOMAK I NAZIVNIK

$$\lim_{x \rightarrow \infty} \left(\frac{x^2+5}{x^2} \right) = \lim_{x \rightarrow \infty} \frac{x^2+5}{x^2} \cdot \frac{1 \cdot x^2}{1 \cdot x^2} = \frac{5}{1} = 5$$

X

$$2. g(x) = \sqrt{x^2+3}$$

$$g'(x) = \frac{1}{2\sqrt{x^2+3}} \cdot 2x = \frac{x}{\sqrt{x^2+3}}$$

$$\frac{x}{\sqrt{x^2+3}} = 0 \Rightarrow x=0$$

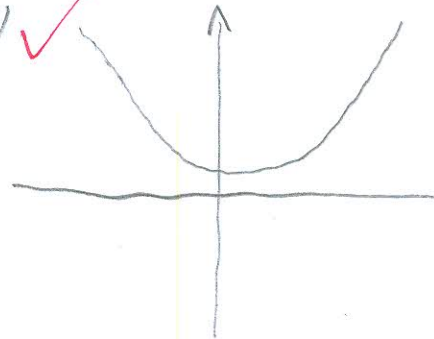
$$g(0) = \sqrt{0+3} = \sqrt{3}$$

$$\lim_{x \rightarrow \infty} g(x) = \lim_{x \rightarrow \infty} \sqrt{x^2+3} = \infty$$

$$\lim_{x \rightarrow -\infty} g(x) = \lim_{x \rightarrow -\infty} \sqrt{x^2+3} = \infty$$

GLOBALNI MINIMUM

$T(0, \sqrt{3})$ ✓



$$3. f(x) = \sqrt{x^2 + 2x}$$

KARLO FRANOV

$$h = \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 2x}}{x} = \lim_{x \rightarrow \infty} \frac{\sqrt{1 + \frac{2}{x}}}{1} = 1 \checkmark \Rightarrow h = 1$$

$$h = \lim_{x \rightarrow \infty} (f(x) - hx) = \lim_{x \rightarrow \infty} (\sqrt{x^2 + 2x} - x) = ?$$

$$h = \lim_{x \rightarrow \infty} (f(x) - hx) = -1$$

$y = x + 1$ NEMA KOSSE ASIMPTOTE ?

$y = -x - 2$ NEMA KOSSE ASIMPTOTE ?

DOMENA

$$x^2 + 2x \geq 0$$

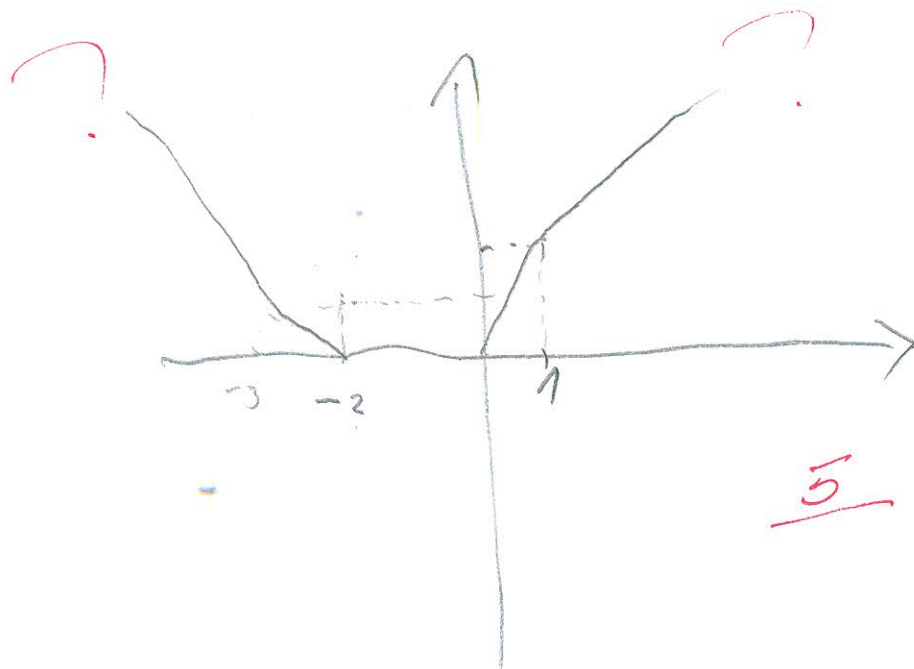
$$x(x+2) = 0$$

$$x_1 = 0$$

$$x_2 = -2$$

$$D_f = \langle -\infty, -2 \rangle \cup [0, +\infty)$$

PO



$$5. \quad 4x - y + z + 2w = -1$$

$$2x + y + z - 3w = 4$$

$$x - y + 2z + w = 2$$

$$2x + y + z - 4w = 7$$

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \begin{array}{l} \times \\ \div 2 \\ \times \\ \times \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -5 & 1 & -3 \\ 0 & 3 & -4 & -5 & 0 \\ 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & -5 & -1 & -7 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & -3 & 7 & 2 & 1 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & -5 & -1 & -7 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 3 & -3 & 0 \\ 0 & 3 & -4 & -5 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -5 & -1 & -7 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 1 & -3 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 3 & -9 \\ 0 & 1 & 0 & -3 & 4 \\ 0 & 0 & 1 & -1 & 9 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right] \begin{array}{l} x \\ y \\ z \\ w \end{array}$$

$$x = 0$$

$$x = 3w = 4$$

$$z = w = 3$$

$$y = 4 + 3w$$

$$z = 3 + 4$$

PROVJERATI?

IME I PREZIME: *LUKA BILUŠIĆ*

BROJ INDEKSA: *17-2-003-2010*

1. Riješi jednačbu među kompleksnim brojevima: $z^3 - 5 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3

2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2 + 3}$ 10

3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2 + 2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote)
20(graf)

4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

(a) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) =$

~~7+2~~

(b) $\lim_{n \rightarrow \infty} \left(\frac{x^2 + 5}{x^2} \right) =$

~~4+2~~

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

~~15+5~~

$$\begin{array}{rcl} 4x - y + z + 2u & = & -1 \\ 2x + y & & - 3u = 4 \\ x - y + 2z + u & = & 2 \\ 2x + y + z - 4u & = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$.

10

Ukupno:

24

6

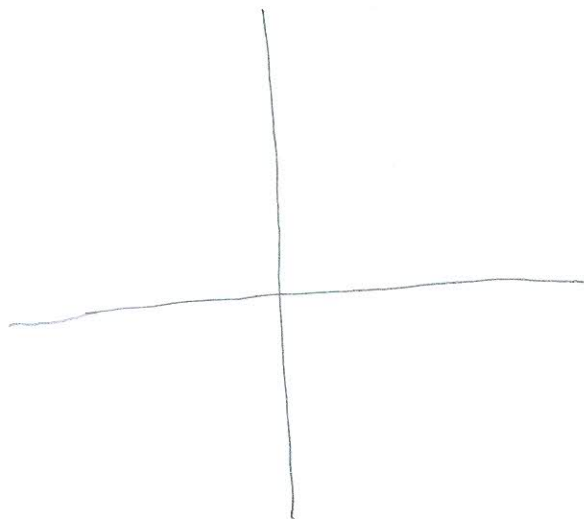
$f(x) = \ln(\cos(2x^2 - 1))$

$f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot (\cos(2x^2 - 1))'$

$f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot (-\sin(2x^2 - 1) \cdot (2x^2 - 1)')$

$f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot (-\sin(2x^2 - 1) \cdot 4x) =$
 $f'(x) = -\frac{4x \sin(2x^2 - 1)}{\cos(2x^2 - 1)}$

1) $z^3 - 5 + 3i = 0$



④

$$b) \lim_{n \rightarrow \infty} \frac{x^2 + 5}{x^2} \stackrel{/: x^2}{=} \lim_{n \rightarrow \infty} \frac{1 + \frac{5}{x^2}}{1} = \frac{1 + 0}{1} = 1 \quad \checkmark$$

$$a) \lim_{x \rightarrow 0} \frac{\sqrt{3+x} - \sqrt{3}}{x} \stackrel{/: x^2}{=} \lim_{x \rightarrow 0} \frac{\sqrt{\frac{3}{x} + 1} - \sqrt{\frac{3}{x}}}{\frac{1}{x}} = \frac{\infty}{\infty} \quad \times$$

$$= \frac{0}{\infty}$$

②

$$g(x) = \sqrt{x^2 + 3}$$

$$g'(x) = \frac{1}{2} (x^2 + 3)^{-\frac{1}{2}}, \quad 2x = 0$$

$$\sqrt{x^2 + 3} = 0 \Rightarrow x = 0$$

x	$-\infty$	0	$\sqrt{3}$	$+\infty$
f'(x)	-	+	-	
f(x)	↘	↗	↘	

globalni minimum je
u točki $(0, \sqrt{3})$ ✓
glob max nema

5

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \sim$$

$$\begin{array}{l} \leftarrow \\ 3r \rightarrow 1r \end{array}$$

$$\begin{array}{l} 1r \cdot (-2) + 2r \\ 1r \cdot (-4) + 3r \\ 1r \cdot (-2) + 4r \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & 0 & -3 & 4 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & 3 & -3 & 9 \end{array} \right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 2 & -2 & 6 \\ 0 & 1 & 0 & -3 & 4 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 1 & 3 \end{array} \right] \sim$$

$$\begin{array}{l} 1r \cdot 2 + 2r \\ 1r \cdot 3 + 3r \\ 1r \cdot 3 + 4r \end{array}$$

$$\begin{array}{l} 1r + 2r \\ 4r \cdot (-3) \\ 3r \cdot (-1) \end{array}$$

$$4r \cdot 2 + 1r$$

$$a \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 12 \\ 0 & 1 & 0 & -3 & 4 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 1 & 3 \end{array} \right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 12 \\ 0 & 1 & 0 & -3 & 4 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 6 \end{array} \right]$$

$$3r + 4r$$

sustav nema
riješivanja ~~X~~

$$\begin{bmatrix} 4 & -1 & 1 & 2 & | & -1 \\ 2 & 1 & 0 & -3 & | & 4 \\ 1 & -1 & 2 & 1 & | & 2 \\ 2 & 1 & 1 & -4 & | & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 2 & 1 & | & 2 \\ 2 & 1 & 0 & -3 & | & 4 \\ 4 & -1 & 1 & 2 & | & 1 \\ 2 & 1 & 1 & -4 & | & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 2 & 1 & | & 2 \\ 0 & 3 & -4 & -5 & | & 0 \\ 0 & 3 & -7 & -2 & | & -9 \\ 0 & 2 & -1 & -5 & | & 5 \end{bmatrix}$$

$3r \leftrightarrow 1r$

$1r \leftrightarrow (-2) + 2r$
 $1r \leftrightarrow (-4) + 3r$
 $4r \leftrightarrow 1r$

$4r \leftrightarrow 1r$

$$\begin{bmatrix} 1 & 2 & -1 & -5 & | & 5 \\ 0 & 3 & -4 & -5 & | & 0 \\ 0 & 3 & -7 & -2 & | & -9 \\ 1 & -1 & 2 & 1 & | & 2 \end{bmatrix}$$

$1r$

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

ε5

IME I PREZIME: DOĐA BUŽUNJA

BROJ INDEKSA: 0268081190

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 5 + 3i = 0$. Prikaži rješenja u kompleksnoj ravnini! 12+3

2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2 + 3}$ 10

3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2 + 2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote)
20(graf)

4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

(a) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) = \lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \cdot \frac{\sqrt{3+x} + \sqrt{3}}{\sqrt{3+x} + \sqrt{3}} \right) = \lim_{x \rightarrow 0} \left(\frac{3+x - 3}{x(\sqrt{3+x} + \sqrt{3})} \right) = \lim_{x \rightarrow 0} \left(\frac{x}{x(\sqrt{3+x} + \sqrt{3})} \right) = \frac{1}{2\sqrt{3}}$

(b) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5}{x^2} \right) = \frac{\infty}{\infty} = \text{L'HOSPITAL} = \lim_{x \rightarrow \infty} \frac{(x^2)' + (5)'}{(x^2)'} = \lim_{x \rightarrow \infty} \frac{2x}{2x} = 1$

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

$4 \cdot \left(-\frac{2}{3}\right) - \frac{5}{6} + 2 + 2 \cdot 0 = -1.5$
 $2 \cdot \left(-\frac{2}{3}\right) + \frac{5}{6} + 2 = \frac{3}{2}$

$$\begin{cases} 4x - y + z + 2u = -1 \\ 2x + y - 3u = 4 \\ x - y + 2z + u = 2 \\ 2x + y + z - 4u = 7 \end{cases}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$.

10

Ukupno:

20

2. $g(x) = \sqrt{x^2 + 3}$

1) MONOTONOST:

$f'(x) = (\sqrt{x^2+3})' = \frac{1}{2\sqrt{x^2+3}} \cdot (x^2+3)' = \frac{2x}{2\sqrt{x^2+3}} = \frac{x}{\sqrt{x^2+3}}$

$f'(x) = 0$

$x = 0$

$A(0, \sqrt{3})$ ✓

$f(0) = \sqrt{0^2 + 3} = \sqrt{3}$

	$-\infty$	0	$+\infty$
$f'(x)$	$-$	$+$	
$f(x)$	\searrow	\nearrow min	

$f''(0) = \frac{3}{(0^2+3)\sqrt{0^2+3}} = \frac{3}{3\sqrt{3}} > 0$

b) $f''(x) = \frac{x' \cdot (\sqrt{x^2+3}) - x \cdot (\sqrt{x^2+3})'}{(\sqrt{x^2+3})^2} = \frac{\sqrt{x^2+3} - \left[x \cdot \frac{1}{2\sqrt{x^2+3}} \cdot (x^2+3)' \right]}{(\sqrt{x^2+3})^2} = \frac{\sqrt{x^2+3} - \frac{x \cdot 2x}{2\sqrt{x^2+3}}}{(\sqrt{x^2+3})^2} = \frac{\sqrt{x^2+3} - \frac{x^2}{\sqrt{x^2+3}}}{(\sqrt{x^2+3})^2}$

$= \frac{\sqrt{x^2+3} - \frac{x^2}{\sqrt{x^2+3}}}{(\sqrt{x^2+3})^2} = \frac{(\sqrt{x^2+3}) \cdot \frac{x^2+3 - x^2}{\sqrt{x^2+3}}}{(\sqrt{x^2+3})^2} = \frac{x^2+3 - x^2}{(\sqrt{x^2+3})^2} = \frac{3}{(\sqrt{x^2+3})^2} = \frac{3}{(x^2+3)\sqrt{x^2+3}}$

$= \frac{3}{(x^2+3)\sqrt{x^2+3}}$

$f'' \neq 0 \Rightarrow 3 = 0 \Rightarrow$ nemoguće, nema točka prevoja

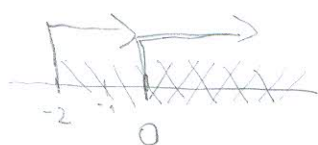
3) $h(x) = \sqrt{x^2+2x}$

1) DOMENA : $x^2+2x \geq 0$

$x(x+2) \geq 0$

$D_f = [0, +\infty)$

1. $x \geq 0$ $x+2 \geq 0$
 $x \geq -2$



2) NULTOČKE : $\sqrt{x^2+2x} = 0/2$ A (-1, 0)

$x^2+2x = 0$

$x_{1,2} = \frac{-2 \pm \sqrt{4-4}}{2} = \frac{-2}{2} = -1$

$x_1 = x_2 = -1$

↳ nije u području definicije f-je

3) ASIMPTOTE :

a) V.A.

$\lim_{x \rightarrow -1} \sqrt{x^2+2x} = \lim_{x \rightarrow -1} \sqrt{x^2+2x} \cdot \frac{\sqrt{x^2+2x}}{\sqrt{x^2+2x}} = \lim_{x \rightarrow -1} \frac{x^2+2x}{\sqrt{x^2+2x}} = \frac{1}{0} = +\infty$

b) K.A. $y = kx + l$

$k = \lim_{x \rightarrow \pm\infty} \frac{f(x)}{x} = \lim_{x \rightarrow \pm\infty} \frac{\sqrt{x^2+2x}}{x} = \frac{\infty}{\infty} = L'HOSPITAL = \lim_{x \rightarrow \pm\infty} \frac{(\sqrt{x^2+2x})'}{x'} = \lim_{x \rightarrow \pm\infty} \frac{\frac{1}{2\sqrt{x^2+2x}} \cdot (x^2+2x)'}{1} =$

$= \lim_{x \rightarrow \pm\infty} \frac{2x+2}{2\sqrt{x^2+2x}} = \lim_{x \rightarrow \pm\infty} \frac{2(x+1)}{2\sqrt{x^2+2x}} = \lim_{x \rightarrow \pm\infty} \frac{x+1}{\sqrt{x^2+2x}} = \frac{1}{1} = 1$

$l = \lim_{x \rightarrow \pm\infty} (f(x) - kx) = \lim_{x \rightarrow \pm\infty} (\sqrt{x^2+2x} - x) = \frac{\infty - \infty}{L'HOSPITAL} = \lim_{x \rightarrow \pm\infty} \left[(\sqrt{x^2+2x})' - x' \right] = \lim_{x \rightarrow \pm\infty} \left(\frac{1}{2\sqrt{x^2+2x}} \cdot (x^2+2x)' - 1 \right) =$

$= \lim_{x \rightarrow \pm\infty} \left(\frac{2x+2}{2\sqrt{x^2+2x}} - x \right) = \lim_{x \rightarrow \pm\infty} \left(\frac{2(x+1)}{2\sqrt{x^2+2x}} - \frac{x}{1} \right) = \lim_{x \rightarrow \pm\infty} \frac{x+1 - x \cdot \sqrt{x^2+2x}}{\sqrt{x^2+2x}} = \frac{1-1 \cdot \sqrt{1}}{\sqrt{1}} = \frac{1-1}{1} =$

$= \frac{0}{1} = 0$

$y = x$

x	0	1	2	B(0,0)
y	0	1	2	C(1,1)

→ nema H.A. jer imamo K.A.

3.4) PARNOST I NEPARNOST

$$f(-x) = \sqrt{(-x)^2 + 2 \cdot (-x)} = \sqrt{x^2 - 2x} \Rightarrow f \text{ - ja nije ni parna ni neparna}$$

5) MONOTONOST:

$$f'(x) = (\sqrt{x^2 + 2x})' = \frac{1}{2\sqrt{x^2 + 2x}} \cdot (x^2 + 2x)' = \frac{2x + 2}{2\sqrt{x^2 + 2x}} = \frac{2(x+1)}{2\sqrt{x^2 + 2x}} = \frac{x+1}{\sqrt{x^2 + 2x}}$$

$$f'(x) = 0 \quad \mathbb{P}(-1, \sqrt{-1})$$

$$x+1 = 0$$

$$x = -1$$

$$f(-1) = \sqrt{(-1)^2 + 2 \cdot (-1)} = \sqrt{1 - 2} = \sqrt{-1}$$

↳ nije u domeni

f je

	$-\infty$	-1	$+\infty$
$f'(x)$		$-$	$+$
$f(x)$		\searrow	\nearrow

6) EKSTREMI:

$$f''(x) = \frac{(x+1)' \cdot (\sqrt{x^2+2x}) - (x+1) \cdot (\sqrt{x^2+2x})'}{(\sqrt{x^2+2x})^2} = \frac{\sqrt{x^2+2x} - (x+1) \cdot \frac{1}{2\sqrt{x^2+2x}} \cdot (x^2+2x)'}{(\sqrt{x^2+2x})^2} = \frac{\sqrt{x^2+2x} - \left[(x+1) \cdot \frac{1}{2\sqrt{x^2+2x}} \cdot (2x+2) \right]}{(\sqrt{x^2+2x})^2}$$

$$= \frac{\sqrt{x^2+2x} - \frac{(x+1) \cdot (2x+2)}{2\sqrt{x^2+2x}}}{(\sqrt{x^2+2x})^2} = \frac{\sqrt{x^2+2x} - \frac{2(x+1)(x+1)}{2\sqrt{x^2+2x}}}{(\sqrt{x^2+2x})^2} = \frac{\sqrt{x^2+2x} - \frac{x^2+x+x+1}{\sqrt{x^2+2x}}}{(\sqrt{x^2+2x})^2} = \frac{\sqrt{x^2+2x} - \frac{x^2+2x+1}{\sqrt{x^2+2x}}}{(\sqrt{x^2+2x})^2}$$

$$= \frac{\cancel{x^2+2x} - \cancel{x^2+2x} - 1}{\sqrt{x^2+2x}} = \frac{-1}{\sqrt{x^2+2x} (\sqrt{x^2+2x})^2}$$

$$f''(x) = 0$$

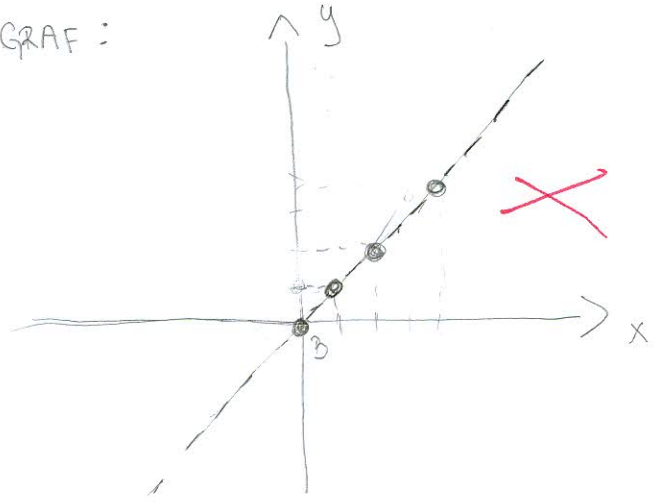
$$-1 = 0$$

↳ nemoguće pa nemamo
točke prevoja

$$f''(-1) = \frac{-1}{\sqrt{1-2} (\sqrt{1-2})^2} = \frac{-1}{\sqrt{-1} \cdot (-1)^2} = \frac{-1}{\sqrt{-1} \cdot 1} = \frac{-1}{\sqrt{-1}}$$

⇒ nema

7) GRAF :



DORA BUŽONJA

6) $f(x) = \ln(\cos(2x^2 - 1))$

$$f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot (\cos(2x^2 - 1))' = \frac{1}{\cos(2x^2 - 1)} \cdot (-\sin(2x^2 - 1)) \cdot (2x^2 - 1)' =$$

$$= \frac{1}{\cos(2x^2 - 1)} \cdot (-\sin(2x^2 - 1)) \cdot 4x$$

5)

$$\begin{bmatrix} 4 & -1 & 1 & 2 & 1 & -1 \\ 2 & 1 & 0 & 3 & 1 & 4 \\ 1 & -1 & 2 & 1 & 2 & 2 \\ 2 & 1 & 1 & -4 & 7 & 7 \end{bmatrix} \begin{matrix} /:4 \\ \\ \\ \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & -\frac{1}{4} \\ 2 & 1 & 0 & 3 & 1 & 4 \\ 1 & -1 & 2 & 1 & 2 & 2 \\ 2 & 1 & 1 & -4 & 7 & 7 \end{bmatrix} \begin{matrix} \\ \leftarrow + \\ \leftarrow + \\ \leftarrow + \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & -\frac{1}{4} \\ 0 & \frac{3}{2} & -\frac{1}{2} & \frac{5}{2} & \frac{3}{2} & \frac{9}{2} \\ 0 & -\frac{3}{4} & \frac{7}{4} & \frac{1}{2} & \frac{7}{4} & \frac{9}{4} \\ 0 & \frac{3}{2} & \frac{1}{2} & -\frac{9}{2} & \frac{11}{2} & \frac{11}{2} \end{bmatrix} \begin{matrix} \\ \\ \leftarrow + \\ \leftarrow + \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & -\frac{1}{4} \\ 0 & 1 & -\frac{1}{2} & \frac{5}{2} & \frac{3}{2} & \frac{9}{2} \\ 0 & -\frac{3}{4} & \frac{7}{4} & \frac{1}{2} & \frac{7}{4} & \frac{9}{4} \\ 0 & 0 & 1 & -7 & \frac{3}{2} & \frac{11}{2} \end{bmatrix} \begin{matrix} \\ \\ \leftarrow + \\ \leftarrow + \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & \frac{5}{2} & \frac{3}{2} & \frac{9}{2} \\ 0 & 0 & 1 & 1 & 3 & 3 \\ 0 & 0 & 0 & -8 & 1 & 1 \end{bmatrix} \begin{matrix} \\ \\ \\ /:(-8) \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 & \frac{5}{2} & \frac{9}{2} \\ 0 & 0 & 1 & 1 & 3 & 3 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \\ \\ \end{matrix}$$

$$x + \frac{1}{3} = 0 \Rightarrow x = -\frac{1}{3}$$

$$y + \frac{5}{3} = \frac{5}{2} \Rightarrow y = \frac{5}{2} - \frac{5}{3} = \frac{5}{6}$$

$$z + 1 = 3 \Rightarrow z = 2$$

$$u = 0$$

PROVERA?

$$z^3 - 5 + 3i = 0$$

$$z^3 = 5 - 3i \quad \sqrt[3]{}$$

$$z = \sqrt[3]{5 - 3i} \Rightarrow w = 5 - 3i \Rightarrow x = 5, y = -3$$

$$\sqrt[n]{z} = \sqrt[n]{\rho} \left(\cos \frac{\theta_1 + 2k\pi}{n} + i \sin \frac{\theta_2 + 2k\pi}{n} \right)$$

$$\rho = \sqrt{x^2 + y^2} = \sqrt{5^2 + (-3)^2} = \sqrt{25 + 9} = \sqrt{34}$$



$$\begin{bmatrix} 2 & 1 & 0 & 3 & 4/2 \\ 4 & -1 & 1 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{3}{2} & 2 \\ 4 & -1 & 1 & 2 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{bmatrix} \begin{array}{l} / \cdot (-4) \\ / \cdot (-1) \\ / \cdot (-2) \\ + \\ + \\ + \end{array}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{3}{2} & 2 \\ 0 & -3 & -3 & -4 & -4 \\ 0 & -\frac{3}{2} & 2 & -\frac{1}{2} & 0 \\ 0 & -2 & 1 & -7 & 3 \end{bmatrix} \begin{array}{l} \\ / \cdot (-3) \\ \\ \end{array}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{3}{2} & 2 \\ 0 & 1 & 1 & \frac{4}{3} & \frac{5}{3} \\ 0 & -\frac{3}{2} & 2 & -\frac{1}{2} & 0 \\ 0 & -2 & 1 & -7 & 3 \end{bmatrix} \begin{array}{l} \\ / \cdot (-\frac{1}{2}) \\ / \cdot (\frac{3}{2}) \\ / \cdot 2 \\ + \\ + \end{array}$$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} & \frac{5}{2} & -\frac{5}{2} \\ 0 & 1 & 1 & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & \frac{7}{2} & \frac{2}{3} & \frac{2}{3} \\ 0 & 0 & 3 & -\frac{13}{3} & \frac{17}{3} \end{bmatrix} \begin{array}{l} \\ \\ / \cdot \frac{2}{7} \\ \\ \end{array}$$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} & \frac{5}{2} & -\frac{5}{2} \\ 0 & 1 & 1 & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & 1 & \frac{2}{3} & \frac{2}{3} \\ 0 & 0 & 3 & -\frac{13}{3} & \frac{17}{3} \end{bmatrix} \begin{array}{l} \\ \\ / \cdot \frac{1}{2} \\ / \cdot (-1) \\ / \cdot (-3) \\ + \\ + \end{array}$$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} & \frac{5}{2} & -\frac{5}{2} \\ 0 & 1 & 1 & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & 1 & \frac{2}{3} & \frac{2}{3} \\ 0 & 0 & 9 & -13 & 17 \end{bmatrix} \begin{array}{l} \\ \\ \\ + \frac{1}{3} \text{ wala} \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{22}{21} & \frac{34}{21} \\ 0 & 1 & 0 & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & 1 & \frac{2}{3} & \frac{2}{3} \\ 0 & 0 & 0 & \frac{-118}{4} & \frac{83}{4} \end{bmatrix} \begin{array}{l} \\ \\ \\ / \cdot (-1) \\ / \cdot (-9) \\ / \cdot (\frac{14}{21}) \\ + \\ + \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{22}{21} & \frac{34}{21} \\ 0 & 1 & 0 & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & 1 & \frac{2}{3} & \frac{2}{3} \\ 0 & 0 & 0 & -118 & 83 \end{bmatrix} \begin{array}{l} \\ \\ \\ / \cdot (-118) \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{22}{21} & \frac{34}{21} \\ 0 & 1 & 0 & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & 1 & \frac{2}{3} & \frac{2}{3} \\ 0 & 0 & 0 & -118 & 83 \end{bmatrix} \begin{array}{l} \\ \\ \\ / \cdot (-\frac{22}{21}) \\ / \cdot (\frac{5}{3}) \\ / \cdot (-\frac{1}{3}) \end{array}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

ε5

IME I PREZIME: *Ankarić Jović*

BROJ INDEKSA: *17-2-0278-2013*

0269076958

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

- (1) Riješi jednadžbu među kompleksnim brojevima: $z^3 - 5 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
- (2) Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2 + 3}$ 10
- (3) Ispitati asimptote funkcije: $h(x) = \sqrt{x^2 + 2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote)
20(graf)
- (4) Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

(a) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) =$

(b) $\lim_{n \rightarrow \infty} \left(\frac{x^2 + 5}{x^2} \right) =$

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

$$4x - y + z + 2u = -1$$

$$2x + y - 3u = 4$$

$$x - y + 2z + u = 2$$

$$2x + y + z - 4u = 7$$

- (6) Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$.

10

Ukupno:

20

5.

$$4x - y + z + 2u = -1$$

$$2x + y - 3u = 4$$

$$x - y + 2z + u = 2$$

$$2x + y + z - 4u = 7$$

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 0 & -1 & 1 & -3 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \begin{array}{l} (-2)+IV \\ (-4)+IV \end{array} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 0 & -1 & 1 & -3 \\ 4 & -1 & 1 & 2 & -1 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right] \begin{array}{l} (-4)+IV \\ :(-3) \end{array} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right] \begin{array}{l} :(-3) \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & -1 & 1 & 2 & -1 \end{array} \right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & -1 & 1 & 2 & -1 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 0 & -1 & 1 & -3 \end{array} \right] \begin{array}{l} (-1)+II \\ (-1)+III \end{array} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 0 & -1 & 1 & -3 \end{array} \right] \begin{array}{l} (-1)+II \\ (-1)+III \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 3 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 0 & -1 & 1 & -3 \end{array} \right] \begin{array}{l} (-3)+II \\ (-3)+III \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 3 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & -4 & 4 & -12 \\ 0 & 0 & -1 & 1 & -3 \end{array} \right] \begin{array}{l} :(-4) \\ (-1)+II \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 3 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 1 & -3 \end{array} \right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 1 & -3 \end{array} \right] \begin{array}{l} (-1)+II \\ (-1)+III \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 & 4 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 1 & -3 \end{array} \right] \begin{array}{l} (-1)+III \end{array}$$

~~$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 1 & -3 \end{array} \right]$$~~

$x=0$ $y=4$ $z+u=-3$
 $y-3u=4$

$z-4=3$ $-z+u=3$

~~$x+y+z+u=7$~~
 ~~$z+z+u=7$~~

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & -1 & 1 & -3 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & -3 \end{array} \right] \begin{array}{l} (-3)+II \end{array}$$

BESKONACNO RESENJA
 PROVJERA ?

Antonio Jović

5.

$$4x - y + z + 2u = -1$$

$$2x + y - 3u = 4$$

$$x - y + 2z + u = 2$$

$$2x + y + z - 4u = 7$$

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \sim$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \xrightarrow{(-1)+I} \sim$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 0 & -1 & -4 & -3 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \xrightarrow{(-2)+III} \sim$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 0 & -1 & -4 & -3 \\ 0 & -3 & -1 & 10 & -15 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \sim$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 0 & -1 & -4 & -3 \\ 0 & -3 & -1 & 10 & -15 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right] \sim$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & -3 & -1 & 10 & -15 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right] \xrightarrow{(-1)} \sim$$

~~$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & -3 & -1 & 10 & -15 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right] \sim$$~~

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & 1 & -10 & 15 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right] \xrightarrow{+I} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 9 & 17 \\ 0 & 1 & 1 & -10 & 15 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right] \xrightarrow{(-3)+II} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 9 & 17 \\ 0 & 1 & 1 & -10 & 15 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & -6 & 24 & -42 \end{array} \right] \xrightarrow{(-7)} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 9 & 17 \\ 0 & 1 & 1 & -10 & 15 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -6 & 24 & -42 \end{array} \right] \xrightarrow{(-6)} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 9 & 17 \\ 0 & 1 & 1 & -10 & 15 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -4 & 7 \end{array} \right] \xrightarrow{\begin{array}{l} (-3)+I \\ (-1)+II \end{array}} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 12 & 8 \\ 0 & 1 & 0 & -6 & 8 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -4 & 7 \end{array} \right] \xrightarrow{(-1)+III} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 12 & 8 \\ 0 & 1 & 0 & -6 & 8 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & -3 & 4 \end{array} \right] \xrightarrow{(-3)} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 12 & 8 \\ 0 & 1 & 0 & -6 & 8 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 & -\frac{3}{4} \end{array} \right] \xrightarrow{\begin{array}{l} (-2)+I \\ (+6)+II \\ (+6)+III \end{array}} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{21}{4} \\ 0 & 1 & 0 & 0 & \frac{7}{2} \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 & -\frac{3}{4} \end{array} \right] \xrightarrow{+III} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{21}{4} \\ 0 & 1 & 0 & 0 & \frac{7}{2} \\ 0 & 0 & 1 & 0 & \frac{9}{4} \\ 0 & 0 & 0 & 1 & -\frac{3}{4} \end{array} \right]$$

Antonio Jovic

3.25

$$\begin{cases} x = \frac{29}{4} \\ y = \frac{7}{2} \\ z = \frac{9}{4} \\ u = -\frac{3}{4} \end{cases}$$

$$\frac{4 \cdot \frac{29}{4} - \frac{7}{2} + \frac{9}{4} + 2 \cdot \left(\frac{3}{4}\right) = -1$$
$$2 \cdot \left(-\frac{21}{4}\right) + \frac{7}{2} - 3 \cdot \left(-\frac{3}{4}\right) = 4$$

6.

$$f(x) = \ln(\cos(2x^2 - 1))$$

$$f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot -\sin(2x^2 - 1) \cdot 4x$$

$$f'(x) = -\tan(2x^2 - 1) \cdot 4x \quad \checkmark$$

7.

a) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right)$

$$\lim_{x \rightarrow 0^+} \left(\frac{\sqrt{3+0^+} - \sqrt{3}}{0^+} \right) = +\infty //$$

$$\lim_{x \rightarrow 0^-} \left(\frac{\sqrt{3+0^-} - \sqrt{3}}{0^-} \right) = +\infty // \quad \times$$

b)

$$\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5}{x^2} \right) = \lim_{x \rightarrow \infty} \frac{x^2 + 5 \cdot \frac{1}{x^2}}{x^2 \cdot \frac{1}{x^2}} = \frac{1 + 5}{1} = 6 \quad \times$$

~~$\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5}{x^2} \right) = \lim_{x \rightarrow \infty} \frac{x^2 + 5}{x^2} = \frac{\infty + 5}{\infty} = \frac{\infty}{\infty} = 1$~~

~~$\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5}{x^2} \right) = \lim_{x \rightarrow \infty} \frac{x^2 + 5}{x^2} = \frac{\infty + 5}{\infty} = \frac{\infty}{\infty} = 1$~~

(2) $g(x) = \sqrt{x^2+3}$
 $g'(x) = \frac{1}{2\sqrt{x^2+3}}$

$x^2+3 \geq 0$
 $x^2 \geq -3$

$D_f \in \mathbb{R}$

$2\sqrt{x^2+3} > 0$
 $x^2+3 \geq 0$

$2\sqrt{x^2+3} = 0 \quad |^2$

$4(x^2+3) = 0$

$4x^2 + 12 = 0$

$4x^2 = 12$

$x^2 = 3$

$x = \pm\sqrt{3}$

$x_1 = \sqrt{3}$

$x_2 = -\sqrt{3}$

~~g'(x) = \frac{1}{2\sqrt{x^2+3}}~~

$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

$g'(x) = \frac{b' \cdot u - b \cdot u'}{u^2}$

$g'(x) = \frac{\frac{1}{2\sqrt{x^2+3}} \cdot 1 - \sqrt{x^2+3} \cdot 0}{1}$

$g'(x) = \frac{1}{2\sqrt{x^2+3}}$ // ~~X~~

~~$y_1 = \frac{1}{2\sqrt{(\sqrt{3})^2+3}}$
 $y_2 = \frac{1}{2\sqrt{(-\sqrt{3})^2+3}}$~~

$y_1 = \frac{1}{2\sqrt{(\sqrt{3})^2+3}} = \frac{1}{2\sqrt{6}}$

$y_2 = \frac{1}{2\sqrt{(-\sqrt{3})^2+3}} = \frac{1}{2\sqrt{6}}$

$T_1 (\sqrt{3}, \frac{1}{2\sqrt{6}})$

$T_2 (-\sqrt{3}, \frac{1}{2\sqrt{6}})$ //

$$3) f(x) = \sqrt{x^2 + 2x}$$

$$x^2 + 2x \geq 0$$

$$x(x+2) \geq 0$$

$$1) D_f = \langle -\infty, -2 \rangle \cup [0, +\infty)$$

kandidatke see vertikálnu asim.
nemá!

$$2) \lim_{x \rightarrow \infty} \sqrt{x^2 + 2x} = +\infty$$

HORIZONTÁLNÁ NE MÁ!

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + 2x}}{x} = \lim_{x \rightarrow +\infty} \sqrt{1 + \frac{2}{x}} = \sqrt{1+0} = 1,$$

$$\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 2x} - x) = \lim_{x \rightarrow +\infty} \frac{2x}{\sqrt{x^2 + 2x} + x} = \lim_{x \rightarrow +\infty} \frac{2}{\sqrt{1 + \frac{2}{x}} + 1} = 1,$$

$y = x + 1$ je kosá asymptota !!



$$\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 2x}}{x} = \lim_{x \rightarrow -\infty} -\sqrt{1 + \frac{2}{x}} = -1$$

$$\lim_{x \rightarrow -\infty} \sqrt{x^2 + 2x} + x = \lim_{x \rightarrow -\infty} \frac{2x}{-\sqrt{x^2 + 2x} - x} = \lim_{x \rightarrow -\infty} \frac{2}{-1-1} = -1$$

$y = x - 1$ je kosá asymptota !!



1.

$$z^3 - 5 + 3i = 0$$

$$z^3 = 5 - 3i$$

$$5 - 3i = w$$

~~z = w^{1/3}~~

$$w = z^3$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{5^2 + 3^2}$$

$$r = \sqrt{34}$$

$$\tan \varphi = \frac{y}{x}$$

$$\tan \varphi = -\frac{3}{5}$$

~~z = w^{1/3}~~

$$\varphi = \arctan\left(-\frac{3}{5}\right)$$

~~z = w^{1/3}~~

$$k_1 = 0$$

$$k_2 = 1$$

$$k_3 = 2$$

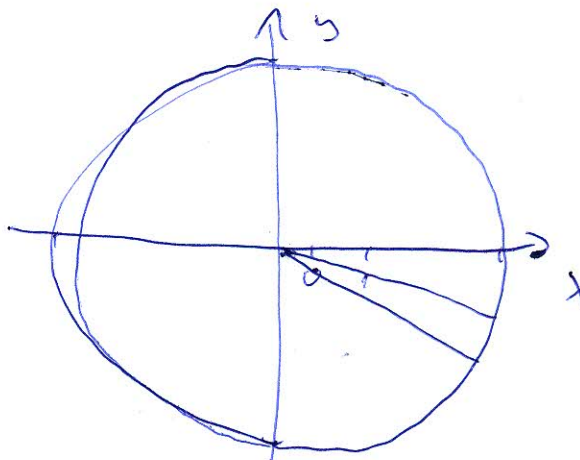
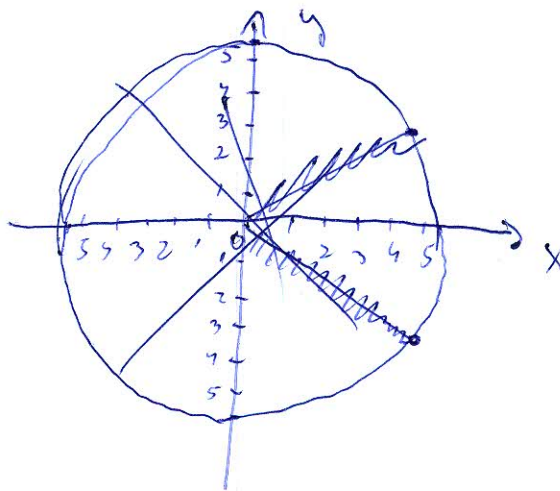
Antonio Jović

$$w = \sqrt[3]{34} \left(\cos \left(\arctan\left(-\frac{3}{5}\right) \right) + i \sin \left(\arctan\left(-\frac{3}{5}\right) \right) \right)$$

$$z_1 = \sqrt[3]{34} \left(\cos \left(\frac{\arctan\left(-\frac{3}{5}\right)}{3} \right) + i \sin \left(\frac{\arctan\left(-\frac{3}{5}\right)}{3} \right) \right)$$

$$z_2 = \sqrt[3]{34} \left(\cos \left(\frac{\arctan\left(-\frac{3}{5}\right) + 2\pi}{3} \right) + i \sin \left(\frac{\arctan\left(-\frac{3}{5}\right) + 2\pi}{3} \right) \right)$$

$$z_3 = \sqrt[3]{34} \left(\cos \left(\frac{\arctan\left(-\frac{3}{5}\right) + 4\pi}{3} \right) + i \sin \left(\frac{\arctan\left(-\frac{3}{5}\right) + 4\pi}{3} \right) \right)$$



3. NASTAVAK

Antonio Jurić

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

3° $x(x+2) = 0$

$$x_1 = 0$$

$$x_2 = -2$$

4° $h'(x) = \frac{b' \cdot n - b \cdot n'}{h^2}$

$$h'(x) = \frac{\frac{1}{2\sqrt{x^2+2x}} \cdot 1 - 0}{1} = \frac{1}{2\sqrt{x^2+2x}}$$

$$2\sqrt{x^2+2x} = 0 \quad |^2$$

$$4(x^2+2x) = 0$$

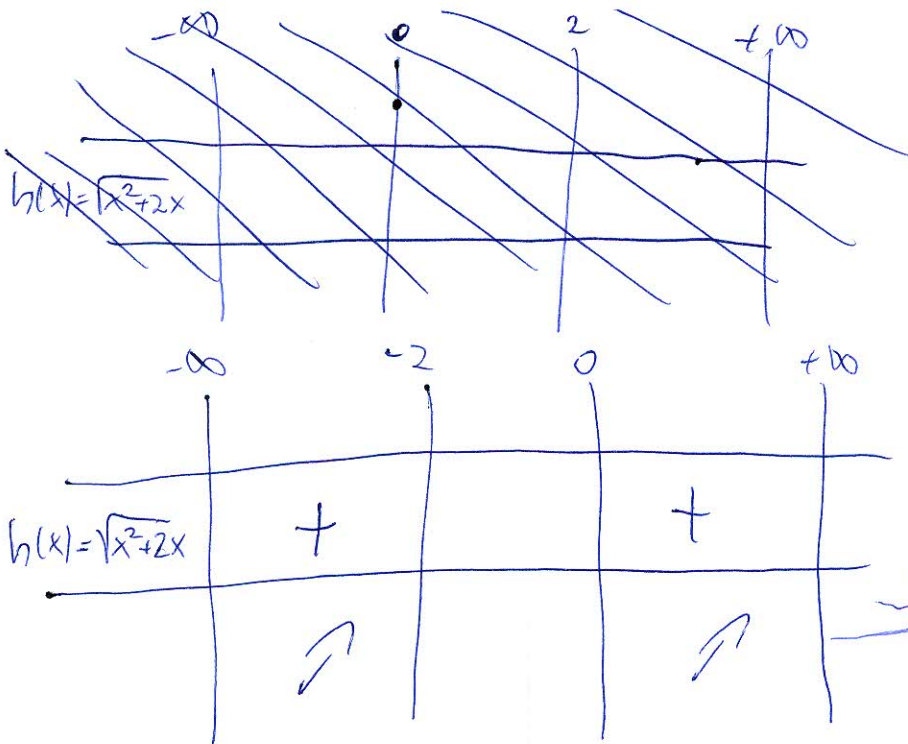
$$4x^2 + 8x = 0$$

$$4x(x+2) = 0$$

$$x_1 = -2$$

$$x_2 = 0$$

$$w_1 = \frac{1}{2\sqrt{4-4}} = \frac{1}{0} = \phi \quad \text{nema ekstremuma!}$$

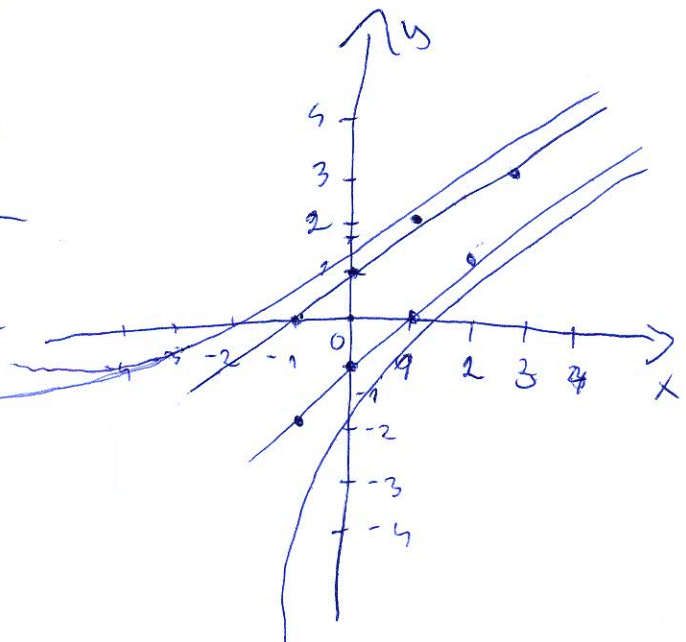


1°

x	-1	0	1	2
y	0	1	2	3

2°

x	-1	0	1	2
y	2	1	0	1





MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

ε5

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **ANDRIJA PAVIN**

BROJ INDEKSA: **17-2-0123-2011**

- Riješi jednačbu među kompleksnim brojevima: $z^3 - 5 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
- Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2 + 3}$ 10
- Ispitati asimptote funkcije: $h(x) = \sqrt{x^2 + 2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote)
20(graf)
- Određiti i uvrštavanjem (kalkulator) provjeriti rezultat
(a) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) =$ 7+2
(b) $\lim_{n \rightarrow \infty} \left(\frac{x^2 + 5}{x^2} \right) =$ 4+2
- Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rcl} 4x & - & y & + & z & + & 2u & = & -1 \\ 2x & + & y & & & - & 3u & = & 4 \\ x & - & y & + & 2z & + & u & = & 2 \\ 2x & + & y & + & z & - & 4u & = & 7 \end{array}$$

- Određiti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$. 10

Ukupno:

20

$$1) z^3 - 5 + 3i = 0$$

$$z^3 = 5 - 3i \quad x = 5 \quad y = -3$$

$$z = \sqrt[3]{5 - 3i}$$

$$\sqrt[n]{z} = \sqrt[n]{|z|} \cdot \left(\cos \frac{\varphi + 2k\pi}{n} + i \sin \frac{\varphi + 2k\pi}{n} \right)$$

$$k = 0, 1, 2$$

$$k = 0, \quad z_1 = \sqrt[3]{34} \cdot \left(\cos \frac{-0.54 + 2 \cdot 0 \cdot \pi}{3} + i \sin \frac{-0.54 + 2 \cdot 0 \cdot \pi}{3} \right)$$

$$z_1 = \sqrt[3]{34} \cdot (0.98 - 0.18i)$$

$$z_1 = 3.17 - 0.58i$$

$$k = 1, \quad z_2 = \sqrt[3]{34} \cdot \left(\cos \frac{-0.54 + 2 \cdot 1 \cdot \pi}{3} + i \sin \frac{-0.54 + 2 \cdot 1 \cdot \pi}{3} \right)$$

$$z_2 = \sqrt[3]{34} \cdot (-0.34 + 0.94i)$$

$$z_2 = -1.10 + 3.05i$$

$$k = 2, \quad z_3 = \sqrt[3]{34} \cdot \left(\cos \frac{-0.54 + 2 \cdot 2 \cdot \pi}{3} + i \sin \frac{-0.54 + 2 \cdot 2 \cdot \pi}{3} \right)$$

$$z_3 = \sqrt[3]{34} \cdot (-0.65 - 0.76i)$$

$$z_3 = -2.11 - 2.46i$$

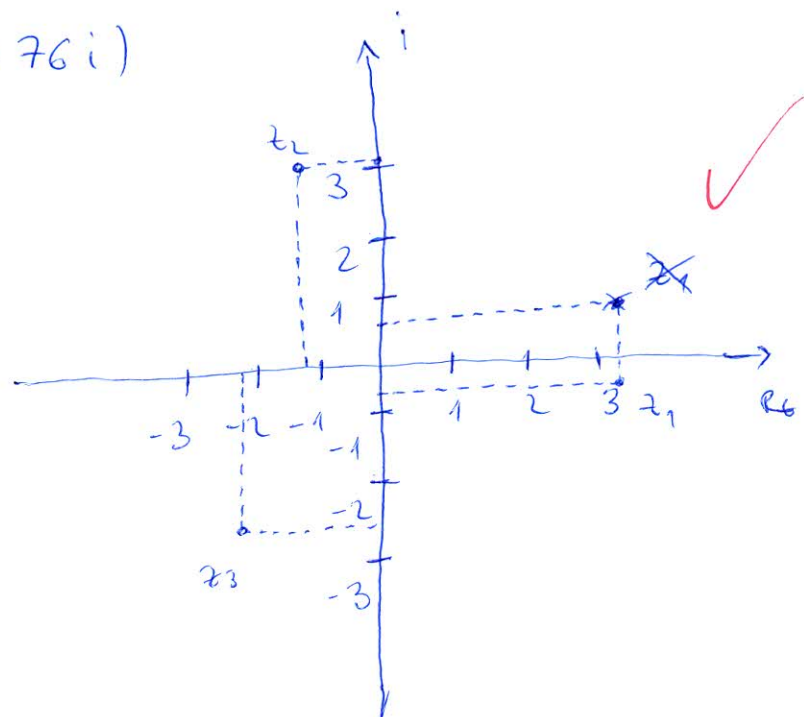
$$|z| = \sqrt{x^2 + y^2} \quad \varphi = \arctan \frac{y}{x}$$

$$|z| = \sqrt{5^2 + (-3)^2} \quad \varphi = \arctan \frac{-3}{5}$$

$$|z| = \sqrt{25 + 9}$$

$$|z| = \sqrt{34}$$

$$\varphi = -0.54$$



$$\begin{aligned} 5) \quad & 4x - y + z + 2u = -1 \\ & 2x + y - 3u = 4 \\ & x - y + 2z + u = 2 \\ & 2x + y + z - 4u = 7 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right] \begin{array}{l} / \cdot (-4) / \cdot (-2) \\ \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right] \begin{array}{l} / \cdot (-1) \\ \text{II} + \text{III} \\ \text{II} + \text{IV} \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 0 & 3 & -3 & 9 \\ 0 & 0 & 4 & -4 & 12 \end{array} \right] \begin{array}{l} \\ \\ / \cdot 4 \\ / \cdot (-3) \end{array} \sim$$

VÍŠE RJEŠENJA ---

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 0 & 12 & -12 & 36 \\ 0 & 0 & -12 & 12 & -36 \end{array} \right] \begin{array}{l} \\ \\ \text{III} + \text{IV} \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 0 & 12 & -12 & 36 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\boxed{u = 0}$$

$$12z = 36 \quad / : 12$$

$$\boxed{z = 3}$$

PROVJERA

$$3y - 7 \cdot 3 = -9$$

$$3y = 12 \quad / : 3$$

$$\boxed{y = 4}$$

$$x + (-1 \cdot 4) + 2 \cdot 3 = 2$$

$$x - 4 + 6 = 2$$

$$\boxed{x = 0}$$

$$1) \quad 4 \cdot 0 - 4 + 3 + 2 \cdot 0 = -1$$

$$-4 + 3 = -1 \quad \boxed{-1 = -1}$$

$$2) \quad 2 \cdot 0 + 4 - 3 \cdot 0 = 4 \quad \boxed{4 = 4}$$

$$3) \quad 0 - 4 + 2 \cdot 3 + 0 = 2$$

$$-4 + 6 = 2 \quad \boxed{2 = 2}$$

$$4) \quad 2 \cdot 0 + 4 + 3 - 4 \cdot 0 = 7 \quad \boxed{7 = 7}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

£5

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME:

BROJ INDEKSA:

KREŠIMIR PIPLIĆ

0269 078324

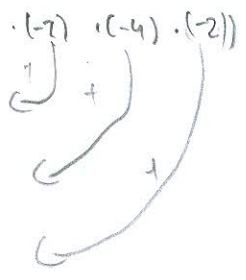
- Riješi jednadžbu među kompleksnim brojevima: $z^3 - 5 + 3i = 0$. Prikaži rješenja u kompleksnoj ravnini! 12+3
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- Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$. 10

Ukupno:

15

$$\begin{bmatrix} 4 & -1 & 1 & 2 & 1 & -1 \\ 2 & 1 & 0 & -3 & 1 & 4 \\ 1 & -1 & 2 & 1 & 1 & 2 \\ 2 & 1 & 1 & -4 & 1 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 1 & 1 & 2 \\ 2 & 1 & 0 & -3 & 1 & 4 \\ 4 & -1 & 1 & 2 & 1 & -1 \\ 2 & 1 & 1 & -4 & 1 & 1 \end{bmatrix}$$



$$\sim \begin{bmatrix} 1 & -1 & 2 & 1 & 1 & 2 \\ 0 & 3 & -4 & -5 & 1 & 0 \\ 0 & 3 & -7 & -2 & 1 & -9 \\ 0 & 3 & -3 & -6 & 1 & 3 \end{bmatrix}$$

③ H.A.

$$h(x) = \sqrt{x^2 + 2x} \quad | :x^2$$

$$h(x) = \sqrt{\frac{x^2}{x^2} + \frac{2x}{x^2}} = \sqrt{1 + \frac{2}{x}} \quad \text{H.A. } g=1$$

K.A. nema jer imamo H.A.

⑤ deskripcija

$$h(x) = \sqrt{x^2 + 2x}$$

$$b) h'(x) = \frac{2x}{\sqrt{x^2 + 2x}} = 0 \quad | \cdot \sqrt{x^2 + 2x}$$

$$2x = 0 \quad | :2$$

 $x = 0 \rightarrow$ stacionarna točka

$$a) h'(x) = \frac{1}{2\sqrt{x^2 + 2x}} \cdot (x^2 + 2x)'$$

$$= \frac{1}{2\sqrt{x^2 + 2x}} \cdot 2x + 2$$

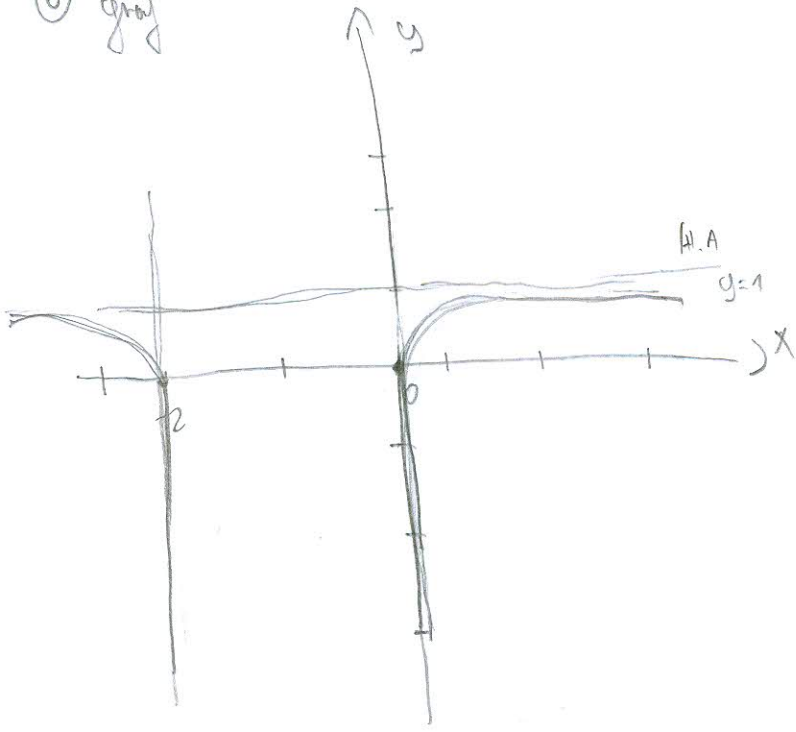
$$= \frac{2x}{\sqrt{x^2 + 2x}} + 2$$

	$-\infty$	0	$+\infty$
$f'(x)$	-	+	
$f(x)$	↘	↓	↗

ODGOVOR NA PITANJE
U ZADATKU
JEST?

5

⑥ graf



4

$$a) \lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) = \frac{+\text{brs}}{0} = +\infty$$

$$b) \lim_{n \rightarrow \infty} \left(\frac{x^2+5}{x^2} \right) =$$

$$6) f(x) = \ln(\cos(2x^2-1))$$

$$= \frac{1}{(\cos(2x^2-1))}$$

cos cos'

$$= \frac{-\sin(2x^2-1)}{-\cos(2x^2-1)} \cdot (2x^2-1)'$$

$$(2x^2-1)' =$$

$$= \frac{-\sin(2x^2-1)}{-\cos(2x^2-1)}$$

$$= \frac{4x^2 \cdot \sin(2x^2-1)}{\cos(2x^2-1)}$$



$$= 4x^2 \tan(2x^2-1)$$

cos cos'

$$⑤ \quad 4x - y + z + 2u = -1$$

$$2x + y - 3u = 4$$

$$x - y + 2z + u = 2$$

$$2x + y + z - 4u = 7$$

$$\begin{bmatrix} 4 & -1 & 1 & 2 & -1 & -1 \\ 2 & 1 & 0 & -3 & 1 & 4 \\ 1 & -1 & 2 & 1 & 1 & 2 \\ 2 & 1 & 1 & -4 & 1 & 7 \end{bmatrix} \xrightarrow{2} \begin{bmatrix} 1 & -1 & 2 & 1 & 1 & 2 \\ 2 & 1 & 0 & -3 & 1 & 4 \\ 4 & -1 & 1 & 2 & -1 & -1 \\ 2 & 1 & 1 & -4 & 1 & 7 \end{bmatrix} \begin{array}{l} \cdot (-2) \\ + \\ \cdot (-2) \\ + \end{array}$$

$$\xrightarrow{2} \begin{bmatrix} 1 & -1 & 2 & 1 & 1 & 2 \\ 0 & 3 & -4 & -5 & 1 & 0 \\ 0 & 3 & -4 & -2 & 1 & 9 \\ 0 & 3 & -3 & -6 & 1 & 3 \end{bmatrix}$$

② $g(x) = \sqrt{x^2+3}$

Domain

$$\sqrt{x^2+3} \geq 0 \quad |^2$$

$$D(g) = \mathbb{R}$$

$$x^2+3 \geq 0$$

$$x^2 \geq -3 \quad |^{\vee}$$

$$x \geq \sqrt{-3}$$

lim $g(x) = \sqrt{x^2+3}$

$$g'(x) = \frac{1}{\sqrt{x^2+3}} \cdot (x^2+3)' = \frac{2x}{\sqrt{x^2+3}}$$

$$g'(x) = \frac{2x}{\sqrt{x^2+3}} = 0 \quad | \sqrt{x^2+3}$$

$$2x = 0 \quad | :2$$

$$x = 0 \quad \checkmark$$

	$-\infty$	0	$+\infty$
$f'(x)$		-	+
$f(x)$		↓	↗

5

$$\begin{cases} 4x - y + 2z + 2u = -1 \\ 2x - y - 3u = 4 \\ x - y + 2z + u = 2 \\ 2x + y - z - 4u = 7 \end{cases}$$

$$\begin{bmatrix} 4 & -1 & 2 & 2 & -1 \\ 2 & -1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 1 & 2 \\ 2 & -1 & 0 & -3 & 4 \\ 4 & -1 & 2 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{bmatrix} \begin{matrix} \cdot (-2) \\ \cdot (-4) \\ \cdot (-2) \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -4 & -5 & 10 \\ 0 & 3 & -6 & -2 & -9 \\ 0 & 3 & -3 & -6 & -3 \end{bmatrix} \begin{matrix} \cdot (-3) \\ \cdot (-3) \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -4 & -5 & 10 \\ 0 & 0 & 6 & 13 & -9 \\ 0 & 0 & 9 & 9 & 13 \end{bmatrix}$$

$$\begin{matrix} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -4 & -5 & 10 \\ 0 & 0 & 6 & 13 & -9 \\ 0 & 0 & 9 & 9 & 13 \end{matrix}$$

$$③ h(x) = \sqrt{x^2 + 2x}$$

① Domain

$$\sqrt{x^2 + 2x} \geq 0 \quad | \quad ?$$

$$x^2 + 2x \geq 0$$

$$a = 1$$

$$b = 2$$

$$c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 0}}{2}$$

$$= \frac{-2 \pm 2}{2}$$

$$x_1 = \frac{0}{2} = 0$$

$$x_2 = -2$$

$$D_f \left\langle -\infty, -2 \right] \cup \left[0, +\infty \right)$$

② Nulltočke

$$\sqrt{x^2 + 2x} = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm 2}{2}$$

$$x_1 = 0$$

$$x_2 = -2$$

$$y_0 = \sqrt{0^2 + 2 \cdot 0} = 0$$

$$(0, 0)$$

$$(-2, 0)$$

GRAF MIJE DOBIL

③ parna ili neparna

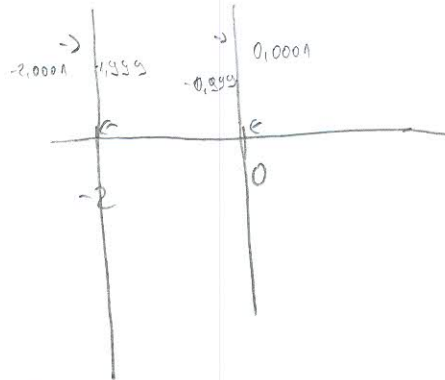
$$h(x) = \sqrt{x^2 + 2x}$$

$$h(-x) = \sqrt{(-x)^2 + 2 \cdot (-x)} = \sqrt{x^2 - 2x}$$

Funkcija nije ni parna ni neparna

④ Asimptote

N.A.



$$\lim_{x \rightarrow -2^+} \frac{+\text{broj}}{+0} = +\infty$$

$$\lim_{x \rightarrow -2^-} \frac{+\text{broj}}{+0} = +\infty$$

V.A. = -2

$$\lim_{x \rightarrow 0^-} \frac{+\text{broj}}{0} = +\infty$$

$$\lim_{x \rightarrow 0^+} \frac{+\text{broj}}{0} = +\infty$$

V.A. = 0

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

ε5

IME I PREZIME: DRAGAN ASIĆ

BROJ INDEKSA: 17-2-0286-2013

- Riješi jednačbu među kompleksnim brojevima: $z^3 - 5 + 3i = 0$. Prikaži rješenja u kompleksnoj ravnini! 12+3
- Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2 + 3}$ 10
- Ispitati asimptote funkcije: $h(x) = \sqrt{x^2 + 2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote)
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- Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat 7+2
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$$\begin{array}{rcl} 4x - y + z + 2u & = & -1 \\ 2x + y & & - 3u = 4 \\ x - y + 2z + u & = & 2 \\ 2x + y + z - 4u & = & 7 \end{array}$$

- Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$. 10

Ukupno:

10

$$\begin{array}{l} \textcircled{5} \left. \begin{array}{l} 4x - y + z + 2u = -1 \\ 2x + y - 3u = 4 \\ x - y + 2z + u = 2 \end{array} \right\} \\ 2x + y + z - 4u = 7 \end{array}$$

$$A: \begin{bmatrix} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 7 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{bmatrix} \begin{array}{l} \text{I} - (2 \cdot \text{I}) \\ \text{II} - (4 \cdot \text{I}) \\ \text{IV} - (2 \cdot \text{I}) \end{array}$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -1 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{bmatrix} \begin{array}{l} \text{II} - \text{III} \\ \text{IV} - \text{III} \end{array} \sim \begin{bmatrix} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 0 & 2 & 4 & -12 \\ 0 & 0 & -2 & -4 & 12 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

NEMA RIJEŠENJA X

⑥ $\lim_{x \rightarrow \infty} (\cos(2x^2 - 1))$

$$f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot (-\sin(2x^2 - 1)) \cdot 4x$$

$$f'(x) = \frac{1}{\cos(x^2 - 1)} \cdot \frac{-\sin(2x^2 - 1)}{1} \cdot \frac{4x}{1}$$

$$f'(x) = \frac{-4x \cdot \sin(2x^2 - 1)}{\cos(2x^2 - 1)} \quad \checkmark$$

$$f'(x) = \frac{-4x \cdot \sin x}{\cos x} \quad \times$$

③ $h(x) = \sqrt{x^2 + 2x}$

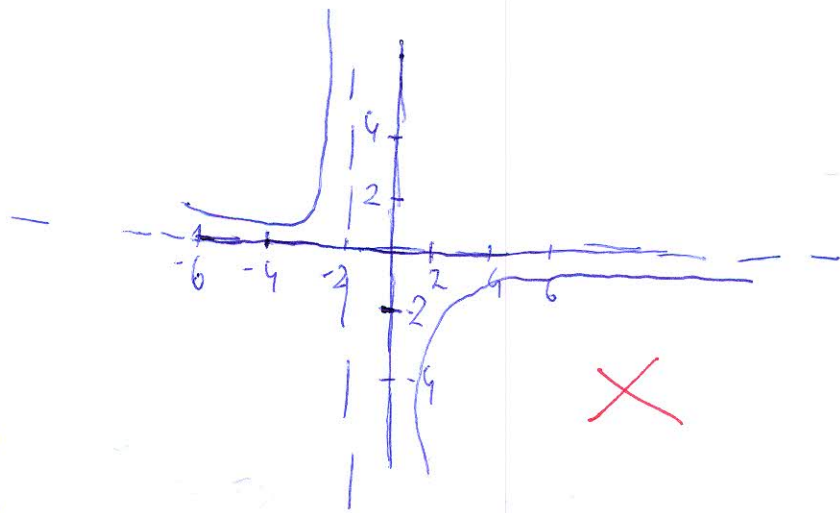
DOMENA $x^2 + 2x > 0$

NA = $\mathbb{R} \setminus [2, 0]$ $x \cdot (x+2) > 0$

$$x \cdot (x+2) = 0$$

$$x_1 = 0 \quad x_2 = x + 2 = 0$$

$$x_2 = x = -2$$



ASIMPTOTE

NULTÖCKE NEMA

$$\lim_{x \rightarrow 0^+} \sqrt{x^2 + 2x} = +\infty$$

$$\lim_{x \rightarrow 0^-} \sqrt{x^2 + 2x} = \frac{0}{-0} \quad x = 0 \text{ V.A.}$$

$$\lim_{x \rightarrow -2^-} \sqrt{x^2 + 2x} = -\infty \quad x = -2 \text{ Vert. ASIMPTOTA) Vertikalna asimptota}$$

H. ASIMPTOTE

$$\lim_{x \rightarrow \pm\infty} \sqrt{x^2 + 2x} = \pm\infty \quad y = 0 \text{ H.A.}$$

①

③ $h(x) = \sqrt{x^2 + 2x}$

$h(x) =$

PARNOST NEPARNOST

~~$h(-x) = \sqrt{(-x)^2 + 2(-x)}$~~

FUNKCIJA JE PARNA

~~$h(-x) = \sqrt{x^2 - 2x}$~~

$h(x) = \sqrt{x^2 + 2x}$

$h^2(x) = (x^2 + 2x)^{\frac{1}{2}}$

② ~~$\sqrt{x^2 + 3}$~~

~~$g(x) = \sqrt{x^2 + 3}$~~

~~$g(x) = (x^2 + 3)^{\frac{1}{2}}$~~

~~$g'(x) = x^2 + 3$~~

~~$E_1 = x^2 + 3 = 0$~~

~~$x^2 = -3$~~

~~$x = \sqrt{-3}$~~

~~$E_1 = (-\sqrt{3}, 0)$~~ $E_1 = [\sqrt{3}, 2.5]$

$E_2 = [\sqrt{-3}, 1.1]$

?

?

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

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NASTAVNIK
Broj ↓
bodova

IME I PREZIME: *GORAN BASIOLI*

BROJ INDEKSA: *17-1-0031-2010*

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 5 + 3i = 0$. Prikaži rješenja u kompleksnoj ravnini! 12+3
2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2 + 3}$ 10
3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2 + 2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote)
20(graf)
4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

(a) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) =$ 7+2

(b) $\lim_{n \rightarrow \infty} \left(\frac{x^2 + 5}{x^2} \right) =$ 4+2

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{aligned} 4x - y + z + 2u &= -1 \\ 2x + y - 3u &= 4 \\ x - y + 2z + u &= 2 \\ 2x + y + z - 4u &= 7 \end{aligned}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$. 10

4) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5}{x^2} \right) \stackrel{/:x^2}{=} \frac{5}{1} = 5$ X

$\lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) = \frac{0}{0} = \phi = \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{x+3}}}{1} = \frac{1}{\sqrt{x+3}}$ X

Ukupno:

10

$$6) f(x) = \ln(\cos(2x^2 - 1)):$$

$$f'(x) = \frac{1}{(\cos(2x^2 - 1))} + (\cos(2x^2 - 1))$$

$$f'(x) = \frac{-\cos(2x^2 - 1) - 4x}{\cos(2x^2 - 1)}$$

$$2) g(x) = \sqrt{x^2 + 3}$$

$$g'(x) = \frac{1}{2\sqrt{x^2 + 3}} + 2x = \frac{x}{\sqrt{x^2 + 3}}$$

$$g'(x) = \frac{x}{\sqrt{x^2 + 3}} = 0 \Rightarrow x = 0$$

$$g(0) = \sqrt{0 + 3} = \sqrt{3}$$

$$\lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} \sqrt{x^2 + 3} = \sqrt{3}$$

$$\lim_{x \rightarrow \infty} g(x) = \lim_{x \rightarrow \infty} \sqrt{x^2 + 3} = \infty$$

GONAN BASIOLI

$$1) \quad z^3 - 5 + 3i = 0$$

$$z^3 = 5 - 3i$$

$$|z| = \sqrt{5^2 + (-3)^2} = \sqrt{25+9} = \sqrt{34}$$

$$\operatorname{tg} \varphi = \frac{y}{x} = \frac{-3}{5} = -0,6 \Rightarrow \varphi \approx 330^\circ$$

$$z = |z| \cdot (\cos \varphi + i \sin \varphi)$$

$$k=0$$

$$w_0 = \sqrt[3]{|z|} \cdot \left(\frac{\cos \varphi + 2\pi \cdot k}{3} + i \frac{\sin \varphi + 2\pi \cdot k}{3} \right) = \sqrt[3]{34} \cdot \left(\frac{\cos 330^\circ + 2\pi \cdot 0}{3} + i \frac{\sin 330^\circ + 2\pi \cdot 0}{3} \right)$$

$$w_1 = 1,8 \cdot \left(\frac{\cos 330^\circ}{3} + i \frac{\sin 330^\circ}{3} \right) = \underline{1,8 \cdot (0,29 - 0,16i)}$$

$$k=1$$

$$w_2 = \sqrt[3]{|z|} \cdot \left(\frac{\cos \varphi + 2\pi \cdot k}{3} + i \frac{\sin \varphi + 2\pi \cdot k}{3} \right) = \sqrt[3]{34} \cdot \left(\frac{\cos 330^\circ + 360^\circ \cdot 1}{3} + i \frac{\sin 330^\circ + 360^\circ \cdot 1}{3} \right)$$

$$= \underline{1,8 \cdot (0,29 - 0,16i)}$$

$$k=2$$

$$w_2 = \sqrt[3]{|z|} \cdot \left(\frac{\cos \varphi + 2\pi \cdot k}{3} + i \frac{\sin \varphi + 2\pi \cdot k}{3} \right) = \sqrt[3]{34} \cdot \left(\frac{\cos 330^\circ + 360^\circ \cdot 2}{3} + i \frac{\sin 330^\circ + 360^\circ \cdot 2}{3} \right)$$

$$1,8 \cdot (0,29 - 0,16i)$$

ISTO?

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!

ε5

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: HRVOJE DIJAN

BROJ INDEKSA:

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(a) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) =$ ~~7+2~~

(b) $\lim_{n \rightarrow \infty} \left(\frac{x^2 + 5}{x^2} \right) =$ 4+2

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: ~~15+5~~

$$\begin{aligned} 4x - y + z + 2u &= -1 \\ 2x + y & - 3u &= 4 \\ x - y + 2z + u &= 2 \\ 2x + y + z - 4u &= 7 \end{aligned}$$

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6. $f(x) = \ln(\cos(2x^2 - 1))$

$f'(x) = \frac{1}{\cos(2x^2 - 1)} \cdot \cos(2x^2 - 1) + (-\sin(2x^2 - 1)) \cdot 4x$ ~~10~~

Ukupno:

0

5. b) $\lim_{n \rightarrow \infty}$

$$\left| \begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right| \sim \left| \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array} \right| \begin{array}{l} -2I \\ -4I \\ -2I \end{array}$$

$$\left| \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{array} \right| \begin{array}{l} \\ \\ /:3 \\ \end{array} \sim \left| \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 1 & -1 & -2 & 1 \end{array} \right| \begin{array}{l} \\ \\ \\ \end{array} \sim \left| \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -4 & -5 & 0 \end{array} \right| \begin{array}{l} \\ \\ -3II \\ -3II \end{array}$$

$$\left| \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & -4 & 4 & -12 \\ 0 & 0 & -1 & 1 & -3 \end{array} \right| \begin{array}{l} \\ \\ /:4 \\ \end{array} \sim \left| \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & -1 & 1 & -3 \end{array} \right| \begin{array}{l} \\ \\ (-I) \\ \end{array} \sim \left| \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 1 & -3 \end{array} \right| \begin{array}{l} \\ \\ \\ (-III) \end{array}$$

$$\left| \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 2 & -6 \end{array} \right| \sim \left| \begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 1 & -3 \end{array} \right|$$

$$\underline{0} = -3 //$$

$$y - 2 - 2u = 1$$

$$x - y + 2z + u = 2$$

$$z - u = -3$$

$$y - (-6) - 2(-3) = 1$$

$$x - (-11) + 2 \cdot (-6) + (-3) = 2$$

$$\underline{z} - (-3) = -3$$

$$y + 6 + 6 = 1$$

$$x + 11 - 12 - 3 = 2$$

$$\underline{z} = -6 //$$

$$y = -11$$

$$x - 4 = 2$$

$$x = 6$$

$$u = -3$$

$$2x + y + 2z + 4u = 7$$

$$z = -6$$

$$12 - 11 - 6 + 12$$

$$y = -11$$

$$7 = 7$$

$$x = 6$$

$$R = \begin{vmatrix} 6 \\ -11 \\ -6 \\ -3 \end{vmatrix} \quad \times$$

$$4. a) \lim_{x \rightarrow 0} \left(\frac{\sqrt{3-x} - \sqrt{3}}{x} \right) = \left[\frac{0}{0} \right] \xrightarrow{\text{L'H.}} = \lim_{x \rightarrow 0} \frac{1}{2} (3-x)$$

$$= \lim_{x \rightarrow 0} \frac{3}{2} - \frac{1}{2}x = \frac{3}{2}$$

$$f(x) = \sqrt{3-x}$$

$$f'(x) = (3-x)^{-\frac{1}{2}}$$

$$g'(x) = \frac{1}{2} (3-x) \cdot 1$$

~~_____~~

IME I PREZIME: **STIPE PREDOVAN**

BROJ INDEKSA: **0269082252**

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Ukupno:

(Handwritten mark)

1) Riješi jednadžbu među kompleksnim brojevima. Prikaži rješenja u kompleksnoj ravnini.

$$z^3 - 5 + 3i = 0$$

$$z^3 = 5 - 3i \quad \sqrt[3]{}$$

$$z = \sqrt[3]{5 - 3i} \quad \text{ALGEBARSKI ZAPIS}$$

$$\begin{cases} x = 5 \\ y = -3 \end{cases}$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{5^2 + (-3)^2}$$

$$r = \sqrt{25 + 9}$$

$$r = \sqrt{34}$$

$$r = 5.83$$

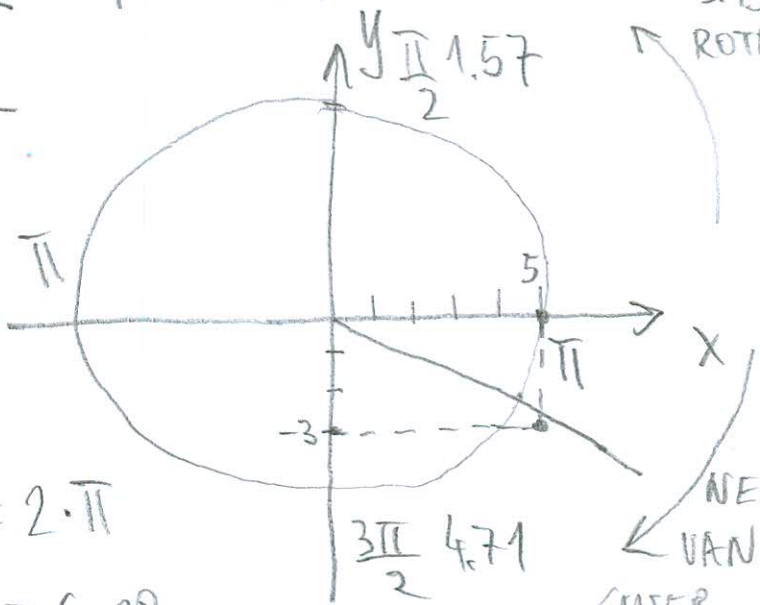
$$\rho = \arctan\left(\frac{y}{x}\right)$$

KUT NEMORE BITI NEGATIVAN

$$\rho = \arctan\left(\frac{-3}{5}\right)$$

$$\rho = -0.54 = \rho_{\text{PRAVI}} = 6.28$$

POZITIVAN SMJER ROTACIJE



$$\rho_{\text{PRAVI}} = 2 \cdot \pi$$

$$\rho_{\text{PRAVI}} = 6.28$$

NEGATIVAN SMJER ROTACIJE

$$z_k = \sqrt[n]{r} \cdot \left[\cos\left(\frac{2k\pi}{n}\right) + i \cdot \sin\left(\frac{2k\pi}{n}\right) \right] \quad k=0,1,2,3$$

$$z_0 = \sqrt[3]{5.83} \cdot \left[\cos\left(\frac{2 \cdot 0 \cdot \pi}{3}\right) + i \cdot \sin\left(\frac{2 \cdot 0 \cdot \pi}{3}\right) \right]$$

$$z_0 = \sqrt[3]{5.83} \cdot [1 + i \cdot 0]$$

$$z_0 = 1.78 + 0i \quad \times$$

$$z_1 = \sqrt[3]{5.83} \cdot \left[\cos\left(\frac{2 \cdot 1 \cdot \pi}{3}\right) + i \cdot \sin\left(\frac{2 \cdot 1 \cdot \pi}{3}\right) \right]$$

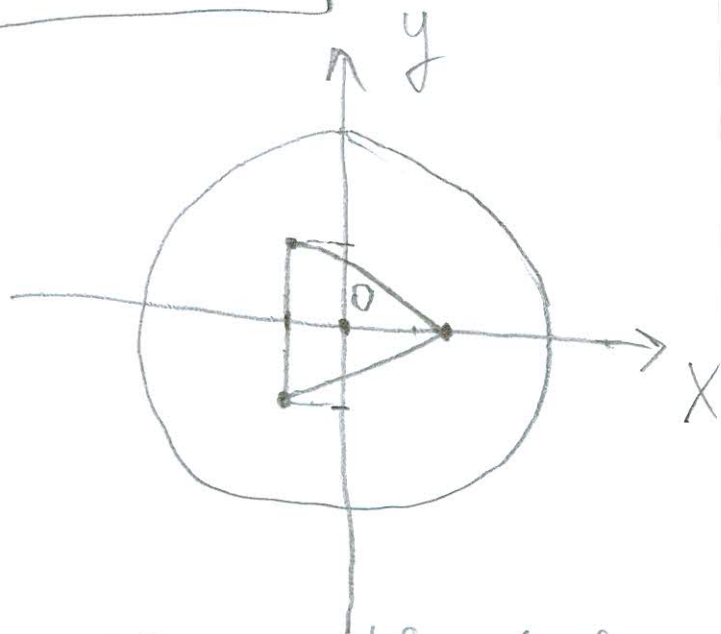
$$z_1 = \sqrt[3]{5.83} \cdot [-0.5 + i \cdot 0.86]$$

$$z_1 = -0.89 + 1.54i$$

$$z_2 = \sqrt[3]{5.83} \cdot \left[\cos\left(\frac{2 \cdot 2 \cdot \pi}{3}\right) + i \cdot \sin\left(\frac{2 \cdot 2 \cdot \pi}{3}\right) \right]$$

$$z_2 = \sqrt[3]{5.83} \cdot [-0.5 + i \cdot (-0.86)]$$

$$z_2 = -0.89 - 1.54i$$



za dobro ruciko treba
troubat.

Ako tražimo
n-ti korjen iz
kompleksnog broja
moramo imati
n rješenja
Kod traženja korjena
kompleksnog broja ako
dobiti je jednostavnije

5.) Rješiti sustav linearnim metodom i dobiti
 vrijednosti x, y, u, z .

$$\boxed{x, y, u, z}$$

$$\left[\begin{array}{cccc|c} 4 & -1 & 2 & 1 & -1 \\ 2 & 1 & -3 & 0 & 4 \\ 4 & -1 & 1 & 2 & 2 \\ 2 & 1 & -4 & 1 & 7 \end{array} \right] \begin{array}{l} \uparrow \\ \downarrow \end{array} \begin{array}{l} \text{ZAMJENA} \\ \text{REDAKA} \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & 2 & 2 \\ 0 & 3 & -5 & -4 & 0 \\ 0 & 3 & -2 & -7 & -9 \\ 0 & 3 & -6 & -3 & 3 \end{array} \right] \begin{array}{l} \\ \text{:(3)} \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & 2 & 2 \\ 2 & 1 & -3 & 0 & 4 \\ 4 & -1 & 2 & 1 & -1 \\ 2 & 1 & -4 & 1 & 7 \end{array} \right] \begin{array}{l} \leftarrow \cdot (-2) \\ \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & 2 & 2 \\ 0 & 1 & -\frac{5}{3} & -\frac{4}{3} & 0 \\ 0 & 3 & -2 & -7 & -9 \\ 0 & 3 & -6 & -3 & 3 \end{array} \right] \begin{array}{l} \leftarrow \cdot (-1) \\ \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & 2 & 2 \\ 0 & 3 & -5 & -4 & 0 \\ 4 & -1 & 2 & 1 & -1 \\ 2 & 1 & -4 & 1 & 7 \end{array} \right] \begin{array}{l} \leftarrow \cdot (-4) \\ \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -\frac{2}{3} & \frac{2}{3} & 2 \\ 0 & 1 & -\frac{5}{3} & -\frac{4}{3} & 0 \\ 0 & 3 & -2 & -7 & -9 \\ 0 & 3 & -6 & -3 & 3 \end{array} \right] \begin{array}{l} \\ \leftarrow \cdot (-3) \\ \leftarrow + \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & 2 & 2 \\ 0 & 3 & -5 & -4 & 0 \\ 0 & 3 & -2 & -7 & -9 \\ 2 & 1 & -4 & 1 & 7 \end{array} \right] \begin{array}{l} \leftarrow \cdot (-2) \\ \\ \\ \leftarrow + \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -\frac{2}{3} & \frac{2}{3} & 0 \\ 0 & 1 & -\frac{5}{3} & -\frac{4}{3} & 0 \\ 0 & 0 & 3 & -3 & -9 \\ 0 & 3 & -6 & -3 & 3 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{2}{3} & \frac{2}{3} & 0 \\ 0 & 1 & -\frac{5}{3} & -\frac{4}{3} & 0 \\ 0 & 0 & 3 & -3 & -9 \\ 0 & 3 & -6 & -3 & 3 \end{array} \right] \begin{array}{l} | \cdot (-3) \\ \leftarrow + \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & -3 & -5 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

BESKONČNO RJEŠENJA.

AKO MIJAN REDKE
PONIŠTIT ĆU ONO ŠTO
SAM DOBRO NAPRAVILA.

$$\begin{cases} x = -2 \\ y = -5 \\ u = -3 \\ z = 0 \end{cases}$$

PROVJERA

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{2}{3} & \frac{2}{3} & 0 \\ 0 & 1 & -\frac{5}{3} & -\frac{4}{3} & 0 \\ 0 & 0 & 3 & -3 & -9 \\ 0 & 0 & -1 & 1 & 3 \end{array} \right] | : (3)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{2}{3} & \frac{2}{3} & 0 \\ 0 & 1 & -\frac{5}{3} & -\frac{4}{3} & 0 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & -1 & 1 & 3 \end{array} \right] \begin{array}{l} \leftarrow + \\ | \cdot \left(\frac{2}{3}\right) \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & -\frac{5}{3} & -\frac{4}{3} & 0 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & -1 & 1 & 3 \end{array} \right] \begin{array}{l} \leftarrow + \\ | \cdot \left(\frac{5}{3}\right) \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & -3 & -5 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & -1 & 1 & 3 \end{array} \right] \begin{array}{l} \leftarrow + \\ | \cdot (-1) \end{array}$$

$$3) h(x) = \sqrt{x^2 + 2\pi}$$

$$h(x) = x^2 + 2\pi^{\frac{1}{2}}$$

$$h'(x) = 2 \cdot x^{2-1} + \frac{1}{2} \cdot 2\pi^{\frac{1}{2}-1}$$

$$h'(x) = 2x + \frac{3.14}{0.5}$$

$$h'(x) = 2x + 2.64 \quad \text{PRVA DERIVACIJA}$$

$$2) g(x) = \sqrt{x^2 + 3} = x^2 + 3^{\frac{1}{2}}$$

$$g'(x) = 2 \cdot x^{2-1} + \frac{1}{2} \cdot 3^{\frac{1}{2}-1}$$

$$g'(x) = 2x + 1.5$$

$$g'(x) = 2x + \frac{1}{1.5^{\frac{1}{2}}}$$

$$g'(x) = 2x + \frac{1}{\sqrt{1.5}}$$

6) Odrediti prvu derivaciju funkcije

$$f(x) = \ln(\cos(2x^2 - 1))$$

FUNKCIJA $2x^2 - 1$ je uvekla kosinusu a ona su je uvekla $\ln - u$, ovo je složena funkcija u funkciji.

$$f'(x) = \frac{1}{(\cos(2x^2 - 1))} \cdot (\cos(2x^2 - 1))'$$

$$f'(x) = \frac{1}{(\cos(2x^2 - 1))} \cdot (-\sin x \cdot (2 \cdot 2x^2 - 0))$$

$$f'(x) = \frac{1}{(\cos(2x^2 - 1))} \cdot (-\sin x \cdot 4x)$$

$$\frac{f' \cdot g - f \cdot g'}{(2 \cdot g)^2}$$

$$f'(x) = \frac{1}{(\cos(2x^2 - 1))} \cdot \frac{(-\sin 4x^2)}{1}$$

$$f'(x) = \frac{-\sin 4x^2}{(\cos(2x^2 - 1))} \quad \times$$

$$f'(x) = \frac{-\sin 4x^2}{1}$$

Ali imamo

$$\frac{1}{\cos} = \sec$$

PIŠEMO
etc