MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ $\mathcal{H}2$

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POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3 7
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2-2}$. 5+15+5 6
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln(4-x^2)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-2x-(2+1)}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf) 6
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x+2y-z+u &= 2 \\ 2x+5y-z+2u &= 2 \\ 3x-y-2z+u &= 3 \\ x-y+3z-5u &= 3 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 78

(1) $z^4 - (4-i)^2 = 0$

$z^4 = (4-i)^2$

$z^4 = 16 - 8i - 1$

$z^4 = 15 - 8i$

$tg\,\varphi = \frac{8}{15}$

$r = \sqrt{15^2+8^2} = 17$

$\varphi_0 = \frac{\varphi}{n} = 82°58'55''$

$\Delta\varphi = 90°$

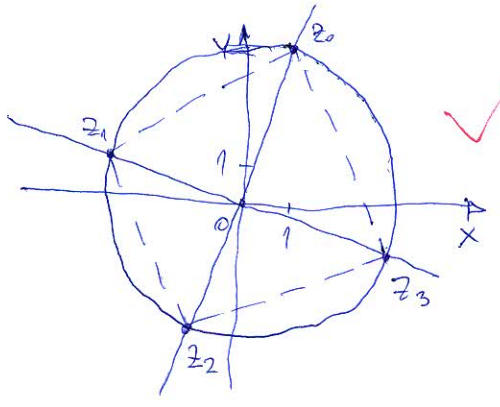
$z^4 = 17(\cos 331°55'39'' + i\sin 331°55'39'')$

$z_0 = \sqrt{17}(\cos 82°58'55'' + i\sin 82°58'55'')$

$z_1 = \sqrt{17}(\cos 172°58'55'' + i\sin 172°58'55'')$

$z_2 = \sqrt{17}(\cos 262°58'55'' + i\sin 262°58'55'')$

$z_3 = \sqrt{17}(\cos 352°58'55'' + i\sin 352°58'55'')$



$f(x) = x - \sqrt{x^2-2}$

OMENA

$x^2-2 \geq 0$

$x^2 \geq 2 \mid \sqrt{\ }$

$|x| \geq \sqrt{2}$

$x \in \langle -\infty, -\sqrt{2}] \cup [\sqrt{2}, +\infty \rangle$ ✓

SIMPTOTE

ERTIKALNE ASIMPTOTE NEMA

ORIZONTALNE

$\lim_{x \to +\infty} \left(x - \sqrt{x^2-2} \right) = [\infty - \infty] = \lim_{x \to +\infty} \frac{\left(x - \sqrt{x^2-2}\right)\left(x + \sqrt{x^2-2}\right)}{x + \sqrt{x^2-2}} =$

$= \lim_{x \to +\infty} \frac{x^2 - x^2 + 2}{x + \sqrt{x^2-2}} = \lim_{x \to +\infty} \frac{2}{x + \sqrt{x^2-2}} = \left[\frac{2}{\infty} \right] = 0$

$\lim_{x \to -\infty} \left(x - \sqrt{x^2-2} \right) = [-\infty - \infty] = -\infty$

HORIZONTALNA ASIMPTOTA S LIJEVA NE POSTOJI

PRAVAC $y = 0$ JE HORIZONTALNA ASIMPTOTA S DESNA ✓ KOSE?

GRAFA FUNKCIJE $f(x) = x - \sqrt{x^2-2}$

DRUGA DERIVACIJA

$f'(x) = 1 - \frac{2x}{2\sqrt{x^2-2}} = 1 - \frac{x}{\sqrt{x^2-2}}$

$f''(x) = -\frac{1 \cdot \sqrt{x^2-2} - x \cdot \frac{2x}{2\sqrt{x^2-2}}}{x^2-2} = -\frac{\sqrt{x^2-2} - \frac{x^2}{\sqrt{x^2-2}}}{x^2-2} = -\frac{\frac{x^2-2-x^2}{\sqrt{x^2-2}}}{x^2-2} = -\frac{2}{(x^2-2)^{\frac{3}{2}}}$ ✓

③ $g(x) = \ln(4-x^2)$ MARINO ĆOSIĆ

DOMENA

$4 - x^2 > 0$

$x^2 < 4 \mid \sqrt{}$

$|x| < 2$

$x \in \langle -2, 2 \rangle$ ✓

(NE)PARNOST

$g(-x) = \ln(4-(-x^2)) = \ln(4-x^2) = g(x)$

FUNKCIJA $g(x) = \ln(4-x^2)$ JE PARNA JER JE $g(-x) = g(x)$. ✓

ZAKRIVLJENOST

$g'(x) = \frac{1}{4-x^2} \cdot (-2x) = \frac{2x}{x^2-4}$

$g''(x) = \frac{2(x^2-4) - 2x \cdot (2x)}{(x^2-4)^2} = \frac{2x^2 - 8 - 4x^2}{(x^2-4)^2} = \frac{-2x^2-8}{(x^2-4)^2} = \frac{-2(x^2+4)}{(x^2-4)^2} = 0$

$x^2 + 4 = 0$

$x^2 = -4$

NEMA REALNIH RJEŠENJA

$(x^2-4)^2 \geq 0 \ \forall x \in \mathbb{R}$

$x^2 + 4 > 0 \ \forall x \in \mathbb{R}$

$\Rightarrow -2\frac{(x^2+4)}{(x^2-4)^2} < 0 \ \forall x \in \mathbb{R}$

FUNKCIJA $g(x) = \ln(4-x^2)$ JE KONKAVNA NA CIJELOM PODRUČJU DEFINICIJE. ✓

$$h(x) = \frac{x^2-2x-(2+1)}{x^2+1} = \frac{x^2-2x-3}{x^2+1}$$

DOMENA

$x^2+1 \neq 0$

$x^2 \neq -1 \quad \forall x \in \mathbb{R}$

$D(f) = \mathbb{R}$

NULTOČKE

$x^2-2x-3=0$

$x_{1,2} = \frac{2 \pm \sqrt{4+12}}{2}$

$x_1 = 3 \quad x_2 = -1$

$$h'(x) = \frac{(2x-2)(x^2+1)-(x^2-2x-3)\cdot(2x)}{(x^2+1)^2} =$$

$$= \frac{2x^3+2x-2x^2-2-2x^3+4x^2+6x}{(x^2+1)^2} =$$

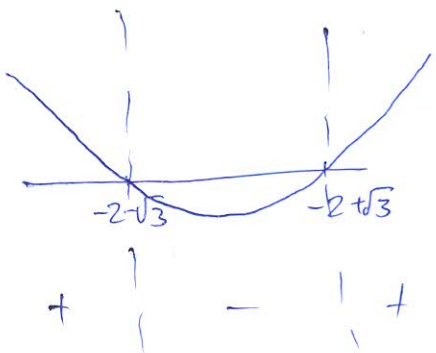
$$= \frac{2x^2+8x+2}{(x^2+1)^2}$$

$$\frac{2x^2+8x+2}{(x^2+1)^2} = 0$$

$2x^2+8x+2=0$

$x^2+4x+1=0$

$x_{1,2} = \frac{-4 \pm \sqrt{12}}{2 \cdot 2} = \frac{-4 \pm 2\sqrt{3}}{2 \cdot 2}$

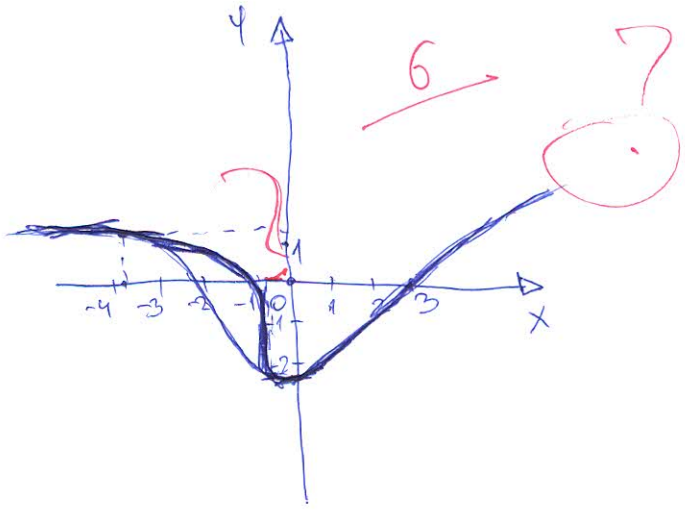


$x_1 = -2+\sqrt{3}$

$x_2 = -2-\sqrt{3}$

} STACIONALNE TOČKE

	$\langle -\infty, -2-\sqrt{3} \rangle$	$\langle -2+\sqrt{3} \rangle$	$\langle -2+\sqrt{3}, +\infty \rangle$
$h'(x)$	+	−	+
$h(x)$	↗	↘	↗



$h(-2-\sqrt{3}) \approx 1.232$

$h(-2+\sqrt{3}) \approx -2.232$

⑥ $\lim_{x\to\infty} \frac{e^x}{x} = \text{l'H} = \lim_{x\to\infty} \frac{e^x}{1} = \left[\frac{\infty}{1}\right] = \infty$

ZA $x = 10$

$\frac{e^x}{x}\Big|_{x=10} \approx 2202.65$

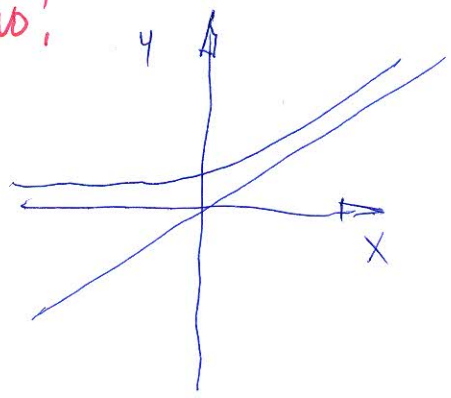
ZA $x = 100$

$\frac{e^x}{x}\Big|_{x=100} \approx 2.68 \cdot 10^{41}$

ZA $x = 1000$

$\frac{e^x}{x}\Big|_{x=1000} \approx 1.97 \cdot 10^{431}$

BRAVO!



e^x BRŽE $\Rightarrow +\infty$

MARINO

$$A_p = \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 2 & 5 & -1 & 2 & 2 \\ 3 & -1 & -2 & 1 & 3 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & -7 & 1 & -2 & -3 \\ 0 & -3 & 4 & -6 & 1 \end{array}\right] \begin{array}{l} \text{II} - 2\text{I} \\ \text{III} - 3\text{I} \\ \text{IV} - \text{I} \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 8 & -2 & -17 \\ 0 & 0 & -7 & -6 & -5 \end{array}\right] \begin{array}{l} \text{III} + 7\text{II} \\ \text{IV} + 3\text{II} \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 8 & -2 & -17 \\ 0 & 0 & 56 & -48 & -40 \end{array}\right] 8 \cdot \text{IV} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 8 & -2 & -17 \\ 0 & 0 & 0 & -34 & 79 \end{array}\right] \text{IV} - 7\text{III}$$

$r(A) = r(A_p) \Rightarrow$ RJEŠENJE POSTOJI

$r(A) = 4$, $u = 4$ } RJEŠENJE JE JEDINSTVENO

$-34u = 79$

$$u = -\frac{79}{34}$$

$8z - 2u = -17$

$$z = -\frac{46}{17}$$

$x + 2y - z + u = 2$

$$x = \frac{7}{34}$$

RJEŠENJE:

$$x = \begin{bmatrix} \frac{7}{34} \\ \frac{12}{17} \\ -\frac{46}{17} \\ -\frac{79}{34} \end{bmatrix}$$

✓