

odgovornosti studenata. **PIŠITE DVOSTRANO!**

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IME I PREZIME: TONI PASTUOVIĆ

BROJ INDEKSA:

17-1-0124-2012

1. Riješiti diferencijalnu jednadžbu:  $y'' - 2y = x^3$  uz početne uvjete  $y(0) = 1$  i  $y'(0) = 0$ .

15

2. Odrediti lokalne ekstreme funkcije:  $f(x, y) = 2y + y\sqrt{x} - y^2 - x$ .

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3. Odredi tangencijalnu ravninu i normalu na plohu  $z = \sqrt{x^2 + y^2}$  u točki  $T(3, 4, z_0)$ .

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4.  $\int_0^{2\pi} \sin^3 x \, dx = ?$

20

5.  $\int_0^1 \frac{\sin(\ln(x^2))}{x} \, dx = ?$

15

6. Neka je  $f(x) = \frac{1}{x^3}$ . Odrediti  $\int_1^{+\infty} f(x) \, dx$ . Skicirati graf funkcije  $f$  i površinu određenu integralom.

20

Ukupno:

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$f$	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
$e^x$	$e^x$
$\alpha^x (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x \, dx = -\ln  \cos x  + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int \frac{dx}{x} = \ln  x  + C$	$\int \cot x \, dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln  x + \sqrt{x^2 \pm a^2}  + C$
$\int e^x \, dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x \, dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[ x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left( x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} \left[ x\sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$	

②  $f(x, y) = 2y + y\sqrt{x} - y^2 - x$

$\frac{\partial f}{\partial x} = y \cdot \frac{1}{2\sqrt{x}} - 1 = 0 \quad | \cdot 2$

$-3y = -4 \quad | : -3$

$y = \frac{4}{3}$

$\sqrt{x} = 2 \cdot \left(\frac{4}{3}\right) - 2$

$\sqrt{x} = \frac{8}{3} - \frac{6}{3} = \frac{2}{3}$

$\sqrt{x} = \frac{2}{3} \quad | \cdot 2$

$x = \frac{4}{9}$

$\frac{\partial f}{\partial y} = 2 + \sqrt{x} - 2y = 0$

$\sqrt{x} = 2y - 2$

$y \cdot \frac{1}{2 \cdot (2y-2)} - 1 = 0$

$\frac{y}{4y-4} - 1 = 0 \quad | \cdot 4y-4$

$y - 4y + 4 = 0$

Stacionarna točka

$T_0 \left( \frac{4}{9}, \frac{4}{3} \right) \checkmark$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \left( y \cdot \frac{1}{2\sqrt{x}} - 1 \right) = y \cdot \frac{1}{2} \cdot x^{-\frac{3}{2}} - 1 \quad \frac{\partial^2 f}{\partial x \partial y} = 0$$

$$= \frac{y}{2} \cdot \frac{-1}{2} \cdot x^{-\frac{3}{2}} = \frac{1}{4} \cdot \frac{1}{\sqrt{(\frac{1}{3})^3}} = \frac{1}{4} \cdot \frac{1}{\sqrt{\frac{1}{27}}} = \frac{1}{4} \cdot \frac{1}{\frac{1}{\sqrt{27}}} = -0.072$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} (2 + \sqrt{x} - 2y) = -2$$

$$\Delta = \begin{vmatrix} \text{NEG.} & 0 \\ 0 & -2 \end{vmatrix} = \text{POSITIVAN}$$

$T_0(\frac{4}{3}, \frac{4}{3})$  stacionarna točka je ekstrem  $\rightarrow$  MAXIMUM  $\checkmark$

②  $z = \sqrt{x^2 + y^2}$

$T(3, 4, z_0)$

$$z^2 = x^2 + y^2$$

$$z - z_0 = f_x(T)(x - x_0) + f_y(T)(y - y_0)$$

$$z_0^2 = 3^2 + 4^2$$

$$z - 5 = \frac{3}{5}x - \frac{3}{5} \cdot 3 + \frac{4}{5}y - \frac{4}{5} \cdot 4$$

$$z^2 = 9 + 16$$

$$z - 5 = \frac{3}{5}x - \frac{9}{5} + \frac{4}{5}y - \frac{16}{5} \dots \dots \text{RE} \quad \checkmark$$

$$z^2 = 25$$

$$z_0 = 5$$

$$\frac{\partial f}{\partial x} = (x^2 + y^2)^{\frac{1}{2}} = \frac{1}{2} (x^2 + y^2)^{-\frac{1}{2}} \cdot 2x = \frac{1}{2} (3^2 + 4^2)^{-\frac{1}{2}} \cdot 6 = \frac{1}{2} \cdot \frac{1}{5} \cdot 6 = \frac{1}{10} \cdot 6 = \frac{3}{5} \quad \checkmark$$

$$\frac{\partial f}{\partial y} = (x^2 + y^2)^{\frac{1}{2}} = \frac{1}{2} (x^2 + y^2)^{-\frac{1}{2}} \cdot 2y = \frac{1}{2} (3^2 + 4^2)^{-\frac{1}{2}} \cdot 8 = \frac{1}{2} \cdot \frac{1}{5} \cdot 8 = \frac{1}{10} \cdot 8 = \frac{4}{5} \quad \checkmark$$

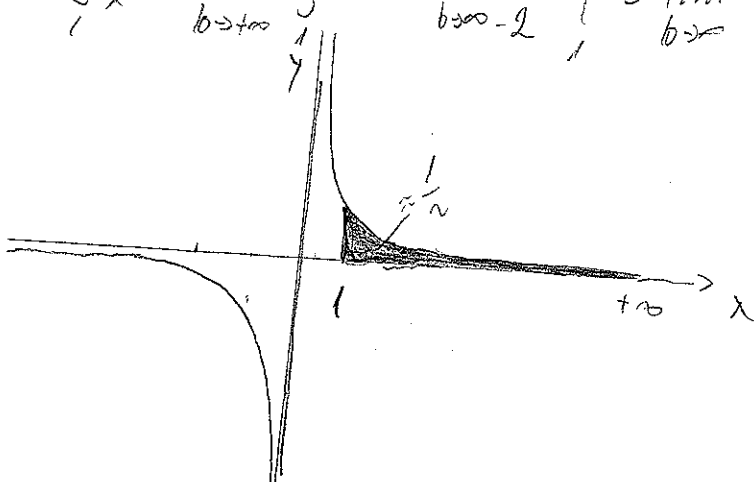
$$\frac{x - x_0}{f_x(T)} = \frac{y - y_0}{f_y(T)} = \frac{z - z_0}{-1}$$

$$\frac{x - 3}{\frac{3}{5}} = \frac{y - 4}{\frac{4}{5}} = \frac{z - 5}{-1}$$

$$\frac{5(x - 3)}{3} = \frac{5(y - 4)}{4} = \frac{z - 5}{-1} \quad \text{NORMAČA}$$

⑥  $f(x) = \frac{1}{x^3}$

$$\int_1^{+\infty} \frac{1}{x^3} dx = \lim_{b \rightarrow +\infty} \int_1^b x^{-3} dx = \lim_{b \rightarrow +\infty} \left[ \frac{1}{b^{-2}} - \frac{1}{1^{-2}} \right] = \lim_{b \rightarrow +\infty} \left[ \frac{1}{b^2} - 1 \right] = \lim_{b \rightarrow +\infty} \left[ \frac{1}{2} \cdot \frac{1}{b^2} + \frac{1}{2} \right] = \frac{1}{2} \quad \checkmark$$



$$\textcircled{5} \int_0^1 \frac{\sin(\ln(x^2))}{x} dx = \left| \begin{array}{l} t = \ln(x^2) \\ dt = \frac{1}{x^2} \cdot 2x dx \\ dt = \frac{2x}{x^2} dx \\ dx = \frac{dt}{\frac{2x}{x^2}} = \frac{x^2 dt}{2x} \end{array} \right| = \int_0^1 \frac{\sin t \cdot x^2}{2x^2} dt$$

$$= \int_0^1 \frac{\sin t}{2} dt = -\frac{1}{2} \cos(\ln x^2) \text{ neprovi integral}$$

$$= \lim_{b \rightarrow \infty} -\frac{1}{2} \cos(\ln(b^2)) - \left(-\frac{1}{2} \cos(\ln(1))\right) = \lim_{b \rightarrow \infty} -\frac{1}{2} \cos(\ln(b^2)) + \frac{1}{2}$$

$$= \underbrace{-\infty}_{>} + \frac{1}{2} = \text{funkcija divergira u beskonačnost } x \text{ nemore se odrediti integral.}$$

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$$\begin{aligned} & \lim_{x \rightarrow 0} \cos(\ln(x^2)) \\ &= \cos\left(\lim_{x \rightarrow 0} \ln(x^2)\right) \\ &= \cos\left(\ln\left(\lim_{x \rightarrow 0} x^2\right)\right) \\ &= \cos(\ln 0) \\ &= \cos(-\infty) \\ &= \text{DIVERGIRA} \end{aligned}$$

# TONI PASTUOVIĆ

①  $y'' - 2y = x^3$        $y(0) = 1$

$r_{1,2} = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$        $y'(0) = 0$

$r_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{2}}$

$r^2 - 2 = 0$

$r_{1,2} = -0,5 \pm 0,707$

$r_1 = 0,207$

$r_2 = -1,207$

$y(x) = 0,428e^{0,207x}$   
 $+ 0,02e^{-1,207x}$

$y(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x}$

$1 = y(0) = C_1 e^0 + C_2 e^0$

$1 = C_1 + C_2$

$C_1 = 1 - C_2$

$y'(x) = C_1 r_1 e^{r_1 x} + C_2 r_2 e^{r_2 x}$

$0 = C_1 \cdot 0,207 - C_2 \cdot 1,207$

$C_1 \cdot 0,207 = C_2 \cdot 1,207$

$0,207 - C_2 \cdot 0,207 = C_2 \cdot 1,207 \quad | : C_2$

$\frac{0,207}{C_2} - 0,207 = 1,207$

$\frac{0,207}{C_2} = 1,234$

$C_2 = 0,02$

$C_1 = 0,978$

④  $\int_0^{2\pi} \sin^3 x = \int_0^{2\pi} \sin^2 x \cdot \sin x = \int_0^{2\pi} \frac{1 - \cos(2x)}{2} \cdot \sin x = \frac{1}{2} \int_0^{2\pi} 1 - \cos(2x) \cdot \sin(x)$

$\frac{1}{2} \int_0^{2\pi} dx - \frac{1}{2} \int_0^{2\pi} \cos(2x) \cdot \sin(x) =$

$\int (\cos(2x) \cdot \sin(x)) dx = \int \frac{t = \sin(x)}{dt = -\cos(x) \cdot dx} \cdot dx = \int \frac{\cos(2x) \cdot t}{-\cos(x)} dt = -\int t dt = -\frac{1}{2} t^2$

$= \frac{1}{2} \int_0^{2\pi} dx + \frac{1}{4} \int_0^{2\pi} \sin^2 x dx$

$= \frac{x}{2} + \frac{x}{8} - \frac{1}{x} \int \sin(2x)$

$= \frac{x}{2} + \frac{x}{8} - \frac{1}{x} \cdot -\frac{1}{2} \cdot \cos(2x)$

$= x + \frac{1}{8} x \cdot \cos(2x)$

$= 2\pi + \pi \cdot \cos(4\pi) - (0 + 0 - 1)$

$= 3\pi \cdot (\cos 4\pi - 1)$

$= 8,2$

$\int \sin^2 x dx = \int \frac{1 - \cos(2x)}{2} = \frac{1}{2} \int dx - \frac{1}{2} \int \cos(2x)$

$= \frac{x}{2} - \frac{\sin(2x)}{4}$

$\int \sin(2x) \quad | \frac{t=2x}{dx = \frac{dt}{2}} = \int \frac{\cos t}{2} dt = \frac{1}{2} \sin(2x)$

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BROJ INDEKSA: **0269078055**

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$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x \, dx = -\ln  \cos x  + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int \frac{dx}{x} = \ln  x  + C$	$\int \cot x \, dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln  x + \sqrt{x^2 \pm a^2}  + C$
$\int e^x \, dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x \, dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[ x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left( x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
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$t = \tan \frac{x}{2}$        $x = \arctan t$        $dx = \frac{2dt}{1+t^2}$   
 $\sin x = \frac{2t}{1+t^2}$        $\cos x = \frac{1-t^2}{1+t^2}$        $\tan x = \frac{2t}{1-t^2}$        $\cot x = \frac{1-t^2}{2t}$

$C_1 e^{k_1 x} + C_2 e^{k_2 x} \quad k_1 \neq k_2$

$e^{k_1 x} (C_1 + C_2 x) \quad k_1 = k_2$

$e^{ax} (C_1 \cos bx + C_2 \sin bx)$

$y = e^{ax} Q_n(x) \quad a \neq 0$

$y = x^r e^{ax} Q_n(x) \quad r=1 \quad r=2$

$e^{ax} \left( \sin(x) \cos bx + \cos(x) \sin bx \right)$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \frac{1}{\cos^2 x}$

$\sin(2x) = 2 \sin x \cos x$

$\sin^2 x = \frac{1 - \cos(2x)}{2}$

$\cos^2 x = \frac{1 + \cos(2x)}{2}$

$\cos(2x) = \cos^2 x - \sin^2 x$

$$1. \quad y'' - 2y = x^3$$

$$y'' - 2y = 0$$

$$k^2 - 2 = 0$$

$$k^2 = 2 /$$

$$k_1 = \sqrt{2}$$

$$k_2 = -\sqrt{2}$$

$$y_H = C_1 e^{\sqrt{2}x} + C_2 e^{-\sqrt{2}x}$$

$$f(x) = x^3 \quad a=0 \quad r=0$$

$$y = Ax^3 + Bx^2 + Cx + D$$

$$y' = A \cdot x \cdot 3 + B \cdot x \cdot 2 + C$$

$$y' = 3Ax^2 + 2Bx + C$$

$$y'' = 6Ax + 2B$$

$$6Ax + 2B - 2Ax^3 - 2Bx^2 - 2Cx - 2D = x^3$$

$$1 = -2A \Rightarrow A = -\frac{1}{2}$$

$$0 = -2B \Rightarrow B = 0$$

$$0 = 6A - 2C \Rightarrow C = 3A = -\frac{3}{2}$$

$$0 = 2B - 2D \Rightarrow D = 0$$

$$0 = 2 \cdot 0 - 2D$$

$$0 = -2D$$

$$D = 0$$

$$0 = 6 \cdot \left(-\frac{1}{2}\right) - 2C$$

$$0 = -3 - 2C$$

$$-2C = -3 / 2$$

$$C = -\frac{3}{2}$$

$$y(0) = 1 \quad y'(0) = 0$$

$$y = C_1 e^{\sqrt{2} \cdot 0} + C_2 e^{-\sqrt{2} \cdot 0} - \frac{1}{2} \cdot 0^3 - \frac{3}{2} \cdot 0$$

$$1 = C_1 + C_2 \Rightarrow C_1 = 1 - C_2 \Rightarrow \boxed{C_1 = 3,828}$$

$$y' = C_1 \cdot e^{\sqrt{2}x} \cdot \sqrt{2} + C_2 \cdot e^{-\sqrt{2}x} \cdot (-\sqrt{2}) - \frac{3}{2}x^2 - \frac{3}{2}$$

$$0 = C_1 \cdot e^{\sqrt{2} \cdot 0} \cdot \sqrt{2} + C_2 \cdot e^{-\sqrt{2} \cdot 0} \cdot (-\sqrt{2}) - \frac{3}{2} \cdot 0^2 - \frac{3}{2}$$

$$0 = \sqrt{2}C_1 - \sqrt{2}C_2 - \frac{3}{2}$$

$$\sqrt{2}C_1 - \sqrt{2}C_2 = \frac{3}{2}$$

$$\sqrt{2} - \sqrt{2}C_2 - \sqrt{2}C_2 = \frac{3}{2}$$

$$\sqrt{2} - 2\sqrt{2}C_2 = \frac{3}{2}$$

$$-2\sqrt{2}C_2 = \frac{3}{2} - \sqrt{2} / -2$$

$$\sqrt{2}C_2 = -3 + \frac{\sqrt{2}}{2} / \sqrt{2}$$

$$C_2 = -3\sqrt{2} + \frac{\sqrt{2}}{2}$$

$$C_2 = -3\sqrt{2} + \sqrt{2}$$

$$C_2 = -2\sqrt{2}$$

$$\boxed{C_2 = -2,828}$$

$$y = -\frac{1}{2}x^3 + 0 \cdot x^2 + \left(-\frac{3}{2}\right)x + 0$$

$$y = -\frac{1}{2}x^3 - \frac{3}{2}x$$

$$y = C_1 e^{\sqrt{2}x} + C_2 e^{-\sqrt{2}x} - \frac{1}{2}x^3 - \frac{3}{2}x$$

$$y = 3,828 e^{\sqrt{2}x} - 2,828 e^{-\sqrt{2}x} - \frac{1}{2}x^3 - \frac{3}{2}x$$

$$\boxed{3} \quad z = \sqrt{x^2 + y^2}$$

$$T(3, 4, z_0)$$

$$z_0 = \sqrt{3^2 + 4^2}$$

$$z_0 = \sqrt{9 + 16}$$

$$z_0 = \sqrt{25}$$

$$\underline{z_0 = 5}$$

$$\left( \left( (x^2 + y^2)^{\frac{1}{2}} \right)' \right) = \frac{1}{2} \cdot (2x + 0)$$

$$\left( \left( (x^2 + y^2)^{\frac{1}{2}} \right)' \right) = x$$

$$= \frac{1}{2} \cdot (0 + 2y) = y$$

$$f_x = x \quad \times$$

$$f_y = y \quad \times$$

$$f_x(T) = 3$$

$$f_y(T) = 4$$

$$z - z_0 = f_x(T)(x - x_0) + f_y(T)(y - y_0)$$

$$z - 5 = 3(x - 3) + 4(y - 4)$$

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$$z - 5 = 3x - 9 + 4y - 16 \quad \times$$

$$z - 5 - 3x + 9 - 4y + 16 = 0$$

$$z - 3x - 4y + 20 = 0$$

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NORMALA.....

$$\frac{(x - x_0)}{f_x(T)} = \frac{(y - y_0)}{f_y(T)} = \frac{z - z_0}{-1}$$

$$\frac{x - 3}{3} = \frac{y - 4}{4} = \frac{z - 5}{-1}$$

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ANTONIO BARBARA

[2.]  $f(x,y) = 2y + y\sqrt{x} - y^2 - x$

$f_x = \frac{y}{2\sqrt{x}} - 1$

$(y\sqrt{x})' = 0 \cdot \sqrt{x} + y \cdot \frac{1}{2\sqrt{x}}$

$(y\sqrt{x})' = \sqrt{x} + y \cdot 0 = \sqrt{x}$

$f_y = 2 + \sqrt{x} - 2y$

$\left(\frac{y}{2\sqrt{x}}\right)' = \frac{0 \cdot 2\sqrt{x} - y \cdot 2 \cdot \frac{1}{2\sqrt{x}}}{(2\sqrt{x})^2} = \frac{-\frac{y}{\sqrt{x}}}{4x}$

$\frac{y}{2\sqrt{x}} - 1 = 0$

$\frac{y}{2 \cdot (2y-2)} = 1$

$2 + \sqrt{x} - 2y = 0$

$\frac{y}{4y-4} = 1 \quad | \cdot (4y-4)$

$\sqrt{x} = 2y - 2$

$y = 4y - 4$

$\sqrt{x} = \frac{8}{3} - 2$

$-3y = -4$

S.T.  $\left(\frac{4}{9}, \frac{4}{3}\right)$

$\sqrt{x} = \frac{8-6}{3}$

$y = \frac{4}{3}$

$\sqrt{x} = \left(\frac{2}{3}\right)^2$

$x = \frac{4}{9}$

$f''_{xx} = \frac{-y}{4x^2}$

$A = f''_{xx}\left(\frac{4}{9}, \frac{4}{3}\right) = \frac{-\frac{4}{3}}{\frac{16}{9}} = \frac{-\frac{12}{3}}{\frac{16}{9}} = \frac{-4}{\frac{16}{9}} = \frac{-36}{16} = \frac{-9}{4}$

$B = f''_{yy}\left(\frac{4}{9}, \frac{4}{3}\right) = -2$

$C = f''_{xy}\left(\frac{4}{9}, \frac{4}{3}\right) = \frac{1}{2 \cdot \frac{2}{3}} = \frac{\frac{1}{2}}{\frac{4}{3}} = \frac{3}{8}$

$f''_{yy} = -2$

$f''_{xy} = \frac{1}{2\sqrt{x}}$

$\Delta = \begin{vmatrix} \frac{-9}{4} & \frac{3}{8} \\ \frac{3}{8} & -2 \end{vmatrix} = \frac{-108}{8} - \frac{9}{16} = \frac{-36-9}{16} = \frac{-45}{16}$

$\Delta < 0$

$A > 0$

točka  $\left(\frac{4}{9}, \frac{4}{3}\right)$  nije ekstrem (X)



$$\textcircled{5} \int_0^1 \frac{\sin(\ln(x^2))}{x} dx = \left[ \begin{array}{l} \ln x^2 = t // \\ \frac{2x}{x^2} dx = dt \\ \frac{2}{x} dx = dt // 2 \\ \frac{dx}{x} = \frac{dt}{2} \end{array} \right]$$

$$= \int_0^1 \sin t \cdot \frac{dt}{2} = \frac{1}{2} \int_0^1 \sin t dt$$

$$= \int_0^1 \sin t \frac{dt}{2} = \frac{1}{2} \int_0^1 \sin t dt = -\frac{1}{2} \cos(\ln(x^2)) \Big|_0^1$$

$$= -\frac{1}{2} \cos(\ln(1)) + \frac{1}{2} \cos(\ln(0)) = -\frac{1}{2} \cos(\ln(1))$$

$\cos(-\infty) = \frac{0}{0}$

$$= -\frac{1}{2} \cos 0 = -\frac{1}{2} \times$$

$$\textcircled{4} \int_0^{2\pi} \sin^3 x dx = \int_0^{2\pi} (1 - \cos^2 x) \cdot \sin x dx = \left[ \begin{array}{l} \cos x = t // \\ -\sin x dx = dt \end{array} \right]$$

$$= -\int_0^{2\pi} 1 - t^2 dt = -\int_0^{2\pi} 1 dt + \int_0^{2\pi} t^2 dt = -t \Big|_0^{2\pi} + \frac{t^3}{3} \Big|_0^{2\pi}$$

$$= -\cos x \Big|_0^{2\pi} + \frac{\cos^3 x}{3} \Big|_0^{2\pi} = -1 + 1 + \frac{1}{3} - \frac{1}{3} = 0 \checkmark$$

ANTONIO

BABADA

$\sin^2(\theta) + \cos^2(\theta) = 1$   
 $\sin^2(\theta) = 1 - \cos^2(\theta)$

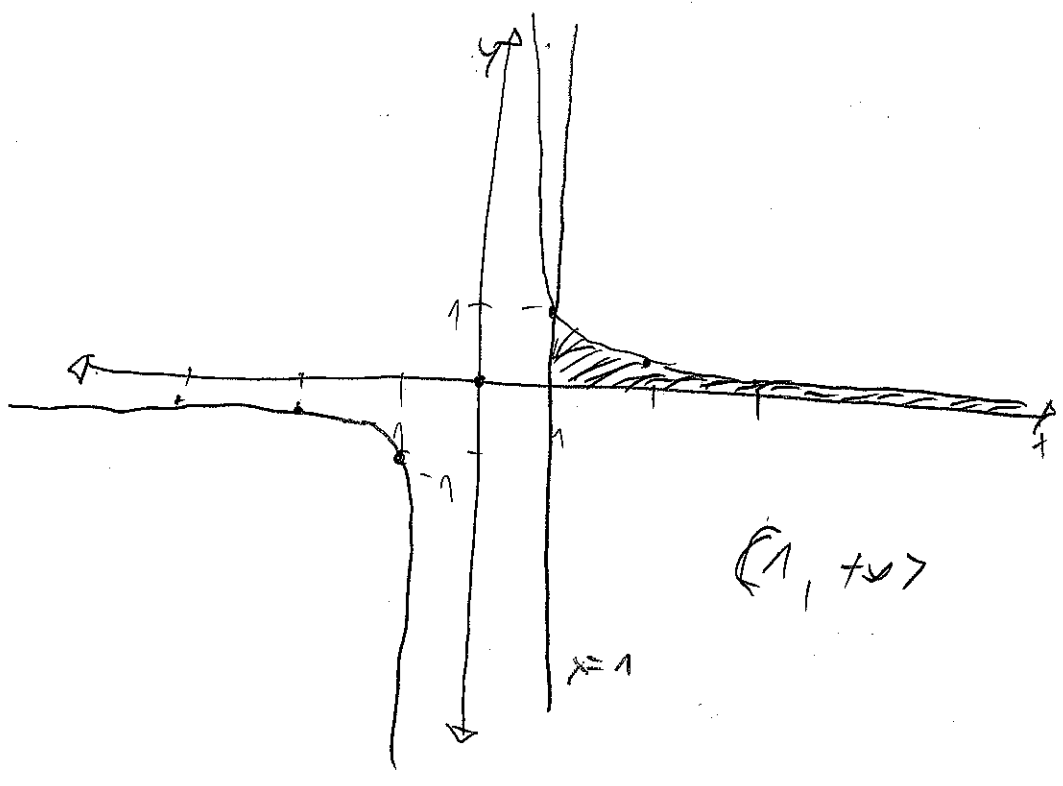
6.  $f(x) = \frac{1}{x^3}$   $\int_1^{\infty} f(x) dx = \int_1^{\infty} \frac{1}{x^3} dx = \int_1^{\infty} x^{-3} dx = \left[ \frac{x^{-2}}{-2} \right]_1^{\infty} = \left[ -\frac{1}{2x^2} \right]_1^{\infty} = 0 - \left( -\frac{1}{2} \right) = \frac{1}{2}$

$\int_1^{\infty} \frac{1}{x^3} dx = \int_1^{\infty} x^{-3} dx = \left[ \frac{x^{-2}}{-2} \right]_1^{\infty} = -\frac{1}{2x^2} \Big|_1^{\infty} = \left( -\frac{1}{2 \cdot \infty^2} \right) - \left( -\frac{1}{2 \cdot 1^2} \right) = 0 + \frac{1}{2} = \frac{1}{2}$

tezi m muler  
 $= \frac{1}{2}$   
 $\frac{1}{2}$  ✓

$y = \frac{1}{x^3}$

x	0	1	-1	2	-2	3	-3
y	E	1	-1	$\frac{1}{8}$	$-\frac{1}{8}$	$\frac{1}{27}$	$-\frac{1}{27}$



$(1, +\infty)$

odgovornosti studenata. **PIŠITE DVOSTRANO!**

xxx

IME I PREZIME: TINO BRASKOVIĆ

BROJ INDEKSA: 12-2-0100-2011

1. Riješiti diferencijalnu jednačbu:  $y'' - 2y = x^3$  uz početne uvjete  $y(0) = 1$  i  $y'(0) = 0$ .

15

2. Odrediti lokalne ekstreme funkcije:  $f(x, y) = 2y + y\sqrt{x} - y^2 - x$ .

15

3. Odredi tangencijalnu ravninu i normalu na plohu  $z = \sqrt{x^2 + y^2}$  u točki  $T(3, 4, z_0)$ .

15

4.  $\int_0^{2\pi} \sin^3 x \, dx = ?$

20

5.  $\int_0^1 \frac{\sin(\ln(x^2))}{x} \, dx = ?$

15

6. Neka je  $f(x) = \frac{1}{x^3}$ . Odrediti  $\int_1^{+\infty} f(x) \, dx$ . Skicirati graf funkcije  $f$  i površinu određenu integralom.

20

Ukupno:

55

$f$	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
$e^x$	$e^x$
$a^x (\alpha > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln  \cos x  + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int \frac{dx}{x} = \ln  x  + C$	$\int \cot x dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln  x + \sqrt{x^2 \pm a^2}  + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin(\frac{x}{a})] + C$	

~~$y'' - 2y = x^3$   $y(0) = 1$   $y'(0) = 0$~~

~~$f(x, y) = 2y + y\sqrt{x} - y^2 - x$~~

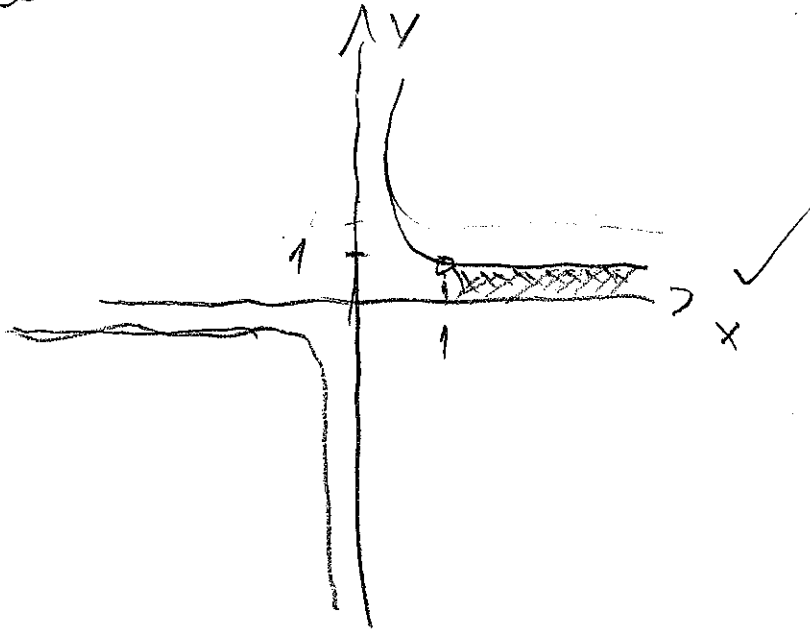
~~$y'' - 2y = x^3$   $y' = 4y^2 - x^2$   $y = 4 \int \frac{y^2}{2} dx = \frac{2y^3}{3} + C$   
 $y' - 2 \int \frac{y^2}{2} dx = \frac{2y^3}{3} + C$   $4 \int dy = 4 \int \frac{y^3}{3} - \int \frac{y^3}{3}$   
 $y' - 2 \frac{y^2}{2} = \frac{x^3}{4} + C$   $y = 4 \cdot \frac{y^3}{3} - \frac{x^3}{3}$   
 $y' = y^2 - \frac{x^3}{4} + C$   $4 = \frac{4y^3}{3} - \frac{x^3}{3}$   
 $4y' = 4y^2 - x^3$   $\frac{-4y^3}{3} + \frac{y^3}{3} = -4 + C$  (1)  
 $0 = 4 - y^4$   $\frac{4y^3}{3} - \frac{y^3}{3} = 4$   
 $y^4 = 4$~~



$$6. f(x) = \frac{1}{x^3} \quad \int_1^{+\infty} f(x) dx$$

$$f(x) = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x^3} dx = \lim_{b \rightarrow +\infty} \left( \frac{x^{-2}}{-2} \right) \Big|_1^b = \lim_{b \rightarrow +\infty} \left( -\frac{1}{2x^2} \right) \Big|_1^b =$$

$$\lim_{b \rightarrow +\infty} \left( -\frac{1}{2b^2} + \frac{1}{2} \right) = \frac{1}{2} \checkmark$$



$$4 \quad \int_0^{2\pi} \sin^3 x dx = \int_0^{2\pi} \sin^2 x \sin x dx = \int_0^{2\pi} (1 - \cos^2 x) \sin x dx$$

$$\left. \begin{array}{l} \cos x = t \\ -\sin x dx = dt \end{array} \right|_{x=2\pi}^{x=1} = \int_1^{-1} (t^2 - 1) dt = 0 \checkmark$$

$$2. f(x, y) = 2y + y\sqrt{x - y^2} - x$$

TIWO  
BRAJKOVIĆ

17-2-0100-2011

$$\frac{\partial f}{\partial x} = \frac{y}{2\sqrt{x}} - 1 = 0 \Rightarrow y = 2\sqrt{x}$$

$$\frac{\partial f}{\partial y} = 2 + \sqrt{x} - 2y = 0$$

$$2 + \sqrt{x} - 4\sqrt{x} = 0$$

$$3\sqrt{x} = 2$$

$$\sqrt{x} = \frac{2}{3} \Rightarrow x = \frac{4}{9} \quad y = \frac{4}{3}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{y}{2} \left(-\frac{1}{2}\right) x^{-\frac{3}{2}} = -\frac{y}{4\sqrt{x}^3} = -\frac{3}{8}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{1}{2\sqrt{x}} = \frac{3}{4}$$

$$\Delta = -\frac{3}{8} \cdot \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 = -\frac{27}{16} - \frac{9}{16} = -\frac{36}{16} < 0$$

$$\frac{\partial^2 f}{\partial x^2} < 0 \rightarrow \underline{\underline{\text{MAXIMUM}}} \checkmark$$

$$1. \quad y'' - 2y = x^3 \quad y_u = C_1 C$$

$$r^2 - 2 = 0$$

$$r^2 = 2$$

$$r_u = 3\sqrt{2}$$

~~0~~

odgovornosti studenata. **PIŠITE DVOSTRANO!**

xxx

IME I PREZIME: ~~MATEA~~ ČULINA

BROJ INDEKSA: 17-2-0206-2012

Zadar 11.09.2014.

✗ Riješiti diferencijalnu jednadžbu:  $y'' - 2y = x^3$  uz početne uvjete  $y(0) = 1$  i  $y'(0) = 0$ .

15

2. Odrediti lokalne ekstreme funkcije:  $f(x, y) = 2y + y\sqrt{x} - y^2 - x$ .

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✗ Odredi tangencijalnu ravninu i normalu na plohu  $z = \sqrt{x^2 + y^2}$  u točki  $T(3, 4, z_0)$ .

15

✗  $\int_0^{2\pi} \sin^3 x \, dx = ?$

20

5.  $\int_0^1 \frac{\sin(\ln(x^2))}{x} \, dx = ?$

15

✗ Neka je  $f(x) = \frac{1}{x^3}$ . Odrediti  $\int_1^{+\infty} f(x) \, dx$ . Skicirati graf funkcije  $f$  i površinu određenu integralom.

20

Ukupno:

70

$f$	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
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Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln  \cos x  + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
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~~W. ...~~

4.  $\int_0^{2\pi} \sin^3 x \, dx = \int_0^{2\pi} \sin^2 x \cdot \sin x \, dx = \int_0^{2\pi} (1 - \cos^2 x) \cdot \sin x \, dx =$

$\left. \begin{array}{l} \cos x = p \quad x = 2\pi \quad p = 1 \\ -\sin x \, dx = dp \\ \sin x \, dx = -dp \quad x = 0 \quad p = -1 \end{array} \right\}$

$= \int_{-1}^1 (1 - t^2) (-dt) = 0 \quad \checkmark$

$$3. \frac{df}{dx} = \frac{x}{\sqrt{x^2+y^2}} \quad (3,4) = \frac{3}{5}$$



$$= \frac{df}{dy} = \frac{y}{\sqrt{x^2+y^2}}$$

$$\frac{df}{dy}(3,4) = \frac{4}{5}$$

$$z_0 = \sqrt{3^2+4^2} = 5$$

$$e \dots z-5 = \frac{3}{5}(x-3) + \frac{4}{5}(y-4) \quad \checkmark$$

$$z-5 = \frac{3}{5}x - \frac{9}{5} + \frac{4}{5}y - \frac{16}{5} \quad / \cdot 5$$

$$5z-25 = 3x-9+4y-16$$

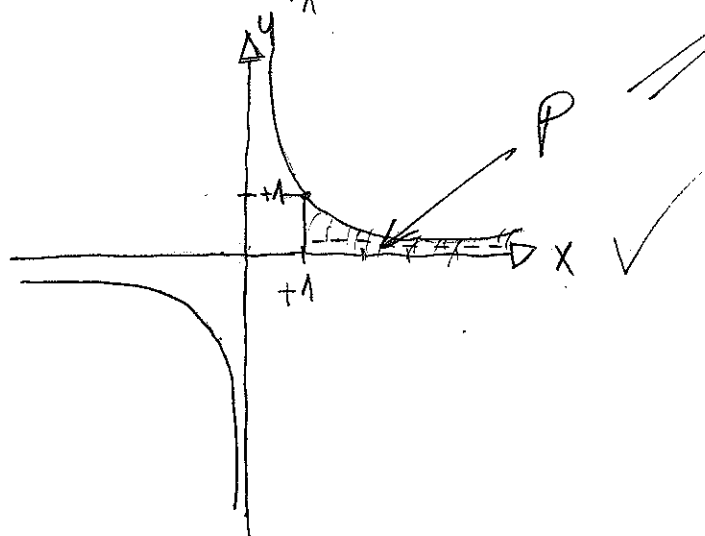
$$3x+4y-5z=0$$





6.)  $\int_1^{+\infty} \frac{1}{x^3} \cdot dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x^3} \cdot dx = \lim_{b \rightarrow +\infty} \left( \frac{x^{-2}}{-2} \right) \Big|_1^b$  b Matea Călinescu  
11.09.2014.

$$= \lim_{b \rightarrow +\infty} \left( -\frac{1}{2x^2} \right) \Big|_1^b = \lim_{b \rightarrow +\infty} \left( -\frac{1}{2b^2} + \frac{1}{2} \right) = \frac{1}{2}$$



1.)  $y'' - 2y = x^3$

$$r^2 - 2 = 0$$

$$r^2 = 2$$

$$r_{1,2} = \pm \sqrt{2}$$

$$y_H = C_1 e^{-\sqrt{2}x} + C_2 e^{\sqrt{2}x}$$

$$g(x) = x^3 \quad y_P = Ax^3 + Bx^2 + Cx + D$$

$$y_P' = 3Ax^2 + 2Bx + C$$

$$y_P'' = 6Ax + 2B$$

$$6Ax + 2B - 2Ax^3 - 2Bx^2 - 2Cx - 2D = x$$

$$-2A = 1 \rightarrow A = -\frac{1}{2}$$

$$-2B = 0 \rightarrow B = 0$$

$$6A - 2B = 0 \rightarrow C = 3A = -\frac{3}{2}$$

$$2B - 2C = 0 \rightarrow D = 0$$

$$y = C_1 e^{-\sqrt{2}x} + C_2 e^{\sqrt{2}x} - \frac{1}{2}x^3 - \frac{3}{2}x$$

$$y' = -\sqrt{2}C_1 e^{-\sqrt{2}x} + \sqrt{2}C_2 e^{\sqrt{2}x} - \frac{3}{2}x^2 - \frac{3}{2}$$

$$y(0) = 1 \rightarrow C_1 + C_2 = 1 \rightarrow C_1 = 1 - C_2$$

$$y'(0) = 0 \rightarrow -\sqrt{2}C_1 + \sqrt{2}C_2 - \frac{3}{2} = 0$$

$$-\sqrt{2} + \sqrt{2}C_2 + \sqrt{2}C_2 - \frac{3}{2} = 0$$

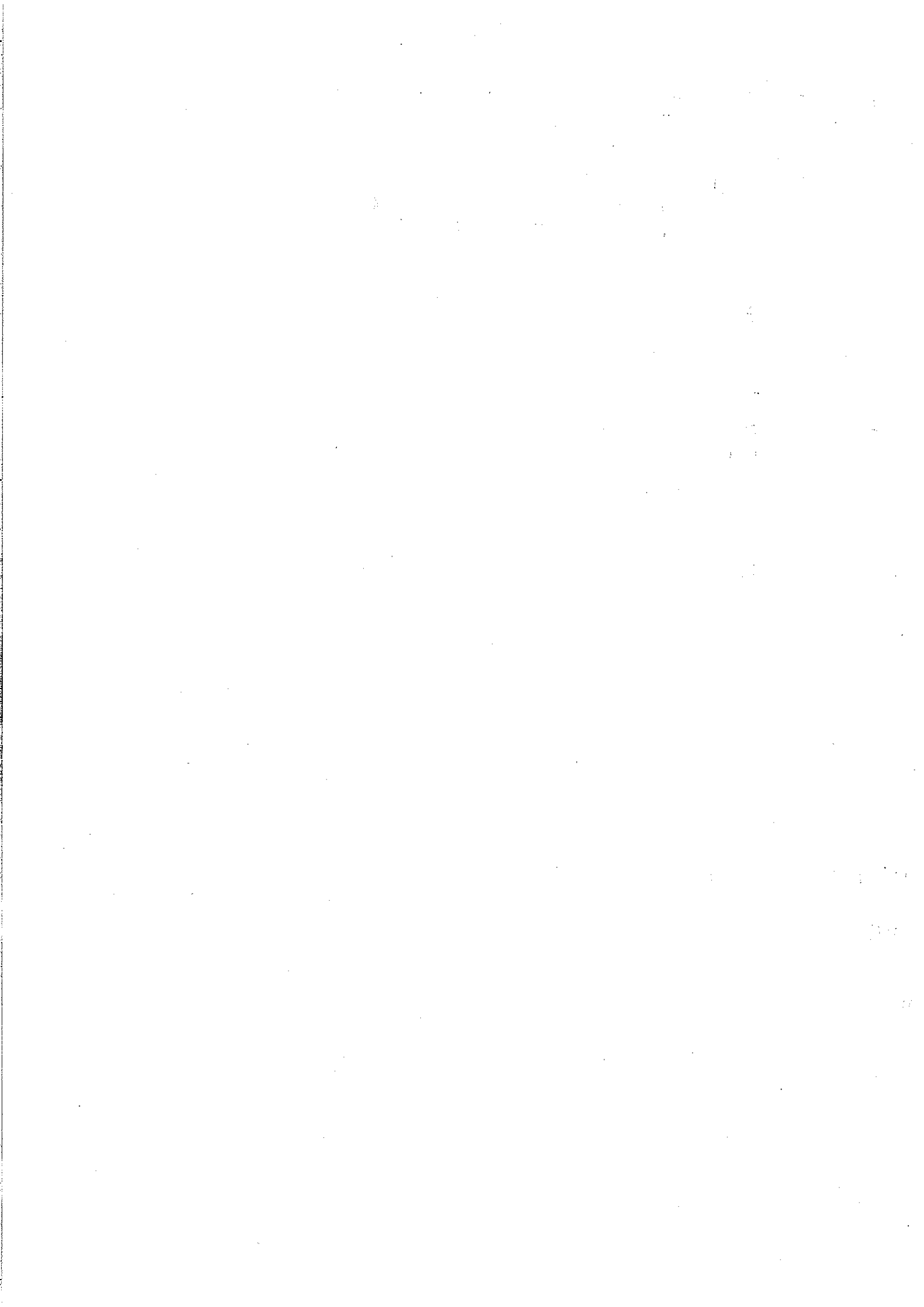
$$2\sqrt{2}C_2 = \frac{3}{2} + \sqrt{2}$$

$$C_2 = \frac{3}{4\sqrt{2}} + \frac{1}{2}$$

$$C_1 = \frac{1}{2} - \frac{3}{4\sqrt{2}}$$

$$y = \left( \frac{1}{2} - \frac{3}{4\sqrt{2}} \right) e^{-\sqrt{2}x} + \left( \frac{3}{4\sqrt{2}} + \frac{1}{2} \right) e^{\sqrt{2}x}$$

$$- \frac{1}{2}x^3 - \frac{3}{2}x //$$



IME I PREZIME:

Marin Dušević

BROJ INDEKSA:

0269081811

1. Riješiti diferencijalnu jednačbu:  $y'' - 2y = x^3$  uz početne uvjete  $y(0) = 1$  i  $y'(0) = 0$ .

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5)  $\int_0^1 \frac{\sin(\ln(x^2))}{x} \, dx =$

$= \lim_{a \rightarrow 0} \int_a^1 \frac{\sin(\ln(x^2))}{x} \, dx =$

$$= \left| \begin{array}{ll} \ln x^2 = t & x=1 \rightarrow t=0 \\ \frac{2}{x} dx = dt & x=a \rightarrow t=\ln a^2 \\ dx = \frac{1}{2} dt & \end{array} \right|$$



$$= \lim_{a \rightarrow 0} \frac{1}{2} \int_{\ln a^2}^0 \sin t \, dt =$$

$$= \lim_{a \rightarrow 0} \left( -\frac{1}{2} \cos t \right) \Big|_{\ln a^2}^0 =$$

$$= \lim_{a \rightarrow 0} \left( -\frac{1}{2} + \frac{1}{2} \cos(\ln a^2) \right)$$

$$= -\frac{1}{2} + \frac{1}{2} \cos(-\infty) \quad \text{DIVERGIRA} \quad \checkmark$$

$$\textcircled{3} \quad \frac{\partial f}{\partial x} = \frac{2x}{2\sqrt{x^2+y^2}} \quad \frac{\partial f}{\partial x}(3,4) = \frac{3}{5}$$

$$\frac{\partial f}{\partial y} = \frac{2y}{2\sqrt{x^2+y^2}} \quad \frac{\partial f}{\partial y}(3,4) = \frac{4}{5}$$

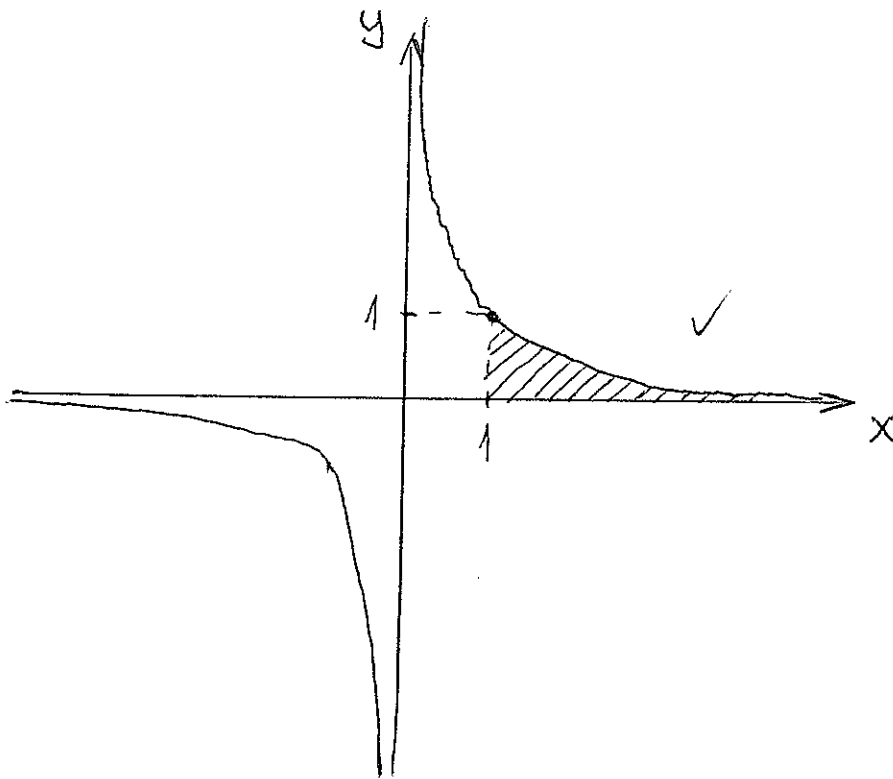
$$Z_0 = \sqrt{3^2+4^2} = 5$$

$$T \dots Z - 5 = \frac{3}{5}(x-3) + \frac{4}{5}(y-4) \quad \checkmark$$

Marin Dušević

$$\textcircled{6} \int_1^{+\infty} \frac{1}{x^3} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x^3} dx =$$

$$= \lim_{b \rightarrow +\infty} \left( -\frac{1}{2x^2} \right) \Big|_1^b = \lim_{b \rightarrow +\infty} \left( -\frac{1}{2b^2} + \frac{1}{2} \right) = \frac{1}{2} \quad \checkmark$$



$$\textcircled{2} \quad x \geq 0$$

$$\frac{\partial f}{\partial x} = \frac{y}{2\sqrt{x}} - 1 = 0 \Rightarrow y = 2\sqrt{x}$$

$$\frac{\partial f}{\partial y} = 2 + \sqrt{x} - 2y = 0 \quad \checkmark$$

---

$$2 + \sqrt{x} - 4\sqrt{x} = 0$$

$$3\sqrt{x} = 2$$

$$\sqrt{x} = \frac{2}{3} \rightarrow x = \frac{4}{9} \quad y = \frac{4}{3}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{y}{2} \cdot \left(-\frac{1}{2}\right) x^{-\frac{3}{2}} = -\frac{y}{4\sqrt{x}^3} = -\frac{9}{8}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{1}{2\sqrt{x}} = \frac{3}{4}$$

$$\frac{\partial^2 f}{\partial y^2} = -2$$

$$\Delta = -\frac{9}{8} \cdot (-2) - \left(\frac{3}{4}\right)^2 = \frac{9}{4} - \frac{9}{16} = \frac{27}{16} > 0$$

$$\frac{\partial^2 f}{\partial x^2} < 0 \rightarrow \text{MAKSIMUM}$$

$$f_{\text{MAX}} = \frac{4}{3} \quad \checkmark$$

odgovornosti studenata. **PIŠITE DVOSTRANO!**

xxx

IME I PREZIME: MAURO MIŠLOV

BROJ INDEKSA: 17-2-0170-2012

1. Riješiti diferencijalnu jednadžbu:  $y'' - 2y = x^3$  uz početne uvjete  $y(0) = 1$  i  $y'(0) = 0$ .

15

2. Odrediti lokalne ekstreme funkcije:  $f(x, y) = 2y + y\sqrt{x} - y^2 - x$ .

~~15~~ 10

3. Odredi tangencijalnu ravninu i normalu na plohu  $z = \sqrt{x^2 + y^2}$  u točki  $T(3, 4, z_0)$ .

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4.  $\int_0^{2\pi} \sin^3 x \, dx = ?$

20

5.  $\int_0^1 \frac{\sin(\ln(x^2))}{x} \, dx = ?$

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6. Neka je  $f(x) = \frac{1}{x^3}$ . Odrediti  $\int_1^{+\infty} f(x) \, dx$ . Skicirati graf funkcije  $f$  i površinu određenu integralom.

20

Ukupno:

45

$f$	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
$e^x$	$e^x$
$a^x (\alpha > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln  \cos x  + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
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$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[ x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left( x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[ x\sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$	

2.  $f(x, y) = 2y + y\sqrt{x} - y^2 - x$

$2 + t - 4t = 0$

$-3t = -2$

$t = \frac{2}{3} \rightarrow x = t^2 = \frac{4}{9}$

3.  $\frac{\partial f}{\partial x} = -1 + \frac{1}{2\sqrt{x}} y$

$y = 2t = \frac{4}{3}$

$\frac{\partial f}{\partial y} = 2 + \sqrt{x} - 2y$

$\left. \begin{aligned} -1 + \frac{y}{2\sqrt{x}} &= 0 \\ 2 + \sqrt{x} - 2y &= 0 \end{aligned} \right\} t = \sqrt{x}$

$-1 + \frac{y}{2t} = 0$

$y = 2t$

$$200. \frac{d^2 f}{dx^2} = (-1 + \frac{1}{2} y = x^{-\frac{1}{2}})^{-1}$$

$$= -\frac{1}{4} y x^{-\frac{3}{2}}$$

$$\frac{d^2 f}{dy^2} = -2 \quad \Delta \left( \frac{4}{9}, \frac{4}{3} \right) = \begin{vmatrix} -\frac{9}{8} & \frac{3}{4} \\ \frac{3}{4} & -2 \end{vmatrix} =$$

$$\frac{d^2 f}{dx dy} = \frac{1}{2\sqrt{x}} = +\frac{18}{8} - \frac{9}{16} > 0$$

POSI EKSTREM?

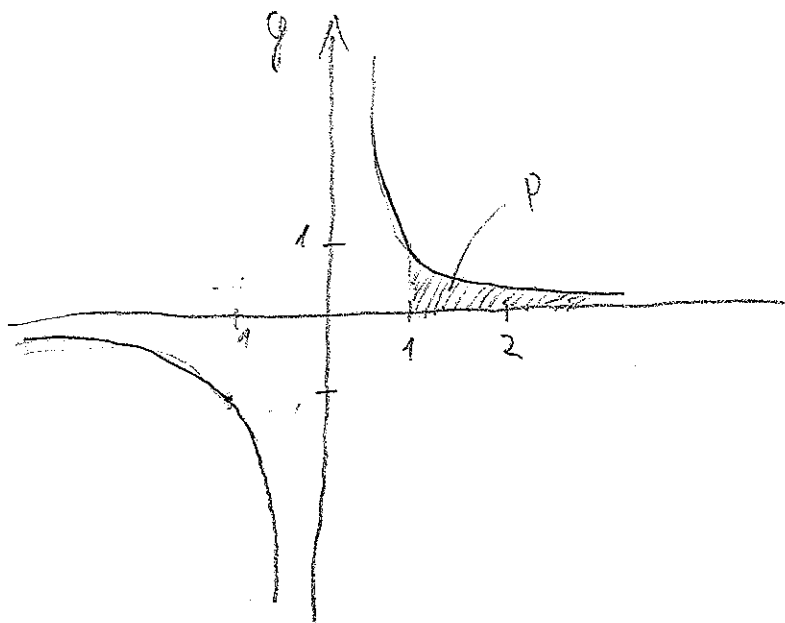
$$\frac{d^2 f}{dx^2} < 0 \text{ i } D \left( \frac{4}{9}, \frac{4}{3} \right) < 0$$

$$f \left( \frac{4}{9}, \frac{4}{3} \right) = \frac{4}{3} //$$

6.  $f(x) = \frac{1}{x}$

$$\int_1^{+\infty} \frac{1}{x^2} dx = \frac{-1}{2x^2} \Big|_1^{+\infty} =$$

$$= \lim_{x \rightarrow +\infty} \frac{-1}{2x^2} - \frac{-1}{2} = 0 + \frac{1}{2} = \frac{1}{2} //$$





$$3. \quad T(3, 4, z_0)$$

MAURCO MISLOV

$$z = \sqrt{x^2 + y^2}$$

$$z_0 = \sqrt{9 + 16} = 5 \quad \checkmark$$

$$T(3, 4, 5)$$

$$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x$$

$$\frac{\partial z}{\partial x} (3, 4) = \frac{6}{10} = \frac{3}{5}$$

$$\frac{\partial z}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y$$

TANGENCIALNA RAVNINA

$$RT = z - 5 = \frac{3}{5}(x - 3) + \frac{4}{5}(y - 4) \quad \checkmark$$

$\parallel$   
 $\checkmark$

$$ax + by + cz + d = 0$$

$$\frac{3}{5}x - \frac{9}{5} + \frac{4}{5}y - \frac{16}{5} + 5 - z$$

$$\vec{n} = \left( \frac{3}{5}, \frac{4}{5}, -1 \right) \parallel$$



odgovornosti studenata. **PIŠITE DVOSTRANO!**

xxx

IME I PREZIME: Duje Mitrović

BROJ INDEKSA: 17-2-0205-2012

1. Riješiti diferencijalnu jednadžbu:  $y'' - 2y = x^3$  uz početne uvjete  $y(0) = 1$  i  $y'(0) = 0$ .

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20

Ukupno:

~~80~~

50

Kosa

$f$	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
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②  $f(x, y) = 2y + y\sqrt{x} - y^2 - x$

$\frac{\partial f}{\partial x} = y \cdot \frac{1}{2\sqrt{x}} - 1$

$T_0 \left( \frac{4}{9}, \frac{4}{3} \right)$

STACIONARNA  
TOČKA

$\frac{\partial f}{\partial y} = 2 + \sqrt{x} - 2y$

$\frac{\partial^2 f}{\partial x^2} = \frac{2\sqrt{x} - y \cdot (2\sqrt{x})'}{(2\sqrt{x})^2} = \frac{2\sqrt{x} - 2 \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x}}}{4x} =$

$= \frac{2\sqrt{x} - \frac{1}{\sqrt{x}}}{4x} \Rightarrow \frac{-y}{4x} = \frac{-\frac{4}{3}}{\frac{16}{9}} = \frac{-2}{16} = \frac{-1}{8} = A$

$\frac{-y}{4x} = \frac{-\frac{4}{3}}{\frac{16}{9}} = \frac{-2}{16} = \frac{-1}{8} = A$

$\frac{y}{2\sqrt{x}} - 1 = 0$

$\frac{y}{2\sqrt{x}} = 1 \quad | \cdot 2\sqrt{x}$

$y = 2\sqrt{x}$

$2\sqrt{x} - 2y = 0$

$\sqrt{x} - 4\sqrt{x} = -2$

$-3\sqrt{x} = -2 \quad | : -3$

$\sqrt{x} = \frac{2}{3} \quad | ^2$

$x = \frac{4}{9}$

$y = 2\sqrt{\frac{4}{9}}$

$y = 2 \cdot \frac{2}{3} = \frac{4}{3}$

$$\frac{\partial^2 z}{\partial y^2} = -2 \quad \boxed{= -2}$$

$$\Delta = \begin{vmatrix} -\frac{9}{8} & \frac{3}{4} \\ \frac{3}{4} & -2 \end{vmatrix} = \frac{9}{4} - \frac{9}{16} = \frac{36-9}{16} = \frac{27}{16}$$

$$\frac{\partial z}{\partial x \partial y} = \frac{1}{2\sqrt{x}} = \frac{1}{\frac{4}{3}} = \frac{3}{4} //$$

$$\Delta > 0 \text{ IMAMO EKSTREMUM} = \frac{27}{16}$$

$A < 0$  To je lokalni maksimum

③  $z = \sqrt{x^2 + y^2}$

$T(3, 4, z_0)$

$$z_0 = \sqrt{9+16}$$

$T(3, 4, 5)$

$$z_0 = \sqrt{25} = 5 //$$

$$\frac{\partial z}{\partial x} = \frac{1}{2} \cdot (x^2 + y^2)^{-\frac{1}{2}} \cdot 2x = \frac{1}{2} \cdot 2x \cdot \frac{1}{\sqrt{x^2 + y^2}} = \frac{x}{\sqrt{x^2 + y^2}} \Rightarrow \frac{3}{\sqrt{9+16}} = \frac{3}{5} //$$

$$\frac{\partial z}{\partial y} = \frac{1}{2} \cdot (x^2 + y^2)^{-\frac{1}{2}} \cdot 2y = \frac{1}{2} \cdot 2y \cdot \frac{1}{\sqrt{x^2 + y^2}} = \frac{y}{\sqrt{x^2 + y^2}} \Rightarrow \frac{4}{\sqrt{25}} = \frac{4}{5}$$

$$z - z_0 = \frac{3}{5}(x - x_0) + \frac{4}{5}(y - y_0)$$

$$z - 5 = \frac{3}{5}(x - 3) + \frac{4}{5}(y - 4)$$

TAN. RAV. ✓

$$\frac{x-3}{\frac{3}{5}} = \frac{y-4}{\frac{4}{5}} = \frac{z-5}{-1}$$

NORNALA

$$\textcircled{4} \int_0^{2\pi} \sin x \sin^2 x \, dx = \left[ \begin{array}{l} u = \sin x \quad v = -2\sin x \cos x \\ du = -\cos x \, dx \quad dv = \sin^2 x \, dx \end{array} \right] =$$

$$= -2\sin^2 x \cos x \Big|_0^{2\pi} - \int_0^{2\pi} \sin x \cos^2 x \, dx = *$$

II  $\Rightarrow$

$$\text{II} = \left[ \begin{array}{l} \cos x = t \\ dt = -\sin x \, dx \\ dx = \frac{dt}{-\sin x} \end{array} \right] = 2 \int_0^{2\pi} t^2 \, dx = \left[ \frac{2t^3}{3} \right]_0^{2\pi} = \left[ \frac{2\cos^3 x}{3} \right]_0^{2\pi}$$

$$* = \left[ -2\sin^2 x \cos x \right]_0^{2\pi} - \left[ \frac{2t^3}{3} \right]_0^{2\pi} =$$

$$= \left[ (-2\sin^2 2\pi \cos 2\pi) - (-2\sin^2 0 \cos 0) \right] - \left[ \frac{2\cos^2 2\pi}{3} - \frac{2\cos^2 0}{3} \right]$$

$$= 0 - 0 - \frac{2}{3} - \frac{2}{3} = -\frac{4}{3} \quad \text{X} \quad \emptyset$$



~~$$\int_0^{2\pi} \sin^3 x dx = \int_0^{2\pi} \sin^2 x \sin x dx = \int_0^{2\pi} (1 - \cos^2 x) \sin x dx = \int_0^{2\pi} \sin x dx - \int_0^{2\pi} \cos^2 x \sin x dx$$~~

4 je na drugom papiru

~~$$= \int_0^{2\pi} \sin x dx - \int_0^{2\pi} \cos^2 x \sin x dx$$~~

~~$$= \int_0^{2\pi} \sin x dx - \int_0^{2\pi} \cos^2 x \sin x dx$$~~

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Duje Mitrović 17-2-0205-2012

$$\textcircled{5} \int_0^1 \frac{\sin(\ln(x^2))}{x} dx = \lim_{a \rightarrow 0} \int_a^1 \frac{\sin(\ln(x^2))}{x} dx = \left[ \begin{array}{l} \ln x^2 = t \\ dt = \frac{1}{x^2} \cdot 2x dx \\ dt = \frac{2}{x} dx \Rightarrow \frac{dt}{2} = \frac{dx}{x} \end{array} \right]$$

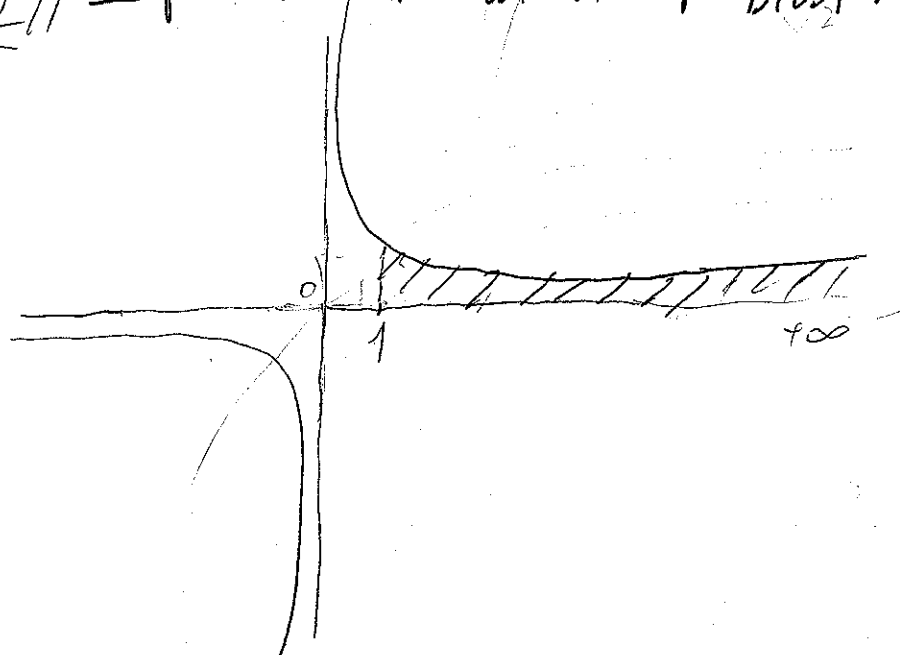
$$= \lim_{a \rightarrow 0} \frac{1}{2} \int_a^1 \sin t dt = \lim_{a \rightarrow 0} \left[ -\frac{1}{2} \cos t \right]_a^1 = \lim_{a \rightarrow 0} \left[ -\frac{1}{2} \cos(\ln x^2) \right]_a^1 =$$

$$= \left[ -\frac{1}{2} \cos(\ln 1) \right] - \left[ -\frac{1}{2} \cos(\ln 0) \right] = -\frac{1}{2} + \infty = \infty + 2k\pi$$

$$\textcircled{6} \int_1^{\infty} \frac{1}{x^3} dx$$

$$\int_1^{\infty} x^{-3} dx = \lim_{b \rightarrow \infty} \left[ \frac{x^{-2}}{-2} \right]_1^b = \lim_{b \rightarrow \infty} \left[ -\frac{1}{2x^2} \right]_1^b = \left[ -\frac{1}{\infty} - \left( -\frac{1}{2} \right) \right] =$$

$$= \underline{-\frac{1}{2}} = P \text{ ISPADA DA BI P BILA NEGATIVNA ???}$$



Boje Hrtovic 17-2-0205-2012



odgovornosti studenata. **PIŠITE DVOSTRANO!**

xxx

IME I PREZIME: JENI MILETIĆ

BROJ INDEKSA: 57143

1. Riješiti diferencijalnu jednačinu:  $y'' - 2y = x^3$  uz početne uvjete  $y(0) = 1$  i  $y'(0) = 0$ .

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$y(0) = 1$   
 $y'(0) = 0$

$f(x) = x^3$   
 $y_p = Ax^3 + B \rightarrow -\frac{x^2}{2}$   
 $y_p'' = 3Ax^2$   
 $y_p'' = 6Ax$

1.  $y'' - 2y = x^3$   
 $\lambda^2 - 2 = 0$   
 $\lambda^2 = 2 / \sqrt{}$   
 $\lambda_{1,2} = \pm \sqrt{2}$

$y_0 = C_1 e^{-\sqrt{2}x} + C_2 e^{\sqrt{2}x}$   
 $y_0' = -\sqrt{2}C_1 e^{-\sqrt{2}x} + \sqrt{2}C_2 e^{\sqrt{2}x}$   
 $1 = C_1 e^{-\sqrt{2} \cdot 0} + C_2 e^{\sqrt{2} \cdot 0} \Rightarrow 1 = C_1 + C_2 \Rightarrow C_1 = 1 - C_2$   
 $0 = -\sqrt{2} \cdot C_1 e^0 + \sqrt{2} \cdot C_2 e^0 \Rightarrow 0 = -\sqrt{2}C_1 + \sqrt{2}C_2$

$6Ax - 2Ax^2 - 2B = x^3$   
 $2A \cdot (-1) = 0 \Rightarrow A = -\frac{1}{2}$   
 $2A \cdot 1 = 6A \Rightarrow 0$   
 $2A \cdot (-2B) = 0 \Rightarrow B = 0$

$y_0 = \frac{1}{2} e^{-\sqrt{2}x} + \frac{1}{2} e^{\sqrt{2}x}$

$\Rightarrow -\sqrt{2} + \sqrt{2}C_2 + \sqrt{2}C_2 = 0$

$2\sqrt{2}C_2 = \sqrt{2} / \sqrt{2}$

$y = \frac{1}{2} e^{-\sqrt{2}x} + \frac{1}{2} e^{\sqrt{2}x} - \frac{x^3}{2}$

$2C_2 = 1$   
 $C_2 = \frac{1}{2}$

2.  $f(x,y) = 2y + y\sqrt{x} - y^2 - x$

$T(\frac{4}{9}, \frac{4}{3})$

-Δ je manji od nule, stoga nemamo inamo li ekstrema u točki  $T(\frac{4}{9}, \frac{4}{3})$

$f_x = y \cdot \frac{1}{2\sqrt{x}} - 1 \Rightarrow y \cdot \frac{1}{2\sqrt{x}} = 1$

$2 + \sqrt{x} - 4\sqrt{x} = 0$

$f_y = 2 + \sqrt{x} - 2y$

$y = 2\sqrt{x}$

$3\sqrt{x} = 2$

$y = 2 \cdot \frac{2}{3} = \frac{4}{3}$

$\sqrt{x} = \frac{2}{3} / 2$

$x = \frac{4}{9}$

$f_{xx} = y \cdot (-\frac{1}{4\sqrt{x}}) = \frac{4}{3} \cdot (-\frac{1}{4 \cdot \frac{2}{3}}) = -\frac{9}{8} < 0$

$f_{yy} = \sqrt{x} - 2 = \frac{2}{3} + \frac{4}{3} - 2 = \frac{2}{3}$

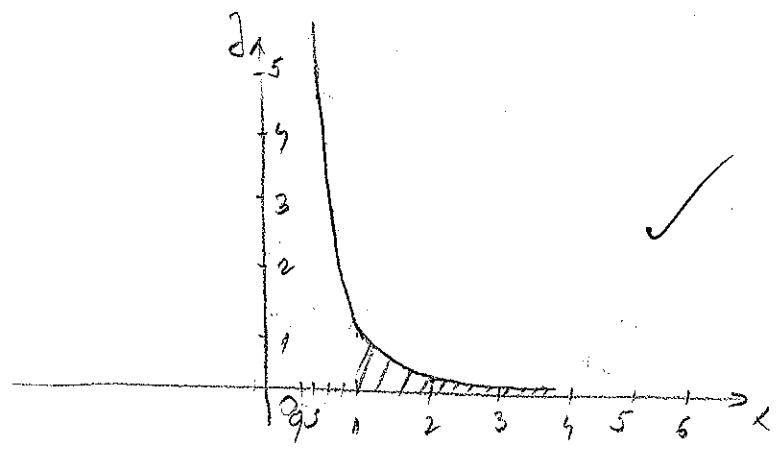
$f_{xy} = 0$

$\Delta = A \cdot C - B^2 = [\frac{9}{8} \cdot (-2)] - (\frac{5}{3})^2 = \frac{9}{4} - \frac{25}{3} = -\frac{73}{12} < 0$

4.  $\int_0^{2\pi} \sin^3 x dx = \int_0^{2\pi} \sin^2 x \cdot \sin x dx = \int_0^{2\pi} (1 - \cos^2 x) \cdot \sin x dx$   
 $= \int_0^{2\pi} (1 - u^2) du = -(\frac{u}{1} - \frac{u^3}{3}) \Big|_0^{2\pi} = -\cos x + \frac{\cos^3 x}{3} \Big|_0^{2\pi} = (-\cos 2\pi + \frac{\cos^3 2\pi}{3}) - (-\cos 0 + \frac{\cos^3 0}{3}) = (-1 + \frac{1}{3}) - (-1 + \frac{1}{3}) = 0$  ✓

5.  $\int_0^1 \frac{\sin(\ln(x^2))}{x} dx = \int \frac{1}{2} \sin u du = -\frac{1}{2} \cos u \Big|_0^1 = -\frac{1}{2} \cos \ln(x^2) \Big|_0^1 = (-\frac{1}{2} \cos \ln 1) - (-\frac{1}{2} \cos \ln 0) = -\frac{1}{2} + \frac{1}{2} = 0$  X

6.  $\int_1^{+\infty} \frac{1}{x^3} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x^3} dx = \lim_{b \rightarrow +\infty} \int_1^b x^{-3} dx = \lim_{b \rightarrow +\infty} -\frac{x^{-2}}{2} \Big|_1^b = \lim_{b \rightarrow +\infty} -\frac{1}{2x^2} \Big|_1^b = (-\frac{1}{2 \cdot b}) - (-\frac{1}{2 \cdot 1}) = \frac{1}{2}$  ✓



odgovornosti studenata. **PIŠITE DVOSTRANO!**

xxx

IME I PREZIME: MARIO ŽHIRE

BROJ INDEKSA: 026 907 1808

1. Riješiti diferencijalnu jednadžbu:  $y'' - 2y = x^3$  uz početne uvjete  $y(0) = 1$  i  $y'(0) = 0$ .

15

2. Odrediti lokalne ekstreme funkcije:  $f(x, y) = 2y + y\sqrt{x} - y^2 - x$ .

15

3. Odredi tangencijalnu ravninu i normalu na plohu  $z = \sqrt{x^2 + y^2}$  u točki  $T(3, 4, z_0)$ .

15

4.  $\int_0^{2\pi} \sin^3 x \, dx = ?$

20

5.  $\int_0^1 \frac{\sin(\ln(x^2))}{x} \, dx = ?$

15

6. Neka je  $f(x) = \frac{1}{x^3}$ . Odrediti  $\int_1^{+\infty} f(x) \, dx$ . Skicirati graf funkcije  $f$  i površinu određenu integralom.

20

Ukupno:

50

$f$	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
$e^x$	$e^x$
$\alpha^x (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln  \cos x  + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int \frac{dx}{x} = \ln  x  + C$	$\int \cot x dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln  x + \sqrt{x^2 \pm a^2}  + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin(\frac{x}{a})] + C$	

1.  $y = 2y - x^3$      $y(0) = 1$      $y'(0) = 0$

$y'' = GAx + 2B$

$GAx + 2B - 2(Ax^3 + Bx^2 + Cx + D) = x^3$

$2B - 2D = 0$

$GAx + 2B - 2Ax^3 + (-2Bx^2) - 2(Cx - 2D) = x^3$

$2D - 2B$

$-2A = 1$

$GA - 2C = 0$

$D = 0$

$A = -\frac{1}{2}$

$2C = GA$

$-2B = 0$

$2C = G^2 \cdot (\frac{1}{2})$

$B = 0$

$2C = -3$

$C = -\frac{3}{2}$

$y_p = -\frac{1}{2}x^3 - \frac{3}{2}x$

$y = C_1 + C_2 e^{2x} - \frac{1}{2}x^3 - \frac{3}{2}x$

$y' = 2C_2 e^{2x} - \frac{3}{2}$

$0 = 2C_2 e^{2 \cdot 0} - 0 - \frac{3}{2}$

$2C_2 = \frac{3}{2} \quad | :2 \quad C_2 = \frac{3}{4}$

$y - y^2 = 0$   
 $y(1-y) = 0$   
 $y_1 = 0$      $y_2 = 1$

$y_H = C_1 e^0 + C_2 e^{2x} = C_1 + C_2 e^{2x}$

$q_1(x) = x^3 \rightarrow \alpha = 0 \quad \beta = 0 \quad k = 0$

$n = 3$

$y_p = Ax^3 + Bx^2 + Cx + D$

$y_p' = 3Ax^2 + 2Bx + C$

$y_p'' = 6Ax + 2B$

$k = 0$

$$C_2 = \frac{3}{2} \quad | : 2$$

$$C_2 = \frac{3}{4}$$

$$y = C_1 - C_2 e^{2x} - \frac{1}{2}x^3 - \frac{3}{2}x$$

$$1 = C_1 + C_2 e^0 - 0 - 0$$

$$C_1 = 1 - C_2$$

$$C_1 = 1 - \frac{3}{4}$$

$$\boxed{C_1 = \frac{1}{4}} \quad y = \frac{1}{4} + \frac{3}{4} e^{2x} - \frac{1}{2}x^3 - \frac{3}{2}x$$

$$\frac{d^2 f}{dx^2} = \frac{-y \cdot 2 \cdot \frac{1}{2\sqrt{x}}}{4x} = \frac{-y}{4x} = \frac{-9}{4x\sqrt{x}}$$

$$\frac{d^2 f}{dy^2} = -9$$

$$\frac{d^2 f}{dx dy} = \frac{d}{dx} \left( \frac{df}{dy} \right) = \frac{d}{dx} (2 + \sqrt{x} - 2y) = \frac{1}{2\sqrt{x}}$$

$$\Delta \left( \frac{4}{3}, \frac{4}{3} \right) = \begin{vmatrix} \frac{-y}{4x\sqrt{x}} & \frac{1}{2\sqrt{x}} \\ \frac{1}{2\sqrt{x}} & -2 \end{vmatrix} = \begin{vmatrix} -\frac{9}{8} & \frac{3}{4} \\ \frac{3}{4} & -2 \end{vmatrix}$$

$$= \frac{9}{4} - \frac{9}{16} = \frac{36-9}{16} = \frac{27}{16} > 0$$

$$f \left( \frac{4}{3}, \frac{4}{3} \right) = 2 \cdot \frac{4}{3} + \frac{4}{3} \cdot \sqrt{\frac{4}{9}} - \left( \frac{4}{3} \right)^3 - \frac{4}{3} \cdot 2$$

$$= \frac{8}{3} + \frac{4}{3} \cdot \frac{2}{3} - \frac{16}{9} - \frac{4}{3} = \frac{8}{3} + \frac{8}{9} - \frac{16}{9} - \frac{4}{3} = \frac{24+8-16-12}{9} = \frac{4}{9} = \frac{4}{3}$$

•  $T \left( \frac{4}{3}, \frac{4}{3}, \frac{4}{3} \right)$  ist Maximum ✓

$$(2) \quad f(x,y) = 2y + 3\sqrt{x} - y^2 - x$$

$$\frac{df}{dx} = \frac{y}{2\sqrt{x}} - 1 = 0 \rightarrow y = 2\sqrt{x}$$

$$\frac{df}{dy} = 2 + \sqrt{x} - 2y = 0$$

$$2 + \sqrt{x} - 2 \cdot 2\sqrt{x} = 0$$

$$2 + \sqrt{x} - 4\sqrt{x} = 0$$

$$-3\sqrt{x} = -2$$

$$\sqrt{x} = \frac{2}{3}$$

$$x = \frac{4}{9}$$

$$y = 2\sqrt{\frac{4}{9}} = 2 \cdot \frac{2}{3}$$

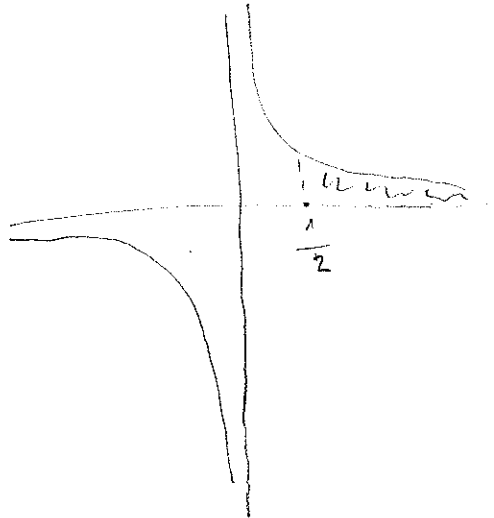
$$y = \frac{4}{3}$$

$$(6) f(x) = \frac{1}{x^3} \quad \int_1^{+\infty} f(x) dx$$

graf i pou  
od. int

$$\int_1^{+\infty} \frac{1}{x^3} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x^3} dx = \lim_{b \rightarrow +\infty} \left( -\frac{1}{2x^2} \right) \Big|_1^b$$

$$= \lim_{b \rightarrow +\infty} \left( -\frac{1}{2b^2} + \frac{1}{2} \right) = \underline{\underline{\frac{1}{2}}} \quad \checkmark$$



(3.)  $z = \sqrt{x^2 + y^2}$   $T(3, 4, z_0)$

$z_0 = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

$\frac{dz}{dx} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$

$\frac{df}{dx}(3, 4) = \frac{3}{\sqrt{3^2 + 4^2}} = \frac{3}{5}$

$\frac{df}{dx}(3, 4) = \frac{3}{\sqrt{3^2 + 4^2}} = \frac{3}{5}$

$\frac{df}{dy} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}}$

$\frac{df}{dy}(3, 4) = \frac{4}{\sqrt{3^2 + 4^2}} = \frac{4}{5}$

At...  $z - 5 = \frac{3}{5}(x - 3) + \frac{4}{5}(y - 4)$  ✓

(4.)  $\int_0^{2\pi} \sin^3 x \, dx = \int_0^{2\pi} \sin^2 x \cdot \sin x \, dx = \int_0^{2\pi} (1 - \cos^2 x) \cdot \sin x \, dx =$

$= \int_0^{2\pi} \begin{cases} \cos x = t \\ -\sin x \, dx = dt \\ \sin x \, dx = -dt \end{cases} = \int_0^{2\pi} (1 - t^2) \cdot (-dt) = - \int_0^{2\pi} (1 - t^2) dt =$

$= - \int_0^{2\pi} (dt - t^2 dt) = - \left( t - \frac{t^3}{3} \right) \Big|_0^{2\pi} = - \left( 2\pi - \frac{8\pi^3}{3} \right) = \underline{\underline{2\pi + \frac{8\pi^3}{3}}}$

(5.)  $f(x, y) = 2y + y\sqrt{x - y^2 - x}$

$\frac{df}{dx} = \frac{y}{2\sqrt{x}} - 1 = 0 \rightarrow y = 2\sqrt{x}$

$\frac{df}{dy} = 2 + \sqrt{x} - 2y = 0$

$2 + \sqrt{x} - 2 \cdot 2\sqrt{x} = 0$

$2 + \sqrt{x} - 4\sqrt{x} = 0$

$-3\sqrt{x} = -2$

$\sqrt{x} = \frac{2}{3}$

$x = \frac{4}{9}$

$x = \frac{4}{9} \rightarrow$

$y = 2\sqrt{\frac{4}{9}} = 2 \cdot \frac{2}{3}$

$y = \frac{4}{3}$

odgovornosti studenata. **PIŠITE DVOSTRANO!**

xxx

IME I PREZIME: **VEJNA ŠARIĆ**

BROJ INDEKSA: **17-2-0222-2012**

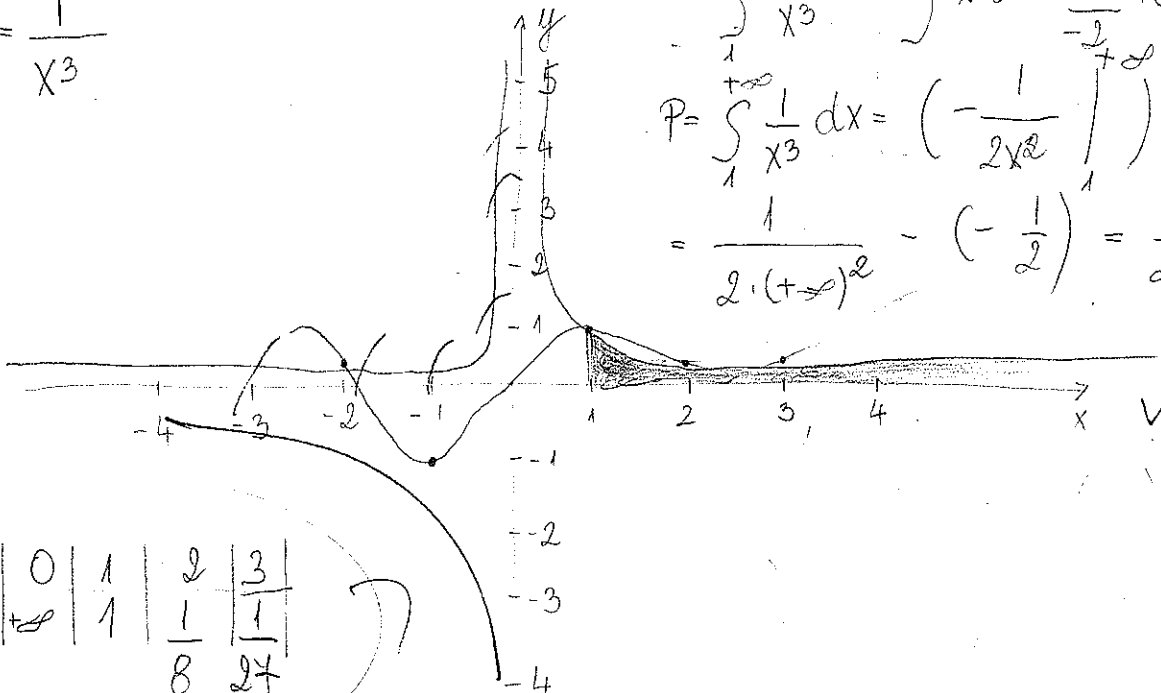
1. Riješiti diferencijalnu jednačbu:  $y'' - 2y = x^3$  uz početne uvjete  $y(0) = 1$  i  $y'(0) = 0$ . 15
2. Odrediti lokalne ekstreme funkcije:  $f(x, y) = 2y + y\sqrt{x} - y^2 - x$ . 15
3. Odredi tangencijalnu ravninu i normalu na plohu  $z = \sqrt{x^2 + y^2}$  u točki  $T(3, 4, z_0)$ . 15
4.  $\int_0^{2\pi} \sin^3 x \, dx = ?$  20
5.  $\int_0^1 \frac{\sin(\ln(x^2))}{x} \, dx = ?$  ~~15~~
6. Neka je  $f(x) = \frac{1}{x^3}$ . Odrediti  $\int_1^{+\infty} f(x) \, dx$ . Skicirati graf funkcije  $f$  i površinu određenu integralom. 20

Ukupno:

40

$f$	$\frac{df}{dx}$	Tablica nekih integrala		
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\ln x$	$\frac{1}{x}$	$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x \, dx = -\ln  \cos x  + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$	$\int \frac{dx}{x} = \ln  x  + C$	$\int \cot x \, dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln  x + \sqrt{x^2 \pm a^2}  + C$
$e^x$	$e^x$	$\int e^x \, dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\alpha^x (\alpha > 0)$	$\alpha^x \ln \alpha$	$\int a^x \, dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\sin x$	$\cos x$	$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[ x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2}) \right] + C$	
$\cos x$	$-\sin x$	$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} \left[ x\sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$	
$\tan x$	$\frac{1}{\cos^2 x}$			
$\cot x$	$-\frac{1}{\sin^2 x}$			
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$			
$\arctan x$	$\frac{1}{1+x^2}$			

6.  $f(x) = \frac{1}{x^3}$



x	-2	-1	0	1	2	3
y	-1/8	-1	+	1	1/8	1/27

$$\textcircled{6} \int_0^1 \frac{\sin(\ln(x^2))}{x} dx = ?$$

$$\int \frac{\sin(\ln(x^2))}{x} dx =$$

$$u = \ln(x^2)$$

$$\begin{aligned} dv &= \sin x \, dx \\ v &= \int \sin x \, dx \\ v &= -\cos x + C \end{aligned}$$

$$= u \cdot v - \int v \cdot du$$

$$= \ln(x^2) / (-\cos x) +$$

$$=$$

$$\textcircled{5} \int_0^1 \sin^3 x \, dx = ?$$

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

$$\int \sin^2 x \cdot \sin x \, dx = \int (1 - \cos^2 x) \cdot \sin x \, dx$$

$$= \int \sin x \, dx - \int \sin x \cos^2 x \, dx$$

$$= -\cos x + \frac{\cos^3 x}{3} + C$$

$$= -\cos x + \frac{\cos^3 x}{3} + C$$

$$\int \sin x \cos^2 x \, dx =$$

$$\begin{aligned} t &= \cos x \\ dt &= -\sin x \, dx \end{aligned}$$

$$= -\int t^2 \, dt$$

$$= -\frac{\cos^3 x}{3} + C$$

$$= -\cos x + \frac{\cos^3 x}{3}$$

$$= \int_0^1 \sin x \, dx$$

$$\approx 2.3195586 \times 10^{-3} = 0$$



odgovornosti studenata. **PIŠITE DVOSTRANO!**

xxx

IME I PREZIME: Alen Bura

BROJ INDEKSA: 17-2-0095-2011

1. Riješiti diferencijalnu jednačinu:  $y'' - 2y = x^3$  uz početne uvjete  $y(0) = 1$  i  $y'(0) = 0$ .

~~15~~

2. Odrediti lokalne ekstreme funkcije:  $f(x, y) = 2y + y\sqrt{x} - y^2 - x$ .

15

3. Odredi tangencijalnu ravninu i normalu na plohu  $z = \sqrt{x^2 + y^2}$  u točki  $T(3, 4, z_0)$ .

~~15~~

4.  $\int_0^{2\pi} \sin^3 x \, dx = ?$

20

5.  $\int_0^1 \frac{\sin(\ln(x^2))}{x} dx = ?$

~~15~~

6. Neka je  $f(x) = \frac{1}{x^3}$ . Odrediti  $\int_1^{+\infty} f(x) dx$ . Skicirati graf funkcije  $f$  i površinu određenu integralom.

~~20~~

Ukupno:

15

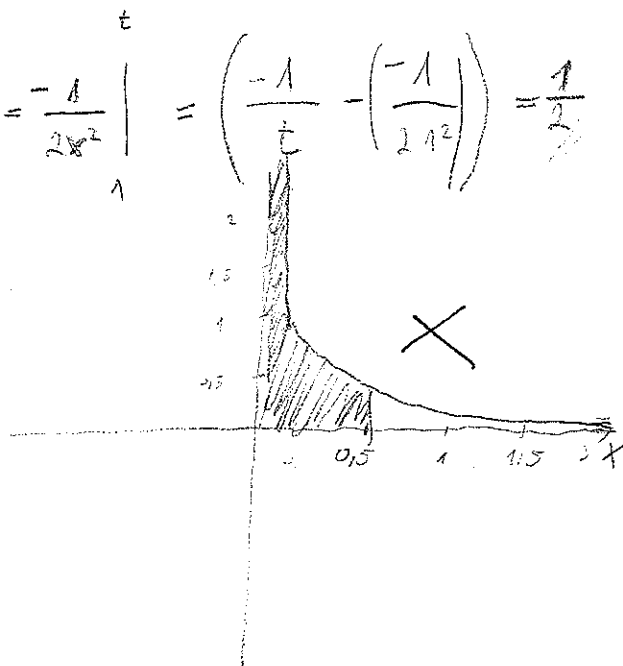
$f$	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
$e^x$	$e^x$
$a^x (\alpha > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
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$\int \frac{dx}{x} = \ln  x  + C$	$\int \cot x dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln  x + \sqrt{x^2 \pm a^2}  + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[ x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left( x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[ x\sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$	

6.  $f(x) = \frac{1}{x^3}$

$$\int_1^{+\infty} \frac{1}{x^3} dx = \lim_{t \rightarrow +\infty} \int_1^t x^{-3} dx = \lim_{t \rightarrow +\infty} \left[ \frac{x^{-2}}{-2} \right]_1^t = \lim_{t \rightarrow +\infty} \left( \frac{-1}{2t^2} - \left( \frac{-1}{2 \cdot 1^2} \right) \right) = \frac{1}{2}$$

x	1/2	0	1	1
f(x)	1/8	1/2	1	1



②  $f(x,y) = 2y + y\sqrt{x} - y^2 - x$

$\frac{df}{dx} = \frac{y}{2\sqrt{x}} - 1$

$\frac{y}{2\sqrt{x}} - 1 = 0 \quad \frac{y}{2\sqrt{x}} = 1 \quad / \cdot 2\sqrt{x}$

$y = 2\sqrt{x} \quad y = 2 \cdot \frac{\sqrt{4}}{3} = \frac{4}{3}$

$y = \frac{4}{3}$

$\frac{df}{dy} = 2 + \sqrt{x} - 2y$

$2 + \sqrt{x} - 2y = 0$

$2 + \sqrt{x} - 2 \cdot (2\sqrt{x}) = 0$

$2 + \sqrt{x} - 4\sqrt{x} = 0$

$-4\sqrt{x} + \sqrt{x} = -2$

$-3\sqrt{x} = -2 \quad / : 2$

$3\sqrt{x} = 1 \quad / : 3$

$\sqrt{x} = \frac{1}{3}$

$T_1 \left( \frac{4}{3}, \frac{4}{3} \right)$

$\frac{df}{dx^2} = \frac{\frac{y}{2\sqrt{x}} - y \cdot \frac{1}{2\sqrt{x}}}{(2\sqrt{x})^2} = \frac{-2y}{2\sqrt{x} \cdot (2\sqrt{x})^2}$

$\frac{df}{dy^2} = -2$

$\begin{vmatrix} -\frac{2}{3} & \frac{3}{4} \\ \frac{3}{4} & -2 \end{vmatrix} = \frac{27}{16}$

$\frac{df}{dx dy} = \frac{1}{2\sqrt{x}}$

$= -\frac{3}{8} < 0 \quad \boxed{\text{max}}$



$$z = \sqrt{x^2 + y^2} \quad T(3, 4, z_0) \quad (3)$$

$$z_0 = \sqrt{3^2 + 4^2} = 5 \quad z_0 = 5$$

$$\frac{dz}{dx} = \frac{1}{2\sqrt{x^2 + y^2}} + 2x \Rightarrow f_x(T) = \frac{61}{10}$$

$$\frac{dz}{dy} = \frac{1}{2\sqrt{x^2 + y^2}} + 2y \Rightarrow f_y(T) = \frac{81}{10}$$

$$\frac{x - x_0}{f_x(T)} = \frac{y - y_0}{f_y(T)} = \frac{z - z_0}{-1}$$

$$\left( \frac{x - 3}{\frac{61}{10}} = \frac{y - 4}{\frac{81}{10}} \right) = \frac{z - 5}{-1}$$

$$\frac{10x - 30}{61} = \frac{10y - 40}{81} = \frac{z - 5}{-1} \quad \leftarrow \text{normal}$$

$$5. \int_0^1 \frac{\sin(\ln(x^2))}{x} dx$$

$$= \int_0^1 -\sin(t) dt$$

$$= 2 \int_0^1 \sin(t) dt$$

$$= -2 \cos(t) \Big|_0^1$$

$$= -2 \cos(\ln(x^2)) \Big|_0^1 = -2 \cos(\ln(1)) - 2 \cos(\ln) = ?$$

$$z - z_0 = f_x(T)(x - x_0) + f_y(T)(y - y_0)$$

$$z - 5 = \frac{61}{10}(x - 3) + \frac{81}{10}(y - 4)$$

$$z - 5 = \frac{61}{10}x - \frac{183}{10} + \frac{81}{10}y - \frac{162}{5}$$

$$z - 5 - \frac{61}{10}x - \frac{81}{10}y + \frac{183}{10} + \frac{162}{5} = 0$$

$$-\frac{61}{10}x - \frac{81}{10}y + z + \frac{457}{10} = 0$$

↑  
tangentenvektor normal

$$\left. \begin{aligned} t &= \ln(x^2) \\ dt &= \frac{1}{x^2} \cdot 2x dx \\ dt &= \frac{2x}{x^2} dx \\ dt &= \frac{2}{x} dx \end{aligned} \right\}$$

$$y'' - 2y = x^3 \quad y(0) = 1 \quad y'(0) = 0 \quad (1)$$

$$k^2 - 2 = 0$$

$$k^2 = 2 / \sqrt{\phantom{x}}$$

$$k = \pm \sqrt{2}$$

$$y_h = c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x}$$

$$y_p = Ax^3 + Bx^2 + Cx$$

$$y_p' = 3Ax^2 + 2Bx + C$$

$$y_p'' = 6Ax + 2B$$

$$6Ax + 2B - 2(Ax^3 + Bx^2 + Cx) = x^3$$

$$\cancel{6Ax} + 2B - \cancel{2Ax^3} - \cancel{2Bx^2} - \cancel{2Cx} = x^3$$

$$\text{uz } x^3: \rightarrow -2A = 1 \quad | : (-2)$$

$$A = -\frac{1}{2}$$

$$\text{uz } x^2: \rightarrow -2B = 0 \quad | : (-2)$$

$$B = 0$$

$$\text{uz } x^1: \rightarrow 6A - 2C = 0$$

$$6 \cdot \left(-\frac{1}{2}\right) - 2C = 0$$

$$-3 - 2C = 0$$

$$-2C = 3 \quad | : (-2)$$

$$C = -\frac{3}{2}$$

$$\text{uz } x^0: \rightarrow 2B = 0$$

$$B = 0$$

$$y(x) = c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x} - \frac{1}{2}x^3 - \frac{3}{2}x$$

1. unjet

$$1 = c_1 e^{\sqrt{2} \cdot 0} + c_2 e^{-\sqrt{2} \cdot 0} - \frac{1}{2} \cdot 0^3 - \frac{3}{2} \cdot 0$$

$$c_1 + c_2 = 1$$

2. unjet

$$y'(0) = 0$$

$$y' = \frac{1}{2} c_1 e^{\frac{1}{2}x} - \frac{1}{2} c_2 e^{-\frac{1}{2}x} - \frac{1}{2} \cdot 3x^2 - \frac{3}{2}$$

$$y'(0) = \frac{1}{2} c_1 e^{\frac{1}{2} \cdot 0} - \frac{1}{2} c_2 e^{-\frac{1}{2} \cdot 0} - \frac{1}{2} \cdot 3 \cdot 0^2 - \frac{3}{2}$$

$$0 = \frac{1}{2} c_1 - \frac{1}{2} c_2 - \frac{3}{2}$$

$$\frac{1}{2} c_1 - \frac{1}{2} c_2 = \frac{3}{2} \quad | \cdot 2$$

$$c_1 - c_2 = 3$$

odgovornosti studenata. **PIŠITE DVOSTRANO!**

xxx

IME I PREZIME: TOMISLAV GLAVAN

BROJ INDEKSA: 17-0115-2011

1. Riješiti diferencijalnu jednadžbu:  $y'' - 2y = x^3$  uz početne uvjete  $y(0) = 1$  i  $y'(0) = 0$ .

15

2. Odrediti lokalne ekstreme funkcije:  $f(x, y) = 2y + y\sqrt{x} - y^2 - x$ .

~~15~~

3. Odredi tangencijalnu ravninu i normalu na plohu  $z = \sqrt{x^2 + y^2}$  u točki  $T(3, 4, z_0)$ .

~~15~~

4.  $\int_0^{2\pi} \sin^3 x \, dx = ?$

20

5.  $\int_0^1 \frac{\sin(\ln(x^2))}{x} \, dx = ?$

~~15~~

6. Neka je  $f(x) = \frac{1}{x^3}$ . Odrediti  $\int_1^{+\infty} f(x) \, dx$ . Skicirati graf funkcije  $f$  i površinu određenu integralom.

20

Ukupno:

20

$f$	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
$e^x$	$e^x$
$a^x (a > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln  \cos x  + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int \frac{dx}{x} = \ln  x  + C$	$\int \cot x dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln  x + \sqrt{x^2 \pm a^2}  + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[ x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left( x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[ x\sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$	

3) Funkcija je diferencijalna jer je zbir kompozicija diferencijabilnih jednadžbi

$$z_0 = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$\frac{dz}{dx} = \sqrt{x^2 + y^2} = 15 \quad \times$$

$$\frac{dz}{dy} = \sqrt{x^2 + y^2} = 20 \quad \times$$

$$\sqrt{x^2} = \left(x^{\frac{2}{2}}\right)' = x^{1-1} = 1 \cdot x^0 = x$$

Rt ...  $z - z_0 = f_x(T)(x - x_0) + f_y(T)(y - y_0)$

$$z - 5 = 20(x - 3) + 15(y - 4)$$

$$z - 5 = 20x - 60 + 15y - 60$$

$$-20x - 15y + z - 5 + (20)(-1)$$

$$20x + 15y - z + 5 = 120$$

$$\dots \frac{x - 3}{f_x(T)} = \frac{y - 4}{f_y(T)} = \frac{z - 5}{-1}$$

$$\dots \frac{x - 3}{15} = \frac{y - 4}{20} = \frac{z - 5}{-1}$$

$$2) f(x,y) = 2y + y\sqrt{x} - y^2 - x$$

$$\frac{\partial f}{\partial x} = 0 + 1 \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x}} - 0 - 1 = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} - 1 = 0$$

$$\begin{aligned} \frac{1}{2\sqrt{x}} - 1 = 0 &\Rightarrow \frac{1}{2\sqrt{x}} = 1 \quad | \cdot 2 \\ &= \frac{1}{2 \cdot (-1)} = 1^2 \end{aligned}$$

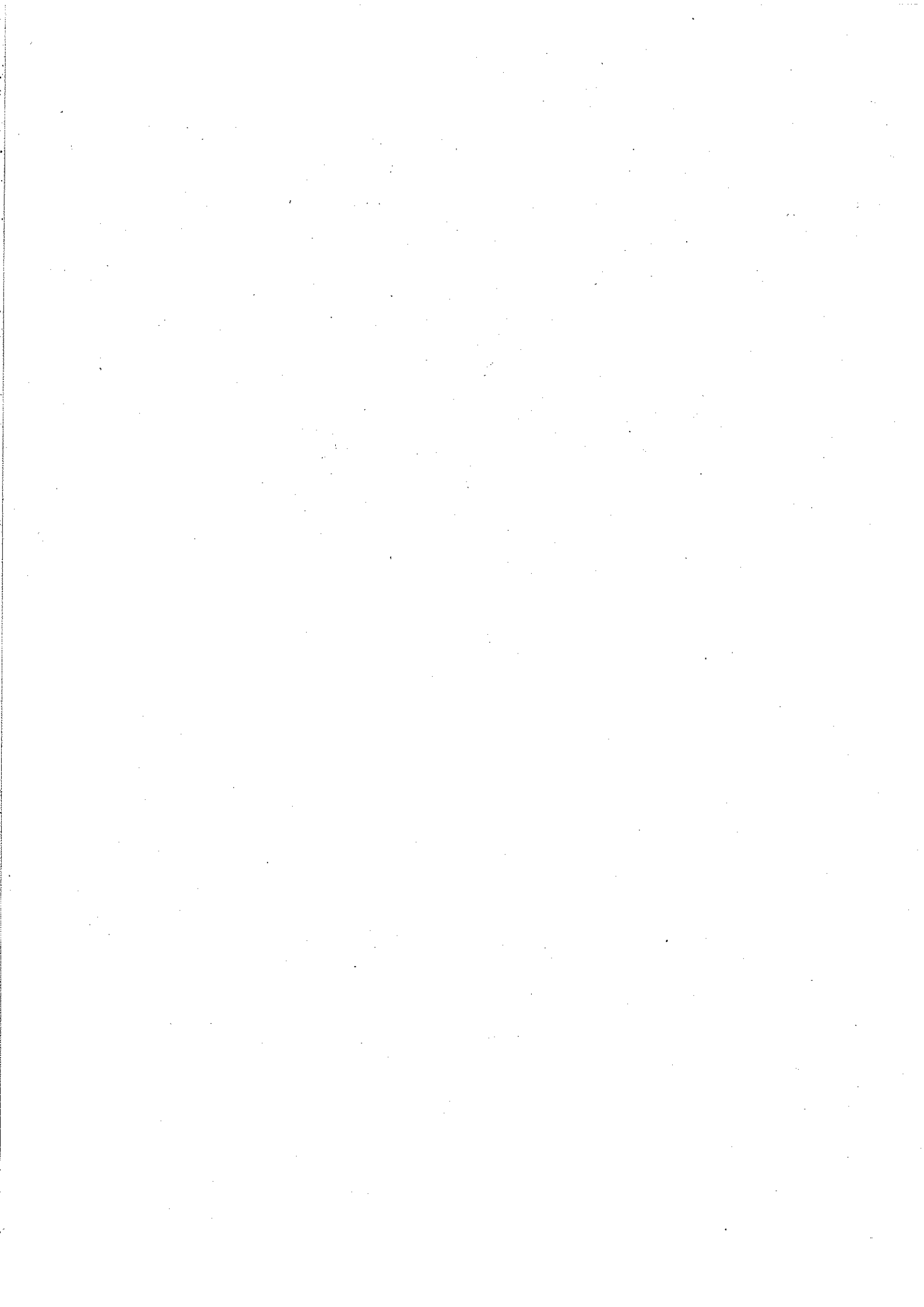
$$\begin{aligned} \sqrt{x} &= x^{\frac{1}{2}} \\ &= \frac{1}{2} \cdot x^{\frac{1}{2}-1} \\ &= \frac{1}{2} \cdot x^{-\frac{1}{2}} \\ &= \frac{1}{2} \cdot \frac{1}{\sqrt{x}} \end{aligned}$$

$$\begin{aligned} \frac{\partial f}{\partial y} &= 2 + 1\sqrt{x} - 2y - 0 \\ &= 2 + 1\sqrt{x} \end{aligned}$$

?

$$\begin{aligned}
 4) \int_0^{2\pi} \sin^3 x \, dx &= \sin^2 x \cdot \sin x \, dx \\
 &= \int_0^{2\pi} (1 - \cos^2 x) \sin x \, dx = \left[ \begin{array}{l} t = \cos x \\ dt = -\sin x \, dx \end{array} \right] \quad \begin{array}{l} \text{graničice} \\ \cos(2\pi) = 1 \\ \cos(0) = 1 \end{array} \\
 &= \int_1^1 (1 - t^2) \cdot \sin x \cdot \frac{-1}{\sin x} dt = \left. - \left( t - \frac{t^3}{3} \right) \right|_1^1 \\
 &= - \left( 1 - \frac{1}{3} - \left( 1 - \frac{1}{3} \right) \right) = \underline{0} \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 5) \int_0^1 \frac{\sin(\ln(x^2))}{x} \, dx &= \left[ \begin{array}{l} t = \ln x^2 \\ dt = \frac{1}{x} 2x \, dx \end{array} \right] \\
 & \quad dx = \frac{x}{2} dt \\
 \int_0^1 \frac{\sin t}{x} \cdot \frac{x}{2} dt &= \int_0^1 \sin t \cdot 2 dt \\
 &= \frac{1}{2} \int_0^1 \sin t \, dt = -\frac{1}{2} \sin(\ln x^2) \Big|_0^1 \\
 &= -\frac{1}{2} (\sin \ln 1 - \lim_{x \rightarrow 0} \sin(\ln x^2)) = -\frac{1}{2} (\sin 0 - \sin(\ln 0)) \\
 &= -\frac{1}{2} (0 - \sin(+\infty)) = -\frac{1}{2} \cdot \sin(+\infty) = ?
 \end{aligned}$$





IME I PREZIME: TOMISLA TU TA

BROJ INDEKSA:

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

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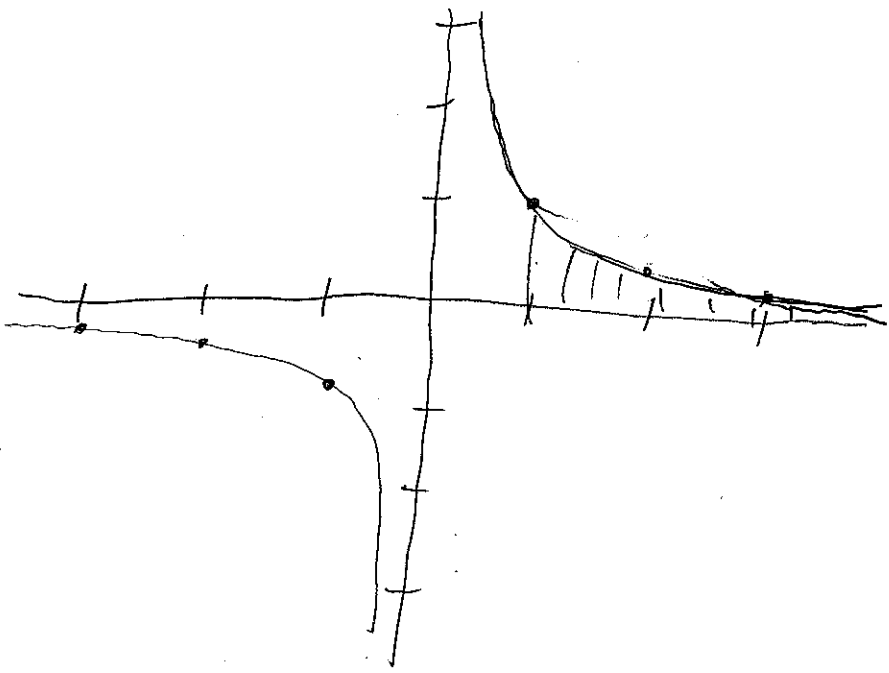
6.  $f(x) = \frac{1}{x^3} \int_1^{+\infty} f(x) dx$

$$\int_1^{+\infty} \frac{1}{x^3} = \int_1^{+\infty} x^{-3} = \left| \frac{x^{-2}}{-2} \right| = -\frac{1}{2} x^{-2}$$

$$\lim_{x \rightarrow \infty} -\frac{1}{2} x^{-2} - \left( -\frac{1}{2} x^{-2} \right) = \frac{1}{2} \checkmark$$



$$f(x) = \frac{1}{x^3}$$



x	f(x)
-3	$-\frac{1}{27}$
-2	$-\frac{1}{8}$
-1	-1
0	/
1	1
2	$\frac{1}{8}$
3	$\frac{1}{27}$

2.)  $f(x,y) = 2y + y\sqrt{x} - y^2 - x$

$$\frac{df}{dx} = \frac{y}{2\sqrt{x}} - 1 \Rightarrow y = 2\sqrt{x}$$

$$\frac{df}{dy} = 2 + \sqrt{x} - 2y$$

$$\frac{d^2f}{dx^2} = \frac{y}{2} \cdot -\frac{1}{2}x^{-\frac{3}{2}}$$

$$\frac{d^2f}{dy^2} = -2$$

$$\frac{df}{dx dy} = \frac{1}{2\sqrt{x}} \quad \begin{matrix} \sqrt{x} = 2 \\ x = 4 \end{matrix}$$

$$\Delta = \begin{vmatrix} \frac{y}{2} \left(-\frac{1}{2}x^{-\frac{3}{2}}\right) & \frac{1}{2\sqrt{x}} \\ \frac{1}{2\sqrt{x}} & -2 \end{vmatrix}$$

$$2 + \sqrt{x} - 2y = 0$$

$$y = 2\sqrt{x}$$

$$2 + \sqrt{x} - 2(2\sqrt{x}) = 0$$

$$y = 2 \cdot \frac{2}{3}$$

$$2 + \sqrt{x} - 4\sqrt{x} = 0$$

$$y = \frac{4}{3} \approx 1.33$$

$$2 - 3\sqrt{x} = 0$$

A = MAX ✓

$$3\sqrt{x} = 2 \Rightarrow \sqrt{x} = \frac{2}{3}$$

$$x = \frac{4}{9} \approx 0.44$$

~~A = 0.341 > 0 EKS~~

~~$$1.807 - \frac{9}{16}$$~~

~~$$= 1.245$$~~

~~$$\frac{2}{3} = 0.8 \times 6$$~~

