

odgovornosti studenata. **PIŠITE DVOSTRANO!**

000

IME I PREZIME: MAJE ĆOSIĆ

BROJ INDEKSA: 55924

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 2y' = 1$, uz $y(0) = 1$ i $y'(0) = 0$.
Na kraju provjeri rješenje.

15

2. Pronađi funkciju koja zadovoljava $xy' = x + y$ i $y(1) = 2$.

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3. Zadana je funkcija a $f(x, y) = \sqrt{(x-1)^2 + (y-1)^2} + 1$. Ispitati domenu, kodomenu i razinske krivulje.

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4. $\int_0^{\pi} \cos(x) \sin^2(x) dx = ?$

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5. $\int_0^2 \frac{x^2 - 1}{x^2 + 1} dx = ?$

20

6. Izračunati površinu lika omeđenog pravcem $y = x + 1$ i parabolom $y = x^2 + 3x - 2$.

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Ukupno:

50

f	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$a^x (\alpha > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
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$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
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$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x \sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

5. $\int_0^2 \frac{x^2 - 1}{x^2 + 1} dx$

$\int \frac{x^2 - 1}{x^2 + 1} dx$
 $x^2 + 1 \neq 0$
 $x^2 = -1$
 $x = \pm 1 \times$ 0 1 2

$\lim_{x \rightarrow 1^-} \int_0^x \frac{x^2 - 1}{x^2 + 1} dx + \lim_{x \rightarrow 1^+} \int_x^2 \frac{x^2 - 1}{x^2 + 1} dx$

$\lim_{x \rightarrow 1^-} x - \arctan x \Big|_0^x + \lim_{x \rightarrow 1^+} x - 2 \arctan x \Big|_x^2$

$\lim_{x \rightarrow 1^-} 1 - \arctan 1 - (0 - \arctan 0) + \lim_{x \rightarrow 1^+} 2 - \arctan 2 - (1 - \arctan 1)$

$0,21 + 0,68 = 0,89$

$\int \frac{x^2 - 1}{x^2 + 1} dx$
 $(x^2 - 1) : (x^2 + 1) = 1$
 $-x^2 - 1$
 -2

$\int dx - 2 \int \frac{dx}{x^2 + 1} = x - 2 \arctan x$

PROVJERA NUMERICKOM
INTEGRACIJOM BI
VAS SPASILA.

$$4.) \int_0^{\pi} \cos(x) \sin^2(x) dx =$$

$$= \frac{\sin^3 x}{3} \Big|_0^{\pi} = \frac{\sin^3 \pi}{3} - \frac{\sin^3 0}{3}$$

$$= 0 - 0 = 0 // \checkmark$$

$$\int \cos x \sin^2 x dx = \left[\begin{array}{l} \sin x = t \\ \cos x dx = dt \end{array} \right]$$

$$\int t^2 dt = \frac{t^3}{3} = \frac{\sin^3 x}{3} // \checkmark$$

$$6.) y = x + 1$$

$$x^2 + 3x - 2 = x + 1$$

$$x^2 + 3x - x - 2 - 1$$

$$x^2 + 2x - 3 = 0$$

$$x_{1,2} = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-3)}}{2}$$

$$x_{1,2} = \frac{-2 \pm 4}{2}$$

$$x_1 = -3 \quad x_2 = 1$$

$$y_1 = -2 \quad y_2 = 2$$

$$S_1(-3, -2) \quad S_2(1, 2)$$

$$y = x^2 + 3x - 2$$

$$x^2 > 0 \quad x^2 + 3x - 2 = 0$$

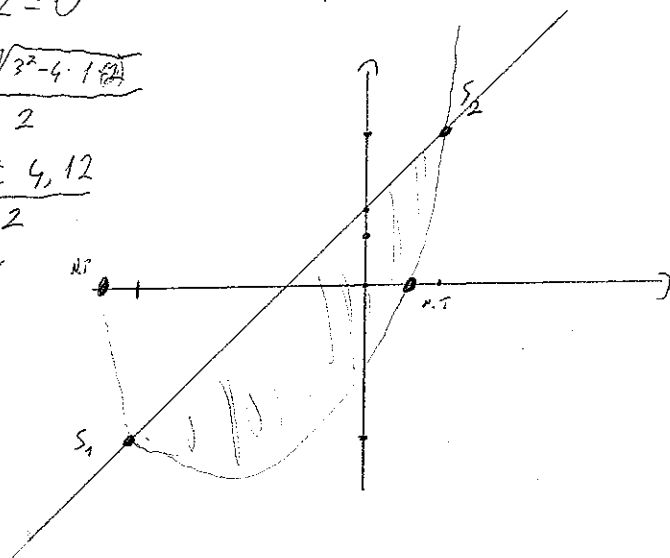
$$x_{1,2} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-2)}}{2}$$

$$x_{1,2} = \frac{-3 \pm 4,12}{2}$$

$$x_1 = -3,56$$

$$x_2 = 0,56$$

x	0	1	2
y = x + 1	1	2	3



$$P = \int_{-3}^1 (x+1) - (x^2 + 3x - 2) dx$$

$$P = \int_{-3}^1 (x+1 - x^2 - 3x + 2) dx = \int_{-3}^1 (-x^2 - 2x + 3) dx = \left[-\frac{x^3}{3} - 2 \frac{x^2}{2} + 3x \right]_{-3}^1$$

$$P = \left[-\frac{1^3}{3} - 2 \frac{1^2}{2} + 3 \cdot 1 \right] - \left[-\frac{(-3)^3}{3} - 2 \frac{(-3)^2}{2} + 3 \cdot (-3) \right] = \left[-\frac{1}{3} - 1 + 3 \right] + 9 = \frac{32}{3} // \checkmark$$

MATE ĆOSIĆ

BR. WD. 55924

1) $y'' - 2y' = 1$

$y(0) = 1, y'(0) = 0$

$y' = \lambda^2 \quad y' = \lambda$

$\lambda^2 - 2\lambda = 0$

$\lambda(\lambda - 2) = 0$

$\lambda_1 = 0 \quad \lambda_2 = 2$

$y_H = C_1 e^{0x} + C_2 e^{2x}$

$y_H = C_1 e^0 + C_2 e^{2x}$

$y_H = C_1 + C_2 e^{2x}$

$1 = e^{\alpha x} (P_m(x) \cos(\beta x) + Q_m(x) \sin(\beta x))$

$\alpha = 0 \quad \beta = 0 \quad \alpha + \beta i = 0 + 0i = \alpha_1 \neq \alpha_2 \Rightarrow \boxed{R=1}$

$P_m = 1 \quad m = 0 \quad Q_m = 0 \quad m = N/P$
 $N = \alpha \beta x \{m, m\}$

$y_P = x^k e^{\alpha x} (S_m(x) \cos(\beta x) + T_m(x) \sin(\beta x))$

$y_P = x S_m(x)$

$y_P = Ax$

$y_P' = A$

$y_P'' = 0$

$y'' - 2y' = 1$

$0 - 2A = 1$

$-2A = 1$

$A = -\frac{1}{2}$

$y_P = -\frac{1}{2}x$

$y = y_H + y_P$

$y = C_1 + C_2 e^{2x} - \frac{1}{2}x$

$y' = 2C_2 e^{2x} - \frac{1}{2}$

$1 = C_1 + C_2 e^0 - \frac{1}{2} \cdot 0$

$0 = 2C_2 e^0 - \frac{1}{2}$

$1 = C_1 + C_2$

$0 = 2C_2 - \frac{1}{2}$

$C_1 + C_2 = 1$

$2C_2 - \frac{1}{2} = 0 \Rightarrow 2C_2 = \frac{1}{2} / 2$

$C_2 = \frac{1}{4}$

$C_1 + \frac{1}{4} = 1$

$C_1 = \frac{3}{4}$

PROVERA
 $y = \frac{3}{4} + \frac{1}{4} e^{2x} - \frac{1}{2}x$

$y' = \frac{1}{2} e^{2x} - \frac{1}{2}$

$y'' = e^{2x}$

$e^{2x} - 2\left(\frac{1}{2} e^{2x} - \frac{1}{2}\right) = 1$

$e^{2x} - e^{2x} + 1 = 1$

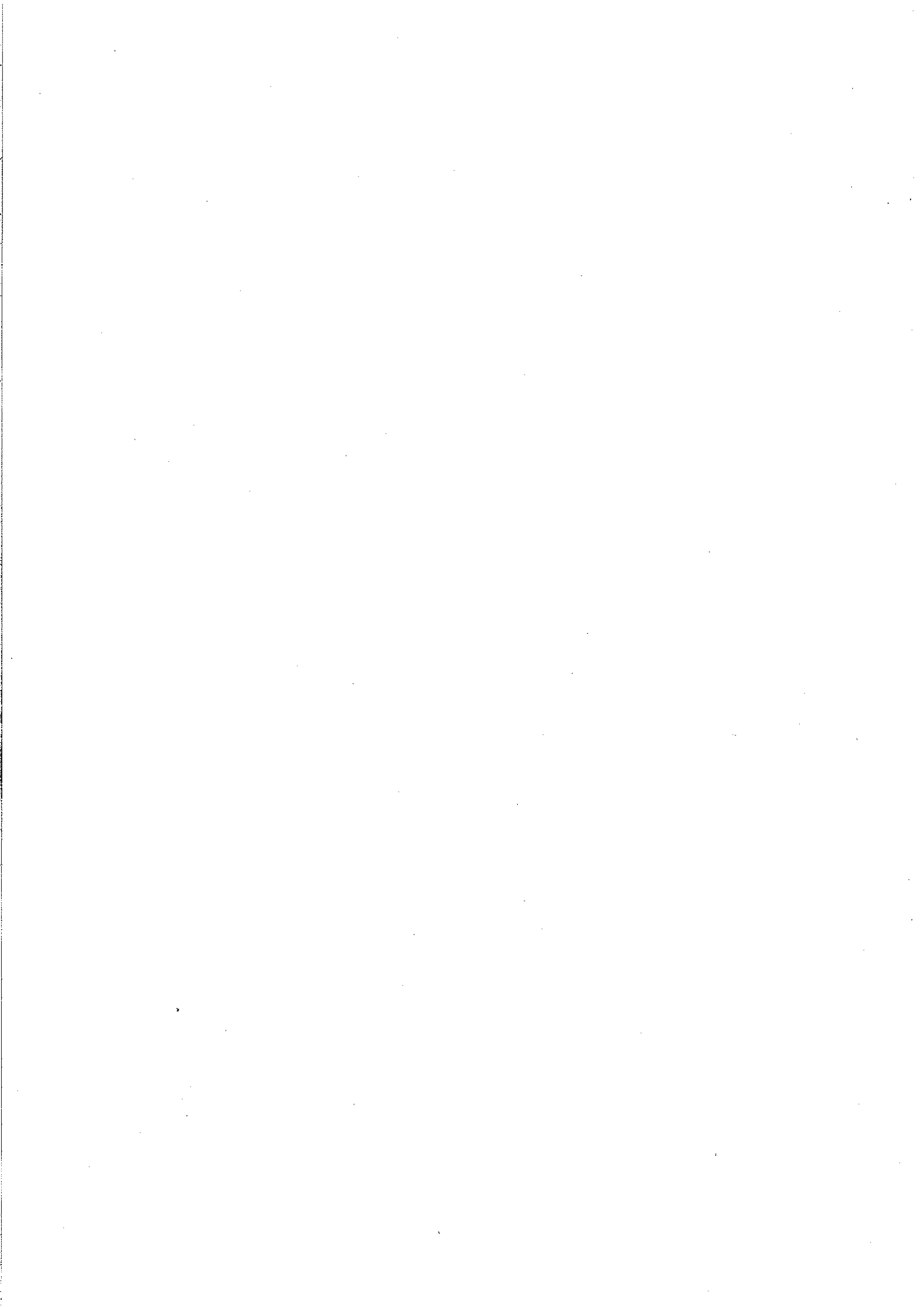
$1 = 1$

$y = \frac{3}{4} + \frac{1}{4} e^{2x} - \frac{1}{2}x$

PROVERA

$y(0) = \frac{3}{4} + \frac{1}{4} = 1 \checkmark$

$y'(0) = \frac{1}{2} - \frac{1}{2} = 0 \checkmark$



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IME I PREZIME: **NIKOLA TOMASOV**

BROJ INDEKSA: **17-2-0161-2012**

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

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Na kraju provjeri rješenje. (15)

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3. Zadana je funkcija a $f(x, y) = \sqrt{(x-1)^2 + (y-1)^2} + 1$. Ispitati domenu, kodomenu i razinske krivulje. 15

4. $\int_0^{\pi} \cos(x) \sin^2(x) dx = ?$ (15)

5. $\int_0^2 \frac{x^2 - 1}{x^2 + 1} dx = ?$ 20

6. Izračunati površinu lika omeđenog pravcem $y = x + 1$ i parabolom $y = x^2 + 3x - 2$. 20

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1. $y'' - 2y' = 1$

$y(0) = 1$ i $y'(0) = 0$

$r^2 - 2r = 0$

$r(r - 2) = 0$

$r_1 = 0$

$r - 2 = 0$

$r_2 = 2$

$y_H = C_1 e^{r_1 x} + C_2 e^{r_2 x}$

$y_H = C_1 + C_2 e^{2x}$

$y_P = K_1 x$

$y_P' = K_1$

$y_P'' = 0$

$y = C_1 + C_2 e^{2x} - \frac{1}{2} x$

$y' = 2C_2 e^{2x} - \frac{1}{2}$

$y'(0) = 0$

$2C_2 - \frac{1}{2} = 0$

$C_2 = \frac{1}{4}$

$-2K_1 = 1$

$K_1 = -\frac{1}{2}$

$y_P = -\frac{1}{2} x$

$y(0) = 1$

$C_1 + C_2 e^0 = 1$

$C_1 + C_2 = 1$

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$C_1 = \frac{3}{4}$

$y = y_H + y_P$
 $y = \frac{3}{4} + \frac{1}{4} e^{2x} - \frac{1}{2} x$ ✓

PROVJERA:

$$y(0) = 1$$

$$y = \frac{3}{4} + \frac{1}{4}e^0 - \frac{1}{2} \cdot 0 = 1$$

$$\underline{\underline{1 = 1}}$$

$$y'(0) = 0$$

$$y' = 2 \cdot \frac{1}{4} \cdot e^0 - \frac{1}{2} = 0$$

$$\underline{\underline{0 = 0}}$$

$$\textcircled{4} \int_0^{\pi} \cos(x) \sin^2(x) dx = \left[\begin{array}{l} t = \sin(x) \\ dt = \cos(x) dx \end{array} \right]$$

$$= \int_0^{\pi} dt \cdot t^2 = \int_0^{\pi} t^2 \cdot dt = \frac{t^3}{3} \Big|_0^{\pi} = \left(\frac{\sin^3(\pi)}{3} - \frac{\sin^3(0)}{3} \right) = \underline{\underline{0}} \checkmark$$

odgovornosti studenata. **PIŠITE DVOSTRANO!**

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IME I PREZIME: **MARTINO ROŽIČAR BUKVIĆ**

BROJ INDEKSA: **0171262202**

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

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IME I PREZIME:

BROJ INDEKSA:

IVAN GAČIKA

0269 06 08 44

POPUNJAVA
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IVSU GEIPIA

1. Odredi partikularno rješenje koje zadovoljava navedenu OΔ i
uvjete $y'' - 2y' = 1$, uz $y(0) = 1$ i $y'(0) = 0$. Prijeti rješenje