MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO

IME I PREZIME: MARGARITA UHODA **BROJ INDEKSA:** 17-2-0175-2012

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \sin(2x)$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2}$. 15
3. Izračunati tangencijalnu ravninu na graf funkcije $f(x,y) = \frac{x^2}{y+1}$ u točki $T(3,1,z_0)$. 15
4. $\int_0^1 xe^x \, dx = ?$ 20
5. $\int_0^2 \frac{3x}{x^2-2x+1} = ?$ 15
6. $\int_2^{+\infty} \frac{dx}{1-x^2} = ?$ 20

Ukupno: 85

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\vert\frac{a+x}{a-x}\right\vert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\vert\cos x\vert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\vert\frac{x-a}{x+a}\right\vert + C$
$\int \frac{dx}{x} = \ln\vert x\vert + C$	$\int \cot x \, dx = \ln\vert\sin x\vert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\vert x + \sqrt{x^2 \pm a^2}\right\vert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

2.

$$f(x,y) = \frac{1}{1+x^2+y^2}$$

$$\frac{\partial f}{\partial x} = \frac{(1+x^2+y^2)0 - 2x}{(1+x^2+y^2)} = \frac{-2x}{(1+x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y} = \frac{-2y}{(1+x^2+y^2)^2}$$

$$\frac{\partial f}{\partial x} = 0 \qquad \frac{-2x}{(1+x^2+y^2)^2} = 0 \Rightarrow \boxed{x=0}$$

$$\frac{\partial f}{\partial y} = 0 \qquad \frac{-2y}{(1+x^2+y^2)^2} = 0 \Rightarrow \boxed{y=0}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{-2(1+x^2+y^2)^2 + 4x(1+x^2+y^2)^2 \cdot 2x}{(1+x^2+y^2)^4}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{[-2\cdot(1+x^2+y^2)+8x]\cancel{(1+x^2+y^2)}}{(1+x^2+y^2)^{\cancel{4}\,3}}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{-2(1+x^2+y^2)\cdot 2 + 4y(1+x^2+y^2)\cdot 2y}{(1+x^2+y^2)^4}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{[-2(1+x^2+y^2)+8y)\cancel{(1+x^2+y^2)}}{(1+x^2+y^2)^{\cancel{4}\,3}}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{-2y}{(1+x^2+y^2)^2}\right)$$

$$= \frac{0\cdot(1+x^2+y^2)^2 + 4y(1+x^2+y^2)\cdot 2x}{(1+x^2+y^2)^2}$$

$$= \frac{8xy(1+x^2+y^2)}{(1+x^2+y^2)^2}$$

$$\frac{\partial^2 f}{\partial x^2} T(0,0) = \frac{-2(1+0+0)}{1} = -2$$

$$\frac{\partial^2 f}{\partial y^2} T(0,0) = \frac{-2}{1} = -2$$

$$\frac{\partial^2 f}{\partial x \partial y} T(0,0) = 0$$

$$\Delta = \begin{vmatrix} 2 & 0 \\ 0 & -2 \end{vmatrix} = 4 >$$ EKSTREM POSTOJI

$$\frac{\partial^2 f}{\partial x^2} = -2 < 0$$

MAX $(0,0,1)$ ✓

MARGARITA UHODA
17-2-0175-2012

3. $f(x,y)\ \dfrac{x^2}{y+1}$ $T(3,1,z_0)$

$$z_0 = \frac{3^2}{1+1} = \frac{9}{2}$$

$$\frac{\partial f}{\partial x} = \frac{2x(y+1) - x^2 \cdot 0}{(y+1)^2} = \frac{2x}{y+1}$$ ✓

$$\frac{\partial f}{\partial x} = \frac{-x^2 \cdot 1}{(y+1)^2} = \frac{-x^2}{(y+1)^2}$$ ✓

$$f_x(T) = \frac{2 \cdot 3}{1+1} = 3$$

$$f_y(T) = \frac{-9}{(1+1)^2} = \frac{-9}{4}$$

TANGENCIJALNA RAVNINA

$$z - z_0 = f_x(T)(x - x_0) + f_y(T)(y - y_0)$$

$$z - \frac{9}{2} = 3(x-3) - \frac{9}{4}(y-1)$$

$$z - \frac{9}{2} = 3x - 9 - \frac{9}{4}y + \frac{9}{4}$$

$$\boxed{-3x + \frac{9}{4}y + z + \frac{9}{4} = 0}$$ ✓

4. $$\int_0^1 x e^x dx = \begin{bmatrix} u = x & dv = e^x dx \\ du = dx & v = e^x \end{bmatrix}$$

$$= \left[x \cdot e^x - \int e^x dx\right]_0^1 = \left[x \cdot e^x - e^x\right]_0^1$$

$$= 1 \cdot e^1 - e^1 - (0 - e^0) = 1$$ ✓

5. $\int_0^2 \frac{3x}{x^2-2x+1} = \int_0^2 \frac{3x\,dx}{(x-1)^2}$

$= \int \frac{A\,dx}{x-1} + \int \frac{B\,dx}{(x-1)^2}$

$\frac{3x}{x^2-2x+1} = \frac{A}{x-1} + \frac{B}{(x-1)^2} \Big/ \cdot (x-1)^2$

$3x = A(x-1) + B$

$\boxed{3 = A}$ $\quad -A + B = 0$

$\boxed{B = 3}$

$\int \frac{3\,dx}{x-1} + \int \frac{3\,dx}{(x-1)^2} = \begin{bmatrix} x-1 = t \\ dx = dt \end{bmatrix}$

$= 3\int \frac{dt}{t} + 3\int \frac{dt}{t^2} = 3\ln|t| \; 3\cdot \frac{t^{-2+1}}{-2+1}$

$= 3\ln|t| - \frac{3}{t}$

$\int_0^2 \frac{3x}{x^2-2x+1}\,dx = \left[3\ln|x-1| - \frac{3}{x-1}\right]_0^2$

PREKID U $x = 1$ ✓

$\lim_{a\to 1}\left[3\ln|x-1| - \frac{3}{x-1}\right]_0^a + \lim_{b\to 1}\left[3\ln|x-1| - \frac{3}{x-1}\right]_b^2$

$\lim_{a\to 1}\left(3\ln|a-1| - \frac{3}{a-1} - 3\ln|-1| + \frac{3}{-1}\right) + \lim_{b\to 1}\Big(3\ln 1 -$

$\frac{3}{1} - 3\ln|b-1| + \frac{3}{b-1}\Big]$

$= \underbrace{3\ln 0 - \frac{3}{0}}_{\infty - \infty} - 0 - 3 + 0 - 3 - \underbrace{3\ln 0 + \frac{3}{0}}_{-\infty + \infty}$ $= \text{NE POSTOJI}$ ✓

6. $\int_2^{+\infty} \frac{dx}{1-x^2} = \lim_{a\to\infty} \int_2^a \frac{dx}{1-x^2}$

$= \lim_{a\to\infty} \left[\frac{1}{2} \ln \left|\frac{1+x}{1-x}\right|\right]_2^a$

$= \lim_{a\to\infty} \left[\frac{1}{2} \ln \left|\frac{1+a}{1-a}\right| - \frac{1}{2} \ln \left|\frac{1+2}{1-2}\right|\right]$

$\lim_{a\to\infty} \frac{1}{2} \ln \left|\frac{1+a:a}{1-a:a}\right| = \lim_{a\to\infty} \frac{1}{2} \ln \left|\frac{\frac{1}{a}+1}{\frac{1}{a}-1}\right| = \frac{1}{2} \cdot 0 = 0$

RIJEŠENJE INTEGRALA

$= -\frac{1}{2} \ln 3 = -0.5493$ ✓

MARGARITA UHODA 17-2-0175-2012

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxo	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: Anđela Ukoda **BROJ INDEKSA:** 17-2-0106-2011	

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \sin(2x)$. 15
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Ukupno: 85

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
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Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
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1. $y'' + 2y' + 5y = \sin(2x)$

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2.) Odrediti lokalne ekstreme funkcije $f(x,y)=\frac{1}{1+x^2+y^2}$

$$f(x,y)=\frac{1}{1+x^2+y^2}$$

$T(0,0)$

$$\frac{\partial f}{\partial x}=\frac{(1+x^2+y^2)0-2x}{(1+x^2+y^2)^2}=\frac{-2x}{(1+x^2+y^2)}$$

$$\frac{\partial f}{\partial y}=\frac{-2y}{(1+x^2+y^2)^2}$$

$$\frac{\partial f}{\partial x}=0 \qquad \frac{-2x}{(1+x^2+y^2)^2}=0 \Rightarrow \boxed{x=0}$$

$$\frac{\partial f}{\partial y}=0 \qquad \frac{-2y}{(1+x^2+y^2)^2}=0 \Rightarrow \boxed{y=0}$$

$$\frac{\partial^2 f}{\partial x^2}=\frac{-2(1+x^2+y^2)^2+4x(1+x^2+y^2)^1\cdot 2x}{(1+x^2+y^2)}$$

$$\frac{\partial^2 f}{\partial x^2}=\frac{[-2\cdot(1+x^2+y^2)+8x]\cancel{(1+x^2+y^2)}}{(1+x^2+y^2)^{\cancel{4}\,3}}$$

$$\frac{\partial^2 f}{\partial y^2}=\frac{-2(1+x^2+y^2)^2+4y(1+x^2+y^2)\cdot 2y}{(1+x^2+y^2)^{\cancel{4}\,3}}$$

$$\frac{\partial^2 f}{\partial y^2}=\frac{[-2(1+x^2+y^2)+8y]\cancel{(1+x^2+y^2)}}{(1+x^2+y^2)^{\cancel{4}\,3}}$$

$$\frac{\partial^2 f}{\partial x\partial y}=\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\right)=\frac{\partial}{\partial x}\left(-\frac{2y}{(1+x^2+y^2)^2}\right)$$

$$=\frac{0\cdot(1+x^2+y^2)^2+4y(1+x^2+y^2)\cdot 2x}{(1+x^2+y^2)^2}$$

$$=\frac{8xy(1+x^2+y^2)}{(1+x^2+y^2)^2}$$

$T(0,0)=0$

$$T(0,0)=\frac{-2(1+0+0)}{1}=-2$$

$$T \qquad =\frac{-2}{1}=-2$$

$$\Delta=\begin{vmatrix}-2 & 0\\ 0 & -2\end{vmatrix}=4>0$$

EKSTREM POSTOJI

$$\frac{\partial^2 f}{\partial x^2}=-2<0$$ MAKSIMALAN $(0,0,1)$ ✓

4. $\int_0^1 xe^x dx = ?$ Amdela Uhoda BROJ INDEKSA: 17-2-0106-2011

$$\int_0^1 xe^x dx = \begin{bmatrix} u = x & dv = e^x dx \\ du = dx & v = e^x \end{bmatrix}$$

$$= \left[x \cdot e^x - \int e^x dx\right]_0^1 = \left[x \cdot e^x - e^x\right]_0^1$$

$$= 1 \cdot e^1 - e^1 - (0 - e^0) = 1 \checkmark$$

5. $$\int_0^2 \frac{3x}{x^2-2x+1} = \int_0^2 \frac{3x}{(x-1)^2} = \int \frac{A\,dx}{x-1} + \int \frac{B\,dx}{(x-1)^2}$$

$$\frac{3x}{x^2-2x+1} = \frac{A}{x-1} + \frac{B}{(x-1)^2} \quad / \cdot (x-1)^2$$

$x=1$ V.A. UNUTAR $[0,2]$

OVO JE NEPRAVI INTEGRAL!!!

$$3x = A(x-1) + B$$

$$\boxed{3 = A} \qquad -A + B = 0 \qquad \boxed{B = 3}$$

$$\int^1 \frac{3dx}{x-1} + \int \frac{3dx}{(x-1)^2} = \begin{bmatrix} x-1 = t \\ dx = dt \end{bmatrix}$$

$$= 3\int \frac{dt}{t} + 3\int \frac{dt}{t^2} = 3\ln|t| + 3 \cdot \frac{t^{-2+1}}{-2+1}$$

$$= 3\ln|t| - \frac{3}{t}$$

$$\int_0^2 \frac{3x}{x^2-2x+1} dx = \left[3\ln|x-1| - \frac{3}{x-1}\right]_0^2 \checkmark$$

prekid u $x=1$ $\checkmark$

$$\lim_{a\to 1}\left[3\ln|x-1| - \frac{3}{x-1}\right]_0^a + \lim_{b\to 1}\left[3\ln|x-1| - \frac{3}{x-1}\right]_b^2$$

$$\lim_{a\to 1}\left(3\ln|a-1| - \frac{3}{a-1} - 3\ln|-1| + \frac{3}{-1}\right) +$$

$$\lim_{b\to 1}\left[3\ln 1 - \frac{3}{1} - 3\ln|b-1| + \frac{3}{b-1}\right]$$

$$= \underbrace{3\ln 0 - \frac{3}{0}}_{\infty - \infty} - 0 - 3 + 0 - 3 - \underbrace{3\ln 0 + \frac{3}{0}}_{-\infty + \infty}$$

$=$ NE POSTOJI $\checkmark$

6.

$$\int_2^{+\infty} \frac{dx}{1-x^2} = \lim_{a\to\infty} \int_2^a \frac{dx}{1-x^2}$$

$$= \lim_{a\to\infty} \left[\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|\right]_2^a$$

$$= \lim_{a\to\infty} \left[\frac{1}{2}\ln\left|\frac{1+a}{1-a}\right| - \frac{1}{2}\ln\left|\frac{1+2}{1-2}\right|\right]$$

$$\lim_{a\to\infty} \cdot \frac{1}{2}\ln\left|\frac{1+a\ /:a}{1-a\ /:a}\right| = \lim_{a\to\infty} \frac{1}{2}\ln\left|\frac{\frac{1}{a}+1}{\frac{1}{a}-1}\right| = \frac{1}{2}\cdot 0 = 0$$

RJEŠENJE INTEGRALA

$$= -\frac{1}{2}\ln 3 = -0,5493 \checkmark$$

3.

$$f(x,y) = \frac{x^2}{y+1} \qquad T(3,1,z_0)$$

$$z_0 = \frac{3^2}{1+1} = \frac{9}{2}$$

$$\frac{df}{dx} = \frac{2x(y+1) - x^2\cdot 0}{(y+1)^2} = \frac{2x}{y+1}$$

$$\frac{df}{dx} = \frac{-x^2\cdot 1}{(y+1)^2} = \frac{-x^2}{(y+1)^2} \checkmark$$

$$f_x(T) = \frac{2\cdot 3}{1+1} = 3$$

$$f_y(T) = \frac{-9}{(1+1)^2} = \frac{-9}{4}$$

TANGENCIJALNA RAVNINA

$$z - z_0 = f_x(T)(x-x_0) + f_y(T)(y-y_0)$$

$$z - \frac{9}{2} = 3(x-3) - \frac{9}{4}(y-1)$$

$$z - \frac{9}{2} = 3x - 9 - \frac{9}{4}y + \frac{9}{4}$$

$$\boxed{-3x + \frac{9}{4}y + z + \frac{9}{4} = 0} \checkmark$$

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IME I PREZIME: Vedran Ivanković **BROJ INDEKSA:** 17-2-0235-2012.

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \sin(2x)$. 15
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4. $\int_0^1 xe^x \, dx = ?$ 20
5. $\int_0^2 \frac{3x}{x^2 - 2x + 1} = ?$ ~~15~~
6. $\int_2^{+\infty} \frac{dx}{1 - x^2} = ?$ 20

Ukupno: 70

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
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$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha \, dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln \lvert x \rvert + C$	$\int \cot x \, dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int e^x \, dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
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$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2} \, dx = \frac{1}{2} \left[x\sqrt{a^2-x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

4.
$$\int_0^1 xe^x \, dx = \left[\begin{array}{ll} u = x & dv = e^x dx \\ du = dx & v = e^x \end{array} \right]$$
$$= xe^x \Big|_0^1 - \int_0^1 e^x \, dx = (xe^x - e^x)\Big|_0^1$$
$$= (e - e) - (0 - 1) = 1 \ \checkmark$$

5.) $$\int_0^2 \frac{3x}{x^2-2x+1} = \int_0^2 \frac{3x}{(x-1)^2} = \begin{bmatrix} t = x-1 & t \text{ GORNJA} = 1 \\ dt = dx & t \text{ DONJA} = -1 \end{bmatrix}$$

$$= \int_{-1}^{1} \frac{3t+3}{t^2}\,dt = 3\int_{-1}^{1} \frac{1}{t}\,dt + 3\int_{-1}^{1} \frac{1}{t^2}\,dt$$

NEPRAVI INTEGRAL!!!

$$= \left(3\ln|t| - \frac{3}{t}\right)\Big|_{-1}^{1} = (3\ln 1 - 3) - (3\ln 1 + 3)$$

$$= -6$$

6.) $$\int_2^{+\infty} \frac{dx}{1-x^2} = \lim_{b\to+\infty} -\int_2^b \frac{dx}{(x^2-1)} = \lim_{b\to+\infty} -\left(\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|\right)\Big|_2^b$$

$$= \lim_{b\to\infty} \frac{-1}{2}\left(\ln\left|\underbrace{\frac{b-1}{b+1}}_{\to 1}\right| - \ln\left|\frac{1}{3}\right|\right)$$ ✓

$$= -\frac{1}{2}\left[\ln 1 - \ln 1 + \ln 3\right] = -\frac{1}{2}\ln 3$$ ✓

2.) $f(x,y) = \dfrac{1}{1+x^2+y^2}$

$f'_x = \dfrac{-2x}{(1+x^2+y^2)^2}$ $\qquad \dfrac{-2x}{(1+x^2+y^2)^2} = 0$ ✓

$f'_y = \dfrac{-2y}{(1+x^2+y^2)^2}$ $\qquad \dfrac{-2y}{(1+x^2+y^2)^2} = 0$ ✓

LOKALNI MAKSIMUM $T(0,0)$ ✓ $\qquad x_1 = 0 \qquad y_1 = 0$ ✓

$$f''_{xx} = \frac{-2(1+x^2+y^2)^2 + 2x \cdot 2(1+x^2+y^2) \cdot 2x}{(1+x^2+y^2)^4} \Rightarrow (f''_{xx})_T = -2$$

$$f''_{xy} = \frac{2x \cdot 2(1+x^2+y^2) \cdot 2y}{(1+x^2+y^2)^4} \Rightarrow (f''_{xy})_T = 0$$

$$f''_{yy} = \frac{-2(1+x^2+y^2)^2 + 2y \cdot 2(1+x^2+y^2) \cdot 2y}{(1+x^2+y^2)^4} \Rightarrow (f''_{yy})_T = -2$$

$$\begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix} = 4 > 0 \qquad (f''_{xx})_T < 0$$

1. $y'' + 2y' + 5y = \sin(2x)$

Vedran Ivanković

1. HOMOGENI DIO

$$y'' + 2y' + 5y = 0$$

$$r^2 + 2r + 5 = 0$$

$$r_{1/2} = \frac{-2 \pm \sqrt{4-20}}{2}$$

$$r_{1/2} = \frac{-2 \pm \sqrt{16}}{2}$$

$$r_{1/2} = \frac{-2 \pm 4i}{2}$$

$$r_1 = -1 + 2i$$

$$r_2 = -1 - 2i$$

$$\alpha = -1 \quad \beta = 2$$

$$\boxed{y_h = C_1 e^{-x} \cos 2x + C_2 e^{-x} \sin 2x}$$

2. PARTIKULARNO RJEŠENJE

$$y_p = k_1 \cos 2x + k_2 \sin 2x$$

$$y_p' = -2k_1 \sin 2x + 2k_2 \cos 2x$$

$$y_p'' = -4k \cos 2x - 4k_2 \sin 2x$$

$$-4k_1 \cos 2x - 4k_2 \sin 2x - 4k_1 \sin 2x + 4k_2 \cos 2x + 5k_1 \cos 2x - 5k_2 \sin 2x = \sin 2x$$

$$-4k_1 + 4k_2 + 5k_1 = 0$$

$$-4k_2 - 4k_1 + 5k_2 = 1$$

$$k_1 + 4k_2 = 0 / \cdot 4$$

$$k_2 - 4k_1 = 1$$

$$\left.\begin{aligned} \cancel{4k_1} + 16k_2 &= 0 \\ -\cancel{4k_1} + k_2 &= 1 \end{aligned}\right\} +$$

$$17k_2 = 1$$

$$\boxed{k_2 = \frac{1}{17}} \quad \boxed{k_1 = \frac{-4}{17}}$$

$$y_p'' + 2y_p' + 5y_p = \frac{16 + \cancel{4} - 20}{17} \cos 2x + \frac{-4+16+5}{17} \sin 2x$$ ✓ ✓

PROVJERA:

$$y_p' = \frac{8}{17} \sin 2x + \frac{2}{17} \cos 2x$$

$$y_p'' = \frac{16}{17} \cos 2x - \frac{4}{17} \sin 2x$$

$$y_p = \frac{-4}{17} \cos 2x + \frac{1}{17} \sin 2x$$

$y = y_h + y_p$ OPĆE RJEŠENJE

$$\boxed{y = C_1 e^{-x} \cos 2x + C_2 e^{-x} \sin 2x - \frac{4}{17} \cos 2x + \frac{1}{17} \sin 2x}$$ ✓

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxo	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: MARIN BERAM — BROJ INDEKSA: 17-2-0289-2013 0269079117	

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \sin(2x)$. — 15
2. Odrediti lokalne ekstreme funkcije $f(x, y) = \frac{1}{1+x^2+y^2}$. — 15
3. Izračunati tangencijalnu ravninu na graf funkcije $f(x, y) = \frac{x^2}{y+1}$ u točki $T(3, 1, z_0)$. — 15
4. $\int\limits_0^1 x e^x \,\mathrm{d}x = ?$ — 20
5. $\int\limits_0^2 \frac{3x}{x^2 - 2x + 1} = ?$ — 15
6. $\int\limits_2^{+\infty} \frac{\mathrm{d}x}{1 - x^2} = ?$ — 20

Ukupno: 30

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\|\cos x\| + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x \,\mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

2. $f(x,y) = \frac{1}{1+x^2+y^2}$

$$dx = \frac{0\cdot(1+x^2+y^2) - 1\cdot(1+x^2+y^2)'}{(1+x^2+y^2)^2} = \frac{-2x}{(1+x^2+y^2)^2}$$

$$dy = \frac{-1\cdot(1+x^2+y^2)'}{(1+x^2+y^2)^2} = \frac{-2y}{(1+x^2+y^2)^2}$$

$$\frac{-2x}{(1+x^2+y^2)^2} = 0 \,/\cdot (1+x^2+y^2)^2 \qquad \frac{-2y}{(1+x^2+y^2)^2} = 0$$

$$-2x = 0$$

$$\boxed{x=0} \qquad \boxed{y=0}$$

$T(0,0) \rightarrow$ KANDIDAT

$$d_{xx} = \frac{-2\cdot(1+x^2+y^2)^2 + 2x(2(1+x^2+y^2)\cdot 2x}{(1+x^2+y^2)^4}$$

$$d_{xy} = \frac{+2x(2(1+x^2+y^2)\cdot 2y}{(1+x^2+y^2)^4}$$

$$d_{yy} = \frac{-2(1+x^2+y^2)^2 + 2y(2(1+x^2+y^2)\cdot 2y}{(1+x^2+y^2)^4}$$

$$d_{xx} = \frac{-2\cdot(1+0+0)^2 + 2\cdot 0(2(1+0+0)\cdot 2\cdot 0}{(1+0^2+0^2)^4}$$

$$d_{xx} = \frac{-2+0}{1} = -2$$

$$d_{xy} = \frac{2\cdot 0(2\cdot(1+0^2+0^2)\cdot 2\cdot 0}{(1+0^2+0^2)^4} = \frac{0}{1} = 0$$

$$d_{yy} = \frac{-2\cdot(1+0+0)^2 + 2\cdot 0(2(1+0+0)\cdot 2\cdot 0}{(1+0+0)^4} = \frac{-2}{1} = -2$$

$$\begin{bmatrix} d_{xx} & d_{yx} \\ d_{xy} & d_{yy} \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} = 4 - 0 = 4 > 0 \qquad \begin{matrix} -2 < 0 \\ 4 > 0 \end{matrix}$$

$T(0,0) \Rightarrow$ maksimum funkcije ✓

3) $f(x,y) = \dfrac{x^2}{y+1}$ $\qquad T(3,1,z_0)$

$$z_0 = \frac{3^2}{1+1} = \frac{9}{2}$$

$$z = z_0 + d_x(x - x_1) + d_y(y - y_1)$$

$$d_x = \frac{2x\cdot(y+1) - x^2\cdot 0}{(y+1)^2} = \frac{2xy + 2x}{(y+1)^2} = \frac{2\cdot 3\cdot 1 + 2\cdot 3}{(1+1)^2} = \frac{12}{4} = 3$$

$$d_y = \frac{0\cdot y+1 - x^2\cdot 1}{(y+1)^2} = \frac{-x^2}{(y+1)^2} = -\frac{9}{4}$$

$$z = \frac{9}{2} + 3(x-3) - \frac{9}{4}(y-1)$$

4) $\displaystyle\int_0^1 x e^x \, dx =$

$$\int x e^x dx = \begin{Bmatrix} u = x & dv = e^x dx \\ du = dx & v = e^x \end{Bmatrix} = x e^x - \int e^x dx = \boxed{x e^x - e^x}$$

$$= x e^x - e^x \Big|_0^1 =$$

$$= 1\cdot e^1 - e^1 - (0\cdot e^0 - e^0)$$

$$= \cancel{e^1 - e^1} + e^0$$

$$= e^0 = 1 \;✓$$

$$\cancel{\lim_{b\to 0^+} \int_b^1 x e^x dx = x e^x - e^x \Big|_b^1 = 1\cdot e^1 - e^1 - (b\cdot e^b - e^b) = -(0^+\cdot 1 - 1) = -0^+ + 1 = 1}$$

6. $\int_2^{+\infty} \frac{dx}{1-x^2} =$

$\lim\limits_{b\to+\infty} \int_2^b \frac{dx}{1-x^2} =$

$= -\frac{1}{2}\ln|1-x| + \frac{1}{2}\ln|1+x| \Big|_2^b$

$= -\frac{1}{2}\ln|b-x| + \frac{1}{2}\ln|1+b| - \left(-\frac{1}{2}\ln|1-2| + \frac{1}{2}\ln|1+2|\right)$

$= -\infty + \infty - (0 - 0{,}549)$

= N/P NEMA RJEŠENJA

$\lim\limits_{x\to+\infty} \left(-\ln|\cancel{b}-x| + \ln|1+x|\right)$

$\neq \lim\limits_{x\to+\infty} -\ln|1+x| + \lim\limits_{x\to+\infty} \ln|1+x|$

GORNJI LIMES $= \lim\limits_{x\to+\infty} \ln\left|\frac{1+x}{1-x}\right|$

$= \ln\left(\lim\limits_{x\to+\infty}\left|\frac{1+x}{1-x}\right|\right)$

$= \ln 1 = 0$

DONJI LIMES $= -\infty + \infty =$ NEODREĐENI OBLIK

$\int \frac{dx}{1-x^2} = \frac{A}{(1-x)} + \frac{B}{(1+x)} \Big/ \cdot (1-x)(1+x)$

$1 = A(1+x) + B(1-x)$

$1 = A + Ax + B - Bx$

$1 = A + B \Rightarrow B = 1 - A$

$0 = A - B \Rightarrow \boxed{B = 1 - \frac{1}{2} = \frac{1}{2}}$

$0 = A - (1-A)$

$0 = A - 1 + A$

$0 = 2A - 1$

$2A = 1$

$\boxed{A = \frac{1}{2}}$

$\int \frac{dx}{1-x^2} = \int \frac{\frac{1}{2}dx}{(1-x)} + \int \frac{\frac{1}{2}}{(1+x)}dx$

$= \int \frac{1\,dx}{2(1-x)} + \int \frac{1\,dx}{2(1+x)}$

$= \frac{1}{2}\int \frac{dx}{1-x} + \frac{1}{2}\int \frac{1}{1+x}$

$= \left\{ \begin{array}{l} t = 1-x \\ dt = -dx \end{array} \right\}$ $\qquad \left\{ \begin{array}{l} u = 1+x \\ du = 1dx \end{array} \right\}$

$= -\frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t|$ $\qquad = \frac{1}{2}\ln|1+x|$

$= -\frac{1}{2}\ln|1-x|$

MARIN BERAM

5. $\int_0^2 \frac{3x}{x^2-2x+1} =$ U $x=1$ PREKIDA

$x^2-2x+1 = (x-1)^2$

$\int \frac{3x\,dx}{x^2-2x+1} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} \Big/ \cdot (x-1)^2$

$3x = A(x-1)+B$

$3x = Ax - A + B$

$\boxed{3 = A}$

$-A+B=0$

$A = B$

$\boxed{B=3}$

$\int \frac{3x\,dx}{x^2-2x+1} = \int \frac{3\,dx}{(x-1)} + \int \frac{3\,dx}{(x-1)^2}$

$= 3\int \frac{1}{x-1}dx + 3\int \frac{1}{(x-1)^2}dx = \left\{ \begin{matrix} t = x-1 \\ dt = dx \end{matrix} \right.$ $\quad t^{-2} = -t^{-1}$

$= 3\int \frac{dt}{t} + 3\int \frac{dt}{t^2}$

$= 3\ln|x-1| - 3\cdot(x-1)^{-1}$

$= 3\ln|x-1| - \frac{3}{x-1}$

$\int_0^{1^-} \frac{3x}{x^2-2x+1} + \int_{1^+}^2 \frac{3x}{x^2-2x+1}$

$= \lim_{b\to 1^-} \int_0^b \frac{3x}{x^2-2x+1} + \lim_{b\to 1^+} \int_b^2 \frac{3x}{x^2-2x+1}$

$= 3\ln|x-1| - \frac{3}{x-1} \Big|_0^b + 3\ln|x-1| - \frac{3}{x-1} \Big|_b^2$

$= 3\ln|b-1| - \frac{3}{b-1} - \left(3\ln|x-0| - \frac{3}{0-1}\right) + 3\ln|2-1| - \frac{3}{2-1} - \left(3\ln|b-0| - \frac{3}{b-1}\right)$

$= 3\ln|1^- -1| - \frac{3}{1^- -1} - \left(3\ln|0-0| - \frac{3}{0-1}\right) + 3\ln|1| - \frac{3}{1} - \left(3\ln|1^+ - 0| - \frac{3}{1^+ -1}\right) =$

$= -\infty + \infty - \left(-\infty - \frac{3}{1}\right) + 0 - 3 - 0 + \infty$

$=$ N/P NEMA POVRŠINE ✓

MARIN BEKAN

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxo	POPUNJAVA NASTAVNIK
IME I PREZIME: Marko Mustać **BROJ INDEKSA:** 17-2-0135-2011	Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \sin(2x)$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2}$. 15
3. Izračunati tangencijalnu ravninu na graf funkcije $f(x,y) = \frac{x^2}{y+1}$ u točki $T(3,1,z_0)$. 15
4. $\int_0^1 xe^x \, dx = ?$ 20
5. $\int_0^2 \frac{3x}{x^2-2x+1} = ?$ 15
6. $\int_2^{+\infty} \frac{dx}{1-x^2} = ?$ 20

Ukupno: 50

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

2.) $f(x,y) = \frac{1}{1+x^2+y^2}$

$$\frac{\partial f}{\partial x} = \frac{0\cdot(1+x^2+y^2) - 1\cdot(2x)}{(1+x^2+y^2)^2} = \frac{-2x}{(1+x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y} = \frac{0\cdot(1+x^2+y^2) - 1\cdot(2y)}{(1+x^2+y^2)^2} = \frac{-2y}{(1+x^2+y^2)^2}$$

$$\frac{-2x}{(1+x^2+y^2)^2} = 0 \quad / \cdot (1+x^2+y^2)^2$$

$$\frac{-2y}{(1+x^2+y^2)^2} = 0 \quad / \cdot (1+x^2+y^2)^2$$

$-2x = 0$

$x = \frac{0}{-2} = 0$

$x = 0$

$-2y = 0$

$y = \frac{0}{-2} = 0$

$y = 0$

KANDIDAT ZA EKSTREM JE $(0,0)$ ✓

$$\frac{-2x}{(1+x^2+y^2)^2}$$

$$\frac{\partial f}{\partial x \partial x} = \frac{-2 \cdot (1+x^2+y^2)^2 + 2x \cdot 2x}{((1+x^2+y^2)^2)^2}$$

$$\frac{\partial f}{\partial x \partial y} = \frac{0 \cdot (1+x^2+y^2)^2 + 2x \cdot 2y}{((1+x^2+y^2)^2)^2}$$

$$\frac{-2y}{(1+x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y \partial y} = \frac{-2 \cdot (1+x^2+y^2) + 2y \cdot 2y}{((1+x^2+y^2)^2)^2}$$

$$\frac{\partial f}{\partial y \partial x} = \frac{0 \cdot (1+x^2+y^2)^2 + 2y \cdot 2x}{((1+x^2+y^2)^2)^2}$$

2.) NASTAVAK

$$\begin{bmatrix} \partial x \partial x & \partial x \partial y \\ \partial y \partial x & \partial y \partial y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} = \quad \begin{aligned} D_1 &= -1 \\ D_2 &= -1 \cdot (-2) + 0 \cdot 0 \\ &= 4 \end{aligned}$$

TOČKA $(0,0)$ JE LOKALNI MAKSIMUM ✓

3.) $f(x,y) = \frac{x^2}{y+1}$ $\quad T(\overset{x_0}{3}, \overset{y_0}{1}, z_0)$ Marko Mustać

$z_0 = \frac{3^2}{1+1} = \frac{9}{2} = 4.5$ $\quad f(x_0, y_0) = 4{,}5$

$$\frac{\partial f}{\partial x} = \frac{2x \cdot (y+1) - x^2 \cdot (0)}{(y+1)^2} = \frac{2xy + 2x}{(y+1)^2} = 3$$

$$\frac{\partial f}{\partial y} = \frac{0 \cdot (y+1) - x^2 \cdot (1)}{(y+1)^2} = \frac{-x^2}{(y+1)^2} = -2{,}25$$

$$\partial_x f(x_0,y_0) \cdot (x - x_0) + \partial_y f(x_0,y_0) \cdot (y - y_0)$$
$$+ (-1) \cdot (z - f(x_0,y_0)) = 0$$

$$3 \cdot (x-3) - 2{,}25 \cdot (y-1) + (-1) \cdot (z - 4{,}5) = 0 \quad \checkmark$$

$$3x - 9 - 2{,}25y + 2{,}25 - z + 4{,}5 = 0$$

$$3x - 2{,}25y - z - 2{,}25 = 0$$

4.) $\int_0^1 x e^x \, dx$ $\quad \left| \begin{array}{ll} u = x & dv = e^x dx \\ du = (1) dx & v = e^x \end{array} \right.$

Marko Mustać

$$\int_0^1 x e^x \, dx = x \cdot e^x - \int e^x \, dx$$

$$= x \cdot e^x \Big|_0^1 - e^x \Big|_0^1$$

$$= 1 \cdot e^1 - 0 \cdot e^0 - e^1 + e^0$$

$$= 1 \checkmark$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO

IME I PREZIME: MARKO KARLIĆ **BROJ INDEKSA:** 17-2-0179-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \sin(2x)$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2}$. 15
3. Izračunati tangencijalnu ravninu na graf funkcije $f(x,y) = \frac{x^2}{y+1}$ u točki $T(3,1,z_0)$. 15
4. $\int\limits_0^1 xe^x \, dx = ?$ $= 1$ 20
5. $\int\limits_0^2 \frac{3x}{x^2-2x+1} = ?$ $= 6$ ~~15~~
6. $\int\limits_2^{+\infty} \frac{dx}{1-x^2} = ?$ $\approx -0,55$ 20

Ukupno: 40

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4.) $$\int\limits_0^1 x e^x \, dx = \left\{ \begin{matrix} u = x & dv = e^x dx \\ du = dx & v = e^x \end{matrix} \right\}$$

$$u \cdot v - \int v \, du = x \cdot e^x - \int\limits_0^1 e^x dx = x \cdot e^x - e^x \Big|_0^1 = 1 \cdot e^1 - e^1 - (0 \cdot e^0 - e^0)$$

$$= \cancel{e^1} - \cancel{e^1} + e^0 = e^0 = 1$$

6.) $$\int\limits_2^{+\infty} \frac{dx}{1-x^2} = \lim_{b \to +\infty} \int\limits_2^b \frac{dx}{1-x^2} = \lim_{b \to +\infty} \left[\frac{1}{2} \ln\left|\frac{1+x}{1-x}\right|\right]_2^b$$

$$= \frac{1}{2}\ln\left|\frac{1+b}{1-b}\right| - \left(\frac{1}{2}\ln\left|\frac{1+2}{1-2}\right|\right) = \frac{1}{2}\ln\left|\frac{1+b \ /:b}{1-b \ /:b}\right| - \frac{1}{2}\ln\left|\frac{3}{-1}\right|$$

$$= \frac{1}{2}\ln 1 - \frac{1}{2}\ln 3 = -\frac{1}{2}\ln 3 \approx -0,55$$

1.) $y'' + 2y' + 5y = 2\sin(2x)$

$r^2 + 2r + 5 = 0$

$r_{1,2} = \frac{-2 \pm \sqrt{4-4\cdot 1\cdot 5}}{2} = \frac{-1 \pm \sqrt{-16}}{2} = \frac{-1 \pm 4i}{2} = \quad r_1 = \frac{-1+4i}{2} \quad r_2 = \frac{-1-4i}{2}$

$y_H(x) = C_1^{\frac{-1+4i}{2}\cdot x} + C_2^{\frac{-1-4i}{2}\cdot x}$

MARKO KARLIĆ

5.)

$$\int_0^2 \frac{3x}{x^2-2x+1} = 3\int_0^2 \frac{\cancel{x-1}+1}{\cancel{(x-1)}(x-1} = 3\int_0^2 dx + \underbrace{3\int_0^2 \frac{dx}{x-1}}_{X}$$

$$= 3\cdot[x]_0^2 + \underbrace{3\cdot\int_0^2 \frac{dx}{x-1}\, dx}_{*} = 3\cdot(2-0) + 3\cdot(0) = 6$$

NEPRAVI INTEGRAL

POGREŠNO KRAĆENJE U RAZLOMKU

$$\frac{\cancel{2}+\cancel{2}}{\cancel{2}\cdot\cancel{2}} \neq 1+1$$

$$* \int_0^2 \frac{dx}{x-1} = \left\{ \begin{array}{l} t = x-1 \\ dt = 1 \end{array} \right\} = \int_0^2 \frac{dt}{t} = \ln|t| + C = \Big[\ln|x-1|\Big]_0^2$$

$$= \ln|2-1| - (\ln|0-1|) = \ln 1 - \ln|-1| = \ln 1 - \ln 1 = 0$$

3.)

izračunati tangencijalnu ravninu na graf $f(x,y) = \frac{x^2}{y+1}$ u točki $T(3,1,z_0)$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxo	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: Branimir [illegible] **BROJ INDEKSA:** 17-2-008[illegible]	

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \sin(2x)$. 15

2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2}$. 15

3. Izračunati tangencijalnu ravninu na graf funkcije $f(x,y) = \frac{x^2}{y+1}$ u točki $T(3,1,z_0)$. 15

4. $\int\limits_0^1 xe^x\,dx = ?$ 20

5. $\int\limits_0^2 \frac{3x}{x^2-2x+1} = ?$ ~~15~~

6. $\int\limits_2^{+\infty} \frac{dx}{1-x^2} = ?$ ~~20~~

Ukupno: 35

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

4) $\int_0^1 x e^x dx$

$x = u$ $\quad dv = e^x$

$dx = du$ $\quad v = \int e^x$

$v = e^x$

$x \cdot e^x - \int e^x dx$

$x \cdot e^x - e^x \Big|_0^1$

$\cancel{1 \cdot e^1} - \cancel{e^1} - [0 \cdot e^0 - e^0] = +1$ ✓

$$\int_2^{+\infty} \frac{dx}{1-x^2} = \int \frac{dx}{1-x^2} = \int \frac{dx}{\underbrace{1^2-x^2}_{a^2-x^2}}$$

$$\lim_{x\to+\infty} \left|\frac{1+x}{1-x}\right| = ?$$

$$= \frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right|$$

$$= \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|$$

$$\lim_{x\to\infty} \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| \Bigg|_2^b$$

$$\lim_{x\to\infty} \frac{1}{2}\ln\left|\frac{1+b}{1-b}\right| - \frac{1}{2}\ln\left|\frac{1+2}{1-2}\right|$$

Ovaj integral ne postoji

$$\int_0^2 \frac{3x}{x^2-2x+1} =$$

$2x-2$ $2x-2\,dx$

$$x^2-2x+1=0$$

$$x_{1,2} = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = 2\pm\sqrt{4-4}$$

$$x^2-2x+1=t$$
$$(2x-2)\,dx = dt$$

$$\frac{1}{2}(2x-2)+1$$

$x^2-2x+1 = x^2-2x+1^2$

$(x-1)^2$

$$3\int \frac{x\,dx}{x^2-2x+1} =$$

$$3\int \frac{\frac{1}{2}(2x-2)+1}{x^2-2x+1} =$$

$$\int \frac{dx}{(x-1)^2-(1)^2}$$

$$\frac{3}{2}\int \frac{2x-2}{x^2-2x+1} + \int \frac{dx}{x^2-2x+1}$$

Integral je nepravilan?

$$\frac{3}{2}\int \frac{dt}{t} + \int \frac{dx}{(x^2-x+1)-1}$$

$$\frac{3}{2}\ln|x^2-2x+1| + \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$$

$$\frac{3}{2}\ln|x^2-2x+1| + \frac{1}{2(x-1)}\ln\left|\frac{x-1+1}{x-1-1}\right|$$

$$\frac{3}{2}\ln|x^2-2x+1| + \frac{1}{2x-2}\ln\left|\frac{x}{x-2}\right| \Bigg|_0^2$$

$$\frac{3}{2}\ln|2^2-4+1| + \frac{1}{4-2}\ln\left|\frac{2}{2-2}\right| - \left[\frac{3}{2}\ln(0^2-0+1) + \frac{1}{2\cdot 0-2}\ln\left|\frac{0}{0-2}\right|\right]$$

$f(x,y) = \frac{x^2}{y+1}$

$T = (3, 1, 20)$

$T = (3, 1, \frac{9}{2})$

Braninka Pijaca

$z_0 = \frac{3^2}{1+1} =$

$\partial_x f = \frac{(x^2)' \cdot (y+1) - x^2 \cdot (y+1)'}{(y+1)^2}$

$= \frac{2x \cdot (y+1) - x^2 \cdot y^0}{(y+1)^2}$

$= \frac{2xy + 2x - x^2 \cdot 1}{(y+1)^2}$

$\partial_x f = \frac{2 \cdot 3 \cdot 1 + 2 \cdot 3 - 3^2}{(1+3)^2} = \frac{3}{16}$

$f(x,y) = \frac{x^2}{y+1}$

$T\left(3, 1, \frac{9}{2}\right)$

$\frac{1}{x}$

$= -\frac{1}{x^2}$

$\partial_x f = \frac{2x}{y+1}$

$\partial_y f = -\frac{x^2}{(y+1)^2} = -\frac{9}{4}$

$\partial_x f = \frac{6}{1} = 6$

$z - z_0 = f_x T(x - x_0) + f_y T(y - y_0)$

$z - \frac{9}{2} = 6(x-3) - \frac{9}{4}(y-1)$ ✓

$z - \frac{9}{2} = 6x - 18 - \frac{9}{4}y + \frac{9}{4}$

$z = 6x - \frac{9}{4}y - \frac{45}{4}$

$$f(x,y) = \frac{1}{1+x^2+y^2}$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxo	POPUNJAVA NASTAVNIK
IME I PREZIME: KRISTIAN MARTINOVIĆ BROJ INDEKSA: 17-2-0110-2011	Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \sin(2x)$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2}$. ~~15~~
3. Izračunati tangencijalnu ravninu na graf funkcije $f(x,y) = \frac{x^2}{y+1}$ u točki $T(3,1,z_0)$. ~~15~~
4. $\int\limits_0^1 xe^x \, dx = ?$ 20
5. $\int\limits_0^2 \frac{3x}{x^2-2x+1} = ?$ ~~15~~
6. $\int\limits_2^{+\infty} \frac{dx}{1-x^2} = ?$ ~~20~~

Ukupno: 20

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

3. $f(x,y) = \frac{x^2}{y+1}$, $T(3,1,z_0)$

$$z_0 = \frac{3^2}{1+1} = \frac{9}{2}$$

$$\frac{\partial f}{\partial x} = \frac{1}{y+1} \cdot 2x = \frac{2x}{y+1} \qquad f_x(T) = \frac{2 \cdot 3}{1+1} = \frac{6}{2} = 3$$

$$\frac{\partial f}{\partial y} = \frac{x^2}{1+1} = \frac{x^2}{2} \times \qquad f_y(T) = \frac{3^2}{2} = \frac{9}{2}$$

$$z - z_0 = f_x(T)(x - x_1) + f_y(T)(y - y_0) \qquad z = 3x + \frac{9}{2}y - 9$$

$$z - \frac{9}{2} = 3(x-3) + \frac{9}{2}(y-1)$$

$$z - \frac{9}{2} = 3x - 9 + \frac{9}{2}y - \frac{9}{2}$$

$$z = 3x + \frac{9}{2}y - \cancel{\frac{9}{2}} + \cancel{\frac{9}{2}} - 9$$

(2.) $f(x,y) = \frac{1}{1+x^2+y^2}$

$\frac{\partial f}{\partial x} = \frac{1}{1+2x+0}$ $\qquad 1 = \frac{1}{1+2x} = 0 \;/\cdot(1+2x)$

$1 \neq 0$

$\frac{\partial f}{\partial y} = \frac{1}{1+0+2y}$

(4.) $\int_0^1 x e^x dx = \left[\begin{matrix} x = u & e^x dx = dv \\ dx = du & e^x = v \end{matrix}\right]$

$= \int_0^1 \left(u \cdot v - \int v \, du\right)$

$= \int_0^1 \left(x \cdot e^x - \int e^x dx\right)$

$= \left[x \cdot e^x - e^x\right]_0^1$

$= (1 \cdot e^1 - e^1) - (0 \cdot e^0 - e^0)$

$= 0 - (-1)$

$= +1$ ✓

KRISTIAN MARTINOVIĆ

5. $\int_0^2 \frac{3x}{x^2-2x+1} =$

$x^2-2x+1=0$

$x_{1,2} = \frac{2 \pm \sqrt{4-4}}{2} = \frac{2 \pm 0}{2} = 1$

$x_1 = 1 \quad x_2 = 1$

$a(x-x_1)(x-x_2)$

$(x-1)(x-1)$

$= \int_0^2 \frac{3x}{(x-1)(x-1)} = \cancel{\frac{A}{x-1} + \frac{B}{x-1}}$

$\frac{A}{x-1} + \frac{B}{(x-1)^2}$

$\frac{3x}{(x-1)(x-1)} = \frac{A}{x-1} + \frac{B}{x-1} \quad / \cdot (x-1)(x-1)$

$3x = A(x-1) + B(x-1)$

$3x = Ax - A + Bx - B$

$A + B = 3$

$A = 3 - B$

$B = 3 - A$

$-A - B = 0$

$-(3-B) - B = 0$

$-3 + B - B = 0$

$-3 \neq 0$

$-A = B$

$\boxed{A = -B}$

$A = -3 + A$

$3 \neq A - A$

$3 \neq 0$

NEMA RJEŠENJA!

⑥ $\int_2^{+\infty} \frac{dX}{1-x^2} = \int_2^{+\infty} \frac{A}{1-x} dx + \int_2^{+\infty} \frac{B}{1+x} dx$

$= \frac{1}{1-x^2} = \frac{A}{1-x} + \frac{B}{1+x} \quad / \cdot (1-x^2)$

$1 = A(1+x) + B(1-x)$

$1 = A + Ax + B - Bx$

$A + \cancel{B} = 1$

$+ \; A - \cancel{B} = 0$

$2A = 1$

$\boxed{A = \frac{1}{2}}$

$\frac{1}{2} + B = 1$

$B = 1 - \frac{1}{2}$

$\boxed{B = \frac{1}{2}}$

$= \frac{1}{2} \cdot \int_2^{+\infty} \frac{1}{1-x} dx + \frac{1}{2} \cdot \int_2^{+\infty} \frac{1}{1+x} dx$

$\begin{bmatrix} 1-x = t \\ -dx = dt \end{bmatrix}$ $\begin{bmatrix} 1+x = u \\ dx = du \end{bmatrix}$

!

$\emptyset$

$= \frac{1}{2} \int_2^{+\infty} \ln|t| + \frac{1}{2} \int_2^{+\infty} \ln|u|$

$= \frac{1}{2} \left(\ln|1-x| \right)_2^{+\infty} + \frac{1}{2} \left(\ln|1+x| \right)_2^{+\infty}$

$= \quad -\infty \quad -\infty$

$= -\infty$

KRISTIAN MARTINOVIĆ

6.
$$\int_2^{+\infty} \frac{dx}{1-x^2} = \left[\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|\right]_2^{+\infty}$$

$\times$

$$= \left(\frac{1}{2}\ln|-\infty| - \frac{1}{2}\ln|-3|\right)$$

$$= -\infty - 0.54$$

$$= -\infty$$

2.
$$f(x,y) = \frac{1}{1+x^2+y^2}$$

$\times$

$$\frac{\partial f}{\partial x} = \frac{1' \cdot (1+x^2+y^2) - 1(1+2x+0)}{(1+x^2+y^2)^2} = \frac{0-1-2x}{(x^2+y^2+1)^2} = \frac{-2x-1}{(x^2+y^2+1)^2}$$

$$\frac{\partial f}{\partial y} = \frac{1' \cdot (1+x^2+y^2) - 1(1+2y)}{(x^2+y^2+1)^2} = \frac{0-1-2y}{(x^2+y^2+1)^2} = \frac{-2y-1}{(x^2+y^2+1)^2}$$

$$\frac{-2x-1}{(x^2+y^2+1)^2} = 0 \quad / \cdot (x^2+y^2+1)^2$$

$$-2x-1=0$$

$$-2x=1$$

$$\boxed{x=-\frac{1}{2}}$$

$$\frac{-2y-1}{(x^2+y^2+1)^2} = 0 \quad / \cdot (x^2+y^2+1)^2$$

$$-2y-1=0$$

$$-2y=1$$

$$\boxed{y=-\frac{1}{2}}$$

$$T\left(-\frac{1}{2}, -\frac{1}{2}\right)$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{(-2x-1)' \cdot (x^2+y^2+1)^2 - (-2x-1) \cdot (x^2+y^2+1)^{2'}}{(x^2+y^2+1)^4}$$

$$= \frac{-2\,(x^2+y^2+1)^2 - (-2x-1) \cdot 2(x^2+y^2+1) \cdot (2x+2y)}{(x^2+y^2+1)^4}$$

$$= (-2x^2-2y^2-2)^2 - (-2x-1)(\qquad ???$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO

IME I PREZIME: SEBASTIJAN KOŠTA **BROJ INDEKSA:** 17-2-0094-2011

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \sin(2x)$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2}$. 15
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4. $\displaystyle\int_0^1 xe^x \, dx = ?$ ~~20~~
5. $\displaystyle\int_0^2 \frac{3x}{x^2-2x+1} = ?$ ~~15~~
6. $\displaystyle\int_2^{+\infty} \frac{dx}{1-x^2} = ?$ ~~20~~

Ukupno: ~~0~~

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

$uv - \int v\,du$

4. $$\int_0^1 xe^x dx = \begin{bmatrix} u = x & dv = e^x dx \\ du = dx & v = e^x \end{bmatrix} = xe^x - \int_0^1 e^x dx$$

$$= xe^x\Big|_0^1 - e^x\Big|_0^1 = 1 \cdot e^1 - 0 \cdot e^0 - e^1 \underset{+}{-} e^0 = 1 \cdot 2.7183 - 0 - 2.7183 - 1 = \underline{-1}$$

PROVJERA NUMERIČKOM INTEGRACIJOM BI VAS SPASILA...

5) $\int_0^2 \frac{3x}{x^2-2x+1}\,dx = \int_0^2 \frac{3x}{(x-1)^2} = \begin{bmatrix} t = x-1 \\ dt = dx \\ \end{bmatrix} = \int_{-1}^{1} \frac{3x}{t^2}\,dt$

$= \begin{bmatrix} t = x^2-2x+1 \\ dt = 2x-2\,dx \\ dx = \frac{dt}{2x-2} \end{bmatrix} = \int_1^1 \frac{3\cancel{x}}{t}\,\frac{dt}{2\cancel{x}-2} = \int_1^1 \frac{3}{t} = 3\int_1^1 t = 3\cdot\frac{t^2}{2}\Big|_1^1$

$= 3\cdot \frac{1^2}{2} - \frac{1^2}{2} = 3\cdot 0 = 0$ NEPRAVI INTEGRAL !!!

6) $\int_2^{+\infty} \frac{dx}{1-x^2} = \begin{bmatrix} t = 1-x^2 \\ dt = 2x\,dx \\ dx = \frac{dt}{2x} \end{bmatrix} = \int \frac{\frac{dt}{2x}}{t}$ . . . . ?

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxo

IME I PREZIME: Stipe Rebić **BROJ INDEKSA:** 17-2-0226-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \sin(2x)$. ~~15~~
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2}$. 15
3. Izračunati tangencijalnu ravninu na graf funkcije $f(x,y) = \frac{x^2}{y+1}$ u točki $T(3,1,z_0)$. 15
4. $\int_0^1 x e^x \, dx = ?$ ~~20~~
5. $\int_0^2 \frac{3x}{x^2-2x+1} = ?$ ~~15~~
6. $\int_2^{+\infty} \frac{dx}{1-x^2} = ?$ 20

Ukupno: 20

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

6. $\int_2^{+\infty} \frac{dx}{1-x^2} =$

$1-x^2 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

$D_f = \mathbb{R} \setminus \{-1, 1\}$

$\int_2^{+\infty} \frac{dx}{1-x^2} = \lim_{a\to+\infty} \int_2^a \frac{dx}{1-x^2} = -\lim_{a\to+\infty} \int_2^a \frac{dx}{x^2-1} = -\lim_{a\to+\infty} \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| \Big|_2^a =$

$= -\lim_{a\to+\infty} \frac{1}{2}\left(\ln\left|\frac{a-1}{a+1}\right| - \ln\left|\frac{2-1}{2+1}\right|\right) = -\frac{1}{2}\left(\ln\left|\frac{\infty-1}{\infty+1}\right| - \ln\left|\frac{1}{3}\right|\right) =$

$= -\frac{1}{2}\left(\ln 1 - \ln\frac{1}{3}\right) = -\frac{1}{2}\ln\frac{1}{\frac{1}{3}} = -\frac{1}{2}\ln 3$ ✓

$$5.\quad \int_0^2 \frac{3x}{x^2-2x+1}\,dx = \int_0^2 \frac{3x}{(x^2-1)^2}\,dx \qquad D_f = \mathbb{R} \setminus \{\cancel{-1}, 1\}$$

$$(x^2-1)^2 = 0 \Rightarrow x^2-1=0 \Rightarrow x^2=1$$

$$x = \pm 1$$

$$\lim_{\varepsilon \to 0} \int_0^{1-\varepsilon} \frac{3x}{(x^2-1)^2}\,dx + \lim_{\delta \to 0} \int_{1+\delta}^{2} \frac{3x}{(x^2-1)^2}\,dx = \left\{ \begin{array}{l} x^2-1 = t \\ 2x\,dx = dt \\ x\,dx = \frac{dt}{2} \end{array} \right\} =$$

$$= \frac{3}{2} \lim_{\varepsilon \to 0} \int_0^{1-\varepsilon} \frac{dt}{t^2} + \frac{3}{2} \lim_{\delta \to 0} \int_{1+\delta}^{2} \frac{dt}{t^2} =$$

$$= \frac{3}{2} \lim_{\varepsilon \to 0} \frac{-1}{t} \Big|_0^{1-\varepsilon} + \frac{3}{2} \lim_{\delta \to 0} \frac{-1}{t} \Big|_{1+\delta}^{2}$$

$$= \frac{3}{2} \lim_{\varepsilon \to 0} \frac{-1}{x^2-1} \Big|_0^{1-\varepsilon} + \frac{3}{2} \lim_{\delta \to 0} \frac{-1}{x^2-1} \Big|_{1+\delta}^{2}$$

$$= \frac{3}{2} \lim_{\varepsilon \to 0} \left(\frac{-1}{(1-\varepsilon)^2-1} + \frac{1}{0^2-1} \right) + \frac{3}{2} \lim_{\delta \to 0} \left(\frac{-1}{2^2-1} + \frac{1}{(1+\delta)^2-1} \right)$$

$$= \frac{3}{2} \cdot (1-1) + \frac{3}{2} \left(-\frac{1}{3} - 1 \right) = 0 + \frac{3}{2} \cdot \left(-\frac{4}{3} \right) = -\frac{1}{2}$$

Stipe Rebić

1. $y'' + 2y' + 5y = \sin(2x)$

$y'' + 2y' + 5y = 0$

$k = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm \sqrt{16}}{2}$

$k_{1,2} = -1 \pm 2i \qquad \alpha = -1 \qquad \beta = 2$

$y_H = e^{-x} (\cos(2x) \cdot C_1 + \sin(2x) \cdot C_2$

DALJE . .

4. $\int_0^1 xe^x dx = \frac{x}{2} ex \Big|_0^1 = \frac{1}{2} e^1 - 0 = \frac{e}{2}$ X

$\int xe^x = \left\{ \begin{array}{ll} u = x & dv = e^x dx \\ du = dx & v = e^x \end{array} \right\} = xe^x - \underbrace{\int xe^x dx}$ X

$I = \int xe^x = xe^x - \int xe^x dx$

$I = xe^x - I$

$2I = xe^x$

$I = \frac{x}{2} e^x$

2. $f(x,y) = \frac{1}{1+x^2+y^2} =$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO

IME I PREZIME: ANTE PAVLOVIĆ   BROJ INDEKSA: 64959/2007

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \sin(2x)$. 15
2. Odrediti lokalne ekstreme funkcije $f(x, y) = \frac{1}{1+x^2+y^2}$. 15
3. Izračunati tangencijalnu ravninu na graf funkcije $f(x, y) = \frac{x^2}{y+1}$ u točki $T(3, 1, z_0)$. 15
4. $\int_0^1 xe^x \, dx =?$ ~~20~~
5. $\int_0^2 \frac{3x}{x^2 - 2x + 1} =?$ 15
6. $\int_2^{+\infty} \frac{dx}{1 - x^2} =?$ ~~20~~

Ukupno: Ø

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

6. $\int_2^{+\infty} \frac{dx}{1-x^2} = \frac{1}{2\cdot 1}\ln\frac{1+x^2}{1-x^2} =$

$\lim_{x\to+\infty}\frac{1+x^2}{1-x^2} = ?$

$= \frac{1}{2\cdot 1}\ln\frac{1+\infty^2}{1-\infty^2} - \frac{1}{2\cdot 1}\ln\frac{1+2^2}{1-2^2} = \frac{1}{2}\ln\frac{1+\infty^2}{1-\infty^2} - \frac{1}{2}\ln\frac{5}{3}$ Ø

4. $\int_0^1 xe^x \, dx = e^x + C = e^1 - e^0$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	XXO
IME I PREZIME: Goran Kovaček	**BROJ INDEKSA:** 17-1-0219-2013.

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \sin(2x)$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2}$. 15
3. Izračunati tangencijalnu ravninu na graf funkcije $f(x,y) = \frac{x^2}{y+1}$ u točki $T(3,1,z_0)$. 15
4. $\int\limits_0^1 xe^x \, dx = ?$ 20
5. $\int\limits_0^2 \frac{3x}{x^2-2x+1} = ?$ 15
6. $\int\limits_2^{+\infty} \frac{dx}{1-x^2} = ?$ 20

Ukupno: ∅

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
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$\tan x$	$\frac{1}{\cos^2 x}$
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$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
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Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
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$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxo	POPUNJAVA NASTAVNIK
IME I PREZIME: Ante Podišić **BROJ INDEKSA:** 17-1-0114-2012	Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \sin(2x)$. 15
2. Odrediti lokalne ekstreme funkcije $f(x, y) = \frac{1}{1+x^2+y^2}$. ~~15~~
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Ukupno: Ø

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha \, dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln \lvert x \rvert + C$	$\int \cot x \, dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
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2. $f(x,y) = \frac{1}{1+x^2+y^2} = (1+x^2+y^2)^{-1}$

$\frac{\partial f}{\partial x} = \frac{1}{x^2} \cancel{x}$ $\qquad$ $\frac{\partial f}{\partial y} = \frac{1}{y^2} x$

$\frac{1}{x^2} = 0 \,/\cdot x^2$ $\qquad$ $\frac{1}{y^2} = 0 \,/\cdot y^2$

$x^2 = 0$ $\qquad$ $y^2 = 0$

$\boxed{x = 0}$ $\qquad$ $\boxed{y = 0}$ $\qquad T(0,0)$

$\frac{\partial^2 f}{\partial x^2} = \frac{1}{2x}$ $\qquad$ $\frac{\partial^2 f}{\partial y^2} = \frac{1}{2}x$ $\qquad$ $\frac{\partial^2 f}{\partial x^2 \partial y^2} = \frac{1}{2}x + \frac{1}{2}y$

$$\Delta = \begin{vmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} \end{vmatrix} = \frac{1}{4} - \frac{1}{16} = 0{,}18 > 0$$

$$f(x,y) = \frac{1}{1+0+0} = 1 \quad T(0,0,1) - \text{Maksimum}$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: ANTONIO PRIBIL **BROJ INDEKSA:** 57666	

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \sin(2x)$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2}$. 15
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4. $\displaystyle\int_0^1 xe^x \,\mathrm{d}x = ?$ 20
5. $\displaystyle\int_0^2 \frac{3x}{x^2-2x+1} = ?$ 15
6. $\displaystyle\int_2^{+\infty} \frac{\mathrm{d}x}{1-x^2} = ?$ 20

Ukupno: Ø

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
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$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\|\cos x\| + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x \,\mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

$$\int_0^1 x e^x \,dx = e^x + C$$

$$\int_0^2 \frac{3x}{x^2-2x+1} =$$

$$\int_2^{+\infty} \frac{dx}{1-x^2} =$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO

IME I PREZIME: ROKO DUŠEVIĆ **BROJ INDEKSA:** 57351-2009

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \sin(2x)$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2}$. ~~15~~
3. Izračunati tangencijalnu ravninu na graf funkcije $f(x,y) = \frac{x^2}{y+1}$ u točki $T(3,1,z_0)$. 15
4. $\int\limits_0^1 x\mathrm{e}^x \,\mathrm{d}x = ?$ ~~20~~
5. $\int\limits_0^2 \frac{3x}{x^2-2x+1} = ?$ ~~15~~
6. $\int\limits_2^{+\infty} \frac{\mathrm{d}x}{1-x^2} = ?$ 20

Ukupno: $\varnothing$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_a x \ (a > 0)$	$\frac{1}{x \ln a}$
e^x	e^x
$a^x \ (a > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
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Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x \mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4.

$$\int_0^1 x e^x dx = \begin{bmatrix} \cancel{u = x} & \cancel{dv = e^x dx} \\ \cancel{du = dx} & \cancel{v = e^x} \end{bmatrix} = \begin{bmatrix} u = e^x & dv = x\,dx \\ du = e^x dx & v = 1 \end{bmatrix} = e^x \Big|_0^1 - \int_0^1 e^x dx =$$

PROVJERA NUMERIČKOM INTEGRACIJOM?

$$= 1 - e^1 - 1 + e^1 = 0 \quad X$$

5.

$$\int_0^2 \frac{3x}{x^2-2x+1} = \begin{bmatrix} t = x^2 - 2x + 1 \\ dt = 2x - 2\,dx \\ dx = \frac{dt}{2x-2} \end{bmatrix} = ?$$

2.) $f(x,y) = \frac{1}{1+x^2+y^2} =$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO

IME I PREZIME: MATEO BAĆIĆ **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK

	Broj ↓ bodova
1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \sin(2x)$.	15
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4. $\int_0^1 xe^x \, dx = ?$	~~20~~
5. $\int_0^2 \frac{3x}{x^2 - 2x + 1} = ?$	15
6. $\int_2^{+\infty} \frac{dx}{1 - x^2} = ?$	20
Ukupno:	∅

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
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Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
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3) $f(x,y) = \frac{x^2}{y+1}$ $\quad T(3, 1, z_0)$

$z_0 = \frac{x^2}{y+1} = \frac{9}{2}$

$T(\overset{x_0}{3}, \overset{y_0}{1}, \overset{z_0}{\frac{9}{2}})$

4) $\int_0^1 xe^x \, dx = ?$ $\left(x \cdot e^x - e^x \right) \Big|_0^1$

$= (1 \cdot e^1 - e^1) + (0 \cdot e^0 - e^0)$

$= 0 - 1$

$\boxed{= -1}$

$\int xe^x dx = \left[\begin{matrix} e^x = t \\ e^x dx = dt \end{matrix} \right]$

$\int$

5) $\int_0^2 \frac{3x}{x^2 - 2x + 1} \, dx =$