MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓ G3

IME I PREZIME: ANTE VEDRIĆ **BROJ INDEKSA:** 17-2-01948 2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja uvrštavanjem. 12+3
2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcr} 2x_1 & - & x_2 & + & x_3 & - & x_4 = -1 \\ 2x_1 & - & x_2 & & & - & 3x_4 = 1 \\ 3x_1 & & & - & x_3 & + & x_4 = -1 \\ 2x_1 & + & 2x_2 & - & 2x_3 & + & 5x_4 = -1 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2+4x+4} - 4x$. 5+15
4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{x+3}{x^2-4}$. 15(graf) 13
5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^3)$. 2+4+6+8
6. Zadana je funkcija $f(x) = \sqrt{4+3x}$. Kolika je derivacija $f'(2)$? Koji su lokalni ekstremi? 10+5

Ukupno: 60

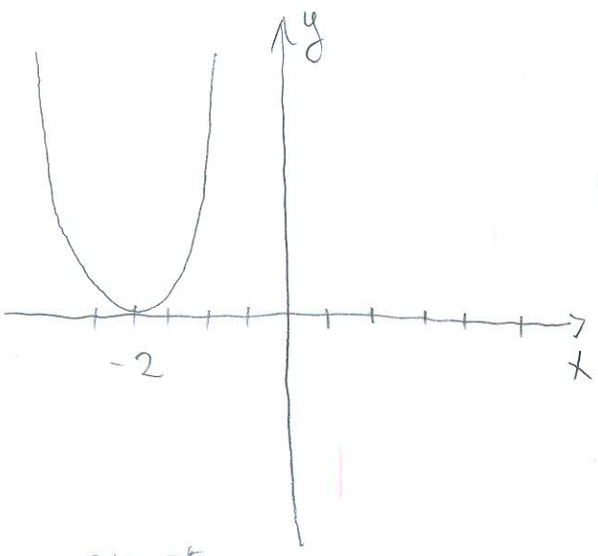
(3) $g(x) = \sqrt{x^2+4x+4} - 4x$

$x^2 + 4x + 4 \geq 0$

$x^2 + 4x + 4 = 0$

$x_{1,2} = \frac{-4 \pm \sqrt{16 - 4 \cdot 1 \cdot 4}}{2}$

$= \frac{-4}{2} = -2$



$x \in \mathbb{R}$ I DOMENA ✓

$\mathcal{D}_g = \mathbb{R}$

ASIMPTOTE

① $\frac{|z|}{z+2i} = 3i$

$x + yi = z$

$\frac{(x+yi)}{x+yi+2i} = 3i$

$\frac{\sqrt{x^2+y^2}}{x+(y+2)i} = 3i$

$\frac{x+(y+2)i}{\sqrt{x^2+y^2}} = \frac{1}{3i} \cdot \frac{3i}{3i}$

$\frac{x+(y+2)i}{\sqrt{x^2+y^2}} = -\frac{3i}{9}$

$\frac{x+(y+2)i}{\sqrt{x^2+y^2}} = 0$

$x = 0$

$\frac{y+2}{\sqrt{x^2+y^2}} = -\frac{1}{3}$

$\frac{\text{Se } x=0}{y+2 = -\frac{\sqrt{x^2+y^2}}{3}}$

$\frac{3}{3}y + 2 = -\frac{y}{3}$

$\frac{4}{3}y = -2$

$4y = -6$

$z = -\frac{6}{4}i$ ✓

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IME I PREZIME: DINO BADŽOKA **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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6. Zadana je funkcija $f(x) = \sqrt{4+3x}$. Kolika je derivacija $f'(2)$? Koji su lokalni ekstremi? 10+5

Ukupno: 35

2.) $2x_1 - x_2 + x_3 - x_4 = -1$

$2x_1 - x_2 - 3x_4 = 1$

$3x_1 - x_3 + x_4 = -1$

$2x_1 + 2x_2 - 2x_3 + 5x_4 = -1$

$$\left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] \cdot(-2) \sim \left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] \cdot(-2) \sim$$

$$\left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -1 & -2 & 2 \\ 3 & 0 & -1 & 1 & -1 \\ 0 & 3 & 1 & 6 & 0 \end{array}\right] \cdot 3 \sim \left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & \frac{1}{3} & 2 & 0 \\ 0 & 0 & 1 & 2 & -2 \\ 3 & 0 & -1 & 1 & -1 \end{array}\right] \cdot(-3) +$$

$$\left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & \frac{1}{3} & 2 & 0 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 3 & -5 & 5 & \frac{1}{2} \end{array}\right] \cdot\left(-\frac{3}{2}\right) + \sim \left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & \frac{1}{3} & 2 & 0 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & -3 & -1 & \frac{1}{2} \end{array}\right] \cdot 3 +$$

$$\left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & \frac{1}{3} & 2 & 0 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & \frac{11}{2} & -\frac{11}{2} \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & \frac{1}{3} & 2 & 0 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \cdot(-2) + \quad +\left(\frac{11}{2}\right) +$$

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IME I PREZIME: IVAN MARETIĆ **BROJ INDEKSA:** 17-1-0216-2013

0269081944

POPUNJAVA NASTAVNIK Broj ↓ bodova

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$$\begin{array}{rcrcrcr} 2x_1 & - & x_2 & + & x_3 & - & x_4 = -1 \\ 2x_1 & - & x_2 & & & - & 3x_4 = 1 \\ 3x_1 & & & - & x_3 & + & x_4 = -1 \\ 2x_1 & + & 2x_2 & - & 2x_3 & + & 5x_4 = -1 \end{array}$$

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Ukupno: 36

3) $g(x) = \sqrt{x^2+4x+4} - 4x$

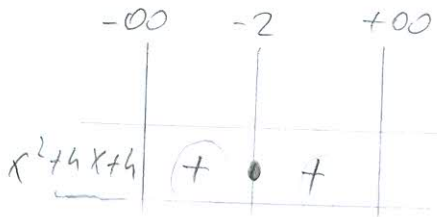
1) domena

$x^2+4x+4 \geq 0$

$x^2+4x+4 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-4 \pm \sqrt{4^2-4\cdot1\cdot4}}{2\cdot1} = \frac{-4 \pm \sqrt{16-16}}{2} = \frac{-4 \pm \sqrt{0}}{2} = \frac{-4 \pm 0}{2}$

$x_1 = x_2 = -\frac{4}{2} = -2$



$D_f : x \in \mathbb{R}$ ✓

$D_g = \{x \in \mathbb{R}\} = \mathbb{R}$

2) asimptote

1) kosa asimptota nema ker nema prekida u domeni ✗

2) horizontalna asimptota

$$\lim_{x\to+\infty} \sqrt{x^2+4x+4} - 4x \cdot \frac{\sqrt{x^2+4x+4}+4x}{\sqrt{x^2+4x+4}+4x} = \lim_{x\to+\infty} \frac{\left(\sqrt{x^2+4x+4}\right)^2-(4x)^2}{\sqrt{x^2+4x+4}+4x} =$$

$$= \lim_{x\to+\infty} \frac{x^2+4x+4-16x^2 \,/:x^2}{\sqrt{x^2+4x+4}+4x \,/:x^2} = \frac{\frac{x^2}{x^2}+\frac{4x}{x^2}-\frac{16x^2}{x^2}}{\sqrt{\frac{x^2}{x^4}+\frac{4x}{x^4}+\frac{4}{x^4}}+\frac{4x}{x^2}} = \frac{1-16}{\sqrt{0}+0} = -\frac{15}{0} = -\infty$$

nema desne horizontalne asimptote

$$\lim_{x\to-\infty} \sqrt{x^2+4x+4} - 4x \cdot \frac{\sqrt{x^2+4x+4}+4x}{\sqrt{x^2+4x+4}+4x} = \lim_{x\to-\infty} \frac{\left(\sqrt{x^2+4x+4}\right)^2-(4x)^2}{\sqrt{x^2+4x+4}+4x} =$$

$$= \lim_{x\to-\infty} \frac{x^2+4x+4-16x^2 \,/:x^2}{\sqrt{x^2+4x+4}+4x \,/:x^2} = \frac{\frac{x^2}{x^2}-\frac{4x}{x^2}+\frac{16x^2}{x^2}}{\sqrt{\frac{x^2}{x^4}+\frac{4x}{x^4}+\frac{4}{x^4}}-\frac{4x}{x^2}} = \frac{-1+16}{0} = \frac{15}{0} = +\infty$$

nema lijeve horizontalne asimptote

3) kosa asimptota

$$k = \lim_{x\to+\infty} \frac{\sqrt{x^2+4x+4}-4x \,/:x}{x \,/:x} = \frac{\sqrt{\frac{x^2}{x^2}+\frac{4x}{x^2}+\frac{4}{x^2}}-\frac{4x}{x}}{\frac{x}{x}} = \frac{1-4}{1} = -\frac{3}{1} = -1$$

$$l = \lim_{x\to+\infty} \left(\sqrt{x^2+4x+4}-4x\right)+3x = \lim_{\left(\sqrt{x^2+4x+4}-4x\right)-3x\to+\infty} \frac{\left(\sqrt{x^2+4x+4}-4x\right)-3x}{\left(\sqrt{x^2+4x+4}-4x\right)-3x} = \lim_{x\to+\infty} \frac{\left(\sqrt{x^2+4x+4}-4x\right)^2-(3x)^2}{\left(\sqrt{x^2+4x+4}-4x\right)-3x}$$

$$= \lim_{x\to+\infty} \frac{x^2+4x+4-16x^2-9x^2}{\left(\sqrt{x^2+4x+4}-4x\right)-3x} = \frac{-15x^2+4x-9x^2 \,/:x^2}{\left(\sqrt{x^2+4x+4}-4x\right)-3x \,/:x^2} = \frac{-\frac{15x^2}{x^2}+\frac{4x}{x^2}-\frac{9x^2}{x^2}}{\left(\sqrt{\frac{x^2}{x^4}+\frac{4x}{x^4}+\frac{4}{x^4}}-\frac{4x}{x^2}\right)-\frac{3x}{x^2}} =$$

$$= \frac{-24}{0} = -\infty \qquad \text{NEMA DESNE KOSE ASIMPTOTE}$$

2)

$$2x_1 - x_2 + x_3 - x_4 = -1$$
$$2x_1 - x_2 \qquad - 3x_4 = 1$$
$$3x_1 \qquad - x_3 + x_4 = -1$$
$$2x_1 + 2x_2 - 2x_3 + 5x_4 = -1$$

$$\left|\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right| \begin{array}{l} \text{II}-\text{I} \\ \text{III}-\text{I} \\ \text{IV}-\text{I} \end{array} \sim \left|\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 0 & 0 & -1 & -2 & 2 \\ 1 & 1 & -2 & 2 & 0 \\ 0 & 3 & -3 & 6 & 0 \end{array}\right| \sim \left|\begin{array}{cccc|c} 1 & 1 & -2 & 2 & 0 \\ 0 & 0 & -1 & -2 & 2 \\ 2 & -1 & 1 & -1 & -1 \\ 0 & 3 & -3 & 6 & 0 \end{array}\right| :3$$

$$\sim \left|\begin{array}{cccc|c} 1 & 1 & -2 & 2 & 0 \\ 0 & 0 & -1 & -2 & 2 \\ 2 & -1 & 1 & -1 & -1 \\ 0 & 1 & -1 & 2 & 0 \end{array}\right| \begin{array}{l} \\ \\ \text{III}-2\text{I} \\ \\ \end{array} \sim \left|\begin{array}{cccc|c} 1 & 1 & -2 & 2 & 0 \\ 0 & 0 & -1 & -2 & 2 \\ 0 & -3 & 5 & -3 & -1 \\ 0 & 1 & -1 & 2 & 0 \end{array}\right| \begin{array}{l} \\ \\ \text{III}+3\text{IV} \\ \\ \end{array} \sim \left|\begin{array}{cccc|c} 1 & 1 & -2 & 2 & 0 \\ 0 & 0 & -1 & -2 & 2 \\ 0 & 0 & 2 & 3 & -7 \\ 0 & 1 & -1 & 2 & 0 \end{array}\right| \begin{array}{l} \\ \\ \text{III}-2\text{II} \\ \\ \end{array}$$

$$\sim \left|\begin{array}{cccc|c} 1 & 1 & -2 & 2 & 0 \\ 0 & 0 & -1 & -2 & 2 \\ 0 & 0 & 0 & 1 & -3 \\ 0 & 1 & -1 & 2 & 0 \end{array}\right| \begin{array}{l} \\ \\ \\ \text{II}+\text{IV} \end{array} \sim \left|\begin{array}{cccc|c} 1 & 1 & -2 & 2 & 0 \\ 0 & 1 & -2 & 0 & 2 \\ 0 & 0 & 0 & 1 & -3 \\ 0 & 1 & -1 & 2 & 0 \end{array}\right| \begin{array}{l} \text{I}-\text{II} \\ \text{I}-\text{III} \\ \text{IV}-\text{II} \sim \\ \text{IV}-2\text{III} \end{array} \left|\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -2 & 0 & 2 \\ 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 1 & 0 & 4 \end{array}\right|$$

$$\sim \left|\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -2 & 0 & 2 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right| \begin{array}{l} \\ \text{II}+2\text{III} \\ \\ \\ \end{array} \sim \left|\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 10 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right|$$

sustav ima jedinstveno rješenje

$x_1 = 1$

$x_2 = 10$

$x_3 = 4$

$x_4 = -3$

PROVJERA

$2 \cdot 1 - 10 + 4 + 3 = -1$

$2 \cdot 1 - 10 + 0 - 3 \cdot (-3) = 1$

$2 \cdot 1 + 2 \cdot 10 - 2 \cdot 4 + 5 \cdot (-3) = -1$

PROVJERA 2., 3. i 4. JEDNADŽBE!

IVAN MARDETKO

VASTAVAK 3. ZADATKA

$$= \lim_{x \to -\infty} \frac{\sqrt{x^2+4x+4} - 4x \;/:x}{x \;/:x} = \frac{1+4}{-1} = \frac{5}{-1} = -5$$

$$= \lim_{x \to -\infty} \frac{(\sqrt{x^2+4x+4} - 4x) + 5x}{\sqrt{x^2+4x+4} - 4x) - 5x} \cdot \frac{(\sqrt{x^2+4x+4} - 4x) - 5x}{(\sqrt{x^2+4x+4} - 4x) - 5x} = \lim_{x \to -\infty} \frac{(\sqrt{x^2+4x+4} - 4x)^2 - (5x)^2}{(\sqrt{x^2+4x+4} - 4x) - 5x} =$$

$$\lim_{x \to -\infty} \frac{x^2+4x+4 - 16x^2 - 25x^2 \;/:x^2}{(\sqrt{x^2+4x+4}) - 4x - 5x \;/:x^2} = \frac{-1-16-25}{0} = \frac{-42}{0} = -\infty$$

NEMA LIJEVE KOSE ASIMPTOTE

1) $\frac{|z|}{2+2i} = 3i$

$$\frac{\sqrt{x^2+y^2}}{x+y_1 i+2i} = 3i \;/\cdot(x+y_1 i+2i)$$

$$\sqrt{x^2+y^2} = 3i(x+y_1 i+2i)$$

$$x^2+y^2 = 3xi + 3yi^2 + 6i^2$$

$$x + y = 3x_1 - 3y - 6$$

$$x+y = 3y-6$$

$$0 = 3x$$

$$x = 0$$

$$0+y = 3y-6$$

$$y-3y = 6$$

$$-2y = -6 \;/\cdot(-\tfrac{1}{2})$$

$$y = 3$$

IVAN MARDETKO

6) $f(x) = \sqrt{4+3x}$

a) $f'(x) = (\sqrt{4+3x})'$

$f'(x) = \frac{1}{2\sqrt{4+3x}} \cdot (4+3x)'$

$f'(x) = \frac{1}{2\sqrt{4+3x}} \cdot 3$

$f'(x) = \frac{3}{2\sqrt{4+3x}}$ ✓

$f'(2) = \frac{3}{2\sqrt{4+2\cdot 2}} = \frac{3}{2\sqrt{4+6}} = \frac{3}{2\sqrt{10}} = \frac{\sqrt{10}}{10}$ ✓

	$-\frac{4}{3}$	$+\infty$
$f'(x)$	+	
$f(x)$	↗	

b) ekstrem

$f'(x) = 0$

$\frac{3}{2\sqrt{4+3x}} = 0 \mid \cdot 2\sqrt{4+3x}$

$3 = 0$ nema lokalnog ekstrema

MIN $(-\frac{4}{3}, 0)$ ✓

1) domena

$4+3x \geq 0$

$3x \geq -4 \mid \cdot \frac{1}{3}$

$x \geq -\frac{4}{3}$

$D_f: [-\frac{4}{3}, +\infty\rangle$

5) $h(x) = \arctan(x^3)$

1) domena $Df: x \in \mathbb{R}$ ✓

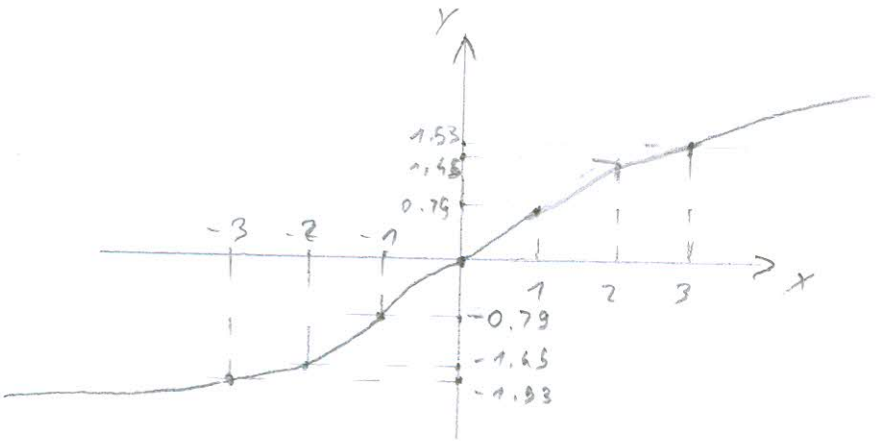
2) periodičnost

3) (ne) parnost

$L(-x) = \arctan(-x^3) = \arctan(-x^3)$

funkcija je parna ✗

	-3	-2	-1	0	1	2	3
$\arctan(x^3)$	-1.53	-1.45	-0.79	0	0.79	1.45	1.53



funkcija ni periodična ✓

4) odvajanje

$f'(x) = (\arctan(x^3))'$

$f'(x) = \frac{1}{1+(x^3)^2} \cdot (x^3)'$

$f'(x) = \frac{1}{1+x^6} \cdot 3x^2$ ✓

$f'(x) = \frac{3x^2}{1+x^6}$

$f''(x) = \left(\frac{3x^2}{1+x^6}\right)'$

$f''(x) = \frac{(3x^2)' \cdot (1+x^6) - 3x^2 \cdot (1+x^6)'}{(1+x^6)^2}$

$f''(x) = \frac{6x(1+x^6) - 3x^2 \cdot 6x^5}{(1+x^6)^2}$ ✓

$f''(x) = \frac{6x+6x^7-18x^7}{(1+x^6)^2} = \frac{-12x^7+6x}{(1+x^6)^2}$

4) $f(x) = \frac{x+3}{x^2-4}$

1) domena

$$D_f : x \in \mathbb{R} \setminus \{-2, 2\}$$

$x^2 - 4 \neq 0$

$x^2 \neq 4 \mid \sqrt{\ }$

$x \neq \pm 2$

2) asimptote

1) navpična asimptota

$\lim_{x \to -2} \frac{-2+3}{(-2)^2-4} = \frac{1}{4-4} = \frac{1}{0} = +\infty$

$x \to -2$ je navpična asimptota

$\lim_{x \to 2} \frac{2+3}{2^2-4} = \frac{5}{4-4} = \frac{5}{0}$ $x \to 2$ je navpična asimptota

2) horizontalna as.

$\lim_{x \to +\infty} \frac{x+3}{x^2-4} \begin{matrix} /:x^2 \\ /:x^2 \end{matrix} = \frac{0}{1} = 0$ $y = 0$

$\lim_{x \to -\infty} \frac{x+3}{x^2-4} \begin{matrix} /:x^2 \\ /:x^2 \end{matrix} = \frac{0}{-1} = 0$ $y = 0$

3) nul točke

$f(x) = 0$

$\frac{x+3}{x^2-4} = 0 \mid \cdot (x^2-4)$

$x + 3 = 0$

$x = -3$ $(-3, 0)$

4) globalna ekstrema

$$f'(x) = \frac{(x+3)' \cdot (x^2-4) - (x+3) \cdot (x^2-4)'}{(x^2-4)^2}$$

$$f'(x) = \frac{1 \cdot (x^2-4) - (x+3) \cdot 2x}{(x^2-4)^2}$$

$$f'(x) = \frac{x^2 - 4 - 2x^2 - 6x}{(x^2-4)^2} = \frac{-x^2 - 6x - 4}{(x^2-4)^2}$$

5) ekstrem

$f'(x) = 0$

$\frac{-x^2-6x-4}{(x^2-4)^2} = 0 \mid \cdot (x^2-4)^2$

$-x^2-6x-4 = 0 \mid \cdot (-1)$

$x^2+6x+4 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-6 \pm \sqrt{6^2-4\cdot 1\cdot 4}}{2\cdot 1} = \frac{-6 \pm \sqrt{36-16}}{2}$

$= \frac{-6 \pm \sqrt{20}}{2} = \frac{-6 \pm 2\sqrt{5}}{2}$

$x_1 = \frac{-6-2\sqrt{5}}{2} = -3-\sqrt{5} \approx -5.24$

$x_2 = \frac{-6+2\sqrt{5}}{2} = -3+\sqrt{5} \approx -0.76$

$f(-5.24) = \frac{-5.24+3}{(-5.24)^2-4} = -0.1$

$f(-0.76) = \frac{-0.76+3}{(-0.76)^2-4} = -0.65$

	$-\infty$		-5.24		-0.76		$+\infty$
$f'(x)$		$-$		$-$		$-$	
$f(x)$		↘		↘		↘	

BOLJE SE SAMO GRAF!

6) $f''(x) = \frac{(-x^2-6x-4)' \cdot (x^2-4)^2 - (-x^2-6x-4)\cdot((x^2-4)^2)'}{(x^2-4)^4}$

$f''(x) = \frac{(-2x-6)(x^2-4) - (x^2-6x-4)\cdot 2(x^2-4)\cdot(x^2-4)'}{(x^2-4)^4}$

$f''(x) = \frac{(-2x-6)(x^2-4) - (x^2-6x-4)\cdot 4x(x^2-4)}{(x^2-4)^3}$

$f''(x) = \frac{(x^2-4)\left((-2x-6) - (x^2-6x-4)\cdot(x^2-4)\right)}{(x^2-4)^2} = \frac{-2x-6-(x^4-4x^2-6x^3+24x-4x^2+16)}{(x^2-4)^3}$

$f''(x) = \frac{-2x-6-x^4+4x^2+6x^3-24x+4x^2-16}{(x^2-4)^3} = \frac{-x^4+6x^3+8x^2-22x-22}{(x^2-4)^3}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!! G3

IME I PREZIME: IVANA FURLAN **BROJ INDEKSA:** 17-2-0173-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja uvrštavanjem. 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcl} 2x_1 & - & x_2 & + & x_3 & - & x_4 = -1 \\ 2x_1 & - & x_2 & & & - & 3x_4 = 1 \\ 3x_1 & & & - & x_3 & + & x_4 = -1 \\ 2x_1 & + & 2x_2 & - & 2x_3 & + & 5x_4 = -1 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2+4x+4} - 4x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{x+3}{x^2-4}$. 15(graf) 13

5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^3)$. 2+4+6+8

6. Zadana je funkcija $f(x) = \sqrt{4+3x}$. Kolika je derivacija $f'(2)$? Koji su lokalni ekstremi? 10+5

Ukupno: 31

2. $\begin{bmatrix} 2 & -1 & 1 & -1 & -1 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{bmatrix} \begin{matrix} \\ \text{II}-\text{I} \\ \sim \\ \\ \text{IV}-\text{I} \end{matrix}$ $\begin{bmatrix} 2 & -1 & 1 & -1 & -1 \\ 0 & 0 & -1 & -2 & 2 \\ 3 & 0 & -1 & 1 & -1 \\ 0 & 3 & -3 & 6 & 0 \end{bmatrix} \begin{matrix} \\ \\ \sim \\ \text{III}-3\text{I}/2 \\ \\ \end{matrix}$ $\begin{bmatrix} 2 & -1 & -1 & -1 & 1 \\ 0 & 0 & -1 & -2 & 2 \\ 0 & 3/2 & -3/2 & 5/2 & 1/2 \\ 0 & 3 & -3 & 6 & 0 \end{bmatrix} \begin{matrix} \\ \\ \sim \\ \\ \text{IV}-2\text{II} \end{matrix}$

$\begin{bmatrix} 2 & -1 & 1 & -1 & -1 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 0 & 0 & 2 & 4 & -1 \end{bmatrix} \text{III}+2\text{II}$ $\begin{bmatrix} 2 & -1 & 1 & -1 & -1 \\ 2 & -1 & 0 & -3 & 1 \\ 0 & 3/2 & -3/2 & 5/2 & 1/2 \\ 0 & 0 & 0 & -3 & -3 \end{bmatrix}$

$-3x_4 = -3$
$x_4 = 1$

$-x_3 - 2x_4 = 2$
$-x_3 - 2 = 2$
$x_3 = -4$

$\frac{3}{2}x_2 - \frac{5}{2}x_3 + \frac{5}{2}x_4 = \frac{1}{2} \quad /\cdot 2$
$3x_2 - 5x_3 + 5x_4 = 1$
$3x_2 + 20 - 5 = 1$
$3x_2 = -20 - 5 + 1$
$3x_2 = -24$
$x_2 = -8$

$2x_1 - x_2 + x_3 - x_4 = -1$
$2x_1 + 8 - 4 - 1 = -1$
$2x_1 = -4$
$x_1 = -2$

PROVJERA
$2\cdot(-2) + 8 + (-4) - 1 = -1$
$-4 + 8 - 4 - 1 = -1$
$-1 = -1$

PROVJERA
$2\cdot(-2) - -8 + -4 - 1 = -1$
$2\cdot(-2) + 2\cdot(-8) - 2\cdot(-4) + 5\cdot 1 = -4 - 16 + 8 + 5$
$= -20 + 13 = -7 \neq -1$

6. $f(x) = \operatorname{arctan}(x^3)$

$f'(x) = \frac{1}{1+(x^3)^2} \cdot 3x^2$

$f'(x) = \frac{3x^2}{1+x^6}$ ✓

$f''(x) = \frac{(3x^2)' \cdot (1+x^6) - 3x^2 \cdot (1+x^6)'}{(1+x^6)^2}$

$f''(x) = \frac{6x \cdot (1+x^6) - 3x^2 \cdot 6x^5}{(1+x^6)^2}$ ✓

4. $f(x) = \frac{x+3}{x^2-4}$

1. DOMENA

$x^2 - 4 = 0$ $\quad Df = x \in \mathbb{R} \setminus \{2, -2\}$

$x^2 = 2$

$x \neq \pm 2$

2. ASIMPTOTE

V.A.

$\lim_{x \to -2^-} \frac{x+3}{x^2-4} = \left[\frac{1}{0^+}\right] = +\infty$

$\lim_{x \to -2^+} \frac{x+3}{x^2-4} = \left[\frac{1}{0^-}\right] = -\infty$

H.A.

$\lim_{x \to \infty} \frac{x+3}{x^2-4} = \lim_{x \to \infty} = \frac{x^2\left(\frac{1}{x} + \frac{3}{x^2}\right)}{x^2\left(1 - \frac{4}{x^2}\right)} = \frac{0}{1} = 0$

$\boxed{y = 0}$

3. EKSTREMI, RAST-PAD

$f(x) = \frac{x+3}{x^2-4}$

$f'(x) = \frac{(x+3)' \cdot (x^2-4) - (x+3) \cdot (x^2-4)'}{(x^2-4)^2}$

$f'(x) = \frac{(x^2-4) - (x+3) \cdot 2x}{(x^2-4)^2}$

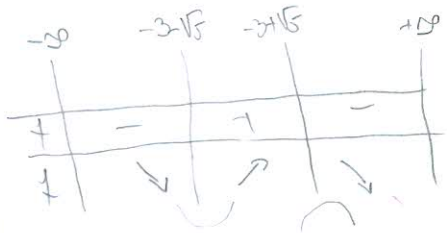
$-x^2 - 6x - 4 = 0$

$x = \frac{6 \pm \sqrt{(+6)^2 - 4 \cdot (-1) \cdot (-4)}}{2 \cdot (-1)}$ $\quad 36 - 16$

$x = \frac{6 \pm \sqrt{20}}{-2}$

$x_1 = -3 - \sqrt{5}$

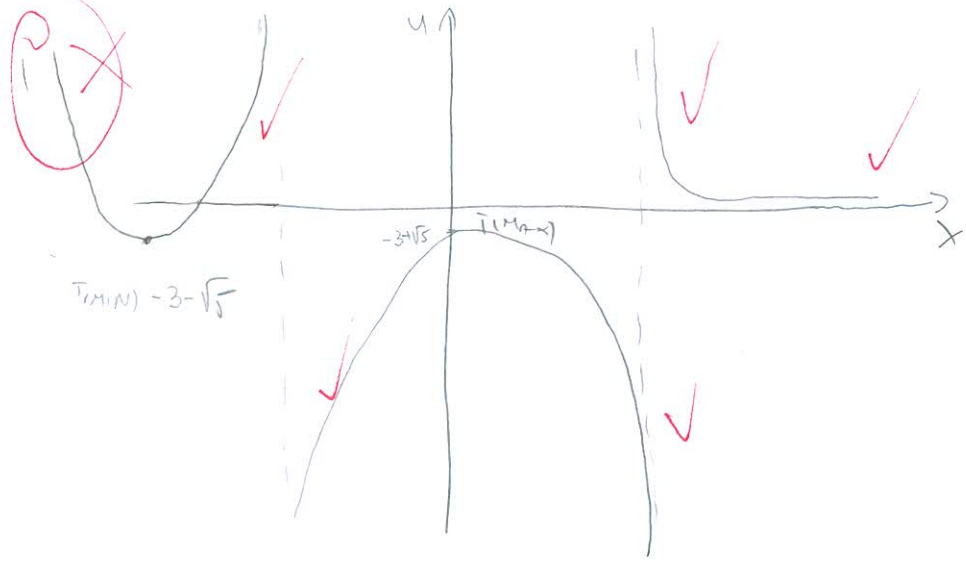
$x_2 = -3 + \sqrt{5}$



$T_{(MIN)} = (-3-\sqrt{5}, -0{,}09)$

$T_{(MAX)} = (-3+\sqrt{5}, -0{,}65)$

GRAF



13

6. $f(x) = \sqrt{4+3x}$

$$f'(x) = \frac{1}{2\sqrt{4+3x}} \cdot 3 = \frac{3}{2} \cdot \frac{1}{\sqrt{4+3x}}$$ ✓

$$f'(2) = \frac{3}{2} \cdot \frac{1}{\sqrt{4+3x}} = \frac{3}{2} \cdot \frac{1}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}$$

$$= \frac{3\sqrt{10}}{20}$$

$f(x) = \frac{3}{2\sqrt{4+3x}} \neq 0$

NEMA LOK. EKSTREMA ✗

$x = -\frac{4}{3}$ JE LOK. MIN.

3. $g(x) = \sqrt{x^2+4x+4} - 4x$

DOM

$x^2+4x+4 \geq 0$

$x = \frac{-4 \pm \sqrt{16-16}}{2} = -2$

$x \in \mathbb{R}$ ✓

ASIMPTOTE

V.A. NEMA

H.A.

$$\lim_{x \to \infty} \sqrt{x^2+4x+4} - 4x = \frac{\sqrt{x^2+4x+4} - 4x}{\sqrt{x^2+4x+4} - 4x}$$

$$\lim_{x \to \infty} \frac{x^2+4x+4-16x^2}{\sqrt{x^2+4x+4} - 4x}$$

$$\lim_{x \to \infty} \frac{-16x^2+4x+4}{\sqrt{x^2+4x+4} - 4x}$$

$$\lim_{x \to \infty} = \frac{x^2\left(-15+\frac{4}{x}+\frac{4}{x^2}\right)}{x^2\left(\sqrt{1+\frac{4}{x}+\frac{4}{x^2}} + \frac{4}{x}\right)} = \infty$$ NEMA H. A.

Κ.Α.

$$\lim_{x\to\infty} \frac{\sqrt{x^2+4x+4}-4x}{x}$$

$$\lim_{x\to+\infty} \frac{x\sqrt{1+\frac{4}{x}+\frac{4}{x^2}}-4}{x} = -3 \checkmark$$

ή $\lim_{x\to\infty} \left(\sqrt{x^2+4x+4}-17x\right) \cdot \frac{\sqrt{x^2+4x+4}-17x}{\sqrt{x^2+4x+4}-17}$

$$\lim_{x\to\infty} \frac{x^2-48\ldots}{x^2\left(\sqrt{\ldots}\ldots\right)} = \infty$$

$\boxed{\gamma=-3}$ ΚΟΣΠ ✗

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓ G3

IME I PREZIME: DUJE SURAĆ **BROJ INDEKSA:** 17-1-0118-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja uvrštavanjem. 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcr} 2x_1 & - & x_2 & + & x_3 & - & x_4 = -1 \\ 2x_1 & - & x_2 & & & - & 3x_4 = 1 \\ 3x_1 & & & - & x_3 & + & x_4 = -1 \\ 2x_1 & + & 2x_2 & - & 2x_3 & + & 5x_4 = -1 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2+4x+4} - 4x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{x+3}{x^2-4}$. 15(graf)

5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^3)$. 2+4+6+8

6. Zadana je funkcija $f(x) = \sqrt{4+3x}$. Kolika je derivacija $f'(2)$? Koji su lokalni ekstremi? 10+5

Ukupno: 5

3.

$$x^2 + 4x + 4 \geq 0$$

$$x_{12} = \frac{-b \pm \sqrt{4 \cdot a \cdot c}}{2a}$$

$$x_{12} = \frac{-4 \pm \sqrt{16 - 4 \cdot 1 \cdot 4}}{2}$$

$$x_{12} = \frac{-4 \pm 0}{2}$$

$$y_{12} = \frac{-4}{2} = -2$$

$D_g = x \in \mathbb{R}$ ✓

$D_g = \mathbb{R}$

4. $f(x) = \frac{x+3}{x^2-4}$

① DOMENA

$x^2 - 4 = 0$

$x^2 = 4$

$x = \sqrt{4}$

$x_1 = 2 \quad , \quad x_2 = -2$

② PARNOST

niti parna, niti neparna

③ PERIODIČNOST

nije periodična, jer nije trigonometrijska

④ ASIMPTOTE

a) V.A.

$\lim_{x \to 2^+} \frac{x+3}{x^2-4} = \frac{5}{0^+} = +\infty$

$\lim_{x \to 2^-} \frac{x+3}{x^2-4} = \frac{5}{0^-} = -\infty$

$\Big\} \; x = 2$

$\lim_{x \to -2^+} \frac{x+3}{x^2-4} = \frac{1}{0} = +\infty$

$\lim_{x \to -2^-} \frac{x+3}{x^2-4} = \frac{1}{0} = -\infty$

$\Big\} \; x = -2$

b) H.A.

$\lim_{x \to +\infty} \frac{x+3 \,/\, x^2}{x^2-4 \,/\, x^2} = \frac{\frac{1}{x} + \frac{3}{x^2}}{1 - \frac{4}{x^2}} = \frac{0}{1} = 0$

$\lim_{x \to -\infty} \frac{x+3}{x^2-4} = \lim_{x \to \infty} \frac{x+3}{x^2-4} = \frac{0}{1} = 0 \qquad y = 0$

c) K.A.

$\lim_{x \to \infty} \frac{x+3}{x^2-4} = \lim_{x \to \infty} \frac{x+3 \,/\, x^3}{x^3-4 \,/\, x^3} = \frac{0}{1} = 0$

Nema kose asimptote.

(5) MONOTONOST

$$f(x) = \frac{x+3}{x^2-4}$$

$$x^2 + 10x + 4 = 0$$

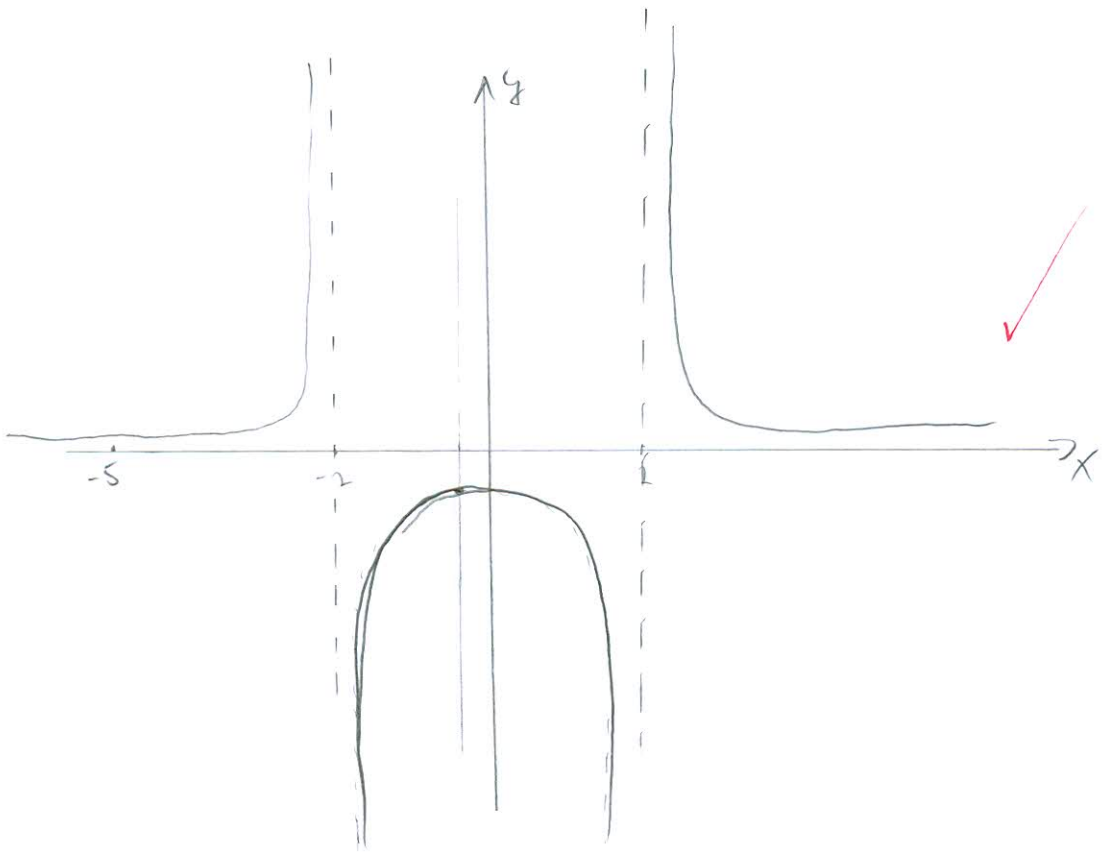
~~$x^2 + 10x$~~

$$x_{1,2} = \frac{-10 \pm \sqrt{100 - 4 \cdot 1 \cdot 4}}{2}$$

$$x_{1,2} = -5 + \sqrt{21} \sim 0,42$$

$$x_2 = -5 - \sqrt{21} \sim 3,58$$

št. točke



$x = -2$
V.A.

$y = 2$
V.A.

6. $f(x) = \sqrt{4+3x}$

$f'(2) = ?$

$f'(x) = \sqrt{4+3\cdot 2}$ ✗

$f'(x) = \sqrt{10}$ ✗

$f'(x) = \frac{1}{2\sqrt{10}}$ ✗

2

$$2x_1 - x_2 + x_3 - x_4 = -1$$
$$2x_1 - x_2 - 3x_4 = 1$$
$$3x_1 - x_3 + x_4 = -1$$
$$2x_1 + 2x_2 - 2x_3 + 5x_4 = -1$$

$$\left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] \quad \left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 0 & 0 & 1 & -4 & -2 \\ 3 & 0 & -1 & 1 & -1 \\ 0 & -3 & 3 & -6 & 0 \end{array}\right]$$

1. $\frac{|z|}{z+2i} = 3i$ $\qquad z = x + yi$

$|z| = 3i\,(z + 2i)$

$|z| = 3i\,(x + yi + 2i)$

$|z| = 3ix - 3y - 2$

$|z| = -(3y + 2) + 3xi$

$|z| = \sqrt{(3y+2)^2 + (3x)^2}$

$= \sqrt{9y^2 + 12y + 4 + 9x^2}$

$= \sqrt{9y^2 + 9y + 4 + 12y}$

5. $h(x) = \arctan(x^3)$

2.

$$\left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 3 & 0 & -1 & 1 & -1 \\ 2 & -1 & 0 & -3 & 1 \\ 2 & -1 & 1 & -1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] \begin{array}{l} \text{II} - \text{III} \\ \text{III} - \text{IV} \\ \text{IV} - \text{II} \end{array}$$

$$\left[\begin{array}{cccc|c} 3 & 0 & -1 & 1 & -1 \\ 0 & 0 & -1 & -2 & 2 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 3 & -2 & 8 & -2 \end{array}\right] \sim \left[\begin{array}{cccc|c} 3 & 0 & -1 & 1 & -1 \\ 0 & 3 & -2 & 8 & -2 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & -1 & -2 & 2 \end{array}\right] \begin{array}{l} \\ \\ \cdot 2 \\ + \end{array} \sim \left[\begin{array}{cccc|c} 3 & 0 & 0 & 3 & -3 \\ 0 & 3 & 0 & 12 & -6 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & -4 \end{array}\right] \begin{array}{l} /:3 \\ /:3 \\ \sim \\ /:(-4) \end{array}$$

~~Zadnji~~ Nema rješenja.

Nema rješenja.

RJEŠENJE JE:

$x_1 = 0$

$x_2 = 2$

$x_3 = 0$

$x_4 = -1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 4 & -2 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ G3

IME I PREZIME: TONI LULIĆ

BROJ INDEKSA: 17-1-0153-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja uvrštavanjem. 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcr} 2x_1 & - & x_2 & + & x_3 & - & x_4 = -1 \\ 2x_1 & - & x_2 & & & - & 3x_4 = 1 \\ 3x_1 & & & - & x_3 & + & x_4 = -1 \\ 2x_1 & + & 2x_2 & - & 2x_3 & + & 5x_4 = -1 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2+4x+4} - 4x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{x+3}{x^2-4}$. 15(graf)

5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^3)$. 2+4+6+8

6. Zadana je funkcija $f(x) = \sqrt{4+3x}$. Kolika je derivacija $f'(2)$? Koji su lokalni ekstremi? 10+5

Ukupno: 0

(3.) $\sqrt{x^2+4x+4} - 4x \geq 0$ ✗

$-4x \geq 0 \quad / \cdot (-1)$

UVJETI:
$x \neq 0$

$4x \leq 0$

$x \leq \frac{0}{4}$

$x = 0 \rightarrow$ nije ispunjen uvjet

$\sqrt{x^2+4x+4} \geq 0 \quad / ^2$

$x^2+4x+4 \geq 0$

$x_{1,2} = \frac{4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$

$x_{1,2} = \frac{4 \pm \sqrt{16-16}}{2}$

$x_{1,2} = \frac{4 \pm 0}{2} = \frac{4}{2} = 2$

$Df: x \in \mathbb{R}$ ✗

4. $h(x) = \arctan(x^3)$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ G3

IME I PREZIME: Ante Jerolimov BROJ INDEKSA: 17-2-0122-2014

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja uvrštavanjem. 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcr} 2x_1 & - & x_2 & + & x_3 & - & x_4 = -1 \\ 2x_1 & - & x_2 & & & - & 3x_4 = 1 \\ 3x_1 & & & - & x_3 & + & x_4 = -1 \\ 2x_1 & + & 2x_2 & - & 2x_3 & + & 5x_4 = -1 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2+4x+4} - 4x$. ~~5+15~~

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{x+3}{x^2-4}$. ~~15(graf)~~

5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^3)$. ~~2+4+6+8~~

6. Zadana je funkcija $f(x) = \sqrt{4+3x}$. Kolika je derivacija $f'(2)$? Koji su lokalni ekstremi? 10+5

Ukupno: ~~0~~

(3) $g(x) = \sqrt{x^2+4x+4} - 4x$

$D_f = \sqrt{x^2+4x+4} \geq 0$

$\frac{-4 \pm \sqrt{8}}{2} \geq 0$

$\frac{-4 \pm \sqrt{16-8}}{2} \geq 0$

$x^2+4x \geq 0$ $D_f = \{-4, +\infty\}$ X

$x(x+4) \geq 0$

$x \geq -4$

VA. $\lim_{x \to \infty} \sqrt{x^2+4x+4} - 4x = \infty - \infty = ?$ -4 je VA. X

HA. $\lim_{x \to 0} \sqrt{x^2+4x+4} - 4x$ $\lim_{x \to 0} \sqrt{0+0+4} - 0$ $\lim_{x \to 0} \sqrt{4} = 2$ H.A. X

KA $y = kx+l$ kose nema.

(5) $h(x) = \arctan(x^3)$

$h'(x) = \frac{1}{1+x^2} \cdot x^3 = \frac{x^3}{1+x^2}$

$h''(x) = \frac{x^3}{1+x^2} = \frac{(1+x^2)' \cdot (x^3) + (1+x^2) \cdot (x^3)'}{2x} = \frac{2x \cdot x^3 + 1 + x^2 \cdot 3x^2}{2x} =$

$= \frac{5x^4+1}{2x}$

④ $f(x) = \frac{x+3}{x^2-4}$

$x^2 - 4 \neq 0$ $\quad D_f = \{-\infty, 2\} \{2, +\infty\}$

$x^2 \neq 4$

$x \neq \sqrt{4}$

$x \neq 2$

$VA = \lim_{x \to 2} \frac{x+3}{x^2-4} = \frac{\infty}{\infty} = 0$ $\quad VA = 0$

$HA = \lim_{x \to \infty} \frac{x+3}{x^2-4} \cdot \frac{1/x^2}{1/x^2} = \frac{1+0}{1-0} = \frac{3}{-3} = -1$ $\quad HA$

Kose mena

$f'(x) = \frac{(x^2-4)' \cdot (x+3) + (x^2-4) \cdot (x+3)'}{2x} = \frac{2x \cdot (x+3) + (x^2-4) \cdot x}{2x}$

$f'(x) = \frac{2x^2+6x+x^3-4x}{2x} = \frac{2x^2+2x+x^3}{2x}$

$f''(x) = \frac{2 \cdot (2x^2+2x+x^3) - (2x) \cdot (4x+2+3x^2)}{2x^2} = \frac{4x^2+4x+2x^3-8x^2-4x-6x^2}{2}$

$f''(x) = \frac{4x^2+2x^3+8x^2+8x+6x^2}{2}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ G3

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: Allen Mišković **BROJ INDEKSA:** 17-2-0057-2016

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja uvrštavanjem. 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcrl} & a & & b & & c & & d & \\ 2x_1 & - & x_2 & + & x_3 & - & x_4 & = -1 & \times \\ 2x_1 & - & x_2 & & & - & 3x_4 & = 1 & y \\ 3x_1 & & & - & x_3 & + & x_4 & = -1 & w \\ 2x_1 & + & 2x_2 & - & 2x_3 & + & 5x_4 & = -1 & z \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2+4x+4} - 4x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{x+3}{x^2-4}$. 15(graf)

5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^3)$. 2+4+6+8

6. Zadana je funkcija $f(x) = \sqrt{4+3x}$. Kolika je derivacija $f'(2)$? Koji su lokalni ekstremi? 10+5

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(2) a b c d

$$\left|\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right| \sim \left|\begin{array}{cccc|c} \boxed{1} & -1 & 2 & -1 & -1 \\ 0 & -1 & 2 & -3 & 1 \\ -1 & 0 & 3 & 1 & -1 \\ -2 & 2 & 2 & 5 & -1 \end{array}\right| \sim \begin{array}{l} R_3+1R_1 \\ R_4+2R_1 \end{array} \left|\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & -1 & 2 & -3 & 1 \\ 0 & -1 & 5 & 0 & -2 \\ 0 & 0 & 6 & 3 & -3 \end{array}\right| \cdot(-1) \sim$$

$$\left|\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & \boxed{1} & -2 & 3 & -1 \\ 0 & -1 & 5 & 0 & -2 \\ 0 & 0 & 6 & 3 & -3 \end{array}\right| \begin{array}{l} R_1+R_2 \\ \sim \\ R_3+R_2 \end{array} \left|\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -2 \\ 0 & 1 & -2 & 3 & -1 \\ 0 & 0 & \boxed{3} & -1 & -3 \\ 0 & 0 & 6 & 3 & -3 \end{array}\right| \sim \cdot\frac{1}{3} \left|\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -2 \\ 0 & 1 & -2 & 3 & -1 \\ 0 & 0 & \boxed{1} & -\frac{1}{3} & -1 \\ 0 & 0 & 6 & 3 & -3 \end{array}\right| \begin{array}{l} R_2+R_3 \\ \sim \\ R_4-6R_3 \end{array} \left|\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -2 \\ 0 & 1 & 0 & \frac{8}{3} & -2 \\ 0 & 0 & 1 & -\frac{1}{3} & -1 \\ 0 & 0 & 0 & 5 & 3 \end{array}\right|$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 2 \\ 0 & 1 & 0 & \frac{8}{3} & -2 \\ 0 & 0 & 1 & -\frac{1}{3} & -1 \\ 0 & 0 & 0 & 5 & 3 \end{array}\right| \cdot(\frac{1}{5}) \sim \left|\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 2 \\ 0 & 1 & 0 & \frac{8}{3} & -2 \\ 0 & 0 & 1 & -\frac{1}{3} & -1 \\ 0 & 0 & 0 & \boxed{1} & \frac{3}{5} \end{array}\right| \begin{array}{l} R_1-2R_4 \\ R_2-\frac{8}{3}R_4 \\ R_3+\frac{1}{3}R_4 \end{array} \sim$$

c b a d

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{4}{5} \\ 0 & 1 & 0 & 0 & -\frac{18}{5} \\ 0 & 0 & 1 & 0 & -\frac{4}{5} \\ 0 & 0 & 0 & 1 & \frac{3}{5} \end{array}\right| \begin{array}{c} x \\ y \\ w \\ z \end{array}$$

$x = \frac{4}{5}$

$y = -\frac{18}{5}$

$w = -\frac{4}{5}$

$z = \frac{3}{5}$

PROVJERA:

$2\cdot\frac{4}{5} + \frac{18}{*5} + \frac{-4}{5} - \frac{3}{5} = \frac{8+18-4-3}{5} = \frac{19}{5} \neq -1$

⑥ $f(x) = \sqrt{4+3x}$