

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

F4

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **MARCO VUKELIĆ**

BROJ INDEKSA: **02 69 080057**

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 3 = 0$. Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem! Dalje izracunaj: $\left(\frac{z_1 - z_2}{z_2 + 3}\right)$.

4+3+8

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

10+5

$$\begin{aligned} x_1 - 2x_2 + 3x_3 - 4x_4 &= 8 \\ x_2 - x_3 + x_4 &= -2 \\ x_1 + 3x_2 - 3x_4 &= 6 \\ -7x_2 + 3x_3 + x_4 &= -2 \end{aligned}$$

3. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(x^2 + 4) + \sin(2x - 3)$.

5+15

4. Odrediti tok funkcije $f(x) = x - \frac{1}{x}$.

15(graf)

5. Odrediti i provjeriti uvrštavanjem: $\lim_{x \rightarrow -4} \frac{x^2 - 3}{x^2 + 8x + 16} =$

4+1

6. Odredi derivaciju funkcije $f(x) = \frac{3}{\sin(5x)}$

10

7. Odrediti tangentu na funkciju $f(x) = \cos x$ tamo gdje je $x = \frac{\pi}{4}$. Nacrtati graf funkcije i nacrtati izračunatu tangentu.

15+3+2

Ukupno:

80

6. $f(x) = \frac{3}{\sin(5x)}$

$$f'(x) = -3 (\sin(5x))^{-2} \cdot \cos(5x) \cdot 5$$

$$= \frac{-15 \cos(5x)}{\sin^2(5x)} \quad \checkmark$$

7. $f(x) = \cos x \quad x = \frac{\pi}{4}$

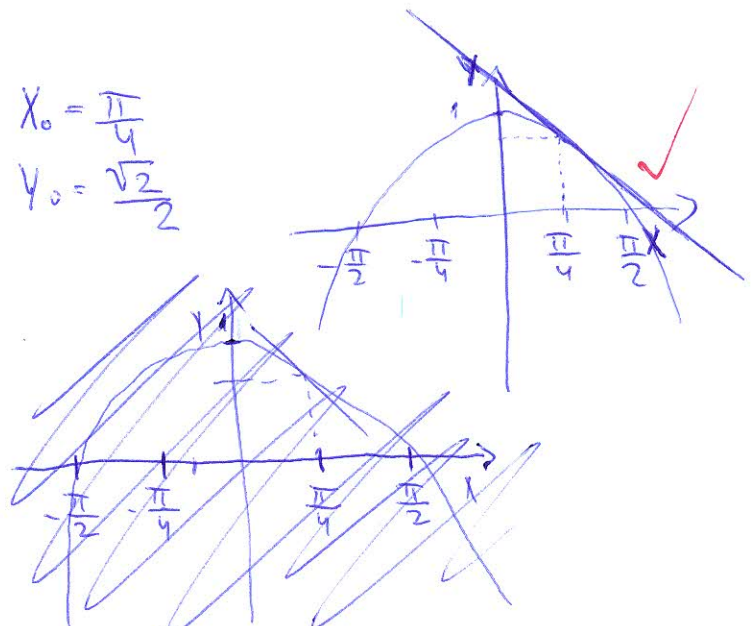
$$f'(x) = -\sin x$$

$$f'\left(\frac{\pi}{4}\right) = \frac{-\sqrt{2}}{2}$$

$$y - y_0 = \frac{\sqrt{2}}{2} \left(x - \frac{\pi}{4}\right)$$

$$y = \frac{\sqrt{2}}{2} x + \frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2}$$

$$x_0 = \frac{\pi}{4} \\ y_0 = \frac{\sqrt{2}}{2}$$



3.

$$f(x) = \ln(x^2 + 4) + \sin(2x - 3)$$

$$x^2 + 4 \geq 0$$

$$x^2 \geq -4$$

$$D(f) = \mathbb{R} \quad \checkmark$$

$$f'(x) = \frac{2x}{x^2 + 4} + \cos(2x - 3) \cdot 2 \quad \checkmark$$

$$1. \quad z^2 - z + 3 = 0$$

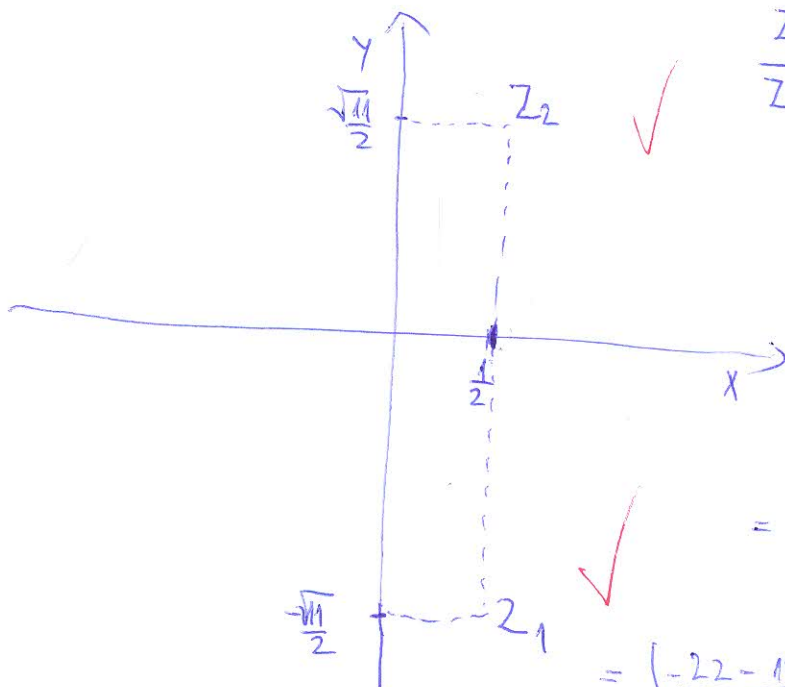
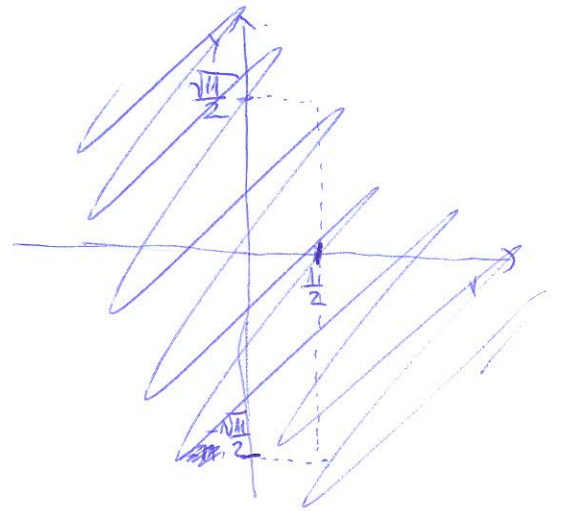
$$z_{1,2} = \frac{1 \pm \sqrt{-11}}{2} = \frac{1}{2} \pm \frac{\sqrt{11}}{2} i$$

$$\left(\frac{1}{2} \pm \frac{\sqrt{11}}{2} i \right)^2 - \left(\frac{1}{2} \pm \frac{\sqrt{11}}{2} i \right) + 3$$

$$\left(\frac{1}{4} + \frac{\sqrt{11}}{2} i - \frac{11}{4} - \frac{1}{2} \pm \frac{\sqrt{11}}{2} i + 3 \right)$$

$$\frac{-5}{2} - \frac{1}{2} + 3 = -\frac{6}{2} + 3$$

$$0 = 0$$



$$\frac{z_1 - z_2}{z_2 + 3} = \frac{\left(\frac{1}{2} - \frac{\sqrt{11}}{2} i - \frac{\sqrt{11}}{2} i \right)}{\left(\frac{1}{2} + \frac{\sqrt{11}}{2} i + 3 \right)}$$

$$= \frac{\left(\frac{1}{2} - \sqrt{11} i - \frac{\sqrt{11}}{2} i \right)}{\frac{1}{2} + \frac{\sqrt{11}}{2} i + 3}$$

$$= \frac{\left(-\sqrt{11} i \right)}{\frac{7 + \sqrt{11} i}{2}}$$

$$= \frac{\left(\frac{-2\sqrt{11} i}{7 + \sqrt{11} i} \cdot \frac{7 - \sqrt{11} i}{7 - \sqrt{11} i} \right)}{\left(\frac{7 - \sqrt{11} i}{7 - \sqrt{11} i} \right)}$$

$$= \left(\frac{-22 - 14\sqrt{11} i}{49 + 11} \right) = -\frac{11}{30} + \frac{7\sqrt{11}}{30} i \quad \checkmark$$

g.

$$f(x) = x - \frac{1}{x} = \frac{x^2 - 1}{x}$$

$$x \neq 0$$

$$D(f) = \mathbb{R} \setminus \{0\}$$

$$x^2 - 1 = 0$$

$$x^2 = 1 \rightarrow x = \pm 1$$

NT₁ (-1, 0)

NT₂ (1, 0)

~~lim_{x→∞} $\frac{x^2-1}{x} \stackrel{!}{:} x^2 = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{1 - \frac{1}{x^2}}{\frac{1}{x}} = \infty$ NETA H.A.~~

~~Asymptote~~

$$\lim_{x \rightarrow 0^-} f(x) = \frac{-1}{0^-} = +\infty$$

$$\lim_{x \rightarrow 0^+} f(x) = \frac{1}{0^+} = -\infty$$

OVA ... x=0

$$\lim_{x \rightarrow \infty} \frac{x^2-1}{x} \stackrel{!}{:} x^2 = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{1 - \frac{1}{x^2}}{\frac{1}{x}} = \infty$$

NETA H.A. ↗

$$k = \lim_{x \rightarrow \infty} \frac{x^2-1}{x^2} \stackrel{!}{:} x^2 = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{1 - \frac{1}{x^2}}{1} = 1$$

$$l = \lim_{x \rightarrow \infty} \left(\frac{x^2-1}{x} - x \right) = \lim_{x \rightarrow \infty} \frac{x^2-1-x^2}{x} = \frac{-1}{\infty} = 0$$

OKA ... y=x

$$k'(x) = \frac{2x \cdot x - (\cancel{x^2} - 1)}{x^2} = \frac{x^2 + 1}{x^2}$$

$$\cancel{x^2 + 1 = 0}$$

$$\cancel{x^2 = -1}$$

