MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ $\mathcal{F}4$

IME I PREZIME: MARKO VUKELIĆ **BROJ INDEKSA:** 02 69080097

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 3 = 0$. *Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem!* Dalje izracunaj: $\left(\frac{z_1 - z_2}{z_2 + 3}\right)$. — 4+3+8

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: — 10+5

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 8 \\ & & x_2 & - & x_3 & + & x_4 = & -2 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 6 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -2 \end{array}$$

3. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(x^2 + 4) + \sin(2x - 3)$. — 5+15

4. Odrediti tok funkcije $f(x) = x - \frac{1}{x}$. — 15(graf)

5. Odrediti i provjeriti uvrštavanjem: $\lim_{x \to -4} \frac{x^2 - 3}{x^2 + 8x + 16} =$ — 4+1

6. Odredi derivaciju funkcije $f(x) = \frac{3}{\sin(5x)}$ — 10

7. Odrediti tangentu na funkciju $f(x) = \cos x$ tamo gdje je $x = \frac{\pi}{4}$. Nacrtati graf funkcije i nacrtati izračunatu tangentu. — 15+3+2

Ukupno: 80

6. $f(x) = \frac{3}{\sin(5x)}$

$f'(x) = -3(\sin(5x))^{-2} \cdot \cos(5x) \cdot 5$

$= \frac{-15\cos(5x)}{\sin^2(5x)}$ ✓

7. $f(x) = \cos x \qquad x = \frac{\pi}{4}$

$f'(x) = -\sin x$

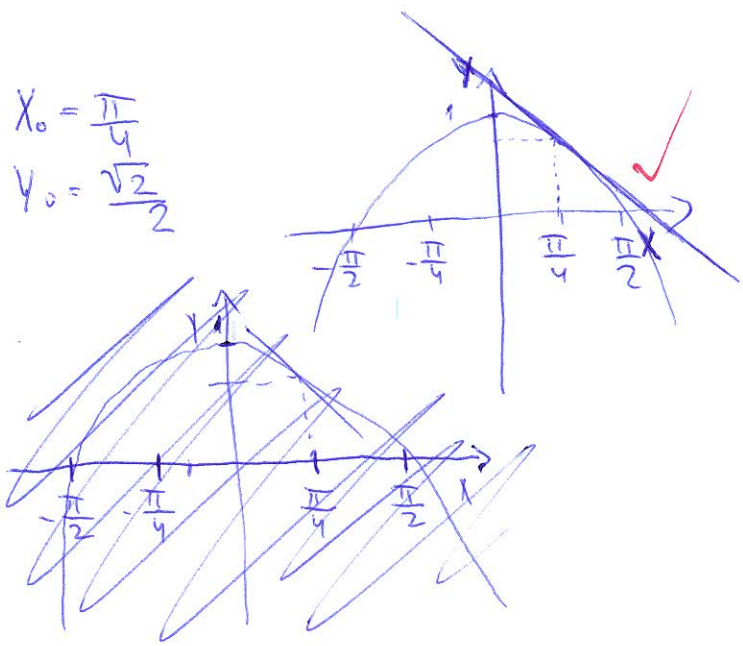
$f'\left(\frac{\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

$t \dots y - \frac{\sqrt{2}}{2} = \frac{-\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

$y = -\frac{\sqrt{2}}{2}x + \frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2}$

$x_0 = \frac{\pi}{4}$

$y_0 = \frac{\sqrt{2}}{2}$



3. $f(x) = \ln(x^2+4) + \sin(2x-3)$

$x^2 + 4 \geq 0$

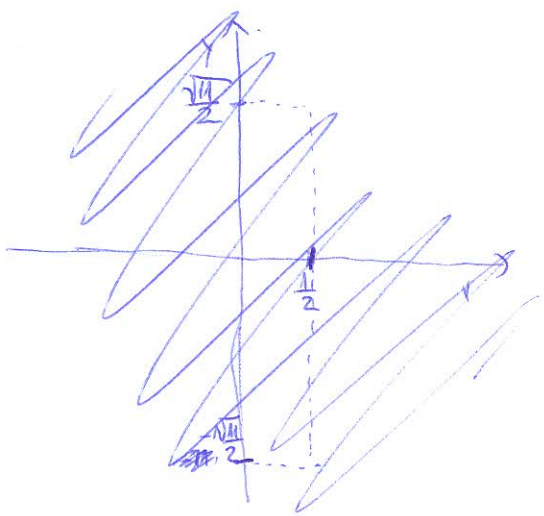
$x^2 \geq -4$

$D(f) = \mathbb{R}$ ✓

$f'(x) = \frac{2x}{x^2+4} + \cos(2x+3) \cdot 2$ ✓

1. $z^2 - z + 3 = 0$

$z_{1,2} = \frac{1 \pm \sqrt{-11}}{2} = \frac{1}{2} \pm \frac{\sqrt{11}}{2} i$

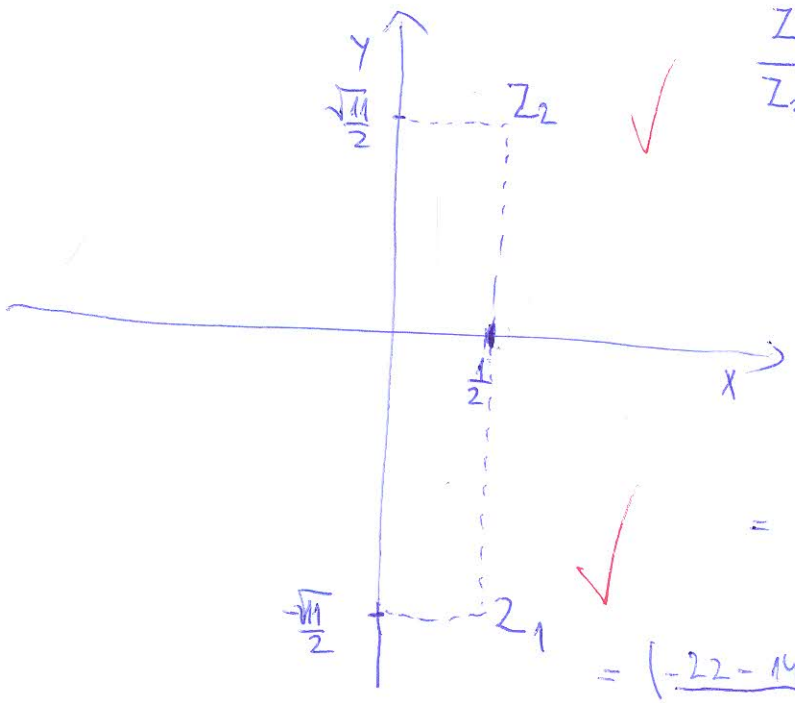


$\left(\frac{1}{2} \pm \frac{\sqrt{11}}{2} i\right)^2 - \left(\frac{1}{2} \pm \frac{\sqrt{11}}{2} i\right) + 3$

$\frac{1}{4} + \frac{\sqrt{11}}{2} i - \frac{11}{4} - \frac{1}{2} \pm \sqrt{\frac{11}{2}} i + 3$

$-\frac{5}{2} - \frac{1}{2} + 3 = -\frac{6}{2} + 3$

$0 = 0$



✓

$\frac{z_1 - z_2}{z_2 + 3} = \left(\frac{\frac{1}{2} - \frac{\sqrt{11}}{2} i - \sqrt{\frac{11}{2}} i}{\frac{1}{2} + \frac{\sqrt{11}}{2} i + 3}\right)$

$= \left(\frac{-\sqrt{11}\, i}{\frac{7 + \sqrt{11}\, i}{2}}\right)$

$= \left(\frac{-2\sqrt{11}\, i}{7 + \sqrt{11}\, i} \cdot \frac{7 - \sqrt{11}\, i}{7 - \sqrt{11}\, i}\right)$

$= \left(\frac{-22 - 14\sqrt{11}\, i}{49 + 11}\right) = -\frac{11}{30} + \frac{7\sqrt{11}}{30} i$ ✓

MARKO VUKELIĆ 0269080097

4.

$f(x) = x - \frac{1}{x} = \frac{x^2-1}{x}$

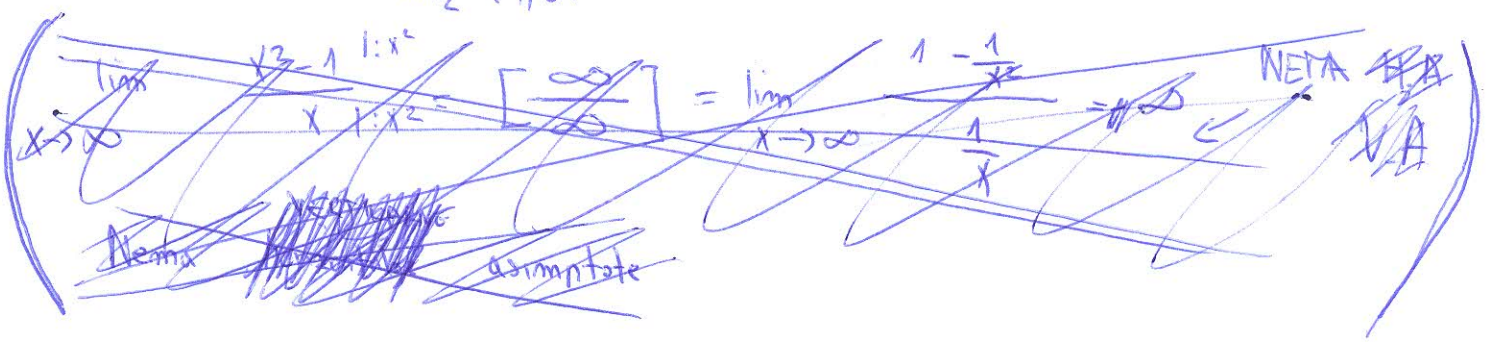
$x \neq 0$

$D(f) = \mathbb{R} \setminus \{0\}$

$x^2 - 1 = 0$

$x^2 = 1 \rightarrow x = \pm 1$

$NT_1(-1,0)$ $NT_2(1,0)$



$\lim_{x \to 0^-} f(x) = \frac{-1}{0^-} = +\infty$

$\lim_{x \to 0^+} f(x) = \frac{1}{0^+} = -\infty$

OVA ... $x = 0$

$\lim_{x \to \infty} \frac{x^2-1 \mid : x^2}{x \mid : x^2} \left[\frac{\infty}{\infty}\right] = \lim_{x \to \infty} \frac{1 - \frac{1}{x^2}}{\frac{1}{x}} = \infty$

NEMA H.A

$k = \lim_{x \to \infty} \frac{x^2-1 \mid : x^2}{x^2 \mid x^2} = \left[\frac{\infty}{\infty}\right] = \lim_{x \to \infty} \frac{1 - \frac{1}{x^2}}{1} = 1$

$l = \lim_{x \to \infty} \left(\frac{x^2-1}{x} - x\right) = \lim_{x \to \infty} \frac{x^2 - 1 - x^2}{x} = \frac{-1}{\infty} = 0$

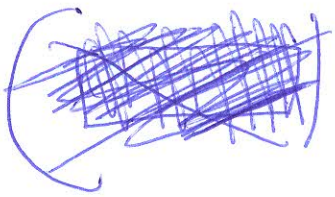
OKA ... $y = x$

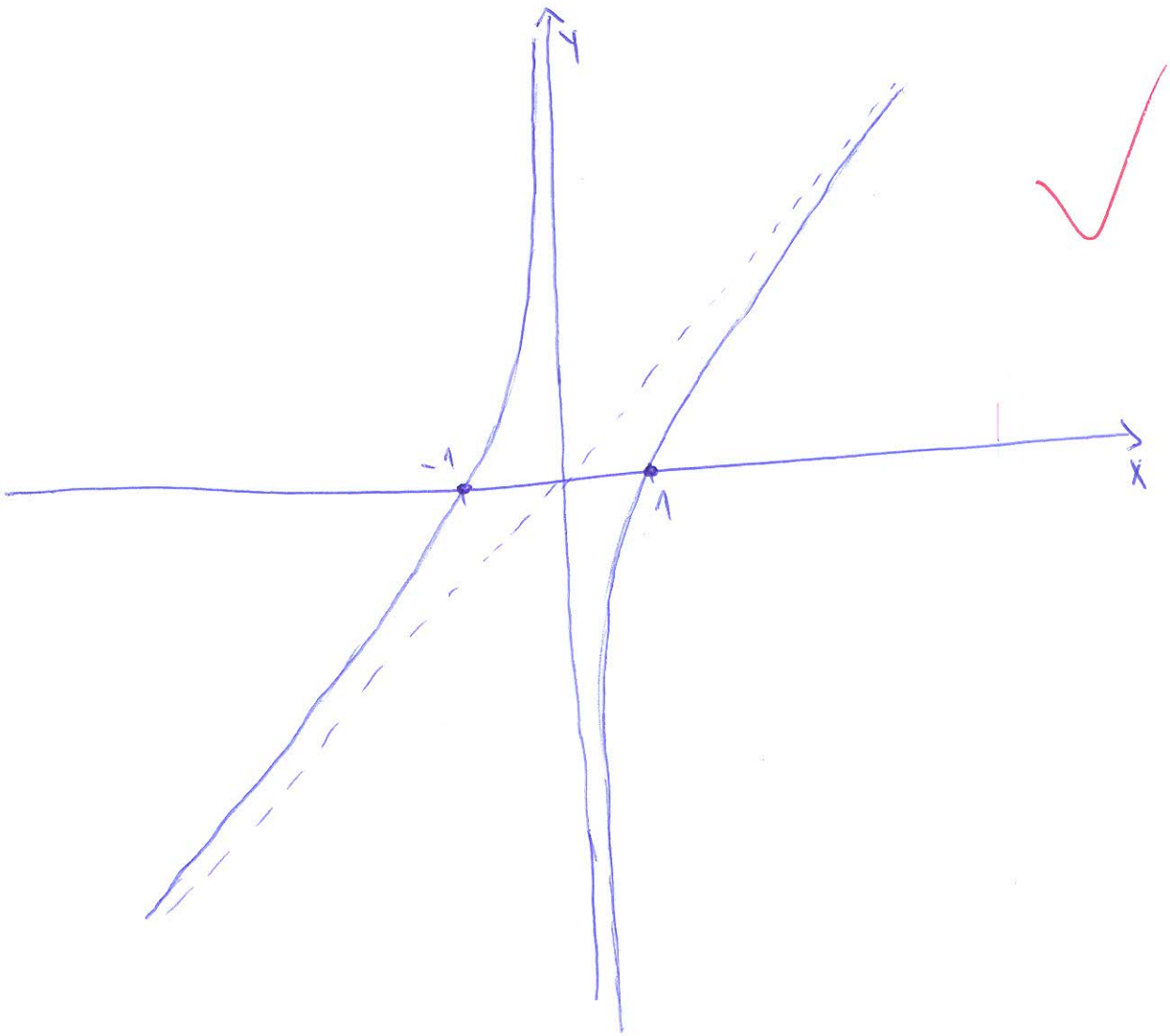
$$f'(x) = \frac{2x \cdot x - (x^2 - 1)}{x^2} = \frac{x^2 + 1}{x^2}$$

$x^2 + 1 = 0$

$x^2 = -1$

	$-\infty$		0		$+\infty$
$f'(x)$	+		+		
	↗		↗		





MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. PIŠITE DVOSTRANO! Obavezno popuniti sva polja ispod⇓⇓ $\mathcal{F}4$

IME I PREZIME: ANDREJ ARAČIĆ

BROJ INDEKSA: 0269082268

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 3 = 0$. *Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem!* Dalje izracunaj: $\overline{\left(\frac{z_1 - z_2}{z_2 + 3}\right)}$. 4+3+8

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$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 8 \\ & & x_2 & - & x_3 & + & x_4 = & -2 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 6 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -2 \end{array}$$

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Ukupno: 55

3. $f(x) = \ln(x^2+4) + \sin(2x-3)$

$f'(x) = \frac{1}{x^2+4} \cdot 2x + \cos(2x-3) \cdot 2$

$f'(x) = \frac{2x}{x^2+4} + 2\cos(2x-3)$

$f'(x) = \frac{2x + 2\cos(2x-3)(x^2+4)}{x^2+4}$

$f'(x) = 2x + 2\cos(2x-3)$

$f(x) = \ln(x^2+4) + \sin(2x-3)$

$x^2+4 > 0$ $\quad -\infty < 2x-3 < \infty$

$x^2 > -4$ $\quad D_{f(x)}: x \in \mathbb{R}$

$D_{f(x)}: x \in \mathbb{R}$

5. $\lim_{x \to -4} \frac{x^2-3}{x^2+8x+16} = \lim_{x \to -4} \frac{x^2-3}{(x-4)(x-4)} = \lim_{x \to -4} \frac{16-3}{(-8)(-8)} = \frac{13}{64}$

$x^2+8x+16 = (x+(-4))(x+(-4)) = (x-4)(x-4)$

$x^2+8x+16 = 0$

$x_{1,2} = \frac{-8 \pm \sqrt{64-64}}{2}$

$x_1 = -4 \quad x_2 = -4$

5. $\lim_{x \to -4} \frac{x^2-3}{x^2+8x+16} = \lim_{x \to -4} \frac{16-3}{16-32+16} = \frac{13}{0} = \infty$

6. $f(x) = \frac{3}{\sin(5x)}$

$f'(x) = \frac{-1\cos(5x)(5)(3)}{(\sin(5x))^2}$

$f'(x) = \frac{-15\cos(5x)}{\sin^2(5x)}$

0,7853981633

4. $f(x) = x - \frac{1}{x}$

1. DOMENA

$x \neq 0$

$D_f = \mathbb{R} \setminus \{0\}$

NULTOČKE

NEMA NULTOČKI

ASIMPTOTE

VERTIKALNA

$x = 0$ (Y OS)

KOSA $y = kx + l$

$k = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{x - \frac{1}{x} : x}{x} = \lim_{x \to \infty} \frac{1 - \frac{1}{x^2}}{1} = \frac{1 - \frac{1}{\infty^2}}{1} = 1$

$k = 1$

$l = \lim_{x \to \infty} [f(x) - kx] = \lim_{x \to \infty} x - \frac{1}{x} - x \overset{\infty}{=} \lim_{x \to \infty} \frac{1}{x^2} = 0$

$y = kx + l = 1 + 0 = 1$

$y = 1$

STAC. TOČKE

$f(x) = x - \frac{1}{x}$

$f'(x) = 1 + \frac{1}{x^2}$ NEMA STAC. TOČKI

$1 + \frac{1}{x^2} = 0$

$\frac{1}{x^2} = -1$

$x^2 = \frac{1}{-1}$

$x = \pm\sqrt{-1}$

$x = \pm i$

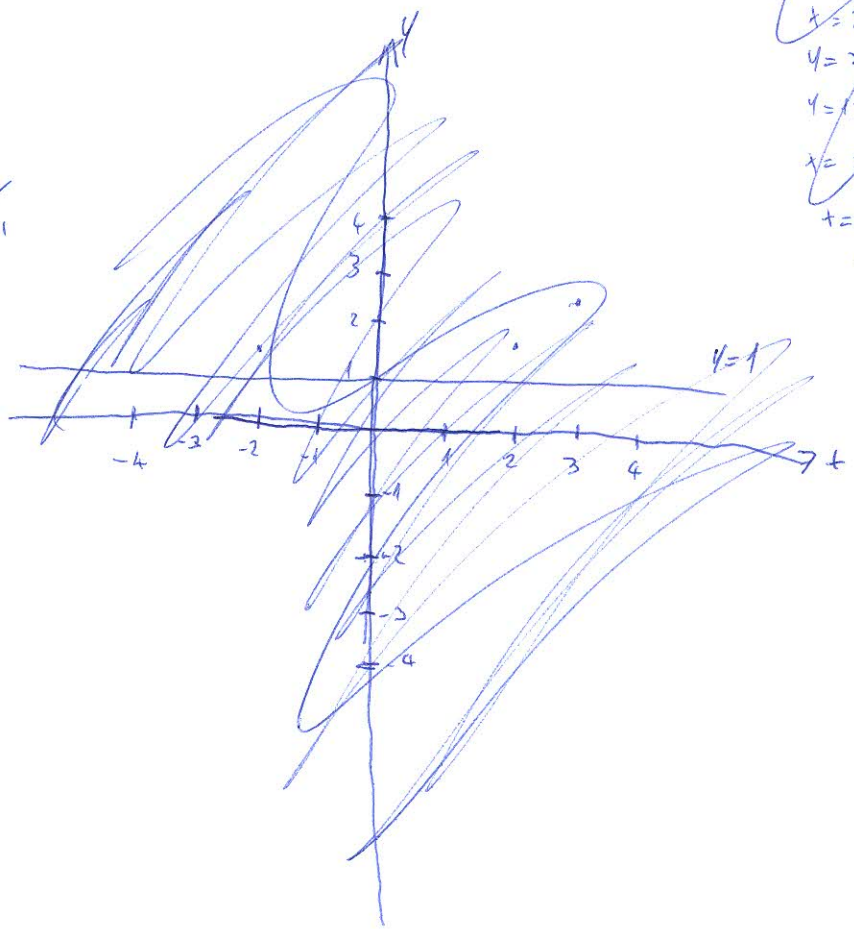
$x = 2$

$y = 2 - \frac{1}{2}$

$y = 1\frac{1}{2}$

$x = 3$ $y = 1\frac{1}{3}$

$x = -2$ $y = \ldots$



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$f'(x) = \frac{1}{x^2+4} \cdot 2x + \cos(2x-3) \cdot 2$ ✓

$f'(x) = \frac{2x}{x^2+4} + 2\cos(2x-3)$

$f'(x) = \frac{2x + 2\cos(2x-3)(x^2+4)}{(x^2+4)}$

$f'(x) = \frac{2x + 2x^2\cos(2x-3) + 8\cos(2x-3)}{x^2+4}$

$f'(x) = \frac{2x^2\cos(2x-3) + 2x + 8\cos(2x-3)}{x^2+4}$

$f(x) = \ln(x^2+4) + \sin(2x-3)$

$x^2+4 > 0$ $\quad -\infty < 2x-3 < \infty$

$x^2 > -4$ $\quad x \in \mathbb{R}$

$x \in \mathbb{R}$

$D_{f(x)}: x \in \mathbb{R}$ ✓

5. $\lim_{x \to -4} \frac{x^2-3}{x^2+8x+16} = \lim_{x \to -4} \frac{16-3}{16-32+16} = \frac{13}{0} = \infty$?

$\frac{13}{0_+} = +\infty$

$\frac{13}{0_-} = -\infty$

6. $f(x) = \frac{3}{\sin(5x)}$

$f'(x) = \frac{-(\cos(5x))(5)(3)}{(\sin(5x))^2}$ ✓

$f'(x) = \frac{-15\cos(5x)}{\sin^2(5x)}$

7. $f(x) = \cos x \qquad x = \frac{\pi}{4}$

$D(\frac{\pi}{4}, 0.707)$

$y_1 = \cos\frac{\pi}{4}$

$y_1 = 0.707$

$f'(x) = -\sin x$

$f'(x) = -\sin\frac{\pi}{4}$

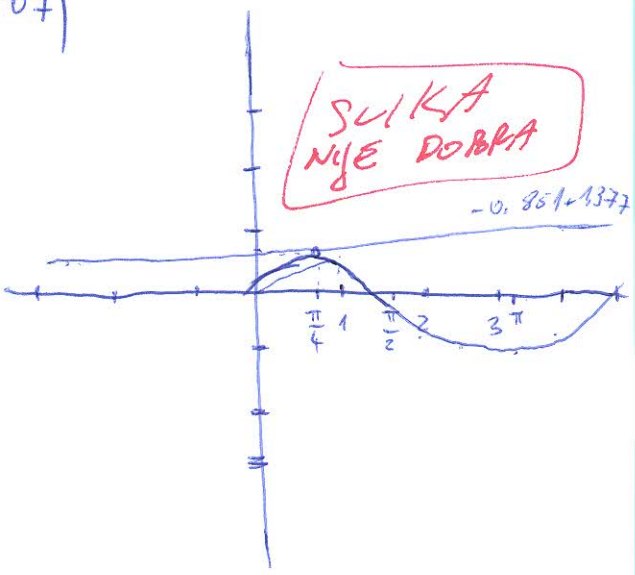
$f'(x) = -0.707$

$y - y_1 = f'(x)(x - x_1)$

$y - 0.525 = -0.851\,(x - \frac{\pi}{4})$

$y - 0.707 = -0.851x + 0.67$

$y = -0.851x + 1.377$



10

4. $f(x) = x - \frac{1}{x} = \frac{x^2-1}{x}$

DOMENA

$x \neq 0$

$D_{f(x)}: x \in \mathbb{R} \setminus \{0\}$

NULTOČKE

$x^2 - 1 = 0$

$x^2 = 1$

$x_1 = 1 \quad x_2 = -1$

ASIMPTOTE VERTIKALNE

$x = 0$ (OS y)

KOSE

$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x^2-1}{x^2} \overset{:x^2}{=} \lim_{x\to\infty} \frac{1-\frac{1}{x^2}}{1} = \frac{1}{1} = 1$

$k = 1$

$L = \lim_{x\to\infty} [f(x) - kx] = \lim_{x\to\infty} \left[\frac{x^2-1}{x} - x\right]$

$L = \lim_{x\to\infty} \frac{x^2-1-x^2}{x} = \lim_{x\to\infty} \frac{-1}{x} \overset{:x}{=} \lim_{x\to\infty} \frac{-1}{x^2} = \frac{-1}{0^-} = 0$

$L = 0$

$y = x + 0$

$y = 1x + 0$

STAL. TOČKE

$f(x) = \frac{x^2-1}{x}$

$f'(x) = \frac{(2x)(x) - (x^2-1)(x)}{x^2}$

$f'(x) = \frac{2x^2 - x^2 + x}{x^2}$

$f'(x) = \frac{x^2+x}{x^2}$

$f'(x) = 0$

$x^2 + x = 0$

$x(x+1) = 0$

$x_1 = 0 \quad x+1 = 0$

$x_2 = -1$

$f'(x) = \frac{(2x+1)(x^2) - (2x)(x^2+x)}{x^4}$

$f''(x) = \frac{2x^3 + x^2 - (4x^2 + 2x^2)}{x^4}$

$f''(x) = \frac{2x^3 + x^2 - 6x^2}{x^4}$

$f''(x) = \frac{2x^3 - 5x^2}{x^4}$

$f''(0) = \frac{0-0}{0} = 0$

$f''(-1) = \frac{-2-5}{-1} = \frac{-7}{-1} = 7$

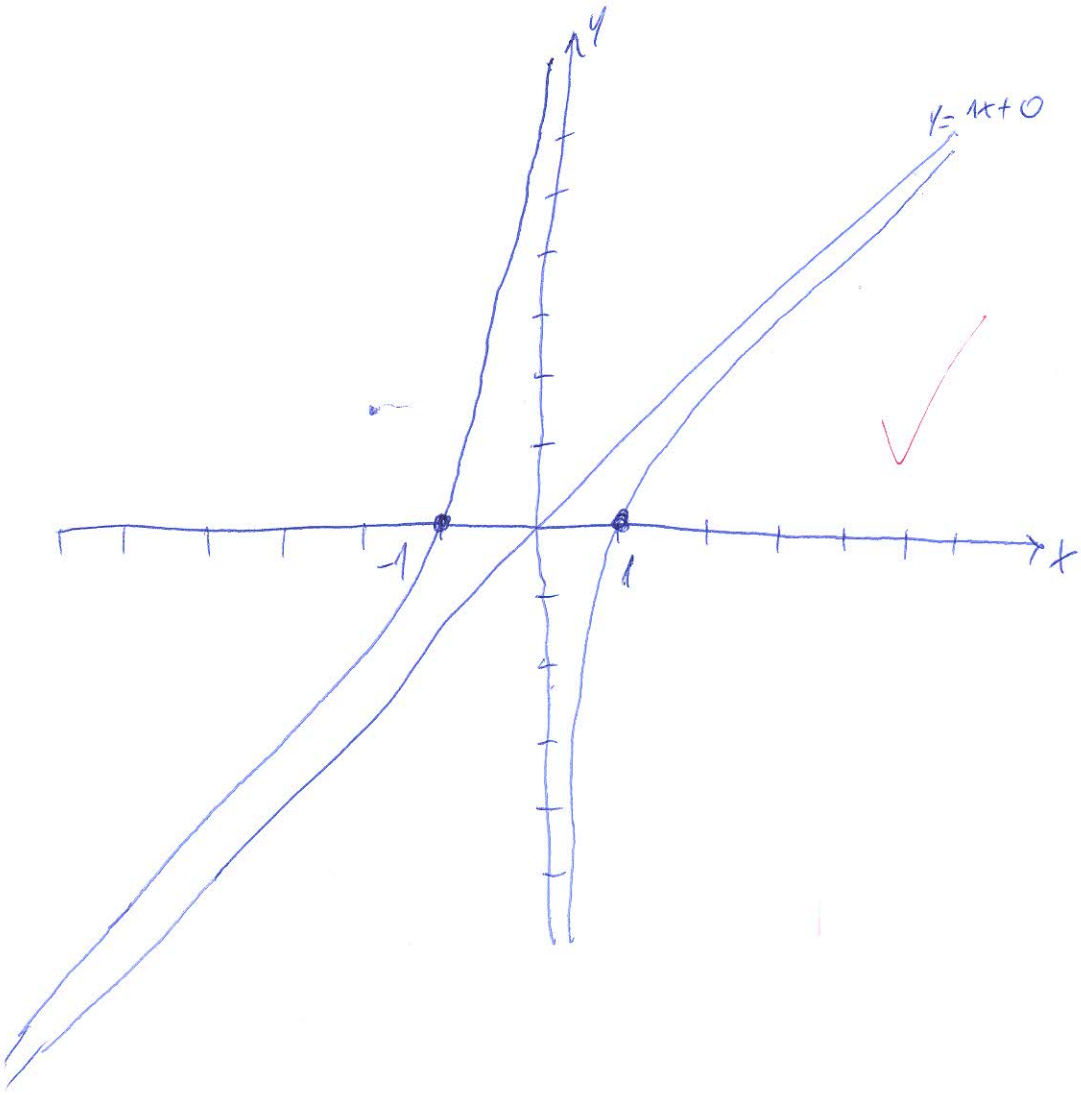
$f(0) = 0 - \frac{1}{0} = 0 - \infty = -\infty$

$f(-1) = -1 - \frac{1}{-1} = -1 + 1 = 0$

ANDREJ ARAČIĆ

4. GRAF

ANDREJ ARAČIČ



ANDREJ ARAČIČ

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IME I PREZIME: LUCIJA IVANDIĆ **BROJ INDEKSA:** 17-2-0109-2011

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Ukupno: 50

(6) $f(x) = \frac{3}{\sin(5x)}$

$$f'(x) = \frac{3'(\sin(5x)) - 3(\sin(5x))'}{(\sin(5x))^2}$$

$$f'(x) = \frac{-3 \cdot \cos(5x) \cdot (5x)'}{(\sin(5x))^2} = -\frac{15\cos(5x)}{(\sin(5x))^2}$$ ✓

(5) $f(x) = x - \frac{1}{x} = \frac{x^2 - 1}{x}$

DOMENA

$x \neq 0$

$D(f) = \mathbb{R} \setminus \{0\}$

PARNOST/NEPARNOST

$f(x) = \frac{x^2 - 1}{x}$

$f(-x) = \frac{(-x)^2 - 1}{-x}$

$f(-x) = \frac{x^2 - 1}{-x}$

Funkcija nije ni parna ni neparna. (- u nazivniku)

PERIODIČNOST

Funkcija nije periodična (ne sadrži sin, cos, tg...)

ASIMPTOTE

NOR (e)

$$\lim_{x\to-\infty}\frac{x^2-1}{x}=\lim_{x\to-\infty}\frac{x^2-1\ /:x^2}{x\ /:x^2}=\lim_{x\to-\infty}\frac{\frac{x^2}{x^2}-\frac{1}{x^2}}{\frac{x}{x^2}}=\frac{1}{0}=\infty$$

(d)

$$\lim_{x\to+\infty}\frac{x^2-1}{x}=\lim_{x\to+\infty}\frac{x^2-1\ /:x^2}{x\ /:x^2}=\lim_{x\to+\infty}\frac{\frac{x^2}{x^2}-\frac{1}{x^2}}{\frac{x}{x^2}}=\frac{1}{0}=\infty$$

- nema horiz. asimptote

KOSA

$y=kx+l$

$$k=\frac{f(x)}{x}=\frac{\frac{x^2-1}{x}}{x}=\frac{x^2-1\ /:x^2}{x^2\ /:x^2}=\frac{\frac{x^2}{x^2}-\frac{1}{x^2}}{\frac{x^2}{x^2}}=\boxed{k=1}$$

$$l=\lim_{x\to\infty}(f(x)-k\cdot x)=\lim_{x\to\infty}\left(\frac{x^2-1}{x}-1\cdot x\right)$$

$$l=\lim_{x\to\infty}\left(\frac{-x^3+x}{x}\right)=\lim_{x\to\infty}\frac{-x^3+x\ /:x^3}{x\ /:x^3}=$$

$$l=\lim_{x\to\infty}\frac{-\frac{x^3}{x^3}+\frac{x}{x^3}}{\frac{x}{x^3}}$$

(5) $\lim_{x\to -4} \frac{x^2-3}{x^2+8x+16} = \lim_{x\to -4} \frac{(-4)^2-3}{(-4)^2+8\cdot(-4)+16} = \lim_{x\to -4} \frac{16-3}{16-32+16}$

$\lim_{x\to -4} \frac{13}{0} = +\infty$, $-4 \Rightarrow$ nije dozvoljeno! – to je uvijek $\frac{x^2-3}{(x+4)^2}$

(3) $f(x) = \ln(x^2+4) + \sin(2x-3)$

$f'(x) = (\ln(x^2+4))' + (\sin(2x-3))'$

$f'(x) = \frac{1}{x^2+4} \cdot (x^2+4)' + \cos(2x-3)\cdot(2x-3)'$

$f'(x) = \frac{1}{x^2+4} \cdot 2x + \cos(2x-3)\cdot 2$

$f'(x) = \frac{2x}{x^2+4} + 2\cos(2x-3)$ ✓

DOMENA

$x^2+4 \neq 0$

$x^2 \neq -4 \ /\sqrt{\ }$

$x \neq \sqrt{-4}$

nema rješenja

$2x-3 \neq 0$

$2x = 3$

$x = \frac{3}{2}$

$D_f \in \mathbb{R}\setminus\{\frac{3}{2}\}$ ✗

LUCIJA IVANDIĆ

(2)

$$\begin{array}{l} x_1 - 2x_2 + 3x_3 - 4x_4 = 8 \\ x_2 - x_3 + x_4 = -2 \\ x_1 + 3x_2 - 3x_4 = 6 \\ -7x_2 + 3x_3 + x_4 = -2 \end{array} \quad (-1)$$

$$\begin{array}{l} x_1 - 2x_2 + 3x_3 - 4x_4 = 8 \\ x_2 - x_3 + x_4 = -2 \\ 0 + 5x_2 - 3x_3 + x_4 = -2 \\ -7x_2 + 3x_3 + x_4 = -2 \end{array} \quad (-5),\ (7)$$

$$\begin{array}{l} x_1 - 2x_2 + 3x_3 - 4x_4 = 8 \\ 0 \quad x_2 - x_3 + x_4 = -2 \\ 0 \quad 0 + 2x_3 - 4x_4 = 8 \\ 0 \quad 0 - 4x_3 + 8x_4 = -16 \end{array} \quad 2$$

$$\begin{array}{l} x_1 - 2x_2 + 3x_3 - 4x_4 = 8 \\ 0 \quad x_2 - x_3 + x_4 = -2 \\ 2x_3 - 4x_4 = 8 \\ 0 + 0 = 0 \end{array}$$

$x_4 = t$

$-4x_3 + 8t = -16 \ /:4$

$-x_3 + 2t = -4$

$x_3 = 2t + 4$

$x_2 - x_3 + x_4 = -2$

$x_2 - (2t+4) + t = -2$

$x_2 - t = 2$

$x_2 = t + 2$

$x_1 = t + 2$
$x_2 = t + 2$
$x_3 = 2t + 4$
$x_4 = t$,
gdje je
t - realan
broj

$x_1 - 2x_2 + 3x_3 - 4x_4 = 8$

$x_1 - 2(t+2) + 3(t+2) - 4t = 8$

$x_1 - 2t - 4 + 3t + 6 - 4t = 8$

$x_1 - 3t = 6$

$x_1 = 6 + 3t \ /:3$

$x_1 = t + 2$

PROVJERA!

$$\left(\frac{z_1 - z_2}{z_2 + z_3}\right) = \left(\frac{\frac{1}{2} + i\frac{\sqrt{11}}{2} - \frac{11}{2} + i\frac{\sqrt{11}}{2}}{\frac{1}{2} - i\frac{\sqrt{11}}{2} + 3}\right) =$$

$$= \frac{(i\sqrt{11})}{\left(\frac{7}{2} - i\frac{\sqrt{11}}{2}\right)} = \frac{-i\sqrt{11}}{\frac{7}{2} + i\frac{\sqrt{11}}{2}} \cdot \frac{\frac{7}{2} - i\frac{\sqrt{11}}{2}}{\frac{7}{2} - i\frac{\sqrt{11}}{2}} =$$

$$= \frac{i\frac{7\sqrt{11}}{2} - \frac{11}{2}}{\frac{49}{4} + \frac{11}{4}} = \frac{-\frac{11}{2} - i\frac{7\sqrt{11}}{2}}{\frac{60}{4} \to 15} = -\frac{11}{30} - i\frac{7\sqrt{11}}{30}$$ ✓

LUCIJA IVANDIĆ

① $z^2 - z + 3 = 0$

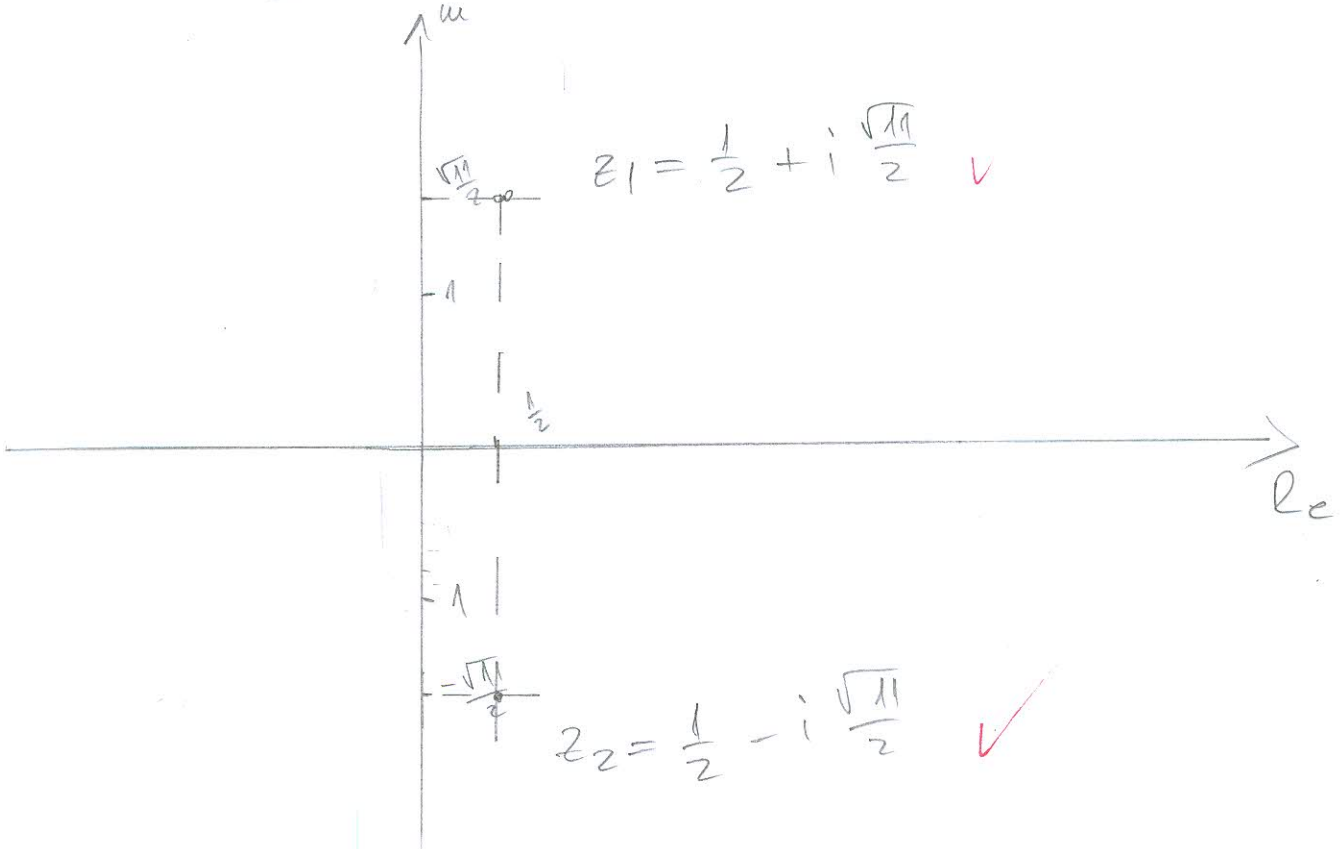
$z_{1,2} = \frac{1 \pm \sqrt{1-12}}{2} = \frac{1 \pm \sqrt{-11}}{2}$

$z_{1,2} = \frac{1}{2} \pm i\frac{\sqrt{11}}{2}$

$\frac{\sqrt{11}}{2} \approx 1.66$

$z_1 = \frac{1}{2} + i\frac{\sqrt{11}}{2}$

$z_2 = \frac{1}{2} - i\frac{\sqrt{11}}{2}$



PROVJERA:

$\left(\frac{1}{2} + i\frac{\sqrt{11}}{2}\right)^2 - \left(\frac{1}{2} + i\frac{\sqrt{11}}{2}\right)^2 =$

$= \frac{1}{4} + i\frac{\sqrt{11}}{2} - \frac{11}{4} - \frac{1}{2} - i\frac{\sqrt{11}}{2} + 3 =$

$= \frac{1 - 11 - 2 + 12}{4} = \frac{0}{4} = 0$, dobro $z_1 = \frac{1}{2} + i\frac{\sqrt{11}}{2}$ je rješenje

Rješenje:

$\left(\frac{1}{2} - i\frac{\sqrt{11}}{2}\right)^2 - \frac{1}{2} - i\frac{\sqrt{11}}{2} + 3 =$

$= \frac{1}{4} - i\frac{\sqrt{11}}{2} - \frac{11}{4} - \frac{1}{2} + i\frac{\sqrt{11}}{2} + 3 =$

$= \frac{1 - 11 - 2 + 12}{4} = \frac{0}{4} = z_2 = \frac{1}{2} - i\frac{\sqrt{11}}{2}$ je rješenje!

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{F}4$

IME I PREZIME: MATE PARAC **BROJ INDEKSA:** 17-1-0179-2013

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 3 = 0$. *Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem!* Dalje izracunaj: $\left(\frac{z_1 - z_2}{z_2 + 3}\right)$. 4+3+8

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 & = & 8 \\ & & x_2 & - & x_3 & + & x_4 & = & -2 \\ x_1 & + & 3x_2 & & & - & 3x_4 & = & 6 \\ & - & 7x_2 & + & 3x_3 & + & x_4 & = & -2 \end{array}$$

3. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(x^2 + 4) + \sin(2x - 3)$. 5+15

4. Odrediti tok funkcije $f(x) = x - \frac{1}{x}$. 15(graf)

5. Odrediti i provjeriti uvrštavanjem: $\lim_{x \to -4} \frac{x^2 - 3}{x^2 + 8x + 16} =$ 4+1

6. Odredi derivaciju funkcije $f(x) = \frac{3}{\sin(5x)}$ 10

7. Odrediti tangentu na funkciju $f(x) = \cos x$ tamo gdje je $x = \frac{\pi}{4}$. Nacrtati graf funkcije i nacrtati izračunatu tangentu. 15+3+2

Ukupno: 50

6. $f(x) = \frac{3}{\sin(5x)}$

$f'(x) = \frac{-3 \cdot \cos(5x) \cdot 5}{\sin^2(5x)}$

$f'(x) = \frac{-15\cos(5x)}{\sin^2(5x)}$ ✓

3. $f(x) = \ln(x^2+4) + \sin(2x-3)$

DOMENA

$x^2 + 4 > 0$

$x^2 + 4 = 0$

$x^2 = -4 \nexists$

$x = \pm\sqrt{-4}$

$D_f : \mathbb{R}$ ✓

$f'(x) = \frac{1}{x^2+4} \cdot (x^2+4)' + \cos(2x-3) \cdot (2x-3)'$

DERIVACIJA

$f'(x) = \frac{2x}{x^2+4} + \cos(2x-3) \cdot 2$ ✓

7. $f(x) = \cos x$

$x_0 = \frac{\pi}{4}$

$y_0 = \frac{\cos \pi}{4}$

$y_0 = \frac{\sqrt{2}}{2}$

$f'(x) = -\sin x$

$f'(x_0) = -\sin \frac{\pi}{4}$

$f'(x_0) = -\frac{\sqrt{2}}{2}$

TANGENTA

$y - y_0 = f'(x_0)(x - x_0)$

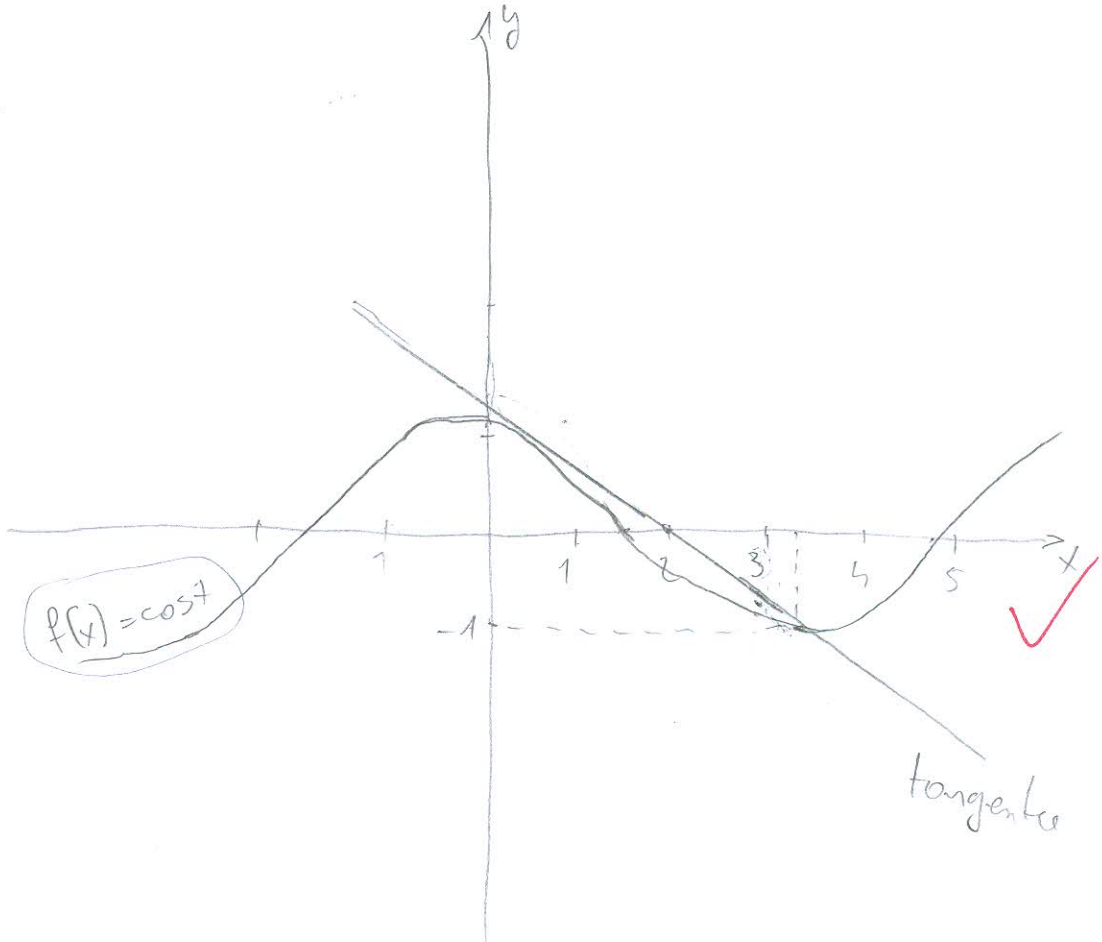
$y - \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

$y - \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}x + \frac{\pi\sqrt{2}}{8}$

$\frac{\sqrt{2}}{2}x + y - \frac{\sqrt{2}}{2} - \frac{\pi\sqrt{2}}{8} = 0$

$0{,}707x + y - 1{,}262 = 0$

x	0	-1
y	1,262	1,969



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{F}4$

IME I PREZIME: Matp Horvat **BROJ INDEKSA:** 17-10229-2013

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 3 = 0$. *Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem!* Dalje izracunaj: $\overline{\left(\frac{z_1 - z_2}{z_2 + 3}\right)}$. 4+3+8

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 8 \\ & & x_2 & - & x_3 & + & x_4 = & -2 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 6 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -2 \end{array}$$

3. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(x^2 + 4) + \sin(2x - 3)$. 5+15

4. Odrediti tok funkcije $f(x) = x - \frac{1}{x}$. 15(graf)

5. Odrediti i provjeriti uvrštavanjem: $\lim_{x \to -4} \dfrac{x^2 - 3}{x^2 + 8x + 16} =$ 4+1

6. Odredi derivaciju funkcije $f(x) = \dfrac{3}{\sin(5x)}$ 10

7. Odrediti tangentu na funkciju $f(x) = \cos x$ tamo gdje je $x = \frac{\pi}{4}$. Nacrtati graf funkcije i nacrtati izračunatu tangentu. 15+3+2

Ukupno: 45

Matěj Hordal

⑦ $f(x) = \cos x \quad x = \frac{\pi}{4}$

$T\left(\frac{\pi}{4}; \frac{\sqrt{2}}{2}\right)$

$f'(x) = -\sin x$

$f'\left(\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = \frac{-\sqrt{2}}{2}$ ... naše tangenta ✓

$y - \frac{\sqrt{2}}{2} = \frac{-\sqrt{2}}{2} \cdot \left(x - \frac{\pi}{4}\right)$

$y - \frac{\sqrt{2}}{2} = \frac{-\sqrt{2}}{2} = \frac{\pi\sqrt{2}}{8}$

$y = \frac{-\sqrt{2}}{2}x + \frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2}$

$y = \frac{-\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}\left(\frac{\pi}{4} + 1\right)$ ⇒ jedna dána tangenta ✓

Vlate Hendel

(2)

$x_1 - 2x_2 + 3x_3 - 4x_4 = 8$

$x_2 - x_3 + x_4 = 2$

$x_1 + 3x_2 - 3x_4 = 6$

$-7x_2 + 3x_3 + x_4 = -2$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 8 \\ 0 & 1 & -1 & 1 & -2 \\ 1 & 3 & 0 & -3 & 6 \\ 0 & -7 & 3 & 1 & -2 \end{array}\right] /(-1)$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 8 \\ 0 & 1 & -1 & 1 & -2 \\ 1 & 3 & 0 & 3 & 6 \\ 0 & -8 & 4 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \\ :3 \\ :4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & 4 & 8 \\ 0 & 1 & -1 & 1 & -2 \\ 1/3 & 0 & 1 & 1 & 2 \\ 0 & -2 & 0 & 1 & 0 \end{array}\right] +$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & 4 & 8 \\ 0 & -1 & 0 & 1 & -2 \\ 1/3 & 1 & 0 & 1 & 2 \\ 0 & -2 & 1 & 0 & 0 \end{array}\right] /\cdot 2$$

$$\left[\begin{array}{cccc|c} 5/3 & 0 & 3 & -2 & 12 \\ 0 & -1 & 0 & 1 & -2 \\ 1/3 & 1 & 0 & 1 & 2 \\ 0 & -2 & 1 & 0 & 0 \end{array}\right] +$$

$$\left[\begin{array}{cccc|c} 5/3 & 0 & 3 & -2 & 12 \\ 0 & -1 & 0 & 1 & -2 \\ 1/3 & 0 & 0 & 2 & 0 \\ 0 & -2 & 1 & 0 & 0 \end{array}\right] +$$

$$\left[\begin{array}{cccc|c} 2 & 0 & 3 & 0 & 12 \\ 0 & -1 & 0 & 1 & -2 \\ 1/3 & 0 & 0 & 2 & 0 \\ 0 & -2 & 1 & 0 & 0 \end{array}\right]$$

$2x_1 + 3x_3 = 12$

$-x_2 + x_4 = -2$

$\frac{1}{3}x_1 + 2x_4 = 0$

$-2x_2 + x_3 = 0$

$\Rightarrow$ $x_2 = x_4 + 2$

$x_1 = 6x_4$ $\Rightarrow$

$x_3 = 2x_2 = 2\cdot(x_4 + 2)$

$2\cdot(-6x_4) + 6\cdot(x_4 + 2 = 12)$

$-12x_4 + 6x_4 + 12 = 12$

$x_4 = 0$ $x_2 = 2$

$x_1 = 0$ $x_3 = 4$

?

PROVJERA!

Mate Hordov

③ $f(x) = \ln(x^2+4) + \sin(2x-3)$

$f'(x) = \frac{x}{x^2+4} \cdot 2 + \cos(3-2x) \cdot 2$ ✓

$f'(x) = 2 \cdot \left(\frac{x}{x^2+4}\right) + \cos(3-2x)$

DER ✓

⑥ $f(x) = \frac{3}{\sin(5x)}$

$f'(x) = \frac{3' \cdot \sin(5x) - 3 \sin'(5x)}{\sin^2(5x)} = \frac{-15\cos(5x)}{\sin^2(5x)} = -\frac{15 \operatorname{tg}(5x)}{\sin(5x)}$ ✓

⑤ $\lim_{x \to -4} \frac{x^2-3}{x^2+8x+16} = \frac{1-\frac{3}{x^2}}{1+\frac{8}{x}-\frac{16}{x^2}} = \frac{1-\frac{3}{16}}{1-2+1} = \frac{0}{0} = \infty$?

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{F}4$

IME I PREZIME: LOVRE RADOVČIĆ

BROJ INDEKSA: 17-1-0177-2013

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 3 = 0$. *Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem!* Dalje izracunaj: $\overline{\left(\frac{z_1 - z_2}{z_2 + 3}\right)}$. — 4+3+8

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: — 10+5

$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 & = & 8 \\ & & x_2 & - & x_3 & + & x_4 & = & -2 \\ x_1 & + & 3x_2 & & & - & 3x_4 & = & 6 \\ & - & 7x_2 & + & 3x_3 & + & x_4 & = & -2 \end{array}$$

3. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(x^2 + 4) + \sin(2x - 3)$. — 5+15

4. Odrediti tok funkcije $f(x) = x - \frac{1}{x}$. — 15(graf)

5. Odrediti i provjeriti uvrštavanjem: $\lim_{x \to -4} \frac{x^2 - 3}{x^2 + 8x + 16} =$ — 4+1

6. Odredi derivaciju funkcije $f(x) = \frac{3}{\sin(5x)}$ — 10

7. Odrediti tangentu na funkciju $f(x) = \cos x$ tamo gdje je $x = \frac{\pi}{4}$. Nacrtati graf funkcije i nacrtati izračunatu tangentu. — 15+3+2

Ukupno: 32

7. $y - y_0 = f(x_0)(x - x_0)$

$f(x) = \cos x \qquad x = \frac{\pi}{4}$

$y_0 = \cos\frac{\pi}{4}$

$y_0 = \frac{\sqrt{2}}{2} = 0{,}7$

$f'(x) = \cos x$

$= -\sin x$

$f'\left(\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$y - y_0 = f(x_0)(x - x_0)$

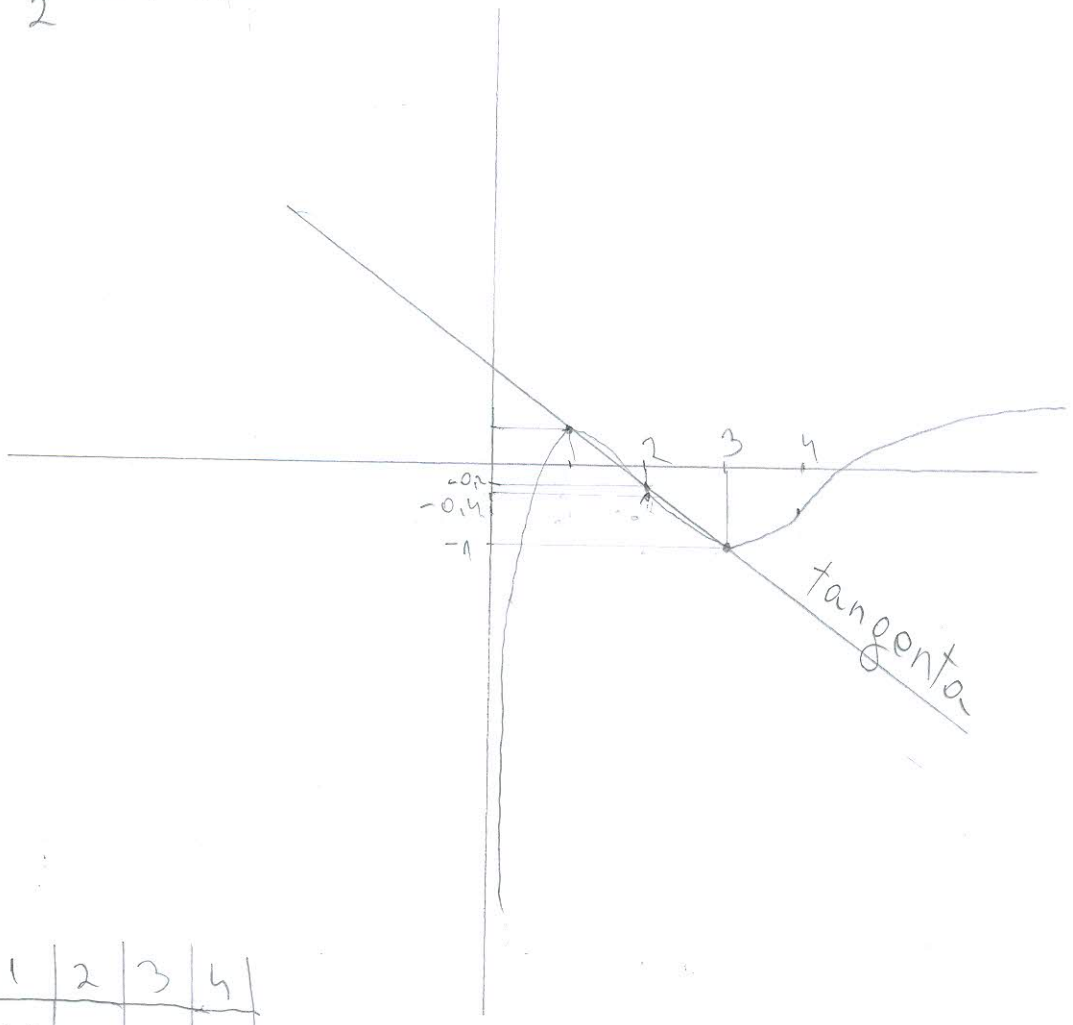
$y - \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}(x - 0{,}78)$

$y - \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}x + 0{,}5$

$y = -\frac{\sqrt{2}}{2}x + 0{,}5 + \frac{\sqrt{2}}{2}$

$y = -\frac{\sqrt{2}}{2}x + 1{,}2$

$$y = -\frac{\sqrt{2}}{2}x + 1{,}2$$



x	1	2	3	4
f(x)	0,5	-0,4	-1	-0,65

$f(x) = \cos x$

x	1	2
y	0,5	-0,2

$$y = -\frac{\sqrt{2}}{2}x + 1{,}2$$

3. $f(x)=\ln(x^2+4)+\sin(2x-3)$ LOVRE RADOVČIĆ

DOMENA

1° $x^2+4>0$

$x^2+4=0$

$x^2=-4$

$x=\pm\sqrt{-4}$

$x\in\mathbb{R}$

2° $-1\leq 2x-3\leq 1$

$2x-3\geq -1$

$2x\geq -1+3$

$2x\geq 2\,|:2$

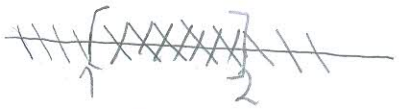
$x\geq 1$

$2x-3\leq 1$

$2x\leq 1+3$

$2x\leq 4\,|:2$

$x\leq 2$



$D_f=[1,2]$

DERIVACIJA

$f'(x)=\left(\ln(x^2+4)+\sin(2x-3)\right)'$

$=\left(\ln(x^2+4)\right)'+\left(\sin(2x-3)\right)'$

$=\frac{1}{x^2+4}(x^2+4)'+\cos(2x-3)(2x-3)'$

$=\frac{1}{x^2+4}2x+\cos(2x-3)(2)$

$=\frac{2x}{x^2+4}+2\cos(2x-3)$

LOVRE RADOVČIĆ

4. $f(x) = x - \frac{1}{x}$

DOMENA

$x \neq 0$

$Df: \mathbb{R} \setminus \{0\}$

ASIMPTOTE

V.A.

$\lim_{x \to 0} 0 - \frac{1}{0} = \infty$ V.A. $x = 0$

H.A.

$\lim_{x \to \infty} x - \frac{1}{x} = \frac{x^2 - 1}{x} \begin{matrix} /:x^2 \\ /:x^2 \end{matrix} = \frac{1}{0} = \infty$ H.A. NE POSTOJI

K.A.

$$k = \lim_{x \to \infty} \frac{x - \frac{1}{x}}{x} = \lim_{x \to \infty} \frac{\frac{x^2-1}{x}}{x} = \lim_{x \to \infty} \frac{x^2-1}{x^2} \begin{matrix} /:x^2 \\ /:x^2 \end{matrix} = 1 \quad k = 1$$

$$l = \lim_{x \to \infty} \frac{x^2-1}{x} - x = \lim_{x \to \infty} \frac{x^2-1-x^2}{x} \begin{matrix} /:x^2 \\ /:x^2 \end{matrix} = \frac{0}{0} = \infty$$

NE(PARNOST)

$f(-x) = -x - \left(-\frac{1}{x}\right) = -x + \frac{1}{x}$ NI PARNA

NI NEPARNA

NULTOČKE

$x - \frac{1}{x} = 0$ $(-1, 0)$

$(1, 0)$

$\frac{x^2-1}{x} = 0 \mid \cdot x$

$x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

DERIVACIJA

$$f'(x)=\left(x-\frac{1}{x}\right)'$$
$$=(x)'-\left(\frac{1}{x}\right)'$$
$$=1-\left(-\frac{1}{x^2}\right)$$
$$=1+\frac{1}{x^2}=\frac{x^2+1}{x^2}$$

KRITIČNE TOČKE

$$\frac{x^2+1}{x^2}=0 \mid \cdot (x^2)$$
$$x^2+1=0$$
$$x^2=-1$$
$$x=\pm\sqrt{-1}$$

	$-\infty$	-1	0	1	$+\infty$
$f'(x)$		+		+	
$f(x)$		↗		↗	

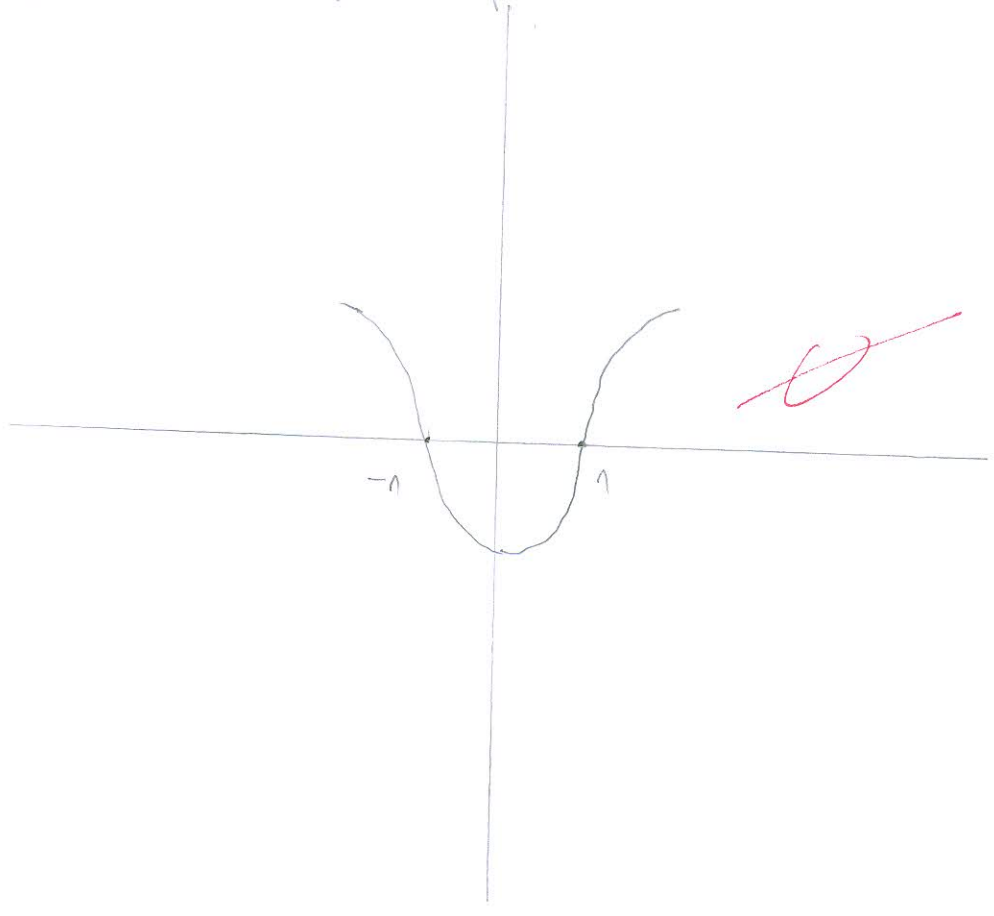
DRUGA DERIVACIJA

$$f''(x)=\left(\frac{x^2+1}{x^2}\right)'$$
$$=\frac{(x^2+1)'(x^2)-(x^2+1)(x^2)'}{(x^2)^2}=\frac{2x(x^2)-(x^2+1)2x}{x^4}=\frac{2x^3-2x^3-2x}{x^4}=\frac{-2x}{x^4}$$

TOČKE INFLEKSIJE

$$\frac{-2x}{x^4}=0 \mid \cdot x^4$$
$$-2x=0 \mid :(-2)$$
$$x=0$$

	$-\infty$	-1	0	1	$+\infty$
$f''(x)$		+		−	
$f(x)$		∪		∩	



5. $\lim_{x \to -4} \frac{x^2-3}{x^2+8x+16} = \lim_{x \to -4} \frac{(-4)^2-3}{(-4)^2+8\cdot(-4)+16} = \left[\frac{13}{0} = \infty\right]$ $\frac{13}{0_+} = +\infty$ $\frac{13}{0_-} = -\infty$

$\lim_{x \to -3{,}999} \frac{x^2-3}{x^2+8x+16} = \lim_{x \to -3{,}999} \frac{(-3{,}999)^2-3}{(-3{,}999)^2+8\cdot(-3{,}999)+16} = \frac{13}{15{,}99-31{,}99+16} = \frac{13}{0}$

$\frac{13}{0} = \infty$

6. $f(x)=\frac{3}{\sin(5x)}$

LOVRE RADOVČIĆ

$f'(x)=\left(\frac{3}{\sin(5x)}\right)'$

$=\frac{(3)'\sin(5x)-5(\sin(5x))'}{(\sin(5x))^2}$

$=\frac{0\cdot\sin(5x)-5(\cos(5x))(5x)'}{(\sin(5x))^2}$

$=\frac{-5\cos(5x)5}{(\sin(5x))^2}=\frac{-10\cos(5x)}{(\sin(5x))^2}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ $\mathcal{F}4$

IME I PREZIME: TINO VUČKOVIĆ **BROJ INDEKSA:** 0269081388

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 3 = 0$. *Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem!* Dalje izracunaj: $\left(\overline{\frac{z_1 - z_2}{z_2 + 3}}\right)$. 4+3+8

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 8 \\ & & x_2 & - & x_3 & + & x_4 = & -2 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 6 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -2 \end{array}$$

3. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(x^2 + 4) + \sin(2x - 3)$. 5+15

4. Odrediti tok funkcije $f(x) = x - \frac{1}{x}$. 15(graf)

5. Odrediti i provjeriti uvrštavanjem: $\lim\limits_{x \to -4} \dfrac{x^2 - 3}{x^2 + 8x + 16} =$ 4+1

6. Odredi derivaciju funkcije $f(x) = \dfrac{3}{\sin(5x)}$ 10

7. Odrediti tangentu na funkciju $f(x) = \cos x$ tamo gdje je $x = \frac{\pi}{4}$. Nacrtati graf funkcije i nacrtati izračunatu tangentu. 15+3+2

Ukupno: 25

$$\lim_{x \to -4} \frac{x^2-3}{x^2+8x-16} = \lim_{x \to -4} \frac{(-4)^2-3}{(-4)^2+8\cdot(-4)-16} \quad | :(-4^2)$$

$$\lim_{x \to -4} \frac{-3}{-12} = \lim_{x \to -4} \frac{3}{12} = \frac{1}{4}$$

(2)

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 8 \\ 0 & 1 & -1 & 1 & -2 \\ 1 & 3 & 0 & -3 & 6 \\ 0 & -7 & 3 & 1 & -2 \end{array}\right] \overset{III-I}{\sim} \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 8 \\ 0 & 1 & -1 & 1 & -2 \\ 0 & 5 & -3 & 1 & -2 \\ 0 & -7 & 3 & 1 & -2 \end{array}\right] \overset{I+2\cdot II}{\sim}$$

$$\overset{I-III}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 4 \\ 0 & 1 & -1 & 1 & -2 \\ 0 & 5 & -3 & 1 & -2 \\ 0 & -7 & 3 & 1 & -2 \end{array}\right] \overset{II-5\cdot 2}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 4 \\ 0 & 1 & -1 & 1 & -2 \\ 0 & 0 & 2 & -4 & 8 \\ 0 & -7 & 3 & 1 & \end{array}\right] :2 \left[\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 4 \\ 0 & 1 & -1 & 1 & -2 \\ 0 & 0 & 1 & -2 & 4 \\ 0 & -7 & 3 & 1 & 1 \end{array}\right] \overset{VI+7I}{\sim}$$

$$\overset{II+III}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 4 & 4 \\ 0 & 1 & -1 & 1 & -2 \\ 0 & 0 & 1 & -2 & 4 \\ 0 & 0 & -4 & 8 & -13 \end{array}\right] \overset{I+2III}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 4 & 4 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & -2 & 4 \\ 0 & 0 & -1 & 2 & \frac{13}{4} \end{array}\right] \overset{I-2III}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 12 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & -2 & 4 \\ 0 & 0 & -1 & 2 & \frac{13}{4} \end{array}\right] \sim$$

$$1:4 \left[\begin{array}{cccc|c} 1 & 0 & 0 & 4 & 4 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & -2 & 4 \\ 0 & 0 & -1 & 2 & \frac{13}{4} \end{array}\right] \overset{IV-II}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 4 & 4 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & -2 & 4 \\ 0 & -1 & -1 & 1 & \frac{5}{4} \end{array}\right] \overset{I+2\cdot III}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 12 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & -2 & 4 \\ 0 & -1 & -1 & 1 & \frac{5}{4} \end{array}\right] \overset{IV+II}{\sim}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 12 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & -2 & 4 \\ 0 & 0 & -1 & 1 & \frac{5}{4} \end{array}\right] \overset{IV+III}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 12 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 6 & \frac{29}{4} \end{array}\right] 1:6 \sim \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 12 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 1 & \frac{29}{24} \end{array}\right] \overset{III-2\cdot IV}{\underset{II-3\cdot IV}{\sim}}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 12 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & \frac{19}{12} \\ 0 & 0 & 0 & 1 & \frac{29}{24} \end{array}\right] \overset{I-2\cdot III}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{56}{6} \\ 0 & 1 & 0 & 0 & -\frac{3}{8} \\ 0 & 0 & 1 & 0 & \frac{19}{12} \\ 0 & 0 & 0 & 1 & \frac{29}{24} \end{array}\right]$$

Prosera --->

$$\begin{bmatrix} 1 & -2 & 3 & -4 & | & 8 \\ 0 & 1 & -1 & 1 & | & -2 \\ 1 & 3 & 0 & -3 & | & 6 \\ 0 & -7 & 3 & 1 & | & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 3 & -4 & | & 8 \\ 1 & 3 & 0 & -3 & | & 6 \\ 0 & 1 & -1 & 1 & | & -2 \\ 0 & -7 & 3 & 1 & | & -2 \end{bmatrix} \; II-I \sim \begin{bmatrix} 1 & -2 & 3 & -4 & | & 8 \\ 0 & 7 & -3 & 1 & | & 2 \\ 0 & 1 & -1 & 1 & | & -2 \\ 0 & -7 & 3 & 1 & | & -2 \end{bmatrix} \; I+2\cdot II$$

$$\begin{bmatrix} 1 & 0 & 1 & -2 & | & 4 \\ 0 & 7 & -3 & 1 & | & 2 \\ 0 & 1 & -1 & 1 & | & -2 \\ 0 & -7 & 3 & 1 & | & -2 \end{bmatrix} \; I+III \; II-6\cdot III \sim \begin{bmatrix} 1 & 0 & -1 & 6 & | \\ 0 & 1 & 3 & -5 & | \\ 0 & 1 & -1 & 1 & | \\ 0 & -7 & 3 & 1 & | \end{bmatrix} \; I-II \; IV+7\cdot III \sim \begin{bmatrix} 1 & 0 & -3 & 4 & | & -8 \\ 0 & 1 & 3 & -5 & | & 14 \\ 0 & 1 & -1 & 1 & | & -2 \\ 0 & 0 & -4 & & | & \end{bmatrix}$$

$$x_1 - 2x_2 + 3x_3 - 4x_4 = 8$$

$$\begin{bmatrix} 1 & -2 & 3 & -4 & | & 8 \\ 0 & 1 & -1 & 1 & | & -2 \\ 1 & 3 & 0 & -3 & | & 6 \\ 0 & -7 & 3 & 1 & | & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 3 & -4 & | & 8 \\ 0 & 1 & -1 & 1 & | & -2 \\ 0 & -7 & 3 & 1 & | & -2 \\ 1 & 3 & 0 & -3 & | & 6 \end{bmatrix} \; IV-I \; II-III \sim \begin{bmatrix} 1 & -2 & 3 & -4 & | & 8 \\ 0 & 8 & -4 & 0 & | & 0 \\ 0 & -2 & 3 & 1 & | & -2 \\ 0 & 5 & -3 & 1 & | & -2 \end{bmatrix} :4 \sim \; III-IV$$

$$\begin{bmatrix} 1 & -2 & 3 & -4 & | & 8 \\ 0 & 2 & -1 & 0 & | & 0 \\ 0 & -12 & 6 & 0 & | & 0 \\ 0 & 5 & -3 & 1 & | & -2 \end{bmatrix} \; I+II \; :6 \sim \begin{bmatrix} 1 & 0 & 2 & -4 & | & 8 \\ 0 & 2 & -1 & 0 & | & 0 \\ 0 & -2 & 1 & 0 & | & 0 \\ 0 & 5 & -3 & 1 & | & -2 \end{bmatrix} \; IV+2\cdot III \sim \begin{bmatrix} 1 & 0 & 2 & -4 & | & 8 \\ 0 & 2 & -1 & 0 & | & 0 \\ 0 & -2 & 1 & 0 & | & 0 \\ 0 & 1 & 1 & 1 & | & -2 \end{bmatrix} \; II-IV \sim$$

$$\begin{bmatrix} 1 & 0 & 2 & -4 & | & 8 \\ 0 & 1 & -2 & -1 & | & 2 \\ 0 & -2 & 1 & 0 & | & \\ 0 & 1 & 1 & 1 & | & -2 \end{bmatrix}$$

PROVJERA?

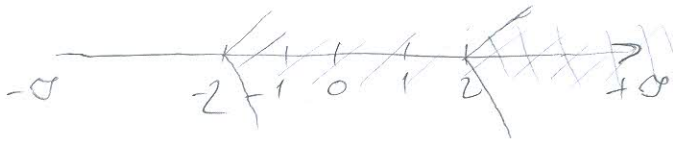
③ $f(x) = \ln(x^2+4) + \sin(2x-3)$

1° wyst ln

$x^2+4 > 0$

$x^2 > -4 \mid \sqrt{}$

$x > \pm 2$



$Df: \langle -2, 2 \rangle \cup \langle 2, +\infty \rangle$ ✗

Derivacja

$f(x) = \ln(x^2+4) + \sin(2x-3)$

$f'(x) = (\ln(x^2+4))' + (\sin(2x-3))'$

$f'(x) = \frac{1}{x^2+4} \cdot (x^2+4)' + \cos(2x-3) \cdot (2x-3)'$

$f'(x) = \frac{1}{x^2+4} \cdot (2x) + \cos(2x-3) \cdot 2$ ✓

① $z^2 - z + 3 = 0$

$$z_1, z_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z_1, z_2 = \frac{z \pm \sqrt{z^2 - 4(z^2)\cdot 3}}{2\cdot(z^2)} = \frac{z \pm \sqrt{z - 12(z^2)}}{2(z)^2} = \text{———}$$

⑥ $f(x) = \frac{3}{\sin(5x)}$

$$f'(x) = \frac{3' \cdot \sin(5x) - 3 \cdot (\sin(5x))'}{(\sin(5x))^2}$$

$$f'(x) = \frac{-3 \cdot \cos(5x) \cdot 5}{\sin^2(5x)} = \frac{-15 \cdot \cos(5x)}{\sin^2(5x)}$$ ✓

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{F}4$

IME I PREZIME: LUKA VIDOV **BROJ INDEKSA:** 17-2-0167/2012

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 3 = 0$. *Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem!* Dalje izracunaj: $\left(\frac{z_1 - z_2}{z_2 + 3}\right)$. — 4+3+8

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: — 10+5

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 8 \\ & & x_2 & - & x_3 & + & x_4 = & -2 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 6 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -2 \end{array}$$

3. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(x^2 + 4) + \sin(2x - 3)$. — 5+15

4. Odrediti tok funkcije $f(x) = x - \frac{1}{x}$. — 15(graf)

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6. Odredi derivaciju funkcije $f(x) = \frac{3}{\sin(5x)}$ — 10

7. Odrediti tangentu na funkciju $f(x) = \cos x$ tamo gdje je $x = \frac{\pi}{4}$. Nacrtati graf funkcije i nacrtati izračunatu tangentu. — 15+3+2

Ukupno: 20

(3.) $f(x) = \ln(x^2+4) + \sin(2x-3)$

$f'(x) = \frac{1}{x^2+4} \cdot (x^2+4)' + (\sin(2x-3))'$

$f'(x) = \frac{1}{x^2+4} \cdot 2x + \cos(2x-3) \cdot (2x-3)'$

$f'(x) = \frac{2x}{x^2+4} + \cos(2x-3) \cdot 2$

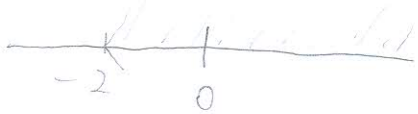
$f'(x) = \frac{2x}{x^2+4} + 2\cos(2x-3)$ ✓

$f(x) = \ln(x^2+4)$

$x^2 + 4 > 0$

$x^2 > -4$ /√

$x > -2$



$D_f: x \in \langle -1, +\infty \rangle$ ✗

6. $f(x) = \dfrac{3}{\sin(5x)}$

$$f(x)' = \frac{3' \cdot (\sin(5x)) - 3 \cdot (\sin(5x))'}{(\sin(5x))^2}$$

$$= \frac{\sin(5x) - 3 \cdot (-\cos(5x)) \cdot 5}{(\sin(5x))^2}$$ ✗

$$= \frac{-15 \cdot (-\cos(5x))}{\sin(5x)}$$

$$= \frac{15\cos(5x)}{\sin(5x)}$$

LUKA VIDOV

(2.)

$$x_1 - 2x_2 + 3x_3 - 4x_4 = 8$$
$$x_2 - x_3 + x_4 = -2$$
$$x_1 + 3x_2 - 3x_4 = 6$$
$$-7x_2 + 3x_3 + x_4 = -2$$

$$\left[\begin{array}{cccc|c} 1 & -2 & +3 & -4 & 8 \\ 0 & 1 & -1 & +1 & -2 \\ 1 & +3 & 0 & -3 & 6 \\ 0 & -7 & +3 & +1 & -2 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -2 & +3 & -4 & 8 \\ 1 & 3 & 0 & -3 & 6 \\ 0 & 1 & -1 & +1 & -2 \\ 0 & -7 & +3 & +1 & -2 \end{array}\right] \begin{array}{l} \text{II} - \text{I} \\ \sim \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & +3 & -4 & 8 \\ 0 & 5 & -3 & 1 & -2 \\ 0 & 1 & -1 & +1 & -2 \\ 0 & -7 & +3 & +1 & -2 \end{array}\right] \sim \left[\begin{array}{cccc|c} \textcircled{1} & -2 & +3 & -4 & 8 \\ 0 & \textcircled{1} & -1 & +1 & -2 \\ 0 & 5 & -3 & 1 & -2 \\ 0 & -7 & +3 & +1 & -2 \end{array}\right] \begin{array}{l} \text{I} + 2\text{II} \\ \text{III} - 5\text{II} \\ \text{IV} + 7\text{II} \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 4 \\ 0 & 1 & -1 & +1 & -2 \\ 0 & 0 & 2 & -4 & 8 \\ 0 & 0 & -4 & 8 & -16 \end{array}\right] \begin{array}{l} (:2) \\ (:4) \end{array} \sim$$

$$\left[\begin{array}{cccc|c} \textcircled{1} & 0 & 1 & -2 & 4 \\ 0 & \textcircled{1} & -1 & +1 & -2 \\ 0 & 0 & \textcircled{1} & -2 & 4 \\ 0 & 0 & -1 & 2 & -4 \end{array}\right] \begin{array}{l} \text{I} - \text{III} \\ \text{II} + \text{III} \\ \\ \text{IV} + \text{III} \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = 0$
$x_2 = 2$
$x_3 = 4$
$x_4 = 0$

?

VIŠE RJEŠENJA...

$$0 - (2 \cdot 2) + (3 \cdot 4) - (4 \cdot 0) = 8$$
$$2 - 4 + 0 = -2$$
$$0 + (3 \cdot 2) - 3 \cdot 0 = 6$$
$$(-7 \cdot 2) + (3 \cdot 4) + 0 = -2$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{F}4$

IME I PREZIME: Petra Dolaček **BROJ INDEKSA:** 0269080611

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 3 = 0$. *Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem!* Dalje izracunaj: $\left(\frac{z_1 - z_2}{z_2 + 3}\right)$. 4+3+8

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7. Odrediti tangentu na funkciju $f(x) = \cos x$ tamo gdje je $x = \frac{\pi}{4}$. Nacrtati graf funkcije i nacrtati izračunatu tangentu. 15+3+2

Ukupno: 15

① $z^2 - z + 3 = 0$

$(x+yi)^2 - x + yi + 3 = 0$ $\quad i^2 = -1$

$x^2 + 2xyi + (yi)^2 - x + yi + 3 = 0$

$x^2 + 2xyi - y - x + yi + 3 = 0$

$x^2 - x + 3 = 0$

$x^2 - x = -3$

$x(x-1) = -3$

① $x = -3$ ② $x - 1 = -3$

$x = -3 + 1$

$x = -2$

$2xy - y + y = 0$

$2xy = 0$

① $x = -3$

$2 \cdot (-3) y = 0$

$-6y = 0 \,/:(-6)$

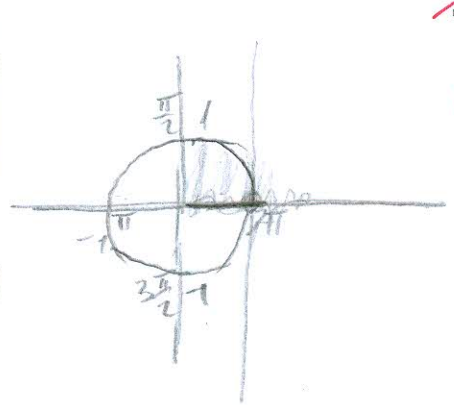
$y = 0$

② $x = -2$

$2 \cdot (-2) y = 0$

$-4y = 0 \,/:(-4)$

$y = 0$



$\operatorname{tg} \varphi = \frac{0}{-2} = 0$

$\varphi = 0$

$z_1 = -3 + 0i$

$z_2 = -2 + 0i$

$$\overline{\left(\frac{z_1 - z_2}{z_1 + 3}\right)} = \frac{-3+0i-(-2)+0i}{-2+0i+3} = \frac{-3+2}{-2+3} = \frac{-1}{1} = -1$$

(3) $f(x) = \ln(x^2+4) + \sin(2x-3)$

doména

$\ln \bigcirc > 0$

$x^2+4 > 0$

$x \in \mathbb{R}$

$x^2+4=0$

$x^2=4 \mid \sqrt{\ }$

$x = \pm 2$

$Df \dots [1,2]$

$-1 \leq 2x-3 \leq 1$

$2x-3 \geq -1$

$2x \geq -1+3$

$2x \geq 2 \mid :2$

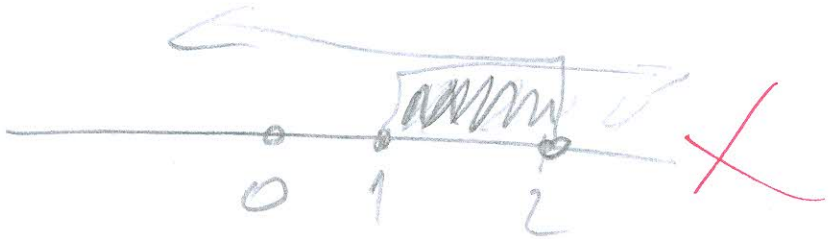
$x \geq 1$

$2x-3 \leq 1$

$2x \leq 1+3$

$2x \leq 4 \mid :2$

$x \leq 2$



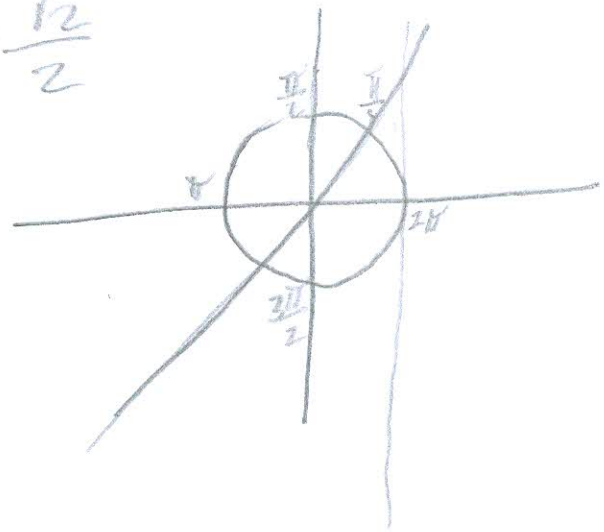
derivace

$\ln(x^2+4)' + \sin(2x-3)'$

$\frac{1}{2x} + \cos 2$

⑦ $f(x) = \cos x = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}$



⑤ $\lim_{x \to -4} \frac{x^2-3}{x^2+8x+16} = \frac{x^2-3}{(x-4)^2} = \frac{(-4)^2-3}{(-4-4)^2} = \frac{16-3}{0} = \frac{13}{0} = +\infty$?

$\frac{13}{0_+} = +\infty$

$\frac{13}{0_-} = -\infty$

⑥ $f(x) = \frac{3}{\sin(5x)'} = \frac{3' \cdot \sin 5x + 3(\sin 5x)'}{(\sin 5x)'} = \frac{3 \cdot \cos 5x}{\cos 5x}$ ✗

PETRA PELALIĆ

② $\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 8 \\ 0 & 1 & -1 & 1 & -2 \\ 1 & 3 & 0 & 3 & 6 \\ 0 & -7 & 3 & 1 & -2 \end{array}\right] \cdot(-) \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 8 \\ 0 & 1 & -1 & 1 & -2 \\ 0 & -5 & 3 & -7 & 2 \\ 0 & -7 & 3 & 1 & -2 \end{array}\right] \cdot 5 \pm \sim$

$\sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 8 \\ 0 & 1 & -1 & 1 & -2 \\ 0 & 0 & -2 & -2 & -8 \\ 0 & -7 & 3 & 1 & -2 \end{array}\right] \cdot 7 \pm \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 8 \\ 0 & 1 & -1 & 1 & -2 \\ 0 & 0 & -2 & -2 & -8 \\ 0 & 0 & -4 & 8 & -16 \end{array}\right] \cdot(-4) \cdot 2 \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 8 \\ 0 & 1 & -1 & 1 & -2 \\ 0 & 0 & -2 & -2 & -8 \\ 0 & 0 & 0 & 24 & 0 \end{array}\right]$

$24x_4 = 0$

$\boxed{x_4 = 0}$

$-2x_3 - 2x_4 = -8$

$-2x_3 = -8 \,/\cdot(-2)$

$\boxed{x_3 = 4}$

$x_2 - x_3 + x_4 = -2$

$x_2 - 4 + 0 = -2$

$x_2 - 4 = -2$

$x_2 = -2 + 4$

$\boxed{x_2 = 2}$

$x_1 - 2x_2 + 3x_3 - 4x_4 = 8$

$x_1 - 2 \cdot 2 + 3 \cdot 4 - 4 \cdot 0 = 8$

$x_1 - 4 + 12 - 0 = 8$

$x_1 + 8 = 8$

$x_1 = 8 - 8$

$\boxed{x_1 = 0}$

PROVJERA

1) $0 - 2 \cdot 2 + 3 \cdot 4 - 4 \cdot 0 = 8$

$0 - 4 + 12 - 0 = 8$

$8 = 8$

2) $0 + 2 - 4 + 0 = -2$

$-2 = -2$

3) $0 + 3 \cdot 2 + 0 - 3 \cdot 0 = 6$

$0 + 6 + 0 - 0 = 6$

$6 = 6$

4) $0 - 7 \cdot 2 + 3 \cdot 4 + 0 = -2$

$0 - 14 + 12 + 0 = -2$

$-2 = -2$

PETRA DELIĆ

④ $f(x) = x - \frac{1}{x}$

$D_f \dots x \in \mathbb{R} \setminus \{0\}$

1) DOMENA $x \neq 0$

2) ~~$x=0$~~ sjecište s y osi
ne može $y = 0$

$x - \frac{1}{x} = 0 \mid \cdot x$

$x^2 - 1 = 0$

$x^2 = 1 \mid \sqrt{}$

$x = \pm 1$

3) PARNOST

$f(-x) = -x - \frac{1}{-x}$

$= -x + \frac{1}{x}$

ni parna ni neparna

4. ASIMPTOSE

V.A.

$\lim_{x \to 0^-} x - \frac{1}{x} = 0.\overline{999} - \frac{1}{0.\overline{999}} = -\infty$ $x=0$ / s lijeve

0 - 0.0001
-0.9999

$\lim_{x \to 0^+} x - \frac{1}{x} = 0.0001 - \frac{1}{0000.1} = +\infty$ $x=0$ s desne

0 + 0.0001
0.0001

H.A

$\lim_{x \to \pm\infty} \frac{x}{1} - \frac{1}{x} = \frac{x^2-1}{x} \overset{LH}{=} \frac{(x^2-1)' \cdot x^2 + (x^2-1) \cdot x'}{x^2} = \frac{2x \cdot x^2 + x^2 - 1}{x^2}$

$= \frac{(2x^3 + x^2 - 1)' / x^2}{x^2 / x^2} = \frac{1+0-0}{0} = \frac{1}{0} = +\infty$ nema horiz.

KOSA.

$y = kx + l$ $k = \lim_{x \to \pm\infty} \frac{f(x)}{x}$ $\lim_{x \to \pm\infty} \frac{x - \frac{1}{x}}{x} = \frac{\frac{x^2-1}{x}}{x} = \frac{x^2-1}{x^2} \mid \cdot x^2 = \frac{1-0}{1} = \frac{1}{1} = 1$

$l = \lim_{x \to \pm\infty} f(x) - kx = x - \frac{1}{x} - x = -\frac{1}{x} \cdot x = -\frac{0}{1} = 0$ $y = 1 + 0 = 1$

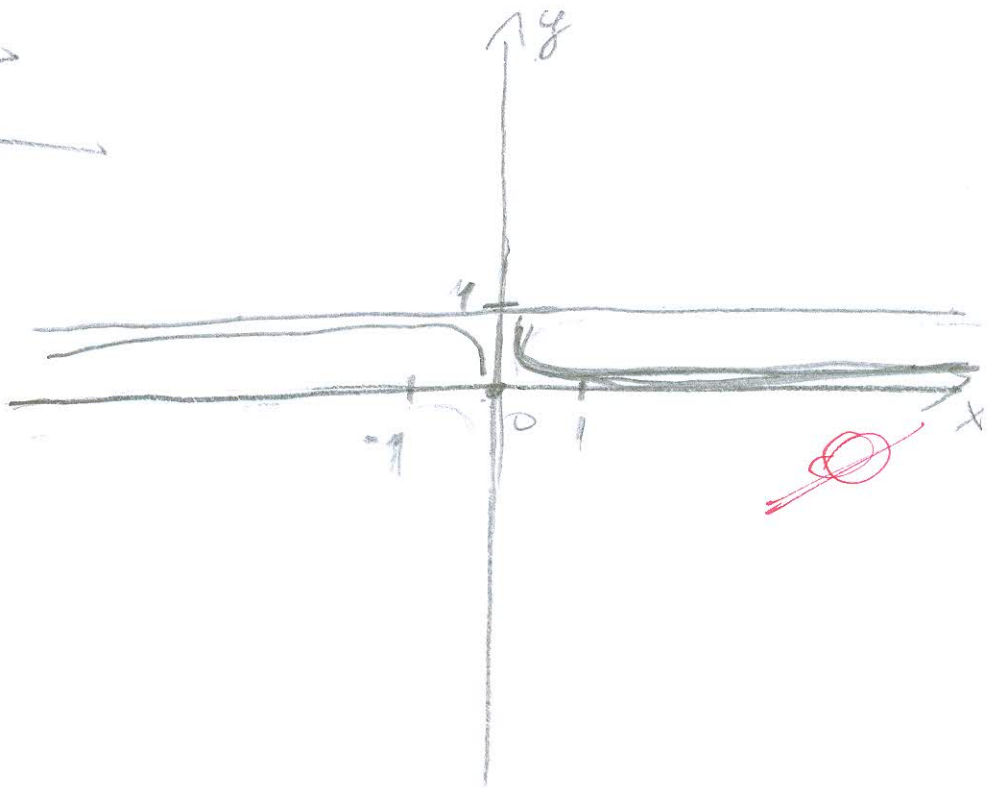
⑤ $f'(x) = x - \left(\frac{1}{x}\right)' = 1 - \frac{1' \cdot x + 1 \cdot x'}{x^2} = 1 - \frac{0+1}{x^2} = 1 - \frac{1}{x^2}$

	$-\infty$		-5		5		$+\infty$
$f'(x)$			$+$		$+$		
$f(x)$		$\nearrow$		$\nearrow$		$\nearrow$	

nima infleksijo

⑥ $f''(x) = 1' - \left(\frac{1}{x^2}\right)' = 0 - \frac{1' \cdot x^2 + 1 \cdot (x^2)'}{(x^2)^2} = \frac{0+2x}{x^4} = \frac{2x}{x^4}$

	$-\infty$		-5		0		5		$+\infty$
$f''(x)$		$-$		$-$		$+$		$+$	
$f(x)$		$\nearrow$		$\nearrow$		$\cup$		$\nearrow$	



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ $\mathcal{F}4$

IME I PREZIME: LUKA ŠATALIĆ **BROJ INDEKSA:** 17-2-0266-2013 02690754 17

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 3 = 0$. *Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem!* Dalje izracunaj: $\overline{\left(\frac{z_1 - z_2}{z_2 + 3}\right)}$. 4+3+8

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 8 \\ & & x_2 & - & x_3 & + & x_4 = & -2 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 6 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -2 \end{array}$$

3. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(x^2 + 4) + \sin(2x - 3)$. 5+15

4. Odrediti tok funkcije $f(x) = x - \frac{1}{x}$. 15(graf)

5. Odrediti i provjeriti uvrštavanjem: $\lim_{x \to -4} \frac{x^2 - 3}{x^2 + 8x + 16} =$ 4+1

6. Odredi derivaciju funkcije $f(x) = \frac{3}{\sin(5x)}$ 10

7. Odrediti tangentu na funkciju $f(x) = \cos x$ tamo gdje je $x = \frac{\pi}{4}$. Nacrtati graf funkcije i nacrtati izračunatu tangentu. 15+3+2

Ukupno: 10

6.) $f(x) = \frac{3}{\sin(5x)}$

$f'(x) = \frac{-3 \cdot \cos(5x) \cdot 5}{\sin^2(5x)}$ ✓

$f'(x) = \frac{-15 \cdot \cos(5x)}{\sin^2(5x)}$

$f'(x) = \frac{-15 \cdot \operatorname{ctg}(5x)}{\sin(5x)}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓ $\mathcal{F}4$

IME I PREZIME: MAJA ŠILIĆ **BROJ INDEKSA:** 17-1-0101-2011

19.09.2014.

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 3 = 0$. *Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem!* Dalje izracunaj: $\overline{\left(\frac{z_1 - z_2}{z_2 + 3}\right)}$. 4+3+8

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 8 \\ & & x_2 & - & x_3 & + & x_4 = & -2 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 6 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -2 \end{array}$$

3. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(x^2 + 4) + \sin(2x - 3)$. 5+15

4. Odrediti tok funkcije $f(x) = x - \frac{1}{x}$. 15(graf)

5. Odrediti i provjeriti uvrštavanjem: $\lim_{x \to -4} \frac{x^2 - 3}{x^2 + 8x + 16} =$ 4+1

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7. Odrediti tangentu na funkciju $f(x) = \cos x$ tamo gdje je $x = \frac{\pi}{4}$. Nacrtati graf funkcije i nacrtati izračunatu tangentu. 15+3+2

Ukupno: 9

4.

1) DOMENA $D(f) = \mathbb{R} \setminus \{0\}$

$f(x) = x - \frac{1}{x}$

$x \neq 0$

2) NUL TOČKE

$\frac{x}{1} - \frac{1}{x} = 0 \quad / \cdot x$

$x^2 - 1 = 0$

$x^2 = 1 \quad / \sqrt{\ }$

$x = \pm 1 = 1$

$T_1(0,1)$

$x_1 = 1$

$x_2 = -1$

$T_2(-1,0)$

$f(0) = \frac{0^2 - 1}{0} = -1$

$T_3(0,-1)$

3) PERIODIČNOST

Funkcija nije periodična jer u sebi ne sadrži sin, cos, tg, ctg....

4) PARNOST I NEPARNOST

$f(x) = x - \frac{1}{x} =$

$f(-x) = \frac{(-x)^2 - 1}{-x} = \frac{x^2 - 1}{-x}$

Funkcija nije ni parna ni neparna!

5) ASIMPTOTE

a) horizontalna

$\lim_{x \to \infty} \frac{x^2 - 1}{x} \Big| :x^2 \Big| :x^2 = \lim_{x \to \infty} \frac{\frac{x^2}{x^2} - \frac{1}{x^2}}{\frac{x}{x^2}} = 1 =$

$= -\frac{1}{1} = -1$

$\lim_{x \to -\infty} \frac{(-x)^2 - 1}{-x} = \lim_{x \to -\infty} \frac{x^2 - 1}{-x} \Big| :x^2 = \lim_{x \to -\infty} \frac{\frac{x^2}{x^2} - \frac{1}{x^2}}{-\frac{x}{x^2}}$

$-x = 1$

OBOSTRANA H.A.: $\boxed{y = -1}$

b) kosa

Nema kose asimptote jer ima obostrane horizontalne!

c) vertikalna -> Nema vertikalne asimptote!

$\lim_{x \to 0} \frac{x^2 - 1}{x} = \lim_{x \to 0} \frac{0^2 - 1}{0} = \frac{1}{0}$

6) STACIONARNE TOČKE

$f(x) = \frac{x^2 - 1}{x}$

$f'(x) = \frac{(x^2 - 1) \cdot x - (x^2 - 1) \cdot (x)'}{x^2}$

$f'(x) = \frac{2x \cdot x - (x^2 - 1) \cdot 1}{x^2}$

$f'(x) = \frac{2x^2 - (x^2 - 1)}{x^2} = \frac{-x^2 - 1}{x^2}$

MAJA ŠIKIĆ
17-1-0101-2011

NAUTIKA I TEHNOLOGIJA POMORSKOG PROMETA

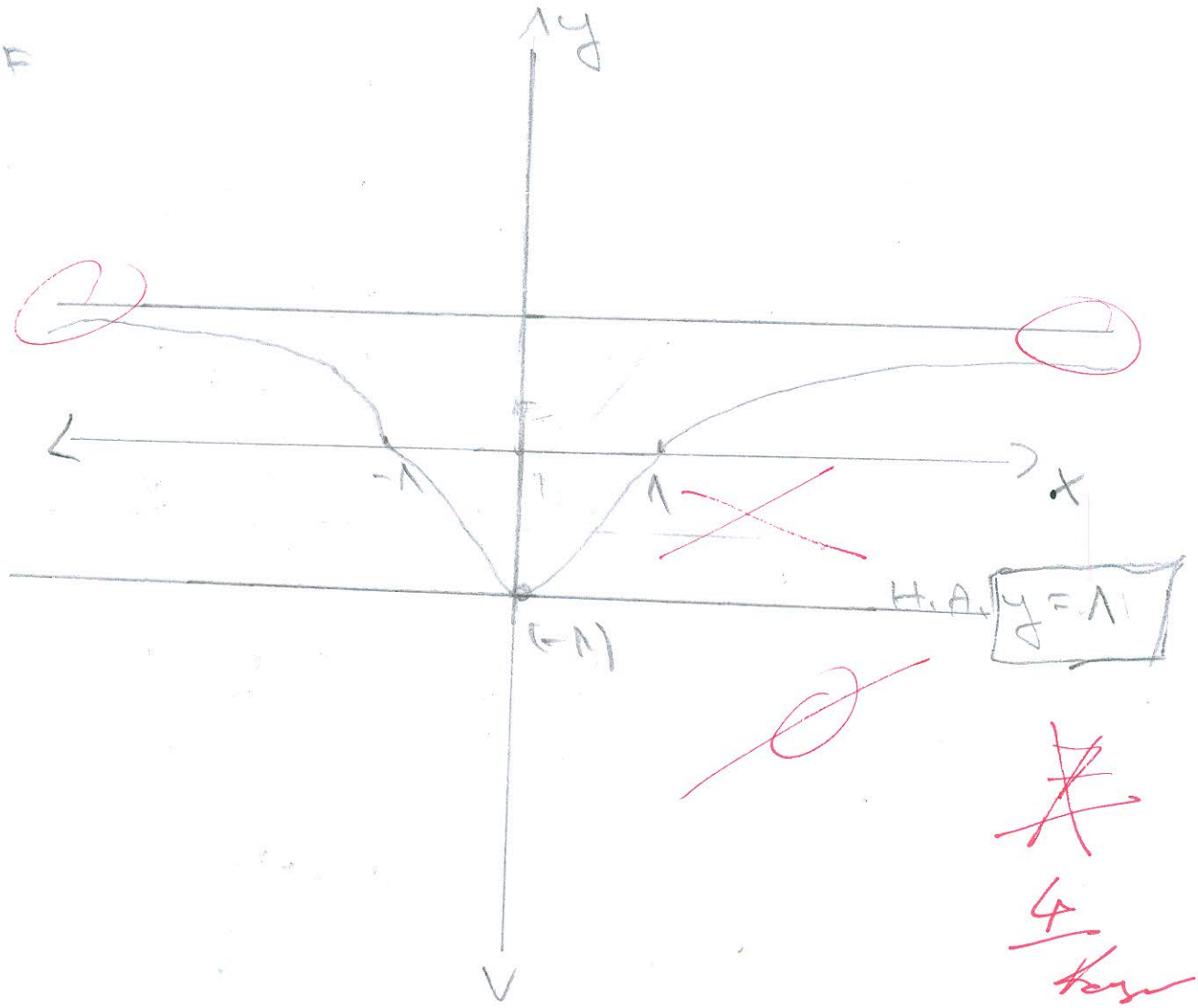
19.09.2014.

7. TOČKE INFLEKSIJE

$$f(x)'' = \frac{x-1}{x^2 g} = f(x)'' = \frac{(x-1)' \cdot x^2 - (x^2)' \cdot (x-1)}{(x^2)^2} =$$

$$f(x)'' = \frac{x-1}{x^4}$$

8. GRAF



4

6. $f(x) = \frac{3}{\sin(5x)}$

$f(x)' = \frac{3' \cdot (\sin(5x)) - (\sin(5x))' \cdot 3}{(\sin(5x))^2}$

$f(x)' = \frac{3\sin(5x) - 3\cos 5x}{(\sin(5x))^2}$ ✗

$f(x)'' = \frac{3\sin(5x) - 3\cos(5x)}{(\sin(5x))^2}$

$f(x)'' = 3\sin$

↓

3. $f(x) = \ln(x^2+4) + \sin(2x-3)$

$Df = R\left\{-\frac{3}{2}\right\}$

a) DOMENA $D(f) = R$ ✓

1° $x^2+4 > 0$

$x^2 > -4$

$x > \sqrt{-4}$

2° $\sin(2x-3)$

$2x-3 \geq 0$

$2x \geq 3 \;/:(2)$

$x \geq -\frac{3}{2}$

$f(x)' = \ln(x^2+4) + \sin(2x-3)$

$f(x)' = \frac{1}{x^2+4} + \cos(2x-3)$ ✗

6. $f(x)'' = \frac{3\sin(5x) - 3\cos(5x)}{(\sin(5x))^2} =$

$f(x)'' = \frac{(3\sin(5x))' \cdot 3\cos(5x) - (3\cos(5x))' \cdot 3\sin(5x)}{((\sin(5x))^2)^2}$

$f(x)'' = \frac{3\cos(5x) \cdot 3\cos(5x) - 3\sin(5x) \cdot 3\sin(5x)}{(\sin(5x))^4}$

$f(x)'' = \frac{9\cos(5x) - 9\sin(5x)}{(\sin(5x))^4}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ $\mathcal{F}4$

IME I PREZIME: KARLO VIDUČIĆ **BROJ INDEKSA:** 17-2-0292-13

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 3 = 0$. *Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem!* Dalje izracunaj: $\overline{\left(\frac{z_1 - z_2}{z_2 + 3}\right)}$. — 4+3+8
2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: — 10+5

$$\begin{array}{rcrcrcrcl} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 & = & 8 \\ & & x_2 & - & x_3 & + & x_4 & = & -2 \\ x_1 & + & 3x_2 & & & - & 3x_4 & = & 6 \\ & - & 7x_2 & + & 3x_3 & + & x_4 & = & -2 \end{array}$$

3. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(x^2 + 4) + \sin(2x - 3)$. — 5+15
4. Odrediti tok funkcije $f(x) = x - \frac{1}{x}$. — 15(graf)
5. Odrediti i provjeriti uvrštavanjem: $\lim_{x \to -4} \frac{x^2 - 3}{x^2 + 8x + 16} =$ — 4+1
6. Odredi derivaciju funkcije $f(x) = \frac{3}{\sin(5x)}$ — 10
7. Odrediti tangentu na funkciju $f(x) = \cos x$ tamo gdje je $x = \frac{\pi}{4}$. Nacrtati graf funkcije i nacrtati izračunatu tangentu. — 15+3+2

Ukupno: 0

6. $f(x) = \frac{3}{\sin(5x)}$

$f'(x) = \frac{3'(\sin(5x)) - 3(\sin(5x))'}{(\sin(5x))^2}$

$f'(x) = \frac{\sin 5x - 3\cos}{(\sin(5x))^2}$ ✗

1. $z^2 - z + 3 = 0$

a) $ax^2 + bx + c = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$z^2 - z + 3 = 0$

$z_{1,2} = \frac{+z \pm \sqrt{(-z)^2 - 4 \cdot z \cdot 3}}{2z}$

$z_{1,2} = \frac{z \pm \sqrt{z^2 - 12z}}{2z}$ ✗

$z_{1,2} = \frac{z \pm z - 12z}{2z}$

$z_1 = \frac{z - z + 12z}{2z}$

$z_1 = \frac{12z}{2z}$

$z_1 = 6$

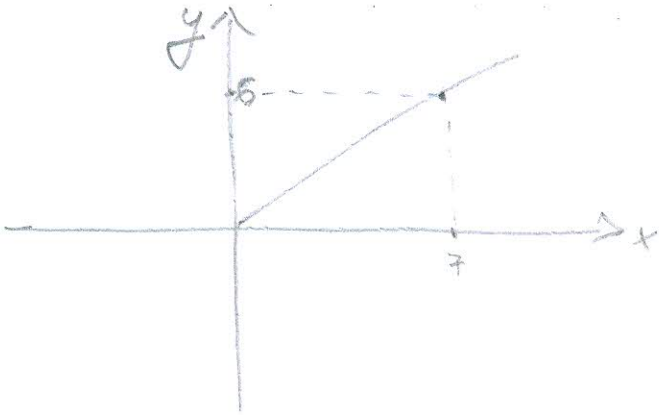
$z_2 = \frac{z \cdot z + 12z}{2z}$

$z_2 = \frac{2z + 12z}{2z}$

$z_2 = \frac{14z}{2z}$

$z_2 = 7$

→ IZ# nastavak



b) $\left(\overline{\frac{z_1 - z_2}{z_2 + 3}}\right) = (\overline{-1-i})$

$= -1+i$

$\frac{z_2}{z_1} = \frac{1-i}{1+i} \cdot \frac{1-i}{1-i}$

$\frac{z_2}{z_1} = \frac{1-3i+i}{3}$

$\frac{z_2}{z_1} = \frac{-3i}{3} = -i$ //

3. $f(x) = \ln(x^2+4) + \sin(2x-3)$

1° UVJET ln

$x^2+4 > 0$

$x^2 > -4$ / $\sqrt{}$

$x >$?

DERIVACIJA:

$f(x) = \ln(x^2+4) + \sin(2x-3)$

$f'(x) = (\ln(x^2+4))' + (\sin(2x-3))'$

$f'(x) = \frac{x}{x} \frac{2x}{4} + 0 + \cos$ ✗

$f'(x) = \frac{2x}{x} + \cos$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ $\mathcal{F}4$

IME I PREZIME: BOŽENO KOLEGA **BROJ INDEKSA:** 17-1-0089-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 3 = 0$. *Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem!* Dalje izracunaj: $\overline{\left(\frac{z_1 - z_2}{z_2 + 3}\right)}$. 4+3+8

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Ukupno: 0

⑤ $\lim_{x \to -4} \frac{x^2-3}{x^2+8x+16} = \frac{(-4)^2-3}{16-32+16} = \frac{16-3}{0} = \frac{13}{0} = +\infty$ X

⑥ $f(x) = \frac{3}{\sin(5x)} = \frac{(3)' \cdot \sin(5x) - 3 \cdot (\sin(5x))'}{\sin(5x)^2}$

$= \frac{-3 \cdot \cos 5}{\sin(5x)} \times = \frac{-3\cos 5}{\sin(5x)}$

$(\ln = \frac{1}{x}$ f' g'

③ $f(x) = \ln(x^2+4) + \sin(2x-3)$ $f' \pm g'$

$= (\ln(x^2+4))' + (\sin(2x-3))'$

$= \frac{1}{x^2+4} + \cos(2)$ X

$\frac{\cos(2)}{x^2+4} = \frac{-\sin}{x}$

BOŽENO KOLEGA

① $z_2 - z + 3 = 0$

$z_1 = 1 + \sqrt{12}$ ✗

$z_2 = 1 - \sqrt{12}$ ✗

$$\left(\frac{z_1 - z_2}{z_2 + 3}\right) = \left(\frac{1+\sqrt{12}i - 1 - \sqrt{12}i}{1-\sqrt{12}i + 3}\right)$$

$$= \left(\frac{0}{1-\sqrt{12}i + 3}\right) /$$ ✗

$$= \left(\sqrt{1-12+3}\right) i$$

$$= \sqrt{-14}\, i$$

$$= -3{,}74$$

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IME I PREZIME: JURE FRŽOP

BROJ INDEKSA: ~~17-2-0269-2013~~ 0269075401

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 3 = 0$. *Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem!* Dalje izracunaj: $\overline{\left(\frac{z_1 - z_2}{z_2 + 3}\right)}$. 4+3+8

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Ukupno: 0

6.) $f(x) = \frac{3}{\sin(5x)}$ $\quad f'(x) = \frac{3' \cdot \sin(5x) + 3 \cdot \sin(5x)'}{(\sin(5x))^2}$

$f'(x) = \left(\frac{3}{\sin(5x)}\right)'$ $\quad f'(x) = \frac{\sin(5x) + 3\cos(5x)}{(\sin(5x))^2}$ ✗

~~$f(x) = \frac{3}{\sin(5x)}$~~ $\quad f'(x) =$

2.)

~~1 2 3~~

1 -2 +3 -4 : 8
① 1 -1 1 : -2
1 3 ⓪ -3 : 6
0 -7 3 1 : -2

3.)

$f(x) = \ln(x^2+4) + \sin(2x-3)$

$f(x) = \frac{1}{\ln(x^2+4)} + \cos(2x-3)$ ✗

$x^2+4=0$ $\quad 2x-3=0$

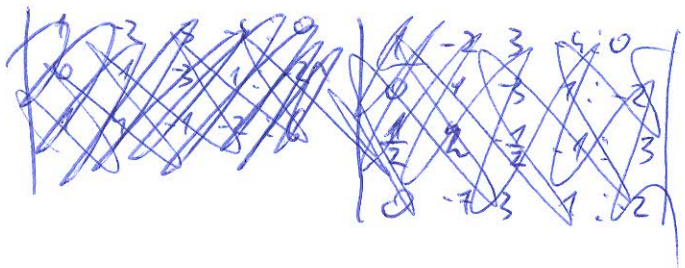
$x^2=-4$ $\quad 2x=3$

$x=\sqrt{-4}$ $\quad x=\frac{3}{2}$

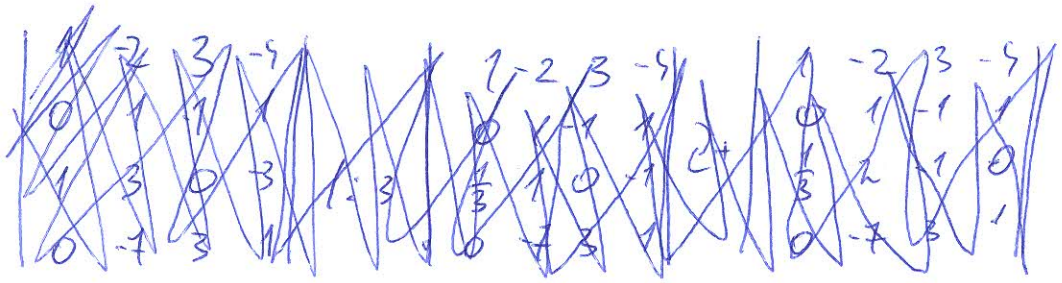
$x=2$

$D_f < \frac{3}{2}, 2)$ ✗

2.)



4.)



1.) $z^2 - z + 3 = 0$

5.) $\lim_{x \to -1} \frac{x^2 - 3}{x^2 + 8x + 16}$