

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

H2

IME I PREZIME: **MARINO ĐOSIĆ**

BROJ INDEKSA: **0269080599**

- Riješiti jednačbu:  $z^4 - (4 - i)^2 = 0$ . Prikaži rješenja u kompleksnoj ravnini!
- Odrediti domenu, sve asimptote i drugu derivaciju funkcije  $f(x) = x - \sqrt{x^2 - 2}$ .
- Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije  $g(x) = \ln(4 - x^2)$ .
- Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije  $h(x) = \frac{x^2 - 2x - (2 + 1)}{x^2 + 1}$ . Ne treba ispitivati zakrivljenost jer se izraz komplicira.
- Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje:

12+3 = 7

5+15+5 = 25

5+5+10 = 20

20(graf) = 6

15

$$\begin{aligned} x + 2y - z + u &= 2 \\ 2x + 5y - z + 2u &= 2 \\ 3x - y - 2z + u &= 3 \\ x - y + 3z - 5u &= 3 \end{aligned}$$

- Izračunati i provjeriti uvrštavanjem:  $\lim_{x \rightarrow \infty} \frac{e^x}{x}$ .

5

Ukupno:

**78**

①  $z^4 - (4 - i)^2 = 0$        $\tan \varphi = \frac{8}{15}$        $\varphi_0 = \frac{\varphi}{n} = 82^\circ 58' 55''$

$z^4 = (4 - i)^2$        $r = \sqrt{15^2 + 8^2} = 17$        $\Delta \varphi = 90^\circ$

$z^4 = 16 - 8i - 1$

$z^4 = 15 - 8i$

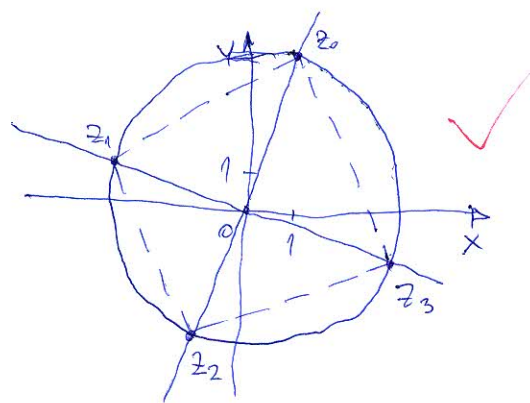
$z^4 = 17(\cos 331^\circ 55' 39'' + i \sin 331^\circ 55' 39'')$

$z_0 = \sqrt[4]{17} (\cos 82^\circ 58' 55'' + i \sin 82^\circ 58' 55'')$

$z_1 = \sqrt[4]{17} (\cos 172^\circ 58' 55'' + i \sin 172^\circ 58' 55'')$

$z_2 = \sqrt[4]{17} (\cos 262^\circ 58' 55'' + i \sin 262^\circ 58' 55'')$

$z_3 = \sqrt[4]{17} (\cos 352^\circ 58' 55'' + i \sin 352^\circ 58' 55'')$



$$f(x) = x - \sqrt{x^2 - 2}$$

DOMENA

$$x^2 - 2 \geq 0$$

$$x^2 \geq 2 \quad | \sqrt{\phantom{x}}$$

$$|x| \geq \sqrt{2}$$

$$x \in (-\infty, -\sqrt{2}] \cup [\sqrt{2}, +\infty) \quad \checkmark$$

ASIMPTOTE

VERTIKALNE ASIMPTOTE NEMA

HORIZONTALNE

$$\lim_{x \rightarrow +\infty} (x - \sqrt{x^2 - 2}) = [\infty - \infty] = \lim_{x \rightarrow +\infty} \frac{(x - \sqrt{x^2 - 2})(x + \sqrt{x^2 - 2})}{x + \sqrt{x^2 - 2}} =$$

$$= \lim_{x \rightarrow +\infty} \frac{x^2 - x^2 + 2}{x + \sqrt{x^2 - 2}} = \lim_{x \rightarrow +\infty} \frac{2}{x + \sqrt{x^2 - 2}} = \left[ \frac{2}{\infty} \right] = 0$$

$$\lim_{x \rightarrow -\infty} (x - \sqrt{x^2 - 2}) = [-\infty - \infty] = -\infty$$

HORIZONTALNA ASIMPTOTA S LIJEVA NE POSTOJI

PRAVAK  $y = 0$  JE HORIZONTALNA ASIMPTOTA S DESNA  $\checkmark$

KOSE?

GRAFA FUNKCIJE  $f(x) = x - \sqrt{x^2 - 2}$

DRUGA DERIVACIJA

$$f'(x) = 1 - \frac{2x}{2\sqrt{x^2 - 2}} = 1 - \frac{x}{\sqrt{x^2 - 2}}$$

$$f''(x) = - \frac{1 \cdot \sqrt{x^2 - 2} - x \cdot \frac{2x}{2\sqrt{x^2 - 2}}}{x^2 - 2} = - \frac{\sqrt{x^2 - 2} - \frac{x^2}{\sqrt{x^2 - 2}}}{x^2 - 2} = - \frac{\frac{x^2 - 2 - x^2}{\sqrt{x^2 - 2}}}{x^2 - 2} = - \frac{2}{(x^2 - 2)^{\frac{3}{2}}} \quad \checkmark$$

$$③ g(x) = \ln(4-x^2)$$

MARINO ĐOSIĆ

### DOMENA

$$4-x^2 > 0$$

$$x^2 < 4 \quad | \sqrt{\quad}$$

$$|x| < 2$$

$$x \in (-2, 2)$$



### (NE) PARNOST

$$g(-x) = \ln(4-(-x^2)) = \ln(4-x^2) = g(x)$$

FUNKCIJA  $g(x) = \ln(4-x^2)$  JE PARNA JER JE  $g(x) = g(x)$



### ZAKRIVLJENOST

$$g'(x) = \frac{1}{4-x^2} \cdot (-2x) = \frac{-2x}{4-x^2}$$

$$g''(x) = \frac{2(x^2-4) - 2x \cdot (-2x)}{(4-x^2)^2} = \frac{2x^2-8+4x^2}{(4-x^2)^2} = \frac{6x^2-8}{(4-x^2)^2} = \frac{-2(x^2+4)}{(4-x^2)^2} = 0$$

$$x^2+4=0$$

$$(x^2-4)^2 > 0 \quad \forall x \in \mathbb{R}$$

$$x^2+4 > 0 \quad \forall x \in \mathbb{R}$$

$$\left. \begin{array}{l} (x^2-4)^2 > 0 \quad \forall x \in \mathbb{R} \\ x^2+4 > 0 \quad \forall x \in \mathbb{R} \end{array} \right\} \Rightarrow -2 \frac{(x^2+4)}{(4-x^2)^2} < 0$$

$x^2 = -4$   
NEMA REALNIH  
RJEŠENJA

FUNKCIJA  $g(x) = \ln(4-x^2)$  JE KONKAVNA NA CJELOM PODRUČJU DEFINICIJE.

$\forall x \in \mathbb{R}$



$$h(x) = \frac{x^2 - 2x - (2+1)}{x^2 + 1} = \frac{x^2 - 2x - 3}{x^2 + 1}$$

DOMENA

$$x^2 + 1 \neq 0$$

$$x^2 \neq -1 \quad \forall x \in \mathbb{R}$$

$$D(f) = \mathbb{R}$$

MULTIPLIKATION

$$x^2 - 2x - 3 = 0$$

$$x_{1,2} = \frac{2 \pm \sqrt{4 + 12}}{2}$$

$$x_1 = 3 \quad x_2 = -1$$

$$h'(x) = \frac{(2x-2)(x^2+1) - (x^2-2x-3) \cdot (2x)}{(x^2+1)^2} =$$

$$= \frac{2x^3 + 2x - 2x^3 - 2 - 2x^3 + 4x^2 + 6x}{(x^2+1)^2} =$$

$$= \frac{2x^2 + 8x + 2}{(x^2+1)^2}$$

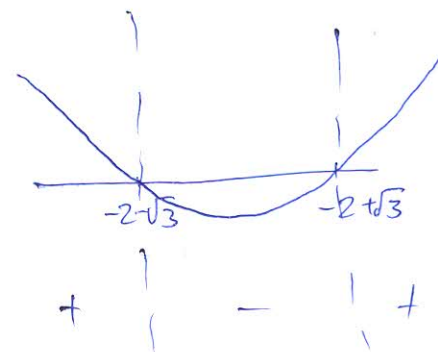
$$\frac{2x^2 + 8x + 2}{(x^2+1)^2} = 0$$

$$2x^2 + 8x + 2 = 0$$

$$x^2 + 4x + 1 = 0$$

$$x_{1,2} = \frac{-4 \pm \sqrt{12}}{2 \cdot 2} = \frac{-4 \pm 2\sqrt{3}}{2 \cdot 2}$$

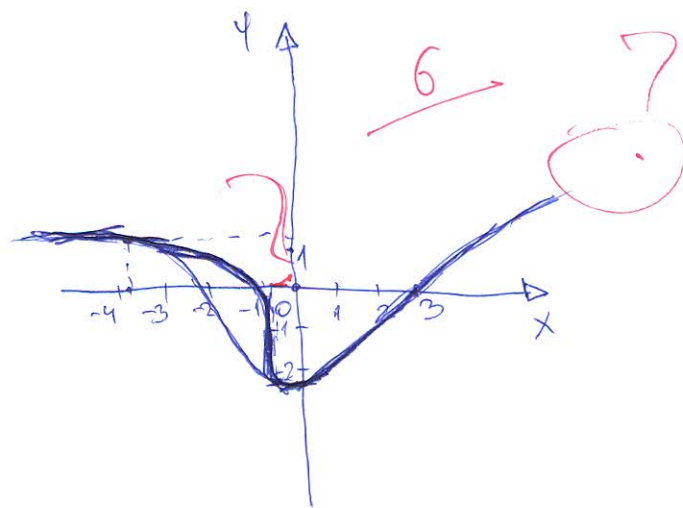
$$\left. \begin{array}{l} x_1 = -2 + \sqrt{3} \\ x_2 = -2 - \sqrt{3} \end{array} \right\} \begin{array}{l} \text{STATIONÄRE} \\ \text{PUNKTE} \end{array}$$



	$\langle -\infty, -2-\sqrt{3} \rangle$	$\langle -2-\sqrt{3}, -2+\sqrt{3} \rangle$	$\langle -2+\sqrt{3}, +\infty \rangle$
$h'(x)$	+	-	+
$h(x)$	↗	↘	↗

$$h(-2-\sqrt{3}) \approx 1.232$$

$$h(-2+\sqrt{3}) \approx -2.232$$



$$(6) \lim_{x \rightarrow \infty} \frac{e^x}{x} = \infty = \lim_{x \rightarrow \infty} \frac{e^x}{1} = \left[ \frac{\infty}{1} \right] = \infty$$

$$ZA \quad x=10$$

$$\frac{e^x}{x} \Bigg|_{x=10} \approx 2202.65$$

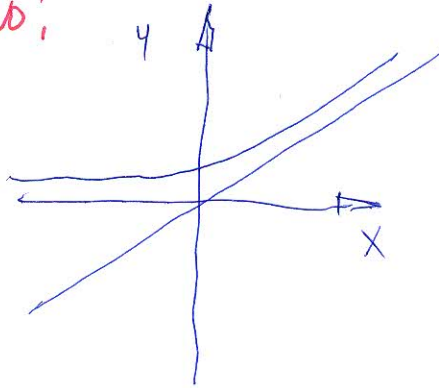
$$ZA \quad x=100$$

$$\frac{e^x}{x} \Bigg|_{x=100} \approx 2.68 \cdot 10^{44}$$

$$ZA \quad x=1000$$

$$\frac{e^x}{x} \Bigg|_{x=1000} \approx 1.97 \cdot 10^{431}$$

BRZU!



$e^x$  BRZE  
 $\rightarrow +\infty$



$$A_p = \begin{bmatrix} 1 & 2 & -1 & 1 & | & 2 \\ 2 & 5 & -1 & 2 & | & 2 \\ 3 & -1 & -2 & 1 & | & 3 \\ 1 & -1 & 3 & -5 & | & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & | & 2 \\ 0 & 1 & 1 & 0 & | & -2 \\ 0 & -7 & 1 & -2 & | & -3 \\ 0 & -3 & 4 & -6 & | & 1 \end{bmatrix} \begin{array}{l} \text{II} - 2\text{I} \\ \text{III} - 3\text{I} \\ \text{IV} - \text{I} \end{array} \sim$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 & | & 2 \\ 0 & 1 & 1 & 0 & | & -2 \\ 0 & 0 & 8 & -2 & | & -17 \\ 0 & 0 & 7 & -6 & | & -5 \end{bmatrix} \begin{array}{l} \\ \text{III} + 7\text{II} \\ \text{IV} + 3\text{II} \end{array} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & | & 2 \\ 0 & 1 & 1 & 0 & | & -2 \\ 0 & 0 & 8 & -2 & | & -17 \\ 0 & 0 & 56 & -48 & | & -40 \end{bmatrix} \begin{array}{l} \\ \\ 8 \cdot \text{IV} \end{array} \sim$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 & | & 2 \\ 0 & 1 & 1 & 0 & | & -2 \\ 0 & 0 & 8 & -2 & | & -17 \\ 0 & 0 & 0 & -34 & | & 79 \end{bmatrix} \begin{array}{l} \\ \\ \text{IV} - 7\text{III} \end{array}$$

$r(A) = r(A_p) \Rightarrow$  RJESENJE POSTOJI

$r(A) = 4$   
 $u = 4$  } RJESENJE JE JEDINSTVENO

$-34u = 79$

$$u = -\frac{79}{34}$$

$8z - 2u = -17$

$$z = -\frac{46}{17}$$

$x + 2y - z + u = 2$

$$x = \frac{7}{34}$$

RJESENJE:

$$x = \begin{bmatrix} \frac{7}{34} \\ \frac{12}{17} \\ -\frac{46}{17} \\ -\frac{79}{34} \end{bmatrix}$$



IME I PREZIME:

BROJ INDEKSA:

DINO MARNOV

0269075721

- Riješiti jednadžbu:  $z^4 - (4 - i)^2 = 0$ . Prikaži rješenja u kompleksnoj ravni! 12+3 ~~3~~ 15
- Odrediti domenu, sve asimptote i drugu derivaciju funkcije  $f(x) = x - \sqrt{x^2 - 2}$ . 5+15+5 25
- Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije  $g(x) = \ln(4 - x^2)$ . 5+5+10 20
- Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije  $h(x) = \frac{x^2 - 2x - (2 + 1)}{x^2 + 1}$ . Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
- Gaussovom metodom riješiti matricni sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x + 2y - z + u &= 2 \\ 2x + 5y - z + 2u &= 2 \\ 3x - y - 2z + u &= 3 \\ x - y + 3z - 5u &= 3 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem:  $\lim_{x \rightarrow \infty} \frac{e^x}{x}$ . 5

Ukupno:

50

3)  $g(x) = \ln(4 - x^2)$

Pamet:  $g(-x) = \ln(4 - (-x)^2)$

$g(-x) = \ln(4 - x^2) = g(x)$  FJA, je parna.

Zakrivljenost:  $g(x) = \ln(4 - x^2)$

$g'(x) = \frac{1}{4 - x^2} \cdot (-2x) = \frac{-2x}{4 - x^2}$

$g''(x) = \frac{-2(4 - x^2) - (-2x)(-2x)}{(4 - x^2)^2}$

$g''(x) = \frac{-8 + 2x^2 - 4x^2}{(4 - x^2)^2} = \frac{-2x^2 - 8}{(4 - x^2)^2}$

Domeno:  $4 - x^2 > 0$

$-x^2 > -4 / \cdot (-1)$

$x^2 < 4$

$x \in (-2, 2) \checkmark$

5)  $\lim_{x \rightarrow \infty} \frac{e^x}{x} = \left[ \frac{e^x}{x} = \frac{\infty}{\infty} \right]^{L'H} = \lim_{x \rightarrow \infty} \frac{e^x}{1} = \lim_{x \rightarrow \infty} e^x = \infty \checkmark$

$\frac{-2x^2 - 8}{(4 - x^2)^2} < 0$  FJA, je konkavna  $\checkmark$

2)  $f(x) = x - \sqrt{x^2 - 2}$

$f'(x) = 1 - \frac{1}{\sqrt{x^2 - 2}} \cdot 2x = 1 - \frac{2x}{2\sqrt{x^2 - 2}} = 1 - \frac{x}{\sqrt{x^2 - 2}}$

$f''(x) = \frac{1 - \sqrt{x^2 - 2} - x \cdot \frac{1}{2\sqrt{x^2 - 2}} \cdot 2x}{x^2 - 2}$

$f''(x) = \frac{-\sqrt{x^2 - 2} + \frac{x^2}{\sqrt{x^2 - 2}}}{x^2 - 2} = \frac{-x^2 + 2 + x^2}{\sqrt{x^2 - 2}(x^2 - 2)} = \frac{2}{(\sqrt{x^2 - 2})^3}$

Domeno:  $f(x) = x - \sqrt{x^2 - 2}$

$x^2 - 2 \geq 0$

$x^2 \geq 2$

$x \in (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$

$\Rightarrow$

Vertikalne asimptote:  $\lim_{x \rightarrow \sqrt{2}} (x - \sqrt{x^2 - 2}) = -\sqrt{2} - 0 = -\sqrt{2}$

Horizontalne:  $\lim_{x \rightarrow +\infty} (x - \sqrt{x^2 - 2}) = \sqrt{2} - 0 = \sqrt{2}$

$$\lim_{x \rightarrow \infty} (x - \sqrt{x^2 - 2}) \cdot \frac{x + \sqrt{x^2 - 2}}{x + \sqrt{x^2 - 2}} = \lim_{x \rightarrow +\infty} \frac{-x^2 - x^2 + 2}{x + \sqrt{x^2 - 2}}$$

$$\lim_{x \rightarrow \infty} \frac{2}{2x} = \frac{0}{2} = 0$$

KOJA JE L.H.A?  
L.H.A?

①  $z^4 - (4 - i)^2 = 0$

$$z^4 - (16 - 8i + i^2) = 0$$

$$z^4 - (16 - 8i - 1) = 0$$

$$z^4 - 15 + 8i = 0$$

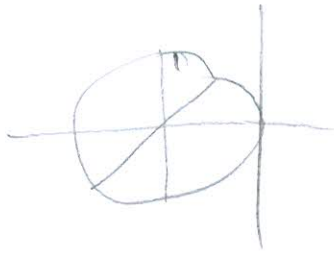
$$z^4 = 15 - 8i$$

$$r = \sqrt{15^2 + (-8)^2} = 17$$

$$\arg z = \frac{-8}{15}$$

$$\theta = 28.1^\circ$$

$$\phi = 208.1^\circ$$



$$z = 17 (\cos 208.1 + i \sin 208.1)$$

$$k = 0$$

$$z_1 = \sqrt[4]{17} (\cos 208.1 + i \sin 208.1)$$

$$= \sqrt[4]{17} (\cos 52.1 + i \sin 52.1)$$

4 RJEŠENJA?



$$2x + 3y - z + 2u = 2$$

$$3x - y - 2z + 4u = 3$$

$$x - y + 3z - 5u = 3$$

Dr. med. ind. 02 69 07521

$$\begin{bmatrix} 1 & 2 & -1 & 1 & : & 2 \\ 2 & 5 & -1 & 2 & : & 2 \\ 3 & -1 & 2 & 1 & : & 3 \\ 1 & -1 & 3 & 5 & : & 3 \end{bmatrix} \begin{array}{l} \text{II} - 2\text{I} \\ \text{III} - 3\text{I} \\ \text{IV} - \text{I} \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 1 & : & 2 \\ 0 & 1 & 1 & -3 & : & -2 \\ 0 & -7 & 1 & -2 & : & -3 \\ 0 & -3 & 4 & -6 & : & 1 \end{bmatrix} \begin{array}{l} \text{III} + 7\text{II} \\ \text{IV} + 3\text{II} \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 1 & : & 2 \\ 0 & 1 & 1 & -3 & : & -2 \\ 0 & 0 & 8 & -2 & : & -17 \\ 0 & 0 & 7 & -6 & : & -5 \end{bmatrix} \begin{array}{l} 1.7 \\ 1.8 \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 1 & : & 2 \\ 0 & 1 & 1 & -3 & : & -2 \\ 0 & 0 & 56 & -18 & : & -119 \\ 0 & 0 & 56 & -48 & : & -40 \end{bmatrix} \text{IV} - \text{III}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 1 & : & 2 \\ 0 & 1 & 1 & -3 & : & -2 \\ 0 & 0 & 56 & -18 & : & -119 \\ 0 & 0 & 0 & -34 & : & -79 \end{bmatrix}$$

$$-34u = -79 \implies u = \frac{-79}{-34} = \frac{79}{34}$$

$$56z + \frac{1106}{34} = -119 \implies 56z = -\frac{2576}{17}$$

$$y + \frac{-46}{17} = -2 \implies y = \frac{12}{17}$$

$$x + 2 \cdot \frac{12}{17} + \frac{46}{17} + \frac{-79}{34} = 2$$

$$x = 2 - \frac{24}{17} - \frac{46}{17} + \frac{79}{34}$$

$$\boxed{x = \frac{7}{34}}$$

Prüfung:  $\frac{7}{34} + \frac{24}{17} + \frac{46}{17} - \frac{79}{34} = 2$

$$\boxed{2=2}$$

9)  $h(x) = \frac{x^2 - 2x - (2+1)}{x^2 + 1}$

1) Domain:  $x \in \mathbb{R}$

2) Nullstelle:  $x^2 - 2x - 3 = 0$

$$x_{1/2} = \frac{2 \pm \sqrt{16}}{2} \implies \boxed{x_1 = 3} \quad \boxed{x_2 = -1}$$

Asymptote, Vertikale keine

Horizontale,  $\lim_{x \rightarrow \infty} \frac{x^2 - 2x - 3}{x^2 + 1} \stackrel{1: x^2}{=} 1$

Derivata:  $f'(x) = \frac{(2x-2) \cdot (x^2+1) - (x^2-2x-3) \cdot (2x)}{(x^2+1)^2}$

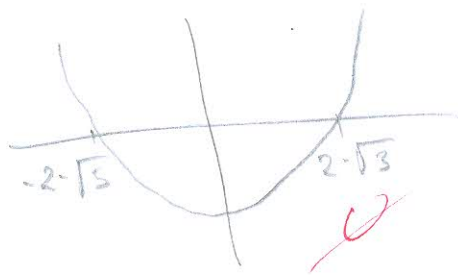
$$f'(x) = \frac{2x^3 + 2x - 2x^3 - 2 - (2x^3 - 4x^2 - 6x)}{(x^2+1)^2} = \frac{2x^3 - 2x - 2x^3 - 2 - 2x^3 + 4x^2 + 6x}{(x^2+1)^2}$$

$$= \frac{2x^2 + 8x - 2}{(x^2+1)^2} = 0$$

$$2x^2 + 8x - 2 = 0$$

$$x_{1/2} = \frac{-8 \pm \sqrt{80}}{4}$$

$$x_1 = -2 + \sqrt{5} \quad x_2 = -2 - \sqrt{5}$$



$x \in (-\infty, -2 - \sqrt{5}) \cup (2 - \sqrt{5}, +\infty)$  Funkcija roste!  
 $x \in (-2 + \sqrt{5}, 2 + \sqrt{5})$  Funkcija pada ↓

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME: *FILIP FLORANI*

BROJ INDEKSA: *17-2-0308-2013*

H2

1. Riješiti jednačbu:  $z^4 - (4 - i)^2 = 0$ . *Prikaži rješenja u kompleksnoj ravnini!*

12+3

2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije  $f(x) = x - \sqrt{x^2 - 2}$ .

~~5+15+5~~ *7*

3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije  $g(x) = \ln(4 - x^2)$ .

~~5+5+10~~

4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije  $h(x) = \frac{x^2 - 2x - (2 + 1)}{x^2 + 1}$ . Ne treba ispitivati zakrivljenost jer se izraz komplicira.

~~20(graf)~~ *15*

5. Gaussovom metodom riješiti matricni sustav i obavezno provjeri rješenje:

15

$$x + 2y - z + u = 2$$

$$2x + 5y - z + 2u = 2$$

$$3x - y - 2z + u = 3$$

$$x - y + 3z - 5u = 3$$

6. Izračunati i provjeriti uvrštavanjem:  $\lim_{x \rightarrow \infty} \frac{e^x}{x}$ .

5

Ukupno:

*47*



$$h(x) = \frac{2x^2 + 8x - 2}{(x^2 + 1)^2}$$

$$E_1 \left( \frac{-8 - 4\sqrt{5}}{4}, -5 - \sqrt{5} \right)$$

$$E_2 \left( \frac{-8 + 4\sqrt{5}}{4}, -1 - \sqrt{5} \right)$$

$$\underline{10 + 6\sqrt{5}}$$

$-x$	$-4,23$	$0,23$	$+x$
$h(x)$	$\nearrow$	$\searrow$	$\nearrow$

$$E_1(-4,23, -7,23)$$

$$E_2(0,23, -3,23)$$

$$h(-2 - \sqrt{5}) = ?$$

$$= \dots \approx \frac{(-4,23)^2 + 4,23 \cdot 2 - 3}{(4,23)^2 + 1}$$

$$\approx 1,012 > 1$$

$$2x^2 + 8x - 2 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{-8 \pm \sqrt{8^2 - 4 \cdot 2 \cdot (-2)}}{4}$$

$$x_{1,2} = \frac{-8 \pm \sqrt{64 + 16}}{4}$$

$$x_{1,2} = \frac{-8 \pm 4\sqrt{5}}{4}$$

$$x_1 = -2 + \sqrt{5} \approx 0,236$$

$$x_2 = -2 - \sqrt{5} \approx -4,23$$

$$\left| \frac{-8 \pm 4\sqrt{5}}{4} \right|$$

$$x_1 = \frac{-8 + 4\sqrt{5}}{4}$$

$$x_2 = \frac{-8 - 4\sqrt{5}}{4}$$

$$x_2 = \frac{-8 - 4\sqrt{5}}{4}$$



$$h(x) = \frac{x^2 - 2x - 3}{x^2 + 1}$$

$$\lim_{x \rightarrow \infty} \frac{x^2 - 2x - 3}{x^2 + 1} \begin{matrix} / : x^2 \\ / : x^2 \end{matrix} = \frac{1 - 0 - 0}{1 + 0} = 1$$

HORIZONTALNA ASIMPTOTA

$$\boxed{y = 1}$$

$$2 \cdot 8^2 + 4 \cdot 3 - 30$$

$$2 \cdot 9 + 12 - 18$$

$$2x^2 + 4x - 30 = 0$$

$$-4 \pm \sqrt{4^2 - 4 \cdot 2 \cdot (-30)}$$

4

$$\frac{-4 \pm \sqrt{16}}{4} = \frac{-4 \pm 4}{4}$$

$$\sqrt{16 + 240}$$

$$\frac{16}{-8} = \frac{16}{4}$$

$$\frac{-4 \pm 16}{4} =$$

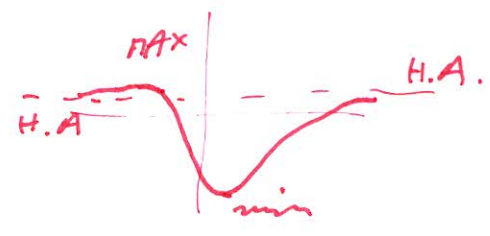
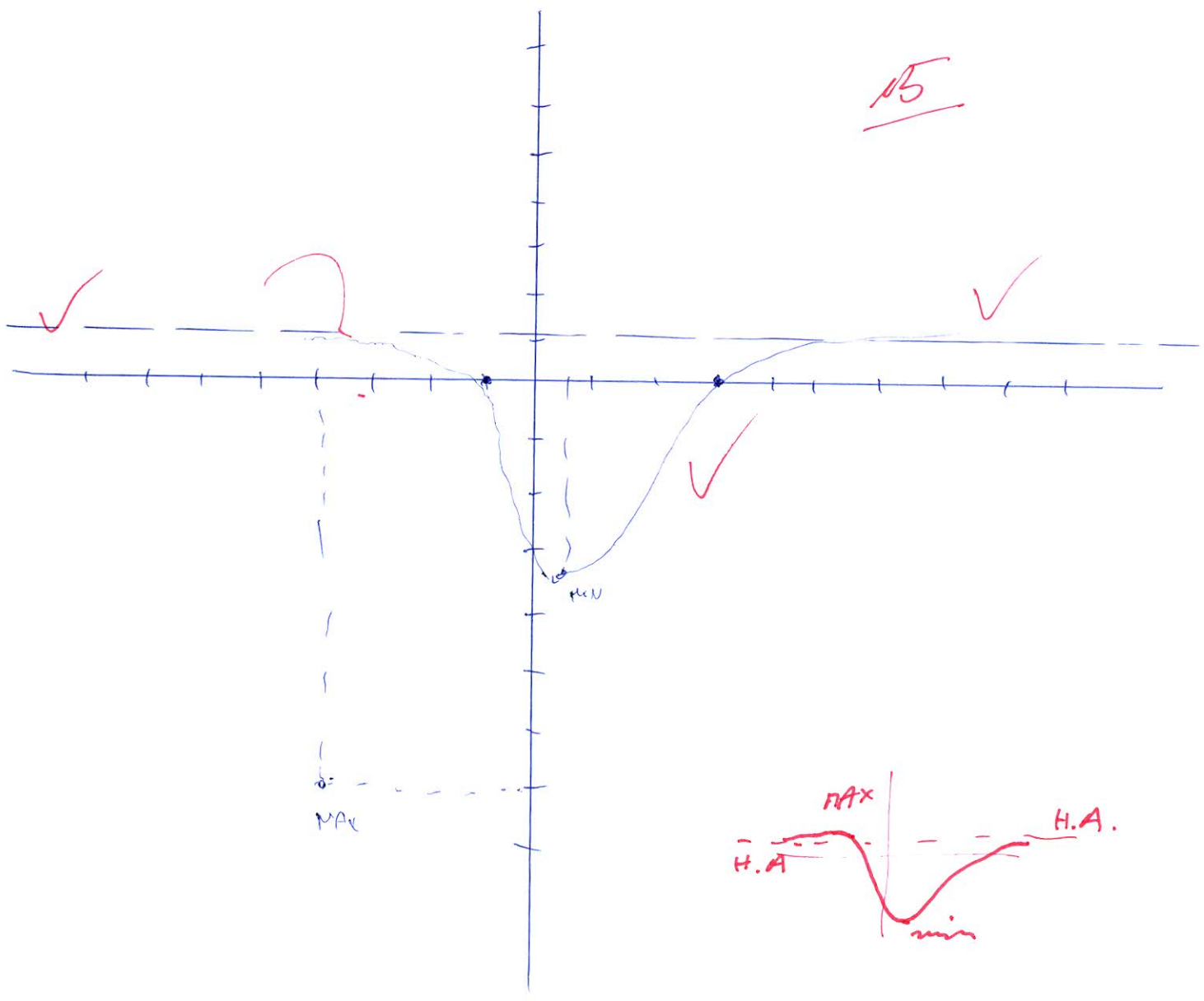


4 •  $D(f) = \mathbb{R}$

- NUL TOČKE  
 $N_1 (3, 0)$   
 $N_2 (-1, 0)$

• ASIMPTOTE  
 $|y = 1|$  HORIZONTALNA  
 OBOSTRANA

- EXTREMI  
 $E_1 \left( \frac{-8 - 4\sqrt{5}}{4}, -5 - \sqrt{5} \right)$   
 $E_2 \left( \frac{-8 + 4\sqrt{5}}{4}, -1 - \sqrt{5} \right)$



FILIP  
 FLORANI

$$\lim_{x \rightarrow \infty} \frac{e^x}{x}$$

3

$$g(x) = \ln(4-x^2)$$

• DOMENA

$$4-x^2 > 0$$

$$-x^2 > -4 \quad | \cdot (-1)$$

$$x^2 < 4 \quad | \sqrt{\quad}$$

~~$x < \pm 2$~~

$$x < \pm 2$$



$$D(g) = \langle -2, 2 \rangle$$

• (NE) PARNOST

$$g(x) = \ln(4-x^2)$$

$$g(-x) = \ln(4-(-x)^2)$$

$$g(-x) = \ln(4-x^2) = g(x)$$

FUNKCIJA JE PARNA ✓

• ZAKRIVČENOST

$$g(x) = \ln(4-x^2)$$

$$g'(x) = \frac{1}{4-x^2} \cdot (-2x)$$


$$g'(x) = \frac{-2x}{4-x^2}$$

$$g''(x) = \frac{(-2x)'(4-x^2) - (-2x)(4-x^2)'}{(4-x^2)^2}$$

$$g''(x) = \frac{-2(4-x^2) - (-2x)(-2x)}{(4-x^2)^2}$$

$$g''(x) = \frac{-8 + 2x^2 - 4x^2}{(4-x^2)^2}$$

$$g''(x) = \frac{-8 - 2x^2}{(4-x^2)^2}$$

	-2		2
$g''$		-	
$g'$			

FUNKCIJA JE KONKAVNA ✓

2

f(x) = x - sqrt(x^2 - 2)

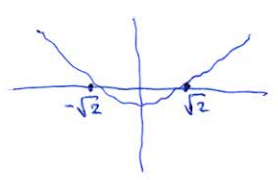
1) DOMENA FUNKCIJE

x^2 - 2 >= 0

x^2 >= 2 | sqrt

x >= +/- sqrt(2)

D(f) = (-inf, -sqrt(2)] union [sqrt(2), +inf) ✓



ASIMPTOTE =>

1) NEMA VERTIKALNIH ASIMPTOTA !

2) HORIZONTALNA DESNA ASIMPTOTA

y = 0 ✓

lim\_{x -> +inf} x - sqrt(x^2 - 2) = inf - inf

lim\_{x -> inf} x - sqrt(x^2 - 2) = lim\_{x -> inf} (x^2 - x^2 - 2) / (x + sqrt(x^2 - 2)) = lim\_{x -> inf} -2 / (x + sqrt(x^2 - 2))

lim\_{x -> inf} (-2/x) = 0 / 2 = 0

3) HORIZONTALNA LIJEVA ASIMPTOTA

y = 0 ✗

lim\_{x -> -inf} x - sqrt(x^2 - 2) [x -> -x, -inf -> +inf] = lim\_{x -> -inf} (-x - sqrt(x^2 - 2)) = -inf

lim\_{x -> inf} (x^2 - x^2 - 2) / (-x + sqrt(x^2 - 2)) = lim\_{x -> inf} -2 / inf = 0

DERIVACIJA =>

f(x) = x - sqrt(x^2 - 2)

f'(x) = 1 - 1/(2\*sqrt(x^2 - 2)) \* (2x)

f'' = ?

f'(x) = 1 - x / sqrt(x^2 - 2)

FILIP FLORANI

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

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JOSIP JANKOVIĆ

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7/2

1. Riješiti jednadžbu:  $z^4 - (4 - i)^2 = 0$ . Prikaži rješenja u kompleksnoj ravnini!

12+3

2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije  $f(x) = x - \sqrt{x^2 - 2}$ .

5+15+5 7

3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije  $g(x) = \ln(4 - x^2)$ .

5+5+10

4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije  $h(x) = \frac{x^2 - 2x - (2+1)}{x^2 + 1}$ . Ne treba ispitivati zakrivljenost jer se izraz komplicira.

20(graf)

5. Gaussovom metodom riješiti matricni sustav i obavezno provjeri rješenje:

15

$$x + 2y - z + u = 2$$

$$2x + 5y - z + 2u = 2$$

$$3x - y - 2z + u = 3$$

$$x - y + 3z - 5u = 3$$

6. Izračunati i provjeriti uvrštavanjem:  $\lim_{x \rightarrow \infty} \frac{e^x}{x}$ .

5

Ukupno:

42

$$\begin{aligned} 1. \quad z^4 - (4 - i)^2 &= 0 \\ &= z^4 - (16 - 8i + i^2) = 0 \\ &= z^4 - (16 - 8i - 1) = 0 \\ &= z^4 - (15 - 8i) = 0 \\ z^4 - 15 + 8i &= 0 \\ z^4 &= 15 - 8i \end{aligned}$$

$$\begin{aligned} x &= 15 \\ y &= -8 \end{aligned}$$

$$\begin{aligned} r &= \sqrt{x^2 + y^2} = \sqrt{15^2 + 8^2} \\ &= \sqrt{289} \\ &= 17 \end{aligned}$$

$$\begin{aligned} \arg z &= \frac{y}{x} = \frac{-8}{15} = -0,49 + 2\pi \\ &= 5,79 \end{aligned}$$

$$z_1 = \sqrt[4]{17} \left( \cos \frac{5,79}{4} + i \sin \frac{5,79}{4} \right)$$

$$z_1 = 2,03 (0,12 + 0,99i)$$

$$z_1 = 0,24 + 2i$$

$$T_1(0,24, 2)$$

$$z_2 = \sqrt[4]{17} \left( \cos \frac{5,79 + 2\pi}{4} + i \sin \frac{5,79 + 2\pi}{4} \right)$$

$$z_2 = 2,03 (-0,99 + 0,14i)$$

$$z_2 = -2 + 0,28i$$

$$T_2(-2, 0,28)$$

$$z_3 = \sqrt[4]{17} \left( \cos \frac{5,79 + 3\pi}{4} + i \sin \frac{5,79 + 3\pi}{4} \right)$$

$$z_3 = 2,03 (-0,11 - 0,99i)$$

$$z_3 = -0,22 - 2i$$

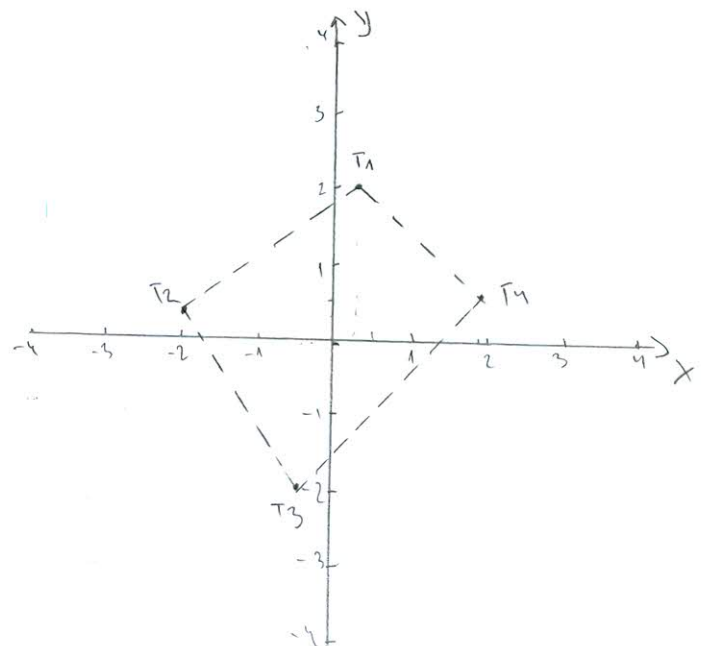
$$T_3(-0,22, -2)$$

$$z_4 = \sqrt[4]{17} \left( \cos \frac{5,79 + 4\pi}{4} + i \sin \frac{5,79 + 4\pi}{4} \right)$$

$$z_4 = 2,03 (0,98 + 0,21i)$$

$$z_4 = 1,99 + 0,43i$$

$$T_4(1,99, 0,43)$$



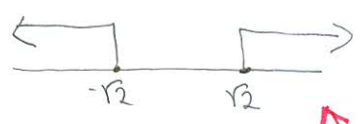


2)  $f(x) = x - \sqrt{x^2 - 2}$

$x^2 - 2 \geq 0$

$x^2 \geq 2$

$x \geq \sqrt{2}$



$D_f \in (-\infty, -\sqrt{2}] \cup [\sqrt{2}, +\infty)$  ✓

1. DER

$f'(x) = x' - (\sqrt{x^2 - 2})'$

$f'(x) = 1 - \frac{1}{2\sqrt{x^2 - 2}} \cdot (x^2 - 2)'$

$f'(x) = 1 - \frac{2x}{2\sqrt{x^2 - 2}}$

$f'(x) = 1 - \frac{x}{\sqrt{x^2 - 2}}$  ✓

2. DER

$f''(x) = 1 - \left( \frac{x}{\sqrt{x^2 - 2}} \right)' = - \left( \frac{x' \sqrt{x^2 - 2} - x (\sqrt{x^2 - 2})'}{(\sqrt{x^2 - 2})^2} \right)$   
 $= - \left( \frac{\sqrt{x^2 - 2} - x \cdot \frac{2x}{2\sqrt{x^2 - 2}}}{x^2 - 2} \right) = - \left( \frac{\sqrt{x^2 - 2} - \frac{x^2}{\sqrt{x^2 - 2}}}{x^2 - 2} \right)$

$= - \left( \frac{\sqrt{x^2 - 2} - x^2}{\sqrt{x^2 - 2} \cdot (x^2 - 2)} \right) = \left( \frac{-2}{\frac{\sqrt{x^2 - 2}}{x^2 - 2}} \right)$  ✓

$f''(x) = \frac{2}{(x^2 - 2)\sqrt{x^2 - 2}} = \frac{2\sqrt{x^2 - 2}}{(x^2 - 2)^2}$

V.A

$= \lim_{x \rightarrow \pm\sqrt{2}} (x - \sqrt{x^2 - 2}) \cdot \frac{x + \sqrt{x^2 - 2}}{x + \sqrt{x^2 - 2}}$

$= \lim_{x \rightarrow \pm\sqrt{2}} \frac{x^2 - x^2 + 2}{x + \sqrt{x^2 - 2}}$

$= \lim_{x \rightarrow \pm\sqrt{2}} \frac{2}{x + \sqrt{x^2 - 2}} \Rightarrow \neq \infty \rightarrow$  nema vertikalne asimptote ✓

$$\pm\infty \quad \left( x - \sqrt{x^2-2} \cdot \frac{x + \sqrt{x^2-2}}{x + \sqrt{x^2-2}} \right)$$

$$\lim_{x \rightarrow \pm\infty} \frac{2}{x + \sqrt{x^2-2}} \cdot x = \frac{0}{1} = 0$$

$\lim_{x \rightarrow -\infty} = \lim_{x \rightarrow +\infty} = 0 \Rightarrow$  pravac  $\frac{x=0}{y=0}$  je horizontalna asimptota, TO JE DESNO, A LIJEVO?

$$g(x) = \ln(4-x^2)$$

$$4-x^2 > 0$$

$$-x^2 > -4 \Rightarrow$$

$$\left\{ \begin{array}{l} D_f \ x \in \mathbb{R} \\ -2 < x < 2 \end{array} \right\}$$

2 | NEPAR

$$g(-x) = \ln(4-(-x)^2)$$

$$g(-x) = \ln(4-x^2)$$

$\Rightarrow g(-x) = g(x) \Rightarrow$  funkcija je parna

$$g'(x) = \frac{-2x}{4-x^2}$$

$$g''(x) = \frac{(-2x)'(4-x^2) - (-2x)(4-x^2)'}{(4-x^2)^2}$$

$$g''(x) = \frac{2 - (4-x^2) - 4x^2}{(4-x^2)^2} = \frac{-8 + 2x^3 - 4x^2}{(4-x^2)^2}$$

$$= \frac{-2x^2 - 8}{(4-x^2)^2} \Rightarrow g''(x) = 0 \Rightarrow \begin{array}{l} -2x^2 - 8 = 0 \\ -2x^2 = 8 \\ x^2 = -4 \end{array}$$

konkavnost i konveksnost

ne mogu odrediti



$$4) h(x) = \frac{x^2 - 2x - 3}{x^2 + 1} = \frac{x^2 - 2x - 3}{x^2 + 1}$$

$$D_f: x^2 + 1 \neq 0$$

$$\Rightarrow D_f \in \mathbb{R}$$

2. Par/impair

$$h(-x) = \frac{x^2 + 2x - 3}{x^2 + 1}$$

$$h(-x) \neq h(x) \neq -h(x) \Rightarrow$$

funkcija nije ni  
parna ni neparna

3.  $h(x) = 0 \Rightarrow$  multivarije

$$\frac{x^2 - 2x - 3}{x^2 + 1} = 0$$

$$x^2 - 2x - 3 = 0$$

$$x_{1,2} = \frac{2 \pm \sqrt{4 + 12}}{2}$$

$$x_{1,2} = \frac{2 \pm 4}{2}$$

$$x_1 = -1 \quad x_2 = 3$$

$$\frac{(x+1)(x-3)}{(x^2+1)} = 0$$

$$x_1 = -1 \quad x_2 = 3$$

$$4) h'(x) = \frac{(x^2 - 2x - 3)'(x^2 + 1)' - (x^2 - 2x - 3)(x^2 + 1)'}{(x^2 + 1)^2}$$

$$h'(x) = \frac{2x \cdot 3 + 2x - 2x^2 - 2 - 2x^3 - 4x^2 - 6x}{(x^2 + 1)^2}$$

$$h'(x) = \frac{-2(3x^2 + 2x + 1)}{(x^2 + 1)^2}$$

$\Downarrow$  nema rješenja u  $\mathbb{R}$

graf

x	-3	-2	-1	0	1	2	3
h(x)	12	1	0	-3	-2	$-\frac{3}{5}$	0

$$h(x) = \frac{x^2 - 2x - 3}{x^2 + 1}$$

$$h(0) = \frac{-3}{1} = -3$$

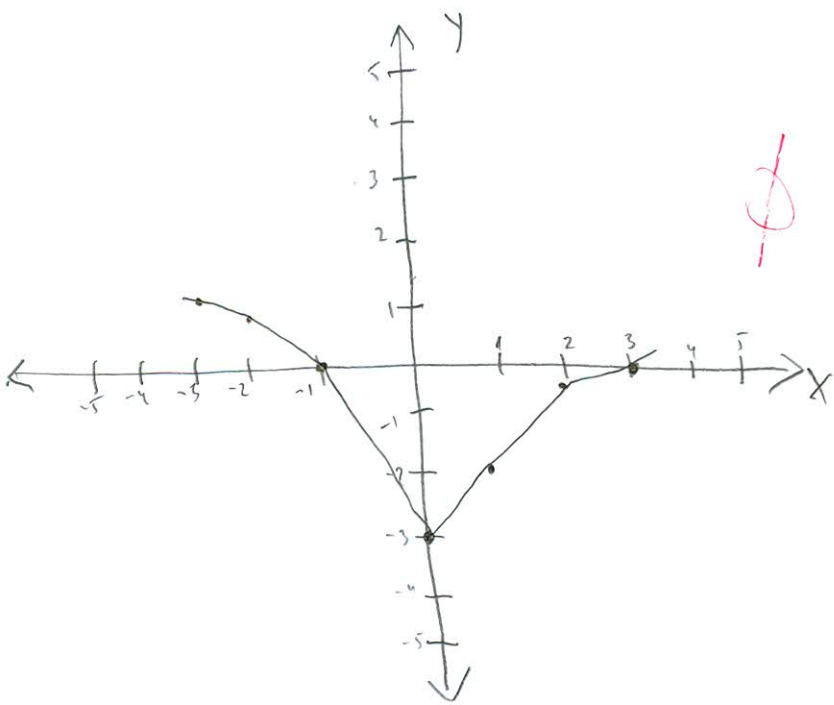
$$h(-2) = \frac{4 + 4 - 3}{5} = \frac{5}{5} = 1$$

$$h(1) = \frac{1 - 2 - 3}{2} = -2$$

$$h(-1) = \frac{1 + 2 - 3}{2} = 0$$

$$h(2) = \frac{4 - 4 - 3}{5} = -\frac{3}{5}$$

$$h(3) = \frac{9 - 6 - 3}{10} = 0$$



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POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

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H2

1. Riješiti jednadžbu:  $z^4 - (4 - i)^2 = 0$ . *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije  $f(x) = x - \sqrt{x^2 - 2}$ . 5+15+5
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije  $g(x) = \ln(4 - x^2)$ . 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije  $h(x) = \frac{x^2 - 2x - (2 + 1)}{x^2 + 1}$ . Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$x + 2y - z + u = 2$$

$$2x + 5y - z + 2u = 2$$

$$3x - y - 2z + u = 3$$

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6. Izračunati i provjeriti uvrštavanjem:  $\lim_{x \rightarrow \infty} \frac{e^x}{x}$ .

~~5~~

Ukupno:

40



$$f''(x) = \frac{(-\sqrt{x^2-2})(\sqrt{x^2-2}) - x^2}{\sqrt{x^2-2}}$$

$$f''(x) = \frac{-(x^2-2) - x^2}{\sqrt{x^2-2} \cdot (x^2-2)}$$

$$f''(x) = \frac{-x^2+2-x^2}{\sqrt{x^2-2} \cdot (x^2-2)}$$

$$f''(x) = \frac{-2x^2+2}{(x^2-2) \cdot \sqrt{x^2-2}}$$

$$f''(x) = \frac{-2x^2+2}{(x^2-2) \cdot \sqrt{x^2-2}}$$

⑥  $\lim_{x \rightarrow \infty} \frac{e^x}{x} =$

$$= \lim_{x \rightarrow \infty} \frac{[\lim_{x \rightarrow \infty} (1 + \frac{1}{x})^x]^x}{x}$$

$$e = \lim_{x \rightarrow \infty} (1 + \frac{1}{x})^x$$

⑦

$$i = 1$$

$$f''(x) = 0$$

$$f''(x) = 0$$

$$f''(x) = 0$$

$$2) f(x) = x - \sqrt{x^2 - 2}$$

Ante Mikelic

$$f'(x) = x' - (\sqrt{x^2 - 2})'$$

$$f'(x) = 1 - (u^{\frac{1}{2}})' \cdot u'$$

$$f'(x) = 1 - ((x^2 - 2)^{\frac{1}{2}})' \cdot (x^2 - 2)'$$

$$f'(x) = 1 - \frac{1}{2} (x^2 - 2)^{\frac{1}{2} - 1} \cdot 2x$$

$$f'(x) = 1 - x \cdot (x^2 - 2)^{-\frac{1}{2}}$$

$$f'(x) = 1 - \frac{x}{(x^2 - 2)^{\frac{1}{2}}}$$

$$f'(x) = 1 - \frac{x}{\sqrt{x^2 - 2}}$$

$$f''(x) = 1' - \left( \frac{x}{\sqrt{x^2 - 2}} \right)'$$

$$f''(x) = \frac{(-x)' \cdot (\sqrt{x^2 - 2}) - x \cdot (\sqrt{x^2 - 2})'}{(\sqrt{x^2 - 2})^2}$$

$$f''(x) = \frac{-1 \cdot \sqrt{x^2 - 2} - x \cdot \left[ ((x^2 - 2)^{\frac{1}{2}})' \cdot (x^2 - 2)' \right]}{x^2 - 2}$$

$$f''(x) = \frac{-\sqrt{x^2 - 2} - x \cdot \frac{1}{2} (x^2 - 2)^{-\frac{1}{2}} \cdot 2x}{x^2 - 2}$$

$$f''(x) = \frac{-\sqrt{x^2 - 2} - x^2 \cdot \frac{1}{(x^2 - 2)^{\frac{1}{2}}}}{x^2 - 2}$$

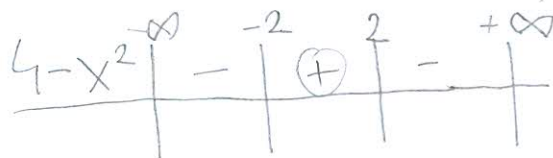
$$f''(x) = \frac{-\sqrt{x^2 - 2} - \frac{x^2}{\sqrt{x^2 - 2}}}{x^2 - 2}$$

$$b) g(x) = \ln(4-x)^2$$

$$4-x^2=0$$

$$x^2 = -4$$

$$x_1 = 2 \quad x_2 = -2$$



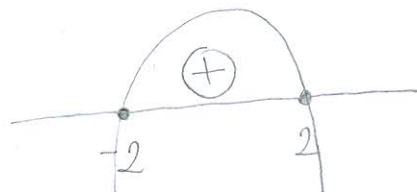
$$-x^2 + 4 = 0$$

$$a = -1$$

$$b = 0$$

$$c = 4$$

$$a = -1 \Rightarrow$$



$$DF: x \in \langle -2, 2 \rangle \checkmark$$

$$f(x) = \ln(4-x^2)$$

$$f(-x) = \ln(4-(-x)^2)$$

$$f(-x) = \ln(4-x^2)$$

$$f(x) = f(-x) \Rightarrow \text{funkcija je parna} \checkmark$$

$$c) f(x) = x - \sqrt{x^2 - 2}$$

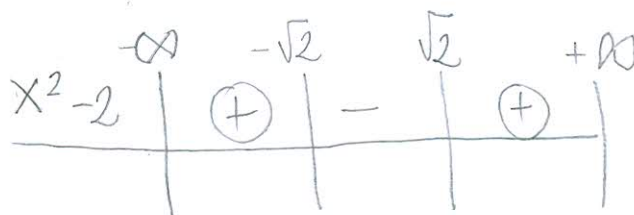
$$x^2 - 2 \geq 0$$

$$x^2 - 2 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{\pm \sqrt{8}}{2} = \pm \frac{2\sqrt{2}}{2}$$

$$x_1 = \sqrt{2} \quad x_2 = -\sqrt{2}$$



$$DF = x \in \langle -\infty, -\sqrt{2} \rangle \cup [\sqrt{2}, +\infty \rangle$$

$$⑤ \quad x + 2y - z + u = 2$$

$$2x + 5y - z + 2u = 2$$

$$3x - y - 2z + u = 3$$

$$x - y + 3z - 5u = 3$$

III-IV

$$A = \left[ \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 2 & 5 & -1 & 2 & 2 \\ 3 & -1 & -2 & 1 & 3 \\ 1 & -1 & 3 & -5 & 3 \end{array} \right] \begin{array}{l} \\ \text{II}-2\text{I} \\ \text{III}-3\text{I} \\ \text{IV}-\text{I} \end{array}$$

$$\sim \left[ \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & -7 & 1 & -2 & -3 \\ 0 & -3 & 4 & -6 & 1 \end{array} \right] \begin{array}{l} \text{I}-2\text{II} \\ \\ \text{III}+7\text{II} \\ \text{IV}+3\text{II} \end{array} \sim \left[ \begin{array}{cccc|c} 1 & 0 & -3 & 1 & 6 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 8 & -2 & -17 \\ 0 & 0 & 7 & -6 & -5 \end{array} \right]$$

$$\sim \left[ \begin{array}{cccc|c} 1 & 0 & -3 & 1 & 6 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 & -12 \\ 0 & 0 & 7 & -6 & -5 \end{array} \right] \begin{array}{l} \text{I}+3\text{III} \\ \text{II}-\text{III} \\ \\ \text{IV}-7\text{III} \end{array}$$

$$\sim \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 13 & -30 \\ 0 & 1 & 0 & -4 & 10 \\ 0 & 0 & 1 & 4 & -12 \\ 0 & 0 & 0 & -34 & 79 \end{array} \right] \begin{array}{l} \\ \\ \\ \text{Z: } (-34) \end{array} \sim \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 13 & 30 \\ 0 & 1 & 0 & -4 & 10 \\ 0 & 0 & 1 & 4 & -12 \\ 0 & 0 & 0 & 1 & -\frac{79}{34} \end{array} \right]$$

$$\sim \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{7}{34} \\ 0 & 1 & 0 & 0 & \frac{12}{17} \\ 0 & 0 & 1 & 0 & -\frac{46}{17} \\ 0 & 0 & 0 & 1 & -\frac{79}{34} \end{array} \right]$$

$$A = \begin{bmatrix} \frac{7}{34} \\ \frac{12}{17} \\ -\frac{46}{17} \\ -\frac{79}{34} \end{bmatrix} \quad \begin{array}{l} x = \frac{7}{34} \\ y = \frac{12}{17} \\ z = -\frac{46}{17} \\ u = -\frac{79}{34} \end{array}$$

$$\begin{array}{l} \text{I}-13\text{IV} \\ \text{II}+4\text{IV} \\ \text{III}-4\text{IV} \end{array}$$
PROVJERA:

$$\frac{7}{34} + 2 \cdot \frac{12}{17} - \left(-\frac{46}{17}\right) + \left(-\frac{79}{34}\right) = 2 //$$

$$2 \cdot \frac{7}{34} + 5 \cdot \frac{12}{17} - \left(-\frac{46}{17}\right) + 2 \cdot \left(-\frac{79}{34}\right) = 2 //$$

$$3 \cdot \frac{7}{34} - \frac{12}{17} - 2 \cdot \left(-\frac{46}{17}\right) + \left(-\frac{79}{34}\right) = 3 //$$

$$\frac{7}{34} - \frac{12}{17} + 3 \cdot \left(-\frac{46}{17}\right) - 5 \cdot \left(-\frac{79}{34}\right) = 3 //$$



**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

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7/2

- Riješiti jednačbu:  $z^4 - (4 - i)^2 = 0$ . Prikaži rješenja u kompleksnoj ravnini! 12+3
- Odrediti domenu, sve asimptote i drugu derivaciju funkcije  $f(x) = x - \sqrt{x^2 - 2}$ . ~~5+15+5~~
- Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije  $g(x) = \ln(4 - x^2)$ . ~~5+5+10~~
- Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije  $h(x) = \frac{x^2 - 2x - (2 + 1)}{x^2 + 1}$ . Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf) ~~7~~
- Gaussovom metodom riješiti matricni sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x + 2y - z + u &= 2 \\ 2x + 5y - z + 2u &= 2 \\ 3x - y - 2z + u &= 3 \\ x - y + 3z - 5u &= 3 \end{aligned}$$

- Izračunati i provjeriti uvrštavanjem:  $\lim_{x \rightarrow \infty} \frac{e^x}{x}$ . 5

Ukupno:

17

④  $h(x) = \frac{x^2 - 2x - (2+1)}{x^2 + 1}$

$x^2 + 1 \neq 0 \quad x \in \mathbb{R}$   
 $x^2 \neq -1/\sqrt{\quad}$

$x \neq \pm\sqrt{-1}$

$\frac{x^2 - 2x - 3}{x^2 + 1} = 0 \quad | \cdot (x^2 + 1)$

$x^2 - 2x - 3 = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-3)}}{2} = \frac{2 \pm \sqrt{4 + 12}}{2} = \frac{2 \pm \sqrt{16}}{2} = \frac{2 \pm 4}{2}$

$x_1 = 3$

$x_2 = -1$

L.T. (3, 0)

L.T. (-1, 0)

L.T. (0, -3)

V.A. ne postoji!

H.A.  $\lim_{x \rightarrow \infty} \frac{x^2 - 2x - 3 \cdot x^2}{x^2 + 1 \cdot x^2} = \lim_{x \rightarrow \infty} \frac{\frac{x^2}{x^2} - \frac{2x}{x^2} - \frac{3}{x^2}}{\frac{x^2}{x^2} + \frac{1}{x^2}}$

$\lim_{x \rightarrow \infty} \frac{1}{1} = 1$

$y = 1$

K.A. ne postoji!

$$h(x) = \frac{x^2 - 2x - 3}{x^2 + 1}$$

$$h'(x) = \frac{(2x-2)(x^2+1) - 2x(x^2-2x-3)}{(x^2+1)^2}$$

$$h'(x) = \frac{2x^3 + 2x - 2x^2 - 2 - 2x^3 + 4x^2 + 6x}{(x^2+1)^2}$$

$$h'(x) = \frac{2x^2 - 8x - 2}{(x^2+1)^2}$$

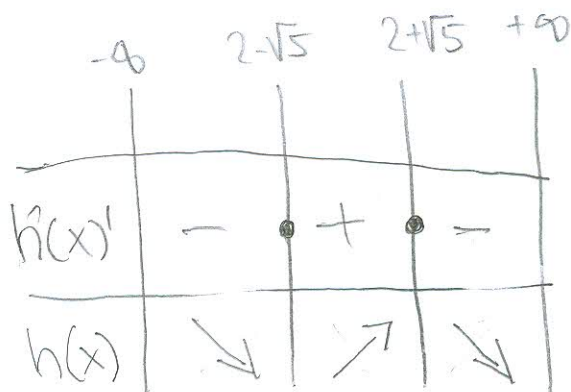
$$\frac{2x^2 - 8x - 2}{(x^2+1)^2} = 0 / (x^2+1)^2$$

$$2x^2 - 8x - 2 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{8 \pm \sqrt{64 - 16}}{4} = \frac{8 \pm 4\sqrt{5}}{4}$$

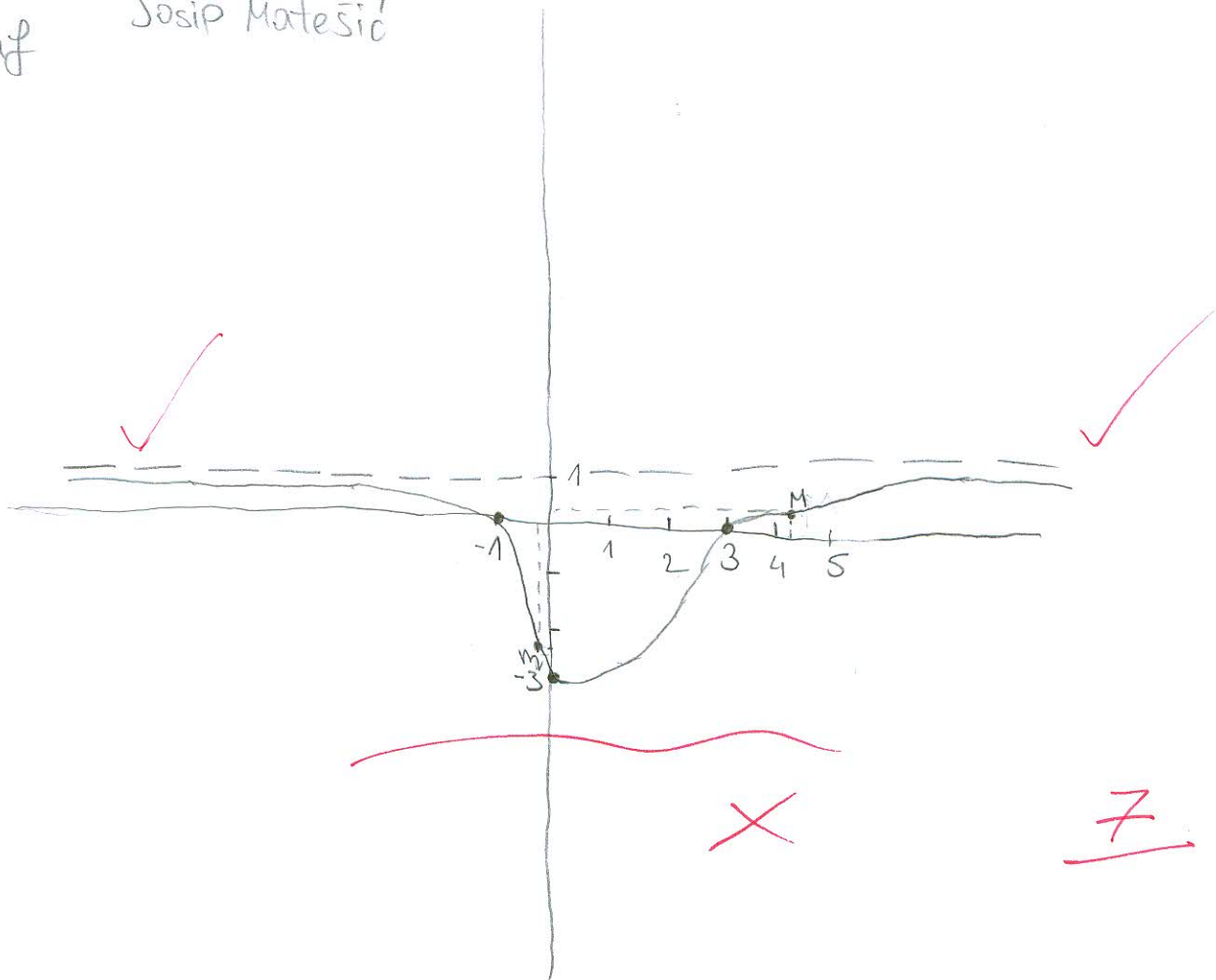
$$x_1 = 2 + \sqrt{5} \quad M(4,24,0,34)$$

$$x_2 = 2 - \sqrt{5} \quad m(-0,24,-2,34)$$



④ Graf

Josip Matesić



③

$$g(x) = \ln(4-x^2)$$

$$x \in \langle -\infty, -2 \rangle \cup \langle 2, +\infty \rangle$$

~~X~~

$$\ln(4-x^2) > 0 / e$$

$$4-x^2 > e^0$$

$$-x^2 > -4 / (-1)$$

$$x^2 > 4 / \sqrt{\quad}$$

$$x > \pm \sqrt{4} \quad \text{X}$$

$$x_1 = 2$$

$$x_2 = -2$$

$$g(x) = \ln(4-x^2)$$

$$g(-x) = \ln(4-(-x)^2)$$

$$g(x) = \ln(4-x^2)$$

Funkcija je PARNA!

✓

2)

$$f(x) = x - \sqrt{x^2 - 2} \quad x \in \langle -\infty, -\sqrt{2} \rangle \cup [\sqrt{2}, +\infty)$$

$$x^2 - 2 \geq 0$$

$$x^2 \geq 2 \quad | \sqrt{\phantom{x}}$$

$$x \geq \pm\sqrt{2}$$

$$\text{V.A.} \quad \lim_{x \rightarrow -\sqrt{2}} x - \sqrt{x^2 - 2} = \lim_{x \rightarrow -\sqrt{2}} -\sqrt{2} - \sqrt{(-\sqrt{2})^2 - 2}$$

$$\lim_{x \rightarrow -\sqrt{2}} = -\sqrt{2}$$

$$\text{V.A.} \quad \lim_{x \rightarrow \sqrt{2}} x - \sqrt{x^2 - 2} = \lim_{x \rightarrow \sqrt{2}} \sqrt{2} - \sqrt{(\sqrt{2})^2 - 2}$$

$$\lim_{x \rightarrow \sqrt{2}} = \sqrt{2}$$

k.A. ne postoji!



H.A.

$$\lim_{x \rightarrow \infty} x - \sqrt{x^2 - 2} \quad | :x$$

$$\lim_{x \rightarrow \infty} \frac{x}{x} - \sqrt{\frac{x^2}{x} - \frac{2}{x}}$$

$$\lim_{x \rightarrow \infty} = 1 \quad \text{X}$$

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

H2

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME: *Josipa Jozeljčić*

BROJ INDEKSA: *0269 07 8354*

1. Riješiti jednačinu:  $z^4 - (4 - i)^2 = 0$ . *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domen, sve asimptote i drugu derivaciju funkcije  $f(x) = x - \sqrt{x^2 - 2}$ . 5+15+5
3. Ispitati domen, (ne)parnost i zakrivljenost grafa funkcije  $g(x) = \ln(4 - x^2)$ . 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije  $h(x) = \frac{x^2 - 2x - (2 + 1)}{x^2 + 1}$ . Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
5. Gaussovom metodom riješiti matricni sustav i obavezno provjeri rješenje: 15

$$\begin{aligned}x + 2y - z + u &= 2 \\2x + 5y - z + 2u &= 2 \\3x - y - 2z + u &= 3 \\x - y + 3z - 5u &= 3\end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem:  $\lim_{x \rightarrow \infty} \frac{e^x}{x}$ .

5

Ukupno:

*10*



2)  $f(x) = x - \sqrt{x^2 - 2}$

Isipia Joselyic

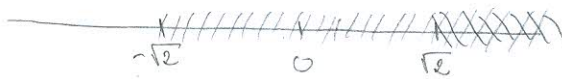
$x \geq \sqrt{2} \quad x \geq -\sqrt{2}$

1) DOMENJA

$x^2 - 2 \geq 0$

$x^2 \geq 2 \quad | \sqrt{\quad}$

$x \geq \pm \sqrt{2}$



$D(f) \subset \mathbb{R} \setminus (\sqrt{2}, +\infty)$  ~~X~~

2) ASIMPTOTE

newa v.a.

H.A.

$\lim_{x \rightarrow \infty} x - \sqrt{x^2 - 2} = (\infty - \infty)$

$\frac{(\sqrt{x^2 - 2})(x^2 + 2)}{-\sqrt{x^4 - 4} \cdot x^2 - 2}$

$\lim_{x \rightarrow \infty} x - \sqrt{x^2 - 2} = \frac{x + \sqrt{x^2 - 2}}{x + \sqrt{x^2 - 2}} = \lim_{x \rightarrow \infty} \frac{(x - \sqrt{x^2 - 2}) \cdot (x + \sqrt{x^2 - 2})}{x + \sqrt{x^2 - 2}} = \lim_{x \rightarrow \infty} \frac{x^2 - (x^2 - 2)}{x + \sqrt{x^2 - 2}} = \frac{2}{x + \sqrt{x^2 - 2}} = \left[ \frac{\infty}{\infty} \right]$

$\lim_{x \rightarrow \infty} \frac{x^2 - \sqrt{x^2 - 2}}{x + \sqrt{x^2 - 2}} \stackrel{1: y}{=} \lim_{x \rightarrow \infty} \frac{\frac{x^2}{x} - \sqrt{1 - \frac{2}{x}}}{1 + \sqrt{1 - \frac{2}{x}}} = \frac{-1}{1 + 1} = \left[ \frac{-1}{2} \right]$

newa kose asimptote ~~X~~

$OHA_{\infty} y = -\frac{1}{2}$  ~~X~~

$f''(x) = ?$

$f'(x) = x - \sqrt{x^2 - 2}$

$f'(x) = (x)' - (\sqrt{x^2 - 2})' = 1 - \frac{1}{2}(x^2 - 2)^{\frac{1}{2} - 1} \cdot (x^2 - 2)'$

$= 1 - \frac{1}{2}(x^2 - 2)^{-\frac{1}{2}} \cdot 2x = 1 - \frac{1}{2\sqrt{x^2 - 2}} \cdot 2x = \left[ 1 - \frac{2x}{2\sqrt{x^2 - 2}} \right]$

Prva derivacija ~~X~~

→ druga derivacija

$$\textcircled{3} \quad g(x) = \ln(4 - x^2)$$

① DOMENA

$$4 - x^2 > 0$$

$$(2-x)(2+x) > 0$$

	$-\infty$	$-2$	$2$	$+\infty$
		$-3$	$1$	$3$
$2-x$		+	+	-
$2+x$		-	+	+
		-	+	-

$$D(g) \mathbb{R} < -2, +2 > \checkmark$$

② NEPARNOST

$$g(x) = g(-x) = \ln(4 - (-x)^2) = \ln(4 - x^2)$$

funkcija je parna.  $\checkmark$

③

→ ② druga derivacija

$$f'(x) = 1 - \frac{2x}{2\sqrt{x^2-2}}$$

$$f''(x) = (1)' - \left( \frac{2x}{2\sqrt{x^2-2}} \right)' = 0 - \frac{2 \cdot (2\sqrt{x^2-2}) - 2x \cdot (x^2-2)^{-\frac{1}{2}}}{(2\sqrt{x^2-2})^2}$$

$$= \frac{4\sqrt{x^2-2} - \frac{2x}{\sqrt{x^2-2}}}{(2\sqrt{x^2-2})^2} = \frac{4\sqrt{x^2-2} - \frac{2x}{\sqrt{x^2-2}}}{(2\sqrt{x^2-2})^2}$$



$$\begin{aligned} (5) \quad & x + 2y - z + u = 2 \\ & 2x + 5y - z + 2u = 2 \\ & 3x - y - 2z + u = 3 \\ & x - y + 3z - 5u = 3 \end{aligned}$$

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 2 & 5 & -1 & 2 & 2 \\ 3 & -1 & -2 & 1 & 3 \\ 1 & -1 & 3 & -5 & 3 \end{array} \right] \begin{array}{l} \cdot (-2) \quad \cdot (-3) \quad \cdot (-1) \\ \leftarrow + \\ \leftarrow + \\ \leftarrow + \end{array} \sim$$

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & -3 & 0 & -2 \\ 0 & -5 & 1 & -2 & -3 \\ 0 & -3 & 4 & -6 & 1 \end{array} \right] \begin{array}{l} \cdot (+5) \quad \cdot (3) \\ \leftarrow + \\ \leftarrow + \end{array} \sim$$

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & -3 & 0 & -2 \\ 0 & 0 & -14 & -2 & -13 \\ 0 & 0 & -5 & -6 & -5 \end{array} \right] \begin{array}{l} \\ \\ \cdot (-1) \\ \\ \end{array}$$

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & -3 & 0 & -2 \\ 0 & 0 & -2 & -2 & -13 \\ 0 & 0 & -5 & -6 & -5 \end{array} \right] \begin{array}{l} \\ \\ \cdot (-1) \quad \cdot (-1) \\ \cdot (-1) \quad \cdot (-1) \end{array}$$

2

4) a)  $h(x) = \frac{x^2 - 2x(-2+1)}{x^2 + 1}$

gledajte s x ovi:

$x^2 + 1 \neq 0$

$x^2 \neq -1/\sqrt{\quad}$

$x \neq \pm\sqrt{-1}$

$D(h) \mathbb{R} \setminus \{\pm\sqrt{-1}\}$

asimptote

$h(x) = 0$

$\frac{x^2 - 2x(-2+1)}{x^2 + 1} = 0 \quad | \cdot x^2 + 1$

$x^2 - 2x(-2+1) = 0$

$x^2 + 4x - 2x = 0$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0$

$x + 2 = 0$

$x = -2$

$NT_1(0, 0)$

$NT_2(-2, 0)$

V.A.  $x_{10} = +\sqrt{-1}$   $x_{20} = -\sqrt{-1}$

$\lim_{x \rightarrow \sqrt{-1}^-} \frac{x^2 - 2x(-2+1)}{x^2 + 1} = \frac{(\sqrt{-1}^-)^2 - 2(\sqrt{-1}^-)(-2+1)}{(\sqrt{-1}^-)^2 + 1} = \frac{-1 - 2\sqrt{-1}^-(-2+1)}{-1 + 1}$

$= \frac{-2^-}{0^-} = \boxed{-\infty}$

$\lim_{x \rightarrow \sqrt{-1}^+} \frac{x^2 - 2x(-2+1)}{x^2 + 1} = \frac{(\sqrt{-1}^+)^2 - 2 \cdot \sqrt{-1}^+(-2+1)}{(\sqrt{-1}^+)^2 + 1} = \frac{-1^+ - 2\sqrt{-1}^+(-2+1)}{-1^+ + 1} = \frac{-2^+}{0^+}$

$= \frac{+\infty}{+\infty}$

$\lim_{x \rightarrow \sqrt{-1}^-} \frac{(\sqrt{-1}^-)^2 - 2(\sqrt{-1}^-)(-2+1)}{(\sqrt{-1}^-)^2 + 1} = \frac{-2^-}{0^-} = \frac{-\infty}{-\infty}$

$\lim_{x \rightarrow \sqrt{-1}^+} \frac{(\sqrt{-1}^+)^2 - 2(\sqrt{-1}^+)(-2+1)}{(\sqrt{-1}^+)^2 + 1} = \frac{-2^+}{0^+} = \frac{+\infty}{+\infty}$

V.A. je obustana

H.A.

$\lim_{x \rightarrow \infty} \frac{x^2 - 2x(-2+1)}{x^2 + 1} = \lim_{x \rightarrow \infty} \frac{x^2 + 4x - 2x}{x^2 + 1} = \lim_{x \rightarrow \infty} \frac{x^2 + 2x}{x^2 + 1} = \lim_{x \rightarrow \infty} \frac{1 + \frac{2}{x}}{1 + \frac{1}{x^2}} = 1$

$= 1$

$\boxed{\text{OHA} \dots y = 1}$

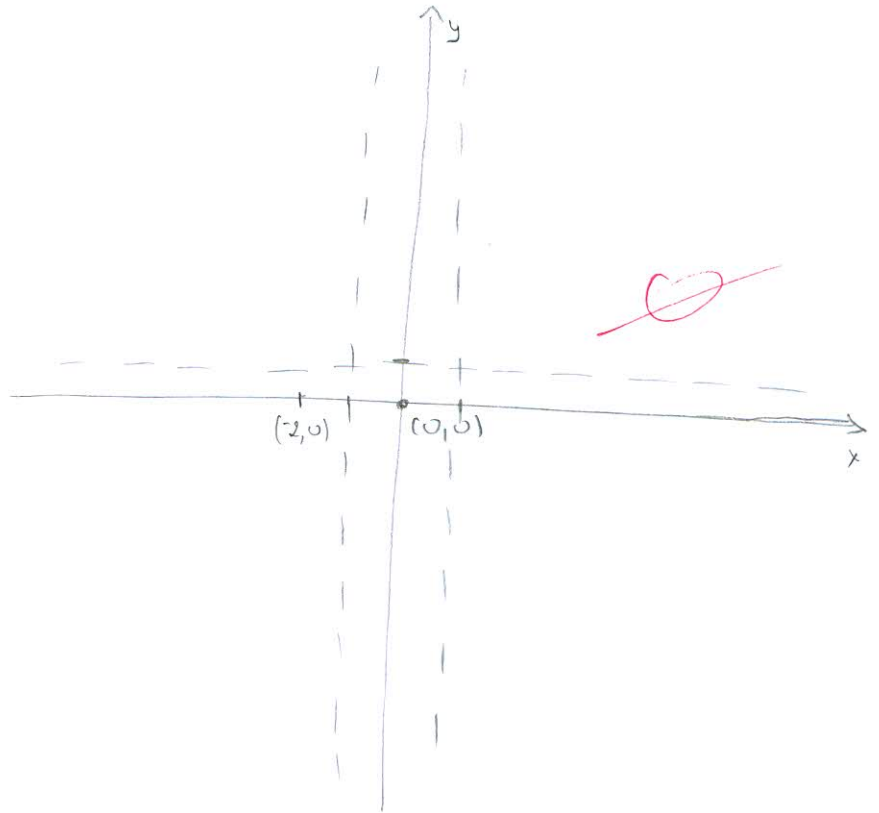
- nema kose asimptote

na drugom papiru (4.b)



$$1. b) f(x) = \frac{1 - 2x(-2+1)}{x^2+1} = \frac{x+2x}{x^2+1}$$

$$f'(x) = \frac{(2x+2)(x^2+1) - (x^2+2)(2x)}{(x^2+1)^2} = \frac{\cancel{2x^3} + 2x + 2x^2 + 2 - (\cancel{2x^3} + 4x)}{(x^2+1)^2} = \frac{2x^2}{(x^2+1)^2}$$





**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME: **IVAN ANDRIJAŠEVIĆ**

BROJ INDEKSA: **0269075298**

H2

1. Riješiti jednačinu:  $z^4 - (4 - i)^2 = 0$ . Prikaži rješenja u kompleksnoj ravni! 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije  $f(x) = x - \sqrt{x^2 - 2}$ . ~~5+15+5~~ **7**
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije  $g(x) = \ln(4 - x^2)$ . ~~5+5+10~~
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije  $h(x) = \frac{x^2 - 2x - (2 + 1)}{x^2 + 1}$ . Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned}x + 2y - z + u &= 2 \\2x + 5y - z + 2u &= 2 \\3x - y - 2z + u &= 3 \\x - y + 3z - 5u &= 3\end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem:  $\lim_{x \rightarrow \infty} \frac{e^x}{x}$ . 5

Ukupno:

**7**

3.)  $4 - x^2 > 0$   
 $\Rightarrow x^2 < 4$   
 $\Rightarrow x < 2$

$D(f) = \langle -\infty, 2 \rangle$

PARNOST

~~$x < 2$~~

~~$x < 2$~~

FUNKCIJA JE PARNA

6.)

$\lim_{x \rightarrow \infty} \frac{e^x}{x} = \frac{\infty}{\infty} = 0$



2.  $f(x) = x - \sqrt{x^2 - 2}$        $D(f) = [\sqrt{2}, +\infty)$

$x^2 - 2 \geq 0$

$x^2 \geq 2/\sqrt{\quad}$

$x \geq \sqrt{2}$  ~~X~~

NEMA VERTIKALNE ASIMPTOTE

$\lim_{x \rightarrow \infty} x - \sqrt{x^2 - 2} = \infty - \sqrt{\infty^2 - 2} = 0$  ~~D.K.A.~~  $= y = 0$  ✓

$x - x - \sqrt{2}$

K.A.

$\lim_{x \rightarrow \pm\infty} \frac{x - \sqrt{x^2 - 2}}{x} = \frac{x - x - \sqrt{2}}{x} = \frac{-\sqrt{2}}{1} = -\sqrt{2} = k$

$b = f_x - kx$

$b = x - \sqrt{x^2 - 2} - 0 \cdot x$

$b = x - \sqrt{x^2 - 2}$

$b = x - x - \sqrt{2}$

$b = -\sqrt{2}$

K.A.  $x = -\sqrt{2} + 0 \cdot x$

K.A.  $k = -\sqrt{2}$

$f(x) = x - \sqrt{x^2 - 2}$        $f' = 1 - \frac{1}{2\sqrt{x^2 - 2}} \cdot 2x$

$f' = 1 - \frac{2x}{2\sqrt{x^2 - 2}}$  ✓  $(\frac{2\sqrt{x^2 - 2}}{2\sqrt{x^2 - 2}} = 2x)$

$f' = \frac{1 \cdot 1 - f \cdot 1}{1 \cdot 1}$

$f'' = \frac{2\sqrt{x^2 - 2} \cdot 0 - 2 \cdot \frac{1}{2\sqrt{x^2 - 2}}}{(2\sqrt{x^2 - 2})^2} = 2 - 2\sqrt{x^2 - 2} - 2x \cdot 2 \cdot \frac{1}{2\sqrt{x^2 - 2}}$

$f'' = \frac{4}{2\sqrt{x^2 - 2}} - 2 - \frac{6\sqrt{x^2 - 2} - 2x}{2\sqrt{x^2 - 2}} = \frac{4 - 6\sqrt{x^2 - 2} - 6\sqrt{x^2 - 2} - 2x}{(2\sqrt{x^2 - 2})^2}$  ~~X~~





5.  $x + 2y - z + u = 2$   
 $2x + 5y - z + 2u = 2$   
 $3x - y - 2z + u = 3$   
 $x - y + 3z - 5u = 3$

$$\left| \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 2 & 5 & -1 & 2 & 2 \\ 3 & -1 & -2 & 1 & 3 \\ 1 & -1 & 3 & 5 & 3 \end{array} \right| \begin{array}{l} \cdot (-2) \\ + \\ + \\ + \end{array} \sim \begin{array}{l} \text{III} + \text{II} \\ \text{IV} + \text{II} \end{array} \left| \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & -7 & 1 & -2 & -3 \\ 0 & -3 & 4 & 4 & 1 \end{array} \right|$$

$$\sim \left| \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 8 & -2 & -17 \\ 0 & 0 & 7 & 4 & -8 \end{array} \right| \begin{array}{l} \leftarrow + \\ \cdot (-1) \end{array}$$

$$\left| \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 1 & -6 & -9 \\ 0 & 0 & 7 & 4 & -8 \end{array} \right| \begin{array}{l} \cdot (-2) \\ \cdot (-7) \\ + \end{array} \sim \left| \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 1 & -6 & -9 \\ 0 & 0 & 0 & 46 & 55 \end{array} \right| \cdot 46$$

$$\sim \left| \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 1 & -6 & -9 \\ 0 & 0 & 0 & 1 & 55/46 \end{array} \right| \begin{array}{l} \cdot (-1) \\ \cdot (-1) \\ + \end{array}$$

$$\left| \begin{array}{cccc|c} 1 & 2 & -1 & 0 & 2,804 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 & -1,826 \\ 0 & 0 & 0 & 1 & 1,195 \end{array} \right| \begin{array}{l} \cdot (-1) \\ \cdot (-1) \\ \cdot (-1) \end{array} \sim \left| \begin{array}{cccc|c} 1 & 2 & 0 & 0 & 2,63 \\ 0 & 1 & 0 & 0 & -0,174 \\ 0 & 0 & 1 & 0 & -1,826 \\ 0 & 0 & 0 & 1 & 1,195 \end{array} \right| \begin{array}{l} \cdot (-1) \\ \cdot (-1) \end{array}$$

~~Handwritten scribbles and crossed-out matrix elements.~~

x y z u

$$\left| \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2,804 \\ 0 & 1 & 0 & 0 & -0,174 \\ 0 & 0 & 1 & 0 & -1,826 \\ 0 & 0 & 0 & 1 & 1,195 \end{array} \right|$$

$$X = \begin{cases} 2,804 \\ -0,174 \\ -1,826 \\ 1,195 \end{cases}$$

$x + 2y - z + u = 2$

$$2,804 - 2 \cdot 0,174 - 1,826 + 1,195 = 2$$

$1,825 \neq 2, \quad \text{O}$

$$h(x) = \frac{x^2 - 2x - (2+1)}{x^2 + 1}$$

$$x^2 + 1 \neq 0$$

$$x^2 \neq -1$$

$$x \neq \pm i$$

$$D(f) = \mathbb{R} \setminus \{i, -i\}$$

NUL TOČKE

$$p(x) = x^2 - 2x - 3$$

$$x_{1/2} = \frac{2 \pm \sqrt{4 - 4 \cdot (-3)}}{2} = \frac{2 \pm \sqrt{16}}{2} = \frac{2 \pm 4}{2}$$

$x_1 = 3$   
 $x_2 = -1$

$$x_1 = 3 \quad x_2 = -1$$

$$h(0) = \frac{-3}{1} = -3$$

VA. 2

$$\lim_{x \rightarrow 2^-} \frac{x^2 - 2x - (2+1)}{x^2 + 1} = \frac{1 + 2 - 3}{2} = \frac{0}{2} = 0$$

$$VA_1 \lim_{x \rightarrow 1^+} \frac{x^2 - 2x - (2+1)}{x^2 + 1} = -2$$

NEMA VERTIKALNE ASIMPTOTE

MA.

$$\lim_{x \rightarrow \pm\infty} \frac{x^2 - 2x - 3}{x^2 + 1} \stackrel{/:x^2}{=} \frac{1 - \frac{2}{x} - \frac{3}{x^2}}{1 + \frac{1}{x^2}} \stackrel{x \rightarrow \infty}{=} \frac{1}{1} = 1 \quad \text{M.A. } y = 1$$

KA.

$$\lim_{x \rightarrow \pm\infty} \frac{x^2 - 2x - 3}{x^2 + 1} \stackrel{/:x^3}{=} \frac{x^2 - 2x - 3/x^3}{x^3 + 1/x} \stackrel{/:x^3}{=} \frac{\frac{1}{x} - \frac{2}{x^2} - \frac{3}{x^3}}{1 + \frac{1}{x^3}} \stackrel{x \rightarrow \infty}{=} \frac{0}{1} = 0 = k$$

$$D = f(x) \rightarrow \mathbb{R}x$$

$$KA = kx + b$$

$$= 0 \cdot x + 1$$

$$= 1$$

$$\lim_{x \rightarrow \pm\infty} \frac{x^2 - 2x - 3}{x^2 + 1} \stackrel{/:x^2}{=} 0 \cdot x = 1$$

GRAF?

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

H2

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME:

STEPAN KOŠIĆA

BROJ INDEKSA:

17-2-0187-2012

- Riješiti jednačbu:  $z^4 - (4-i)^2 = 0$ . Prikaži rješenja u kompleksnoj ravnini! ~~12+3~~
- Odrediti domenu, sve asimptote i drugu derivaciju funkcije  $f(x) = x - \sqrt{x^2 - 2}$ . ~~5+15+5~~
- Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije  $g(x) = \ln(4-x^2)$ . ~~3+5+10~~
- Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije  $h(x) = \frac{x^2 - 2x - (2+1)}{x^2 + 1}$ . Ne treba ispitivati zakrivljenost jer se izraz komplicira. ~~20(graf)~~
- Gaussovom metodom riješiti matricni sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x + 2y - z + u &= 2 \\ 2x + 5y - z + 2u &= 2 \\ 3x - y - 2z + u &= 3 \\ x - y + 3z - 5u &= 3 \end{aligned}$$

~~15~~  
**15**

- Izračunati i provjeriti uvrštavanjem:  $\lim_{x \rightarrow \infty} \frac{e^x}{x}$ . ~~5~~

~~5~~

Ukupno:

**5**

1)  $z^4 - (4-i)^2 = 0$

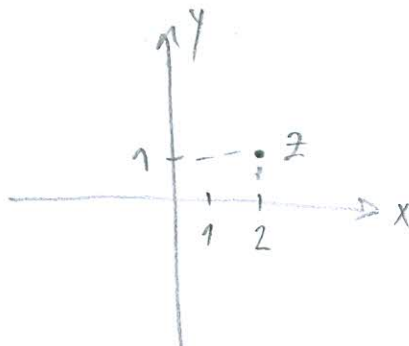
$$z^4 = (4-i)^2 / \sqrt{}$$

$$z^2 = 4-i$$

$$z = \sqrt{4-i}$$

$$z = \sqrt{4} - \sqrt{i}$$

$$x=2 \quad y=1$$



2)  $f(x) = x - \sqrt{x^2 - 2}$

$$x^2 - 2 \neq 0$$

$$x^2 \neq 2$$

$$x \neq \pm\sqrt{2}$$

$$Df \in \mathbb{R} \setminus \{\pm\sqrt{2}\}$$

v.A

$$\lim_{x \rightarrow \sqrt{2}} x - \sqrt{x^2 - 2} = \sqrt{2} - \sqrt{0} = \sqrt{2}$$

O.V.A.

$$\lim_{x \rightarrow -\sqrt{2}} x - \sqrt{x^2 - 2} = -\sqrt{2}$$

H.A

$$\lim_{x \rightarrow +\infty} x - \sqrt{x^2 - 2} \quad | :x^2 = \frac{1}{x} - \sqrt{1 - \frac{2}{x^2}} = -1 \quad \text{~~0~~}$$

$$\lim_{x \rightarrow -\infty} x - \sqrt{x^2 - 2} \quad | :x^2 = \frac{1}{x} - \sqrt{1 - \frac{2}{x^2}} = -1 \quad \text{~~0~~} \quad \boxed{y = -1} = \text{H.A}$$

k.A

$$f(x) = kx + l$$

$$k = \lim_{x \rightarrow \infty} \frac{f(x)}{x} = \frac{x - \sqrt{x^2 - 2}}{x} \quad | :x^2 = \frac{\frac{1}{x} - \sqrt{1 - \frac{2}{x^2}}}{\frac{1}{x}} = 0$$

$$l = \lim_{x \rightarrow \infty} f(x) - kx = x - \sqrt{x^2 - 2} + 0 \quad | :x^2 = \frac{1}{x} - \sqrt{1 - \frac{2}{x^2}} = -1$$

$$\boxed{\text{K.A} = f(x) = -1}$$

$$f'(x) = x - \sqrt{x^2 - 2} \\ = 1 - \frac{1}{2\sqrt{x^2 - 2}} \cdot 2x$$

$$f'(x) = 1 - \frac{x}{\sqrt{x^2 - 2}}$$

$$f''(x) = \frac{1 \cdot \sqrt{x^2 - 2} - x \cdot \frac{1}{2\sqrt{x^2 - 2}} \cdot 2x}{(\sqrt{x^2 - 2})^2}$$

$$f''(x) = \frac{\sqrt{x^2 - 2} - \frac{x^2}{\sqrt{x^2 - 2}}}{x^2 - 2}$$

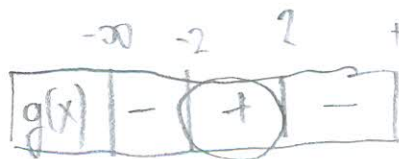
③  $g(x) = \ln(4 - x^2)$

$$4 - x^2 > 0$$

$$x_{1,2} = \frac{0 \pm \sqrt{0 + 16}}{-2}$$

$$x_{1,2} = \frac{\pm 4}{-2}$$

$$\boxed{x_1 = -2} \quad \boxed{x_2 = 2}$$



$$g(-x) = \ln(4 - (-x)^2) = \ln(4 - x^2) = g(x)$$

∩ ∩ ∩ FUNKCIJA JE NEPARNIJA ~~X~~

$$D_f < -2, 2 > \checkmark$$

$$g'(x) = 0 \\ -g' = 2x^2 = 0 \quad (-1)$$

$$x_{1,2} = \frac{0 \pm \sqrt{0 - 32}}{4} = \left[ x_1 = \frac{i\sqrt{32}}{4} \quad x_2 = \frac{-i\sqrt{32}}{4} \right]$$

$$g'(x) = \frac{1}{4 - x^2} \cdot (-2x) = -\frac{2x}{4 - x^2}$$

$$g''(x) = \frac{-2 \cdot (4 - x^2) - (2x) \cdot (-2x)}{(4 - x^2)^2}$$

$$= \frac{-8 + 2x^2 - 4x^2}{(4 - x^2)^2} = \frac{-8 - 2x^2}{(4 - x^2)^2}$$

4.)  $h(x) = \frac{x^2 - 2x - (2+1)}{x^2 + 1}$

1)  $x^2 + 1 \neq 0$   
 $x^2 \neq -1$

DF  $\in \mathbb{R}$

2)  $h(x) = 0$   
 $\frac{x^2 - 2x - 3}{x^2 + 1} = 0$

$x^2 - 2x - 3 = 0$

$x_{1,2} = \frac{2 \pm \sqrt{4 + 12}}{2}$

$x_{1,2} = \frac{2 \pm 4}{2}$

$x_1 = 3$      $x_2 = -1$

$N_1(3, 0)$      $N_2(-1, 0)$

S(0, -3)

4)  $h'(x) = \frac{x^2 - 2x - 3}{x^2 + 1}$

$h'(x) = \frac{(2x - 2) \cdot (x^2 + 1) - (x^2 - 2x - 3) \cdot 2x}{(x^2 + 1)^2}$

$M(-4,236, 4,236) = \frac{2x^3 + 2x - 2x^3 - 2 - 2x^3 + 4x^2 + 6x}{(x^2 + 1)^2}$

$m(0,236, 6,47) = \frac{2x^2 + 8x - 2}{(x^2 + 1)^2}$

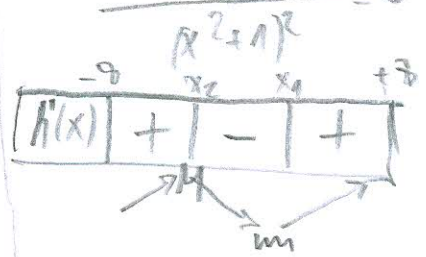
$h''(x) = 0$   
 $2x^2 + 8x - 2 = 0$

$2x^2 + 8x - 2 = 0$

$x_{1,2} = \frac{-8 \pm \sqrt{64 + 16}}{4}$

$x_1 = \frac{-8 + \sqrt{80}}{4} = 0,236$

$x_2 = \frac{-8 - \sqrt{80}}{4} = -4,236$



3) V.A NEMA

H.A  $\lim_{x \rightarrow +\infty}$

$\frac{x^2 - 2x - 3}{x^2 + 1} \Big|_{:x^2} = \frac{1 - \frac{2}{x} - \frac{3}{x^2}}{1 + \frac{1}{x^2}} = 1$

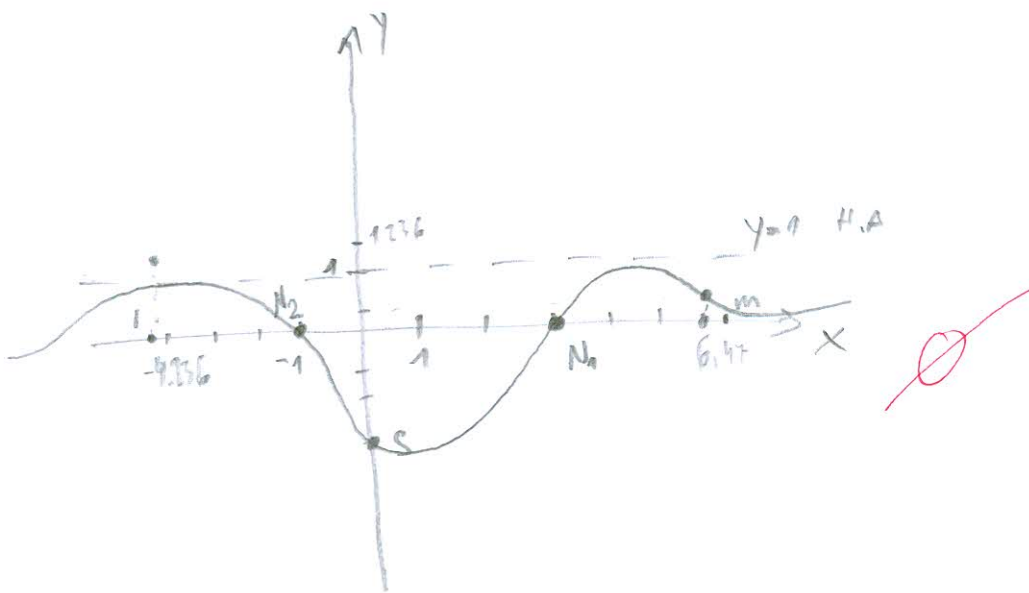
$\lim_{x \rightarrow -\infty}$

$\frac{x^2 - 2x - 3}{x^2 + 1} \Big|_{:x^2} = \frac{1 - \frac{2}{x} - \frac{3}{x^2}}{1 + \frac{1}{x^2}} = 1$

O.H.A

$y = 1$

K.A NEMA



5.)  $x + 2y - z + u = 2$

$2x + 5y - z + 2u = 2$

$3x - y - 2z + u = 3$

$x - y + 3z - 5u = 3$

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 2 & 5 & -1 & 2 & 2 \\ 3 & -1 & -2 & 1 & 3 \\ 1 & -1 & 3 & -5 & 3 \end{array} \right] \begin{array}{l} \leftarrow \begin{array}{l} (-2) \cdot (-3) \quad (-1) \\ + \end{array} \\ \leftarrow \end{array}$$

$x + 2y - z + u = 2$

$7,49 + 2 \cdot (-3,27) - 1,27 + 2,32 = 2$

$2 = 2$

$$\sim \left[ \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & -7 & 1 & -2 & -3 \\ 0 & -3 & 4 & -6 & 1 \end{array} \right] \begin{array}{l} \leftarrow \begin{array}{l} (-7) \cdot (-3) \\ + \end{array} \\ \leftarrow \end{array}$$

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 8 & -2 & -17 \\ 0 & 0 & 7 & -6 & -5 \end{array} \right]$$

$r = 4$

$n - r = 0$  nema param.  $\eta$ .

$n = 4$

iz IV rethva:  $7z - 6u = -5$

$7z = -5 + 6u$

$z = \frac{-5 + 6u}{7}$

$z = 1,27$

iz II rethva:  $y + z = -2$

$y = -2 - 1,27$

$y = -3,27$

iz III rethva:

$8z - 2u = -17$

$34u = 79$

$8 \left( \frac{-5 + 6u}{7} \right) - 2u = -17$

$u = 2,32$

iz I rethva:  $x + 2y - z + u = 2$

$x - 6,54 - 1,27 + 2,32 = 2$

$x = 7,49$

$-\frac{40}{7} + \frac{62u}{7} - 2u = -17$

$-40 + 62u - 14u = 119$

$$6.) \lim_{x \rightarrow +\infty} \frac{e^x}{x} = 0 \quad \text{?}$$

$$\lim_{x \rightarrow -\infty} \frac{e^x}{x} = 0 \quad \text{?}$$





**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

H2

IME I PREZIME: MATEJ SURIC'

BROJ INDEKSA: 017125987-1

1. Riješiti jednačbu:  $z^4 - (4 - i)^2 = 0$ . Prikaži rješenja u kompleksnoj ravnini! 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije  $f(x) = x - \sqrt{x^2 - 2}$ . 5+15+5
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije  $g(x) = \ln(4 - x^2)$ . 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije  $h(x) = \frac{x^2 - 2x - (2 + 1)}{x^2 + 1}$ . Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
5. Gaussovom metodom riješiti matricni sustav i obavezno provjeri rješenje: 15

$$\begin{aligned}x + 2y - z + u &= 2 \\2x + 5y - z + 2u &= 2 \\3x - y - 2z + u &= 3 \\x - y + 3z - 5u &= 3\end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem:  $\lim_{x \rightarrow \infty} \frac{e^x}{x}$ . 5

$$\begin{aligned}1. z^4 - (4 - i)^2 &= 0 \\z^4 &= (4 - i)^2\end{aligned}$$

Ukupno:

~~0~~



$$\sim \begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 2 & 5 & -1 & 2 & 2 \\ 3 & -1 & -2 & 1 & 3 \\ 1 & -1 & 3 & -5 & 3 \end{bmatrix}$$

$\begin{matrix} \text{II} - 2 \cdot \text{I} \\ \text{III} - 3 \cdot \text{I} \\ \text{IV} - \text{I} \end{matrix}$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & -7 & 1 & -2 & -3 \\ 0 & -3 & 4 & -6 & 1 \end{bmatrix}$$

$\begin{matrix} \text{I} + \text{II} \\ \text{III} + 7 \cdot \text{II} \end{matrix}$

$$\sim \begin{bmatrix} 1 & 3 & 0 & 1 & 10 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 8 & -2 & -17 \\ 0 & -3 & 4 & -6 & 1 \end{bmatrix}$$

$\begin{matrix} \text{IV} + 3 \cdot \text{II} \end{matrix}$

$$\sim \begin{bmatrix} 1 & 3 & 0 & 1 & 10 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 8 & -2 & -17 \\ 0 & 0 & 7 & -6 & -5 \end{bmatrix}$$

$\text{III} - \text{IV}$

$$\sim \begin{bmatrix} 1 & 3 & 0 & 1 & 10 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 & -12 \\ 0 & 0 & 7 & -6 & -5 \end{bmatrix}$$

~~II - 3I~~

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 10 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 & -12 \\ 0 & 0 & 0 & -34 & 49 \end{bmatrix}$$

~~I - 3II~~

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 10 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 & -12 \\ 0 & 0 & 0 & -34 & 49 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -3 & 1 & 6 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 & -12 \\ 0 & 0 & 7 & -6 & -5 \end{bmatrix}$$

~~IV + 2I~~

$$\begin{bmatrix} 1 & 0 & -3 & 1 & 6 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 & -12 \\ 0 & 0 & 0 & 1 & -7 \end{bmatrix}$$

$\sim$

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 2 & 5 & -1 & 2 & 2 \\ 3 & -1 & -2 & 1 & 3 \\ 1 & -1 & 3 & -5 & 3 \end{bmatrix} \begin{array}{l} \text{II} - 2\text{I} \\ \text{III} - 3\text{I} \\ \text{IV} - \text{I} \end{array} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & -7 & 1 & -2 & -1 \\ 0 & -3 & 4 & -6 & 1 \end{bmatrix} \begin{array}{l} \text{IV} - 2\text{II} \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & -7 & 1 & -2 & -1 \\ 0 & 11 & 2 & -2 & 3 \end{bmatrix} \begin{array}{l} \text{IV} + 3\text{II} \end{array} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & -7 & 1 & -2 & -1 \\ 0 & 17 & -1 & 1 & 9 \end{bmatrix} \begin{array}{l} \text{IV} + \text{II} \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & -7 & 1 & -2 & -1 \\ 0 & 18 & 0 & 1 & 7 \end{bmatrix} \begin{array}{l} \text{III} + 2\text{IV} \end{array} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 23 & 1 & 0 & 13 \\ 0 & 18 & 0 & 1 & 7 \end{bmatrix} \begin{array}{l} \text{I} - \text{IV} \end{array}$$

$$\sim \begin{bmatrix} 1 & -16 & -1 & 0 & -5 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 23 & 1 & 0 & 13 \\ 0 & 18 & 0 & 1 & 7 \end{bmatrix} \begin{array}{l} \text{I} + \text{II} \end{array} \sim \begin{bmatrix} 1 & -15 & 0 & 0 & -7 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 23 & 1 & 0 & 13 \\ 0 & 18 & 0 & 1 & 7 \end{bmatrix} \begin{array}{l} \text{II} - \text{III} \end{array}$$

$$\sim \begin{bmatrix} 1 & -15 & 0 & 0 & -7 \\ 0 & -28 & 0 & 0 & -15 \\ 0 & 23 & 1 & 0 & 13 \\ 0 & 18 & 0 & 1 & 7 \end{bmatrix} \begin{array}{l} \text{III} + \text{II} \end{array} \sim \begin{bmatrix} 1 & -15 & 0 & 0 & -7 \\ 0 & -28 & 0 & 0 & -15 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 18 & 0 & 1 & 7 \end{bmatrix} \begin{array}{l} 2\text{IV} + 2\text{II} \end{array}$$

$$\sim \begin{bmatrix} 1 & -15 & 0 & 0 & -7 \\ 0 & -28 & 0 & 0 & -15 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 8 & 0 & 1 & -1 \end{bmatrix}$$

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME: *MARIN KALMETA*

BROJ INDEKSA: *17-1-0180-2013*

H2

1. Riješiti jednačinu:  $z^4 - (4 - i)^2 = 0$ . *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije  $f(x) = x - \sqrt{x^2 - 2}$ . 5+15+5
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6. Izračunati i provjeriti uvrštavanjem:  $\lim_{x \rightarrow \infty} \frac{e^x}{x}$ .

5

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Ukupno:

*0*

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