MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: Antonio Marušić **BROJ INDEKSA:** 17-2-0186-2012

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = e^{-x} - 1$. ~~15~~
2. Odrediti lokalne ekstreme funkcije $f(x, y) = e^x - x + y^2$. 15
3. Izračunati tangencijalnu ravninu plohe $z = \sin(x^2 y)$ u točki $(2, 1, \sin(4))$. 15
4. Numeričkom integracijom procijeniti vrijednost $\int_1^2 (x+2)\ln x dx$ i dati procjenu greške ili neku drugu kvalitetnu informaciju o greški. 10+5
5. $\int_0^\pi \frac{dx}{\sin x + 2} = ?$ 20
6. $\int_0^2 \frac{x-1}{x^2+x-2} dx = ?$ 20

Ukupno: 50

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2-x^2} dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

① $y'' - 2y' - 3y = e^{-x} - 1$

$k^2 - 2k - 3 = 0$

$k_{1,2} = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-3)}}{2} = \frac{2 \pm 4}{2}$ $\quad k_1 = 3$, $k_2 = -1$

$y = C_1 e^{2x} + C_2 e^{-x}$

$y_p = Ax e^{-1} + B$

$y_p' = Ae^{-x} + xAe^{-x} \cdot (-1) = Ae^{-x}(1-x)$

$y_p'' = Ae^{-x} \cdot (-1) \cdot (1-x) + Ae^{-x} \cdot (-1) = Ae^{-x}(-1-1+x) = Ae^{-x}(-2+x)$

$= A e^{-x}(-2+x) - 2Ae^{-x}(1-x) + 3xAe^{-x} - 3B = e^{-x} - 1$

$= A(-2+x) - 2A(1-x) - 3Ax = 1$ PROVJERA:

$-2A + Ax - 2A + 2Ax - 3Ax = 1$

$-4A = 1 \qquad -3B = -1$

$A = -\frac{1}{4} \qquad B = \frac{1}{3}$

$y^p = -\frac{1}{4} x e^{-1} + \frac{1}{3}$

$C_1 e^{x} + C_2 e^{-x} = -\frac{1}{4} x e^{-x} + \frac{1}{3}$

$y_p' = \frac{1}{4}xe^{-x} - \frac{1}{4}e^{-x}$

$y_p'' = -\frac{1}{4}xe^{-x} + \frac{1}{2}e^{-x}$

$y_p'' - 2y_p' - 3y_p = \dots$

$\dots = 1 + \underbrace{e^{-x}\left(\frac{1}{2} - 2\cdot\frac{1}{4}\right)}_{=0} + xe^{-x}\underbrace{\left(-\frac{1}{4} - \frac{2}{4} + \frac{3}{4}\right)}_{=0}$

$\varnothing$

② $f(x,y) = e^x - x + y^2$

$\frac{df(x,y)}{dx} = e^x - 1 \qquad e^x - 1 = 0$

$e^x = 1$

$\frac{df(x,y)}{dy} = 2y$

$T(0,0)$ KRITIČNA TOČKA

$2y = 0$

$y = -2$

$\frac{df^2(x,y)}{dx^2} = e^x \qquad \frac{d}{dy} = \left(\frac{df(x,y)}{dx} \right) = 0$

$\frac{df^2(0,0)}{dx^2} = 1 > 0$

$\frac{df^2(x,y)}{dx^2} = 2 \qquad \Delta = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2 > 0$

FUNKCIJA U TOČKI $T(0,0)$ IMA MINIMUM ✓

Antonio Marušić

⑥ $\int_0^2 \frac{x-1}{x^2+x-2}\,dx =$

$x^2+x-2=0$

$\sqrt{9}=\pm 3$

$x_{1,2} = \frac{-1\pm\sqrt{1-(4)\cdot 1\cdot(-2)}}{2} = \frac{-1\pm 3}{2} = x_1 = 1$

$x_2 = -2$

$x^2+x-2 = (x-1)(x+2)$

$\int_0^2 \frac{x-1}{(x-1)(x+2)}\,dx = \int_0^2 \frac{dx}{x+2} = \ln(x+2)\Big|_0^2$

$= \ln 4 - \ln 2 = \ln\frac{4}{2} = \ln 2$ ✓

③

$z = \sin(x^2 y)$

$T((2,1), \sin 4)$

$z_x = \cos(x^2 y)\cdot 2xy$ ✓

$z_x(2,1) = 4\cos(4)$

$z_y = \cos(x^2 y)\cdot x^2$ ✓

$z_y(2,1) = 4\cos(4)$ ✓

$z = 4\cos(4)(x-2) + 4\cos(4)(y-1) + \sin 4$ ✓

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox IME I PREZIME: GABRIJELA BRKIĆ BROJ INDEKSA: 0269069951	POPUNJAVA NASTAVNIK Broj ↓ bodova

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Ukupno: ~~35~~

40

Kozen

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
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$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \mathrm{d}x = -\ln\|\cos x\| + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x \mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

1) $y'' - 2y' - 3y = e^{-x} - 1$

$y'' - 2y' - 3y = 0$

$k = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} \Rightarrow k_1 = \frac{2+4}{2} = \frac{6}{2} = 3 \Rightarrow y_1 = e^{3x}$

$k_2 = -\frac{2}{2} = -1 \Rightarrow y_2 = e^{-x}$

$y_H = C_1 e^{3x} + C_2 e^{-x}$

$y_p = Ax^0 \cdot e^{-x} + C \ /'$

$y_p' = -Ae^{-x} \ /'$

$y_p'' = Ae^{-x}$

$$y_\varphi'' - 2y_\varphi' - 3y_\varphi = e^{x} - 1$$

$$Ae^{-x} - 2(-Ae^{-x}) - 3(Ae^{-x} + C) = e^{x} - 1$$

$$Ae^{-x} + 2Ae^{-x} - 3Ae^{-x} + 3C = e^{x} - 1$$

$$3C = e^{x} - 1$$

$$3C = -1$$

$$C = -\frac{1}{3} \qquad y_\varphi = -\frac{1}{3}$$

$$y = C_1 e^{3x} + C_2 e^{-x} - \frac{1}{3}$$

PROYER 4. –

0

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(5) $$\int_0^{\pi} \frac{dx}{\sin x + 2} = \left\{ \begin{array}{ll} tg\,\frac{x}{2} = t & \sin x = \frac{2t}{1+t^2} \\ x = 2\,\mathrm{arctg}\, t \,/' & \\ dx = \frac{2}{1+t^2}\,dt & \begin{array}{c|c|c} x & \pi & 0 \\ \hline t & \infty & 0 \end{array} \end{array} \right\} =$$

$$= \int_0^{\infty} \frac{\frac{2}{1+t}\,dt}{\frac{2t}{1+t} + 2} = \int_0^{\infty} \frac{\frac{2}{1+t}}{\frac{2t + 2(1+t^2)}{1+t}}\,dt = \int_0^{\infty} \frac{\frac{2}{\cancel{1+t}}}{\frac{2t+2+2t^2}{\cancel{1+t}}}\,dt = \int_0^{\infty} \frac{\cancel{2}}{\cancel{2}(t^2+t+1)}\,dt =$$

$$= \int_0^{\infty} \frac{dt}{t^2+t+1} = \int_0^{\infty} \frac{dt}{\left(t+\frac{1}{2}\right)^2 + \frac{3}{4}} = \left\{ \begin{array}{l} t + \frac{1}{2} = u \,/' \\ dt = du \end{array} \right\} =$$

$$= \int_0^{\infty} \frac{du}{u^2 + \frac{3}{4}} = \lim_{a\to\infty} \int_0^{a} \frac{du}{u^2 + \frac{3}{4}} = \lim_{a\to\infty} \frac{1}{\frac{\sqrt{3}}{2}}\,\mathrm{arctg}\left(\frac{u}{\frac{\sqrt{3}}{2}}\right)\Bigg|_0^a =$$

$$= \lim_{a\to\infty} \frac{2}{\sqrt{3}}\,\mathrm{arctg}\left(\frac{2\left(t+\frac{1}{2}\right)}{\sqrt{3}}\right)\Bigg|_0^a = \lim_{a\to\infty} \cdot \frac{2}{\sqrt{3}} \cdot \left[\mathrm{arctg}\left(\frac{2\cdot\left(a+\frac{1}{2}\right)}{\sqrt{3}}\right) - \mathrm{arctg}\left(\frac{2\cdot\left(0+\frac{1}{2}\right)}{\sqrt{3}}\right)\right]$$

$$= \frac{2}{\sqrt{3}} \cdot \left(\mathrm{arctg}\,\infty - \mathrm{arctg}\,\frac{1}{2\sqrt{3}}\right) = \frac{2}{\sqrt{3}}\left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \frac{2}{\sqrt{3}} \cdot \frac{\pi}{3} = \frac{2\pi}{3\sqrt{3}}$$

GABRIJELA BRKIĆ

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6) $\int_0^2 \frac{x-1}{x^2+x-2}\,dx$, $D_f = \mathbb{R} \setminus \{-2, 1\}$

$$= \underbrace{\int_0^2 \frac{x\,dx}{x^2+x-2}}_{I_1} - \underbrace{\int_0^2 \frac{dx}{(x+\frac{1}{2})^2 - \frac{9}{4}}}_{I_2}$$

$$I_1 = \frac{1}{2}\underbrace{\int_0^2 \frac{2x+1}{x^2+x-2}\,dx}_{I_3} - \frac{1}{2}\underbrace{\int_0^2 \frac{dx}{(x+\frac{1}{2})^2-\frac{9}{4}}}_{I_2}$$

$$I_3 = \int_0^2 \frac{2x+1}{x^2+x-2} = \left\{ \begin{array}{c} x^2+x-2 = t \\ 2x+1/x = dt \end{array} \right\} = \int_0^2 \frac{dt}{t} = \ln t \Big|_0^2$$

$$= \lim_{\varepsilon \to 0} \ln(x^2+x-2) \Big|_0^{1-\varepsilon} + \lim_{\delta \to 0} \ln(x^2+x-2) \Big|_{1+\delta}^2 =$$

$$= -\ln|-2| + \ln 4$$

$$I_2 = \int_0^2 \frac{dx}{(x+\frac{1}{2})^2-\frac{9}{4}} = \lim_{\varepsilon \to 0} \int_0^{1-\varepsilon} \frac{dx}{(x+\frac{1}{2})^2-\frac{9}{4}} + \lim_{\delta \to 0} \int_{1+\delta}^2 \frac{dx}{(x+\frac{1}{2})^2-\frac{9}{4}} =$$

$$= \lim_{\varepsilon \to 0} \frac{1}{3} \ln\left(\frac{x-1}{x+2}\right) \Big|_0^{1-\varepsilon} + \lim_{\delta \to 0} \frac{1}{3} \ln\left|\frac{x-1}{x+2}\right| \Big|_{1+\delta}^2 = -\frac{1}{3}\ln\frac{1}{2} + \frac{1}{3}\ln\frac{1}{4}$$

$$I = (I_1 - I_2) = \frac{1}{2}(I_3 - I_2) - I_2 = \frac{1}{2} I_3 - \frac{1}{2} I_2 - I_2 = \frac{1}{2} I_3 - \frac{3}{2} I_2$$

$$\boxed{I = \frac{-\ln|-2|}{2} + \frac{\ln 4}{2} + \frac{1}{2}\ln\frac{1}{2} - \frac{1}{2}\ln\frac{1}{4}}$$

$$= -\frac{\ln 2}{2} + \frac{2\ln 2}{2} - \frac{1}{2}\ln 2 + \frac{1}{2}(\ln 2)\cdot 2 = \ln 2 \checkmark$$

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IME I PREZIME: IVAN BERAM **BROJ INDEKSA:** 17-1-0130-2012

POPUNJAVA NASTAVNIK
Broj ↓
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Ukupno: 40

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
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Tablica nekih integrala

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$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x \mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

2. $f(x,y) = e^x - x + y^2$

Df:

$\sqrt{-e^x + x} \geq 0$

$-e^x + x \geq 0$

$y^2 = -e^x + x$

$y = \sqrt{-e^x + x}$

$e^x - x + y^2 = C$

$y = \sqrt{C - e^x + x}$

$(x, y) = (x\ \sqrt{C - e^{x} + x} = ?$

$C = 0 \ldots\ldots\ y = \sqrt{-e^x + x}$

$C = 1 \ldots\ldots\ y = \sqrt{1 - e^x + x}$

$C = -1 \ldots\ldots\ y = \sqrt{-1 - e^x + x}$

$C = 2 \ldots\ldots\ y = \sqrt{2 - e^x + x}$

$C = -2 \ldots\ldots\ y = \sqrt{-2 - e^x + x}$

?

3. $z = \sin(x^2 y)$ $\qquad T(2, 1, \sin(4))$

6. $\int_0^2 \frac{x-1}{x^2+x-2}\,dx = \int_0^2 \frac{x-1}{x^2-x+2x-2} = \int_0^2 \frac{\cancel{x-1}}{(x+2)\cancel{(x-1)}} =$

$\int_0^2 \frac{1}{(x+2)} = \Big| _0^2 \ln|x+2| = \ln|4| - \ln|2| =$

$= \ln 4 - \ln 2 = \ln 2$ ✓

$\ln|x+2|' = \frac{1}{x+2} \cdot 1 = \frac{1}{x+2}$

IVAN BERAM

5. $\int_0^{\pi} \frac{dx}{\sin x + 2} = \left[\begin{array}{l} t = tg\frac{x}{2} \\ dx = \frac{2dt}{1+t^2} \end{array} \right.$ $\sin x = \frac{2t}{1+t^2}$

$$\int_0^{\pi} \frac{\frac{2dt}{1+t^2}}{\frac{2t}{1+t^2}+2} = \int_0^{\pi} \frac{\frac{2dt}{1+t^2}}{\frac{2t+2+2t^2}{1+t^2}} = \int_0^{\pi} \frac{2dt}{2t+2+2t^2} = \int_0^{\pi} \frac{2dt}{2(t+1+t^2)} = \int_0^{\pi} \frac{dt}{t^2+t+1} = \int_0^{\pi} \frac{dt}{(t+\frac{1}{2})^2+\frac{3}{4}}$$

$$\left[\begin{array}{l} v = t+\frac{1}{2} \\ dv = dt \end{array} \right] = \int_0^{\pi} \frac{dv}{v^2+\frac{3}{4}} = \int_0^{\pi} \frac{dv}{v^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \Big|_0^{\pi} \frac{2}{\sqrt{3}} \operatorname{arctg} \frac{2v}{\sqrt{3}} = \Big|_0^{\pi} \frac{2}{\sqrt{3}} \operatorname{arctg} \frac{2t+1}{\sqrt{3}} =$$

$$= \frac{2}{\sqrt{3}} \operatorname{arctg} \frac{2tg\frac{x}{2}+1}{\sqrt{3}} = \left(\frac{2}{\sqrt{3}} \cdot \frac{\operatorname{arctg} 2 \cdot tg\frac{\pi}{2}+1}{\sqrt{3}} - \frac{2}{\sqrt{3}} \operatorname{arctg} \frac{2 \cdot tg\, 0+1}{\sqrt{3}} \right)$$

$$= \frac{2}{\sqrt{3}} \operatorname{arctg} \infty - \frac{2}{\sqrt{3}} \operatorname{arctg} \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\pi}{2} - \frac{2}{\sqrt{3}} \cdot \frac{\pi}{6} = \frac{2\pi}{2\sqrt{3}} - \frac{2\pi}{6\sqrt{3}}$$

$$= \frac{2\pi}{3\sqrt{3}} = 1{,}21 \checkmark$$

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xox	Broj ↓ bodova

IME I PREZIME: IVAN RADOVIĆ **BROJ INDEKSA:** 57230

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Ukupno:

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
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Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
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$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
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$f(x,y) = e^x - x + y^2$

$\frac{\partial f}{\partial x} = e^x - 1$

$\frac{\partial f}{\partial y} = 2y$

$\frac{\partial f}{\partial x^2} = e^x$

$\frac{\partial^2 f}{\partial y^2} = 2$

$\frac{\partial f}{\partial x \partial y} = \frac{\partial f}{\partial y \partial x} = 0$

$e^x > 0$ MIN

$e^x < 0$ MAX

$\Delta = \begin{vmatrix} e^x & 0 \\ 0 & 2 \end{vmatrix} = 2e^x - 0 = 2e^x$

$\Delta > 0$ ima extrem $\Delta = 2e^x$

$e^x - 1$ $2y$

$e^x = 1$ $2y =$

$y = \frac{1}{2}$

$1 - x + \left(\frac{1}{2}\right)^2 = 0$

$-x = -\frac{1}{4} - 1$

$-x = -\frac{5}{4}\ /\cdot(-1)$

$x = \frac{5}{4}$

$\Delta = 2 \cdot e^{\frac{5}{4}}$

$\Delta = 3.4903$

Funkcija ima MINIMUM u točkama $\left(\frac{5}{4}, \frac{1}{2}\right)$ X

$$\int_0^{\pi} \frac{dx}{\sin x+2} = \int_0^{\pi} \frac{1}{\sin x+2}\cdot dx = \left[t=\tan\frac{x}{2} \;;\; x=2\arctan t \quad dx=\frac{2dt}{1+t^2} \right]$$

$$= \int_0^{\pi} \frac{1}{\frac{2t}{1+t^2}+2}\cdot\frac{2dt}{1+t^2} = \int_0^{\pi} \frac{1}{\frac{2t+2+2t^2}{1+t^2}}\cdot\frac{2dt}{1+t^2}$$

$$= \int_0^{\pi} \frac{\cancel{1+t^2}\cdot\frac{2dt}{\cancel{1+t^2}}}{2t^2+2t+2} = \int_0^{\pi} \frac{2dt}{2t^2+2t+2} = \int_0^{\pi} \frac{\cancel{2}dt}{\cancel{2}(t^2+t+1)} =$$

$$= \int_0^{\pi} \frac{dt}{t^2+t+2-1} = \int_0^{\pi} \frac{dt}{(t+1)(t+1)-1} = \int_0^{\pi} \frac{dt}{(t+1)(t+1)} - \int_0^{\pi} \frac{dt}{1}$$

$$= \int_0^{\pi} \frac{du}{u^2-(1)^2} = \frac{1}{2}\ln\left|\frac{u-1}{u+1}\right|\Bigg|_0^{\pi} = \frac{1}{2}\ln\left|\frac{t+1-1}{t+1+1}\right| = \frac{1}{2}\ln\frac{t}{t+2}\Bigg|_0^{\pi}$$

$$\frac{dt}{(t+1)(t+1)} = \left[\begin{array}{l} t+1=u \\ dt=du \end{array}\right] =$$

$$= \frac{1}{2}\ln\left(\frac{\pi}{\pi+2}\right) = 0.24631$$

IVAN DACOVIĆ

$$\int_0^2 \frac{x-1}{x^2+x-2}dx = \int_0^2 \frac{x-1}{(x-2)(x+1)}dx = \frac{A}{x-2} + \frac{B}{x+1} \Big/ \cdot (x-2)(x+1)$$

$$x^2+x-2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{1-4\cdot 1(-2)}}{2} = \frac{-1 \pm 3}{2}$$

$$(x-2)(x+1) = x^2+x-2 \qquad \boxed{\begin{matrix} x_1 = -2 \\ x_2 = 1 \end{matrix}}$$

$$x-1 = A(x+1) + B(x-2)$$

$$x-1 = Ax + A + Bx - 2B$$

za $x^0 = -1 = A - 2B$ za x^1 $x = Ax + Bx$

$$-B = A+1 \qquad x = x(A+B) \Big/ :$$

$$B = \frac{A+1}{2} \qquad 1 = A+B$$

$$B = \frac{-B+1+1}{2} \qquad -A = B-1 \Big/ \cdot (-1)$$

$$B = \frac{-B+2}{2} \qquad A = -B+1$$

$$A = -\frac{2}{3}+1$$

$$A = \frac{1}{3}$$

$$2B = -B+2$$

$$2B+B = 2$$

$$3B = 2$$

$$B = \frac{2}{3}$$

$$\int_0^2 \frac{\frac{1}{3}}{x-2}dx + \int_0^2 \frac{\frac{2}{3}}{x+1}dx = \frac{1}{3}\int_0^2 \frac{dx}{x-1} + \frac{2}{3}\int_0^2 \frac{dx}{x+1} = \frac{1}{3}\ln|x-1|\Big|_0^2 + \frac{2}{3}\ln|x+1|\Big|_0^2$$

$$= \frac{1}{3} + 0.7324 = 1.06574$$

$$\left[\left(\frac{1}{3}\ln(2-1)\right) - \left(\frac{1}{3}\ln(0-1)\right)\right] + \left[\left(\frac{2}{3}\ln|2+1|\right) - \left(\frac{2}{3}\ln|0+1|\right)\right]$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox **IME I PREZIME:** RINO KURTIN **BROJ INDEKSA:** 17-2-0112-2011	POPUNJAVA NASTAVNIK Broj ↓ bodova

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1. $y'' - 2y' - 3y = e^{-x} - 1$

$\lambda^2 - 2\lambda - 3 = 0$

$\lambda_{1,2} = \frac{2 \pm \sqrt{4 + 4 \cdot 3 \cdot 1}}{2 \cdot 1}$

$\lambda_{1,2} = \frac{2 \pm 4}{2}$

$\lambda_1 = 3$

$\lambda_2 = -1$

$y_H = C_1 e^{3x} + C_2 e^{-x}$

$y_p = Axe^{-x} - B$

$y'_p = -Axe^{-x}$

$y''_p = Axe^{-x}$

$Axe^{-x} - 2(-Axe^{-x}) - 3Axe^{-x} - B = e^{-x} - 1$

$Axe^{-x} + 2Axe^{-x} - 3Axe^{-x} - B = e^{-x} - 1$

$-B = e^{x} - 1$

$-B = -1$

$B = 1$

$\underline{y = C_1 e^{3x} + C_2 e^{-x} + 1}$ X

2. $f(x,y) = e^x - x + y^2$

$\frac{\partial f}{\partial x} = e^x - 1$ $e^x - 1 = 0$
$e^x = 1$
$x = 0$

$\frac{\partial f}{\partial y} = 2y$

$2y = 0$
$y = 0$

$\frac{\partial f}{\partial xx} = e^x$ $\nearrow x = 0$

$\frac{\partial f}{\partial yy} = 2$

$\frac{\partial f}{\partial yx} = 0$

$\begin{vmatrix} e^x & 0 \\ 0 & 2 \end{vmatrix}$

$\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix}$

$\Delta = 1 \cdot 2 - 0 \cdot 0$
$\Delta = 2$

$T(0,0)$

STACIONARNA TOČKA

IMA EKSTREM I TO LOKALNI MINIMUM ✓

6. $\int_0^2 \frac{x-1}{x^2+x-2}\,dx$

$x_{1,2} = \frac{1 \pm \sqrt{1+4\cdot1\cdot2}}{2\cdot1}$

$x_{1,2} = \frac{1 \pm 3}{2}$

$x_1 = 2$ $(x-2)$
$x_2 = -1$ $(x+1)$

$\frac{x-1}{(x-2)(x+1)} = \frac{A}{(x-2)} + \frac{B}{(x+1)} \quad / (x-2)(x+1)$

$x - 1 = A(x+1) + B(x-2)$

$x - 1 = Ax + A + Bx - 2B$

$Ax + A + Bx - 2B - x + 1 = 0$

$Ax + Bx + A - 2B = x - 1$

$x(A+B) + A - 2B = x - 1$

$A + B = 1$
$A - 2B = -1 \quad / \cdot(-1)$

$A + B = 1$
$-A + 2B = 1$

$3B = 2$
$B = \frac{2}{3}$ ✗ $A = \frac{1}{3}$ ✗

$\int_0^2 \frac{\frac{1}{3}}{\frac{(x-2)}{1}}\,dx + \int_0^2 \frac{\frac{2}{3}}{\frac{(x+1)}{1}}\,dx$

$\int_0^2 \frac{1}{3x-6}\,dx + \int_0^2 \frac{2}{3x+3}\,dx$

$\ln(3x-6)\Big|_0^2 + 2\int \frac{1}{3x+3}\,dx + C$

←

3. $z = \sin(x^2 y)$ $\quad (2, 1, \sin(4))$ RINO KURTIN

$\frac{\partial f}{\partial x} = \cos(x^2 y) \cdot 2x$ ✓ $\Rightarrow f_x(1) = 4$ ✗

$\frac{\partial f}{\partial x}(2,1) = \cos(4) \cdot 4$

$\frac{\partial f}{\partial y}(2,1) = \cos(4)$

$\frac{\partial f}{\partial y} = \cos(x^2 y) \cdot 1$ ✓ $f_y(1) = 1$ ✗

$z - z_0 = f_x(1)(x - x_1) + f_y(1)(y - y_1)$

$z - \sin(4) = 4(x - 2) + y - 1$

$z - \sin(4) = 4x - 8 + y - 1$

$z - \sin(4) = 4x + y - 9$

$4x + y - z + \sin(4) - 9 = 0$ ✗

6. $\ln(0) - \ln(-6) + 2\left(\ln(3x+3)\right)\Big|_0^{2}$

$= 0 + 2\left(\ln(9) - \ln(3)\right) + C$

5. $\displaystyle\int_0^{\pi} \frac{dx}{\sin x + 2}$

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IME I PREZIME: FILIP GOSPIĆ **BROJ INDEKSA:** 58010	

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x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
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2. $f(x,y) = e^x - x + y^2$

$f'(x) = e^x - 1$

$f'(y) = 2y$

$e^x - 1 = 0$ $\qquad 2y = 0 \; /:2$

$e^x = 1$ $\qquad y = 0$

$x = \ln 1$ $\qquad T(0,0)$

$x = 0$

$f''(x) = e^x$

$f''(y) = 2$ $\qquad \Delta = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2 - 0 > 0$

$T(0,0)$ je minimum ✓

5. $\int_0^{\pi} \frac{dx}{\sin x+2} = \underbrace{\int_0^{\pi} \frac{dx}{\sin x} + \int_0^{\pi} \frac{dx}{2}}_{\pi} = \int_0^{\pi} \sin^{-1} x\,dx + \frac{1}{2}\int dx = -\cos^{-1} x \Big|_0^{\pi} + \frac{1}{2} x \Big|_0^{\pi}$

$= -\frac{1}{\cos x} \Big|_0^{\pi} + \frac{1}{2} x \Big|_0^{\pi} = -\left(\frac{1}{1} - \frac{1}{1}\right) + \frac{1}{2}(\pi - 0) = \frac{\pi}{2}$

$$\boxed{\frac{1}{1+1} \neq \frac{1}{1} + \frac{1}{1}}$$

6. $\int_0^2 \frac{x-1}{x^2+x-2}$ $\qquad x_{1,2} = \frac{1 \pm \sqrt{1+8}}{2}$

$x_1 = 2 \qquad x_2 = -1$

$\frac{x-1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1} \quad / \cdot (x-2)(x+1)$

$x - 1 = A(x+1) + B(x-2)$

x_1: $x-1 = A(x+1) = 1 = 3A = 3 \cdot A = 1$
$A = \frac{1}{3}$ $\times$

x_2 $x-1 = B(x-2) = -2 = -3B = B = \frac{3}{2}$ $\times$

$\int_0^2 \left(\frac{\frac{1}{3}}{x-2} + \frac{\frac{3}{2}}{x+1}\right) dx = \frac{1}{3}\int_0^2 \frac{dx}{x-2} + \frac{3}{2}\int_0^2 \frac{dx}{x+1}$

$\frac{1}{3} \ln|x-2| \Big|_0^2 + \frac{3}{2} \ln|x+1| \Big|_0^2$

$\frac{1}{2} \ln|x-2|(x+1) \Big|_0^2$

$\frac{1}{2} \ln|x^2+x-2| \Big|_0^1$

③ $z = \sin(x^2 y)$ $\qquad T(2, 1, \sin(4))$

$R_T \ldots$ $z - z_0 = T_x(x - x_0) + T_y(y - y_0)$

$R_T \ldots$ $z - \sin(4) = 2(x-2) - (y-1)$

$z - \sin(4) = 2x - 4 - y + 1$

$z - \sin(4) - 2x + y + 3 = 0$

$\frac{x - x_0}{T_x} = \frac{y - y_0}{T_y} = \frac{z - z_0}{-1}$

$\frac{x-2}{2} = \frac{y-1}{-1} = \frac{z - \sin(4)}{-1}$

$z'(x) = -2\cos(xy) + 2xy = -2 + 4 = 2$ $\times$

$z'(y) = -\cos(x^2) = -1$ $\times$

FILIP GOSPIĆ 58010

$$\int_1^2 (x+2)\ln x\,dx = \begin{bmatrix} x+2 = t \\ x\,dx = dt \end{bmatrix}$$

$$\int_1^2 t\ln t\,dt$$

$$\int_1^2 t\,dt + \int_1^2 \ln t\,dt$$

$$\int_1^2 \ln t\,dt = \begin{bmatrix} \ln t = m \\ \frac{1}{t}dt = dm \\ t\,dt = \frac{1}{dm} \end{bmatrix}$$

$$\int_1^2 m\frac{1}{dm} = \int_1^2 \frac{m}{dm}$$

$$= t \Big|_1^2 = (2-1) = t = x+2$$

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IME I PREZIME: VESNA MILOŠ **BROJ INDEKSA:** 0269063872

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$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\,dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

2. $f(x,y) = e^x - x + y^2$

$\partial f_x = e^x - 1 \to e^x - 1 = 0$

$\partial f_y = 2y$

$e^x = 1$

$e^x = e^0 \Rightarrow x = 0$

$2y = 0 \ /:2$

$y = 0$

$T(0,0)$

$$\begin{vmatrix} \partial f_{xx} & \partial f_{xy} \\ \partial f_{yx} & \partial f_{yy} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2 > 0$$

LOKALNI MINIMUM u $T(0,0)$

6. $\int_0^2 \frac{x-1}{x^2+x-2}dx$

$$\int \frac{x-1}{x^2+x-2}dx = \frac{A}{x+2} + \frac{B}{x-1} \quad / \cdot (x+2)(x-1)$$

$$= \frac{Ax - A + Bx + 2B}{x^2+x-2}$$

$Ax + Bx = x \Rightarrow A + B = 1$

$-A + 2B = -1 \qquad A = 1 - B$

$-(1-B) + 2B = -1 \qquad \boxed{A = 1}$

$-1 + B + 2B = -1$

$3B = 0 \quad / : 3$

$\boxed{B = 0}$

$$\frac{1}{x+2} + \frac{0}{x-1} = \frac{x-1}{x^2+x-2}$$

$$\int_0^2 \frac{x-1}{x^2+x-2}dx = \underbrace{\left(\frac{2-1}{2^2+2-2}\right) - \left(\frac{0-1}{0^2+0-2}\right)}_{\times} = \frac{1}{4} - \left(\frac{-1}{2}\right) = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4} \quad \cancel{0}$$

$$= \int_0^2 \frac{1}{x+2} = \left[\ln|x+2|\right]_0^2 = \ldots -$$

5. $\int_0^{\pi} \frac{dx}{\sin x + 2}$

$$\int_0^{\pi} \frac{dx}{\sin x+2} = \left\{ \begin{matrix} u = \sin x + 2 & du = \cos x \, dx \\ dv = dx & v = x \end{matrix} \right\} = \left[\sin x + 2 \cdot x\right]_0^{\pi} - \int_0^{\pi} x \cos x \, dx \;\bigstar$$

$$= (\sin\pi + 2\cdot\pi) - (\underbrace{\sin 0 + 2\cdot 0}_{0}) - 0{,}17066795$$

$$= 6{,}33798898 - 0{,}17066795$$

$$= 6{,}167321$$

$$\bigstar \int_0^{\pi} x\cos x \, dx = \left\{ \begin{matrix} u = x & du = dx \\ dv = \cos x \, dx & v = \sin x \end{matrix} \right\} = \left[x \cdot \sin x\right]_0^{\pi} - \int_0^{\pi} \sin x \, dx$$

$$\int_0^{\pi} \sin x \, dx = (-\cos\pi) - (-\cos 0)$$

$$= -\cos\pi + \underbrace{\cos 0}_{1}$$

$$= -0{,}998498 + 1$$

$$= 0{,}00150285$$

$$= (\pi \cdot \sin\pi) - (\underbrace{0 \cdot \sin 0}_{0}) - (0{,}00150285)$$

$$= 0{,}1721708 - 0{,}00150285$$

$$= 0{,}17066795$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: MARIO PAVIN **BROJ INDEKSA:** 57656-2009	

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = e^{-x} - 1$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = e^x - x + y^2$. 15
3. Izračunati tangencijalnu ravninu plohe $z = \sin(x^2 y)$ u točki $(2, 1, \sin(4))$. 15
4. Numeričkom integracijom procijeniti vrijednost $\int_1^2 (x+2)\ln x dx$ i dati procjenu greške ili neku drugu kvalitetnu informaciju o greški. 10+5
5. $\int_0^\pi \frac{dx}{\sin x + 2} = ?$ ~~20~~
6. $\int_0^2 \frac{x-1}{x^2+x-2}dx = ?$ ~~20~~

Ukupno: $\emptyset$

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_a x\ (a > 0)$	$\frac{1}{x \ln a}$
e^x	e^x
$a^x\ (a > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

$$6.\ \int_0^2 \frac{x-1}{x^2+x-2}dx = \left[\begin{array}{ll} x^2+x-2=t & x^2+x-2=t \\ dt = 2x + dx & 0 \mapsto -2 \\ dx = dt - 2x & 2 \mapsto 4 \end{array}\right]$$

$$\int_{-2}^{4} \frac{t-1}{t} \cdot dt - 2x = \int_{-2}^{4}$$

5. $\int_0^{\pi} \frac{dx}{\sin x + 2} = \left[\begin{array}{ll} dx = \frac{2dt}{1+t^2} & \sin x = \frac{2t}{1+t^2} \\ t = tg\frac{x}{2} & 0 \mapsto 0 \\ & \pi \mapsto \infty \end{array} \right]$

MARIO PAVIN
57656-2009

$$\int_0^{\infty} \frac{\frac{2dt}{1+t^2}}{\frac{2t}{1+t^2}+2} =$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox

IME I PREZIME: NEMANJA KORDA **BROJ INDEKSA:** 0269076510

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = \mathrm{e}^{-x} - 1$. 15
2. Odrediti lokalne ekstreme funkcije $f(x, y) = \mathrm{e}^x - x + y^2$. ~~15~~
3. Izračunati tangencijalnu ravninu plohe $z = \sin(x^2 y)$ u točki $(2, 1, \sin(4))$. 15
4. Numeričkom integracijom procijeniti vrijednost $\int_1^2 (x+2)\ln x \mathrm{d}x$ i dati procjenu greške ili neku drugu kvalitetnu informaciju o greški. 10+5
5. $\int_0^\pi \frac{\mathrm{d}x}{\sin x + 2} = ?$ ~~20~~
6. $\int_0^2 \frac{x-1}{x^2+x-2}\mathrm{d}x = ?$ ~~20~~

Ukupno: Ø

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x \mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x \mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

(5) $\int_0^\pi \frac{dx}{\sin x+2} = \left| \begin{matrix} u = \sin x + 2 & dv = dx \\ du = \cos x & v = x \end{matrix} \right|$ $uv - \int v\,du$

$\int_0^\pi \frac{dx}{\sin x+2} = x\sin x + 2 - \int x \cos x\, x$ $\left| \begin{matrix} u = x & dv = \cos x \\ du = dx & v = \sin x \end{matrix} \right|$

$\int_0^\pi \frac{dx}{\sin x+2} = x\sin x + 2 - \left(x\sin x - \int \sin x\, dx\right)$

$\int_0^\pi \frac{dx}{\sin x+2} = x\sin x + 2 - x\sin x - \cos x$

$\Big|_0^\pi x\sin x + 2 - x\sin x - \cos x$

$$\underbrace{\pi \sin \pi + 2 - \pi \sin \pi - \cos \pi}_{1.0015} - \underbrace{\left(0 \sin 0 + 2 - 0 \sin 0 - \cos 0\right)}_{-1} = 1.0015 - 1 = 0.0015$$

6.
$$\int_0^2 \frac{x-1}{x^2+x-2}\,dx = \int_0 \frac{\cancel{x-1}}{x^2+\cancel{x-1}-1}\,dx = \frac{1}{x^2-1}\,dx = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$$

$$\Big|_0^2 \; \frac{1}{2}\ln\left|\frac{2-1}{2+1}\right| - \left(\frac{1}{2}\ln\left|\frac{0-1}{0+1}\right|\right)$$

$$= \frac{1}{2}\ln\left|\frac{1}{3}\right| - \left(\underbrace{\frac{1}{2}\ln|-1|}_{0}\right)$$

$$= \frac{1}{2}\cdot|-1.098| - 0$$

$$= 0.549$$

KRAĆENJE U RAZLOMKU ?

$$\underbrace{\frac{\cancel{2}}{2+\cancel{2}}}_{\frac{2}{4}=\frac{1}{2}} \neq \frac{1}{3}$$

2.
$$f(x,y) = e^x - x + y^2$$

$$y = y'$$
$$y = 2y$$
$$e^x - x = e^0$$
?

$$D = \begin{vmatrix} d_{xx} & d_{xy} \\ d_{yx} & d_{yy} \end{vmatrix} = \begin{vmatrix} e^0 & 0 \\ 0 & 2 \end{vmatrix}$$

$$D = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$$

LOKALNI MINIMUM

GDJE ?

$\emptyset$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: TONI UGLEŠIĆ **BROJ INDEKSA:** 17-1-0065-2011	

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = e^{-x} - 1$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = e^x - x + y^2$. 15
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4. Numeričkom integracijom procijeniti vrijednost $\int_1^2 (x+2)\ln x \, dx$ i dati procjenu greške ili neku drugu kvalitetnu informaciju o greški. 10+5
5. $\int_0^\pi \frac{dx}{\sin x + 2} = ?$ 20
6. $\int_0^2 \frac{x-1}{x^2+x-2} dx = ?$ 20

Ukupno: $\emptyset$

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
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$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

3. $z = \sin(x^2 y)$

$\sin(4) = \sin(1^2 \cdot 1)$

$\sin(4) = \sin(4)$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XOX

POPUNJAVA NASTAVNIK

IME I PREZIME: MARIO GALOŠIĆ **BROJ INDEKSA:** 17-2-0001-2010

Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = e^{-x} - 1$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = e^x - x + y^2$. 15
3. Izračunati tangencijalnu ravninu plohe $z = \sin(x^2 y)$ u točki $(2, 1, \sin(4))$. 15
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5. $\int_0^\pi \frac{dx}{\sin x + 2} = ?$ 20
6. $\int_0^2 \frac{x-1}{x^2+x-2}dx = ?$ 20

Ukupno: $\varnothing$

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
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Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

1. $y'' - 2y' - 3y = e^{-x} - 1$

2. $f(x,y) = e^x - x + y^2$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox

IME I PREZIME: ŠIME-BORNA MAGAŠ **BROJ INDEKSA:** 17-2-0108-2011

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = e^{-x} - 1$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = e^x - x + y^2$. 15
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5. $\int_0^\pi \frac{dx}{\sin x + 2} = ?$ 20
6. $\int_0^2 \frac{x-1}{x^2+x-2}dx = ?$ $\cancel{20}$

Ukupno: Ø

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
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$\cos x$	$-\sin x$
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$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

① $y'' - 2y' - 3y = e^{-x} - 1$

(2) $f(x,y) = e^{x} - x + y^{2}$

(3) $z = \sin(x^{2}y)$ $\quad T(2, 1, \sin(4))$

$$\boxed{\left\{\begin{array}{l} x+2=t \\ dx = dt \end{array}\right\} = \int \frac{dt}{t} \qquad \ln|t| + c = \ln|x+2| + c \,//}$$

(6) $$\int_0^2 \frac{x-1}{x^2+x-2}\,dx = \int \frac{x-1}{(x+1)(x-1)}$$

?

$x^{2} + x = 2$

$x_{1,2} = \frac{-1 \pm \sqrt{1+9?}}{2}$

$x_1 = -2$

$x_2 = 1$

$$\frac{x-1}{(x+2)(x-2)} = \frac{A}{(x+2)} + \frac{B}{(x-1)} \quad \cdot (x+2)(x-1)$$

??

$x - 1 = Ax - A - Bx + 2B$

$A + B - 1 = 9$

$2B - A = -1$

$A = 1 - B$

$2B - 1 + B = -1$

$A = 1 //$

$3B = 0$

DALJE...

$B = 0 //$

$\emptyset$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox

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POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = e^{-x} - 1$. 15
2. Odrediti lokalne ekstreme funkcije $f(x, y) = e^x - x + y^2$. 15
3. Izračunati tangencijalnu ravninu plohe $z = \sin(x^2 y)$ u točki $(2, 1, \sin(4))$. 15
4. Numeričkom integracijom procijeniti vrijednost $\int_1^2 (x+2)\ln x\,dx$ i dati procjenu greške ili neku drugu kvalitetnu informaciju o greški. 10+5
5. $\int_0^\pi \frac{dx}{\sin x + 2} = ?$ 20
6. $\int_0^2 \frac{x-1}{x^2+x-2}dx = ?$ ~~20~~

Ukupno: 0

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

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6. $\int_0^2 \frac{x-1}{x^2+x-2}dx = \int_0^2 \frac{x}{x^2-2+x}dx - \int_0^2 \frac{1}{x^2-2+x}dx = \Bigg/ \begin{matrix} x^2-2+x = t \\ 2x\,dx = dt \\ x\,dx = \frac{dt}{2} \end{matrix}$

$= \int_0^2 \frac{dt}{2\cdot t} = \frac{1}{2}\int_0^2 \frac{dt}{t} = \frac{1}{2}\ln\lvert t\rvert =$