MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xoo

IME I PREZIME: JURE DUNDOVIĆ **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Pronaći opće rješenje ODJ $y'' - y' = x^2$ i provjeriti dobiveno rješenje. 10+5
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \sin(x) \cdot \cos(y)$ na kvadratu $x \in [0, 2\pi]$, $y \in [0, 2\pi]$. 15
3. Pronaći ravninu koja dira graf funkcije $f(x,y) = xy - \ln(xy)$ povučenu u točki $(4, 1, z_0)$ tog grafa. 15
4. $\int\limits_0^2 x \sin x^2 \, dx = ?$ 20
5. $\int_0^1 \frac{2x}{x^2 - x - 2} = ?$ 15
6. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$. 20

Ukupno: ~~50~~

80

Kose

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_a x \ (a > 0)$	$\frac{1}{x \ln a}$
e^x	e^x
$a^x \ (a > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln \lvert x \rvert + C$	$\int \cot x \, dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4) $\int\limits_0^2 x \sin x^2 \, dx = ?$

$(\cos x^2)' = -\sin x^2 \cdot 2x$

$\int\limits_0^2 x \sin x^2 \, dx = -\frac{1}{2} \cos x^2 \Big|_0^2 = -\frac{1}{2} \cos 4 + \frac{1}{2} \cos 0$

$= -\frac{1}{2} \cdot (-0.6536362) + \frac{1}{2}$

$= 0.3268181 + 0,5$

$= 0.8268181$ ✓

3) $f(x,y) = xy - \ln(xy)$ $\qquad T(4, 1, z_0)$

$z = f(x,y)$

$z_0 = 4 \cdot 1 - \ln(4 \cdot 1) = 4 - \ln 4$

$\partial_x f = y - \frac{y}{xy} = y - \frac{y}{xy} = y - \frac{1}{x}$

$\partial_y f = x - \frac{x}{xy} = x - \frac{1}{y}$

$z - 4 + \ln 4 = \partial_x f(4,1)(x-4) + \partial_y f(4,1)(y-1)$

$z - 4 + \ln 4 = \left(1 - \frac{1}{4}\right)(x-4) + \left(4 - \frac{1}{1}\right)(y-1)$

$z - 4 + \ln 4 = \frac{3}{4}(x-4) + 3(y-1)$ ✓

$z - 4 + \ln 4 = \frac{3}{4}x + 3y - 6$

$\frac{3}{4}x + 3y - z - 2 - \ln 4 = 0$ — TANGENCIJALNA RAVNINA

5) $\int_0^1 \frac{2x}{x^2 - x - 2}\,dx = ?$

$x^2 - x - 2 = (x+1)(x-2)$

$\frac{2x}{x^2-x-2} = \frac{A}{x+1} + \frac{B}{x-2}$

$2x = A(x-2) + B(x+1)$

$2 = A + B \qquad 0 = -2A + B$

$\Rightarrow B = 2A$

$2 = A + 2A$

$A = \frac{2}{3} \qquad B = \frac{4}{3}$

$\rightarrow \frac{2x}{x^2-x-2} = \frac{2}{3}\frac{1}{x+1} + \frac{4}{3}\frac{1}{x-2}$

$\int_0^1 \frac{2x\,dx}{x^2-x-2} = \frac{2}{3}\int_0^1 \frac{dx}{x+1} + \frac{4}{3}\int_0^1 \frac{dx}{x-2}$

$= \frac{2}{3}\ln|x+1|\Big|_0^1 + \frac{4}{3}\ln|x-2|\Big|_0^1$

$= \frac{2}{3}(\ln 2 - \ln 1) + \frac{4}{3}(\ln 1^0 - \ln 2) =$

$= \frac{2}{3}\ln 2 - \frac{4}{3}\ln 2 =$

$= -\frac{2}{3}\ln 2 =$

$= -0.46214$ ✓

2) $f(x,y) = \sin(x) \cdot \cos(y)$; $x \in [0, 2\pi]$, $y \in [0, 2\pi]$

$\partial_x f = \cos x \cdot \cos y$

$\partial_y f = -\sin x \cdot \sin y$

$\partial_{xx} f = -\sin x \cos y$ ... A

$\partial_{xy} f = \partial_{yx} f = -\cos x \sin y$ ... B

$\partial_{yy} f = -\sin x \cos y$ ... C

$\cos x \cos y = 0$

1° $\cos x = 0 \Rightarrow x = \frac{\pi}{2} + k\pi$, $k \in \mathbb{Z}$ $\Rightarrow x = \frac{\pi}{2}$ ili $x = \frac{3\pi}{2}$

(na $[0, 2\pi]$)

$-\sin x \sin y = 0$

a) $x = \frac{\pi}{2}$ $\quad -\sin y = 0 \Rightarrow y = 0$ ili $y = \pi$

b) $x = \frac{3\pi}{2}$ $\quad \sin y = 0 \Rightarrow y = 0$ ili $y = \pi$

2° $\cos y = 0 \Rightarrow y = \frac{\pi}{2}$ ili $y = \frac{3\pi}{2}$

$-\sin x \sin y = 0$

a) $y = \frac{\pi}{2}$ $\quad -\sin x = 0 \Rightarrow x = 0$ ili $x = \pi$

b) $y = \frac{3\pi}{2}$ $\quad \sin x = 0 \Rightarrow x = 0$ ili $x = \pi$

TOČKE $T_1\left(\frac{\pi}{2}, 0\right)$, $T_2\left(\frac{\pi}{2}, \pi\right)$, $T_3\left(\frac{3\pi}{2}, 0\right)$, $T_4\left(\frac{3\pi}{2}, \pi\right)$

$T_5\left(0, \frac{\pi}{2}\right)$ $T_6\left(\pi, \frac{\pi}{2}\right)$ $T_7\left(0, \frac{3\pi}{2}\right)$, $T_8\left(\pi, \frac{3\pi}{2}\right)$

$\Rightarrow$ NASTAVAK

1) $y'' - y' = x^2$ JURE DUNDOVIĆ

1° HOMOGENO RJEŠENJE

$r^2 - r = 0$ $\quad r(r-1)=0$

$r_1 = 0 \quad r_2 = 1$

$y_H = C_1 + C_2 e^x \qquad C_1, C_2 \in \mathbb{R}$

2° PARTIKULARNO RJ

$y_p = x(Ax^2 + Bx + C) = Ax^3 + Bx^2 + Cx$

$y_p' = 3Ax^2 + 2Bx + C$

$y_p'' = 6Ax + 2B$

$6Ax + 2B - 3Ax^2 - 2Bx - C = x^2$

$-3A = 1 \quad \Rightarrow A = -\frac{1}{3}$

$6A - 2B = 0 \quad -2 - 2B = 0 \Rightarrow B = -1$

$2B - C = 0 \quad -2 - C = 0 \Rightarrow C = -2$

$y_p = -\frac{1}{3}x^3 - x^2 - 2x$

$y = y_H + y_p = C_1 + C_2 e^x - \frac{1}{3}x^3 - x^2 - 2x$

PROVJERA:

$y' = C_2 e^x - x^2 - 2x - 2$

$y'' = C_2 e^x - 2x - 2$

$\cancel{C_2 e^x} - \cancel{2x} - \cancel{2} - \cancel{C_2 e^x} + x^2 + \cancel{2x} + \cancel{2} = x^2$

$x^2 = x^2$ // ✓

JURE DANDOVIĆ

$T_1 \ldots \quad \Delta_1 = AC - B^2 = 1 - 0 > 0$ ekstr.

$A = -1 \quad B = 0 \quad C = -1 \qquad f(T_1) = 1$

$A < 0$ lokalni maksimum

$T_2 \ldots \quad \Delta_2 = AC - B^2 = 1 - 0 > 0$ ekstr

$A = 1 \quad B = 0 \quad C = 1 \qquad f(T_2) = -1$

$A > 0$ lok. minimum

$T_3 \ldots \quad \Delta_3 = AC - B^2 = 1 - 0 > 0$ ekstr.

$A = 1 \quad B = 0 \quad C = 1 \qquad f(T_3) = -1$ ✓

$A > 0$ lok. minimum

$T_4 \ldots \quad \Delta_4 = AC - B^2 = 1 - 0 > 0$ ekstr.

$A = -1 \quad B = 0 \quad C = -1 \qquad f(T_4) = 1$

$A < 0$ lok. maksimum

$T_5 \ldots \quad \Delta_5 = AC - B^2 = 0 - 1 < 0$ sedlasta točka $f(T_5) = 0$

$A = 0 \quad B = -1 \quad C = 0$

$T_6 \ldots \quad \Delta_6 = AC - B^2 = 0 - 1 < 0$ sedlasta točka $f(T_6) = 0$

$A = 0 \quad B = 1 \quad C = 0$

$T_7 \ldots \quad \Delta_7 = AC - B^2 = 0 - 1 < 0$ sedlasta točka $f(T_7) = 0$

$A = 0 \quad B = 1 \quad C = 0$

$T_8 \ldots \quad \Delta_7 = AC - B_2 = 0 - 1 < 0$ sedlasta točka $f(T_8) = 0$

$A = 0 \quad B = -1 \quad C = 0$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xoo

IME I PREZIME: ANTONIO SEKULA **BROJ INDEKSA:** 17-2-0025-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Pronaći opće rješenje ODJ $y'' - y' = x^2$ i provjeriti dobiveno rješenje. 10+5
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Ukupno: 55

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_a x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
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Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4.) $\int_0^2 x \sin x^2 \, dx = \left| \begin{array}{l} t = x^2 \\ dt = 2x\,dx \\ dx = \frac{dt}{2x} \end{array} \right| = \int x \sin t \frac{dt}{2x} = \frac{1}{2}\int \sin t \, dt =$

$-\frac{1}{2}\cos t = -\frac{1}{2}\cos x^2 \Big|_0^2 = -\frac{1}{2}\cos 2^2 - \left(-\frac{1}{2}\cos 0^2\right) =$

$= 0.3268 + \frac{1}{2} = 0.8268$ ✓

5.) $\int_0^1 \frac{2x}{x^2-x-2} = \frac{2x}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1} \Big/ (x-2)(x+1)$

$x^2 - x - 2 = 0$

$x_{1,2} = \frac{1 \pm \sqrt{1+8}}{2}$

$x_{1,2} = \frac{1 \pm 3}{2}$

$x_1 = \frac{4}{2} = 2$

$x_2 = -\frac{2}{2} = -1$

$2x = A(x+1) + B(x-2)$

$2x = Ax + A + Bx - 2B$

$2x = x(A+B) + A - 2B$

$A + B = 2$

$B = 2 - A$

$B = 2 - \frac{4}{3}$

$B = \frac{2}{3}$

$A - 2B = 0$

$A - 2\cdot(2-A) = 0$

$A - 4 + 2A = 0$

$3A = 4 / :3$

$A = \frac{4}{3}$

$\Rightarrow$

$$\int \frac{\frac{5}{3}}{x-2}dx + \frac{\frac{2}{3}}{x+1}dx = \int \frac{\frac{5}{3}}{x-2}dx + \int \frac{\frac{2}{3}}{x+1}dx = \frac{5}{3}\int \frac{dx}{x-2} + \frac{2}{3}\int \frac{dx}{x+1}$$

$$\frac{5}{3}\ln|x-2| + \frac{2}{3}\ln|x+1| \Big|_0^1 =$$

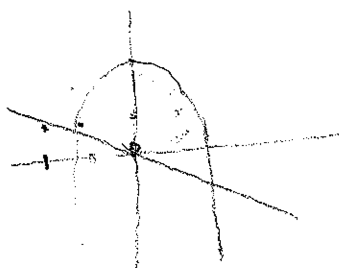
$$\boxed{\begin{aligned} &\int \frac{dx}{x-2} = \left|\begin{matrix} t = x-2 \\ dt = dx \end{matrix}\right| = \int \frac{dt}{t} \\ &\ln|t| = \ln|x-2| \\ &\int \frac{dx}{x+1} = \left|\begin{matrix} t = x+1 \\ dt = dx \end{matrix}\right| = \int \frac{dt}{t} \\ &= \ln|t| = \ln|x+1| \end{aligned}}$$

$$\frac{5}{3}\ln|-1| + \frac{2}{3}\ln|2| - \left(\frac{5}{3}\ln|-2| + \frac{2}{3}\ln|1|\right)$$

$$\frac{5}{3}\ln|-1| + \frac{2}{3}\ln|2| - \frac{5}{3}\ln|-2| - \frac{2}{3}\ln|1| \checkmark =$$

NEPRAVI INT. X

6. $x + y^2 = 6$
$x + y + 1 = 0$



$x = -y^2 + 6$
$x = -y - 1$
$-y - 1 = -y^2 + 6$
$y^2 - y - 7 = 0$

$$y_{1,2} = \frac{1 \pm \sqrt{1 - 4\cdot 1\cdot(-7)}}{2}$$

$$y_{1,2} = \frac{1 \pm \sqrt{29}}{2}$$

$$y_1 = \frac{1+\sqrt{29}}{2} \qquad y_2 = \frac{1-\sqrt{29}}{2}$$

$y_1 = 3.19 \qquad y_2 = -2.19$

$$P = \int_{-2.19}^{3.19} -y^2 + 6 - (-y - 1) =$$

$$= \int -y^2 + 6 + y + 1 = -\int y^2\,dy + 6\int dy + \int y\,dy + \int dy$$

$$= -\frac{y^3}{3} + 6y + \frac{y^2}{2} + y \Big|_{-2.19}^{3.19} =$$

$= 13.1 - (-14.2) = 13.1 + 14.2 =$
$= 27.3 = P \checkmark$

2. $f(x,y) = \sin(x)\cdot\cos(y) \quad x \in [0, 2\pi] \quad y \in [0, 2\pi]$ $\quad \begin{vmatrix} 0^x & 2\pi^x \\ 0^y & 2\pi^y \end{vmatrix}$

$\partial_x f = \cos x \cdot \cos y \quad = \quad X$

$\emptyset$

$\partial_y f = \sin x \cdot (-\sin y) = -$

$$\begin{vmatrix} \partial_{xx} & \partial_{xy} \\ \partial_{yx} & \partial_{yy} \end{vmatrix} = \begin{vmatrix} -\sin x \cdot \cos y & -\cos x \cdot \sin y \\ \cos x \cdot -\sin y & -\sin x \cdot \cos y \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} = \Delta = 0$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xoo	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: MIHOVIL JORDAN **BROJ INDEKSA:** 17-2-0251-2012	

+ 1. Pronaći opće rješenje ODJ $y'' - y' = x^2$ i provjeriti dobiveno rješenje. 10+5
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+ 4. $\int\limits_0^2 x\sin x^2\,\mathrm{d}x = ?$ 20

− 5. $\int_0^1 \frac{2x}{x^2-x-2} = ?$ 15

6. Izračunati površinu područja omeđenog krivuljama $x+y^2=6$ i $x+y+1=0$. 20

Ukupno: 50

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha>0)$	$\alpha^x\ln\alpha$
$\sin x$	$\cos x$
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$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
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Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\| + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x\,\mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\|x+\sqrt{x^2\pm a^2}\right\| + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

3

$f(x,y) = xy - \ln(xy)$
$z_0 = 4 - \ln 4$

$$\frac{\partial f}{\partial x} = y - \frac{1}{x} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\frac{\partial f}{\partial y} = x - \frac{1}{y} = 4 - 1 = 3$$

$T \ldots z - 4 + \ln 4 = \frac{3}{4}(x-4) + 3(y-1)$ ✓

(4) $\int_0^2 x \sin x^2 \, dx = ?$

$\int_0^2 x \sin x^2 \, dx = \left| \begin{array}{l} x^2 = t \\ 2x\,dx = dt \\ x\,dx = \frac{1}{2}\,dt \end{array} \right.$ $\quad x=2 \quad t=4$ $\quad x=0 \quad t=0$

$\int_0^4 \frac{1}{2} \sin \, dt \left(-\frac{1}{2}\cos t\right)\Big|_0^4 =$

$= -\frac{1}{2}\cos 4 + \frac{1}{2}\cos 0 = \frac{1}{2} - \frac{1}{2}\cos 4$ ✓

(1) $r^2 - r = 0$

$r(r-1) = 0$

$r_1 = 0 \quad r_2 = 1$

$y_H = C_1 + C_2 e^x$

$g(x) = x^2 \qquad y_P = Ax^3 + Bx + C\,x$

$y'_P = 3Ax^2 + 2Bx + C$

$y''_P = 6Ax + 2B$

$6Ax + 2B - 3Ax^2 - 2Bx - C = x^2$

$-3A = 1 \qquad A = -\frac{1}{3}$

$6A - 2B = 0 \qquad B = 3A - 1$

$2B - C = 0 \qquad C = 2B = -2$

$y = C_1 + C_2 e^x - \frac{1}{3}x^3 - x^2 - 2x$

PR

$y' = C_2 e^x - x^2 - 2x - 2$

$y'' = C_2 e^x - 2x - 2$

$y'' - y' = C_2 e^x - 2x - 2 - C_2 e^x + x^2 + 2x + 2 = x^2$

✓

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. PIŠITE DVOSTRANO! xoo	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: IVAN VELEMIR — BROJ INDEKSA: 17-2-0067-2010	

1. Pronaći opće rješenje ODJ $y'' - y' = x^2$ i provjeriti dobiveno rješenje. 10+5
2. Odrediti lokalne ekstreme funkcije $f(x, y) = \sin(x) \cdot \cos(y)$ na kvadratu $x \in [0, 2\pi]$, $y \in [0, 2\pi]$. ~~15~~
3. Pronaći ravninu koja dira graf funkcije $f(x, y) = xy - \ln(xy)$ povučenu u točki $(4, 1, z_0)$ tog grafa. 15
4. $\displaystyle\int_0^2 x \sin x^2 \, dx = ?$ ~~20~~
5. $\displaystyle\int_0^1 \frac{2x}{x^2 - x - 2} = ?$ 15
6. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$. 20

Ukupno: 0

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
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Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln \lvert x \rvert + C$	$\int \cot x \, dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
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$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

4.

$$\int_0^2 x \sin x^2 dx =$$

$dv = x\,dx \,/ \int$

$v = \frac{x^2}{2}$

$u = \sin^2 x$ /

$du = 2 \sin x \cdot \cos x$

$du = \sin^2 x$

$\int u\,dv = uv - \int u\,dv$

$$\int_0^2 \sin^2 x \cdot x d = \sin^2 x \cdot \frac{x^2}{2}\Big|_0^2 - \int_0^2 \frac{x^2}{2} = \left[(\sin 2)^2 \cdot \frac{4}{2} - \underbrace{\sin 0^2}_{=0} \cdot \frac{0}{2} - \frac{1}{2}\int_0^2 x^2 \sin\right.$$

(2) $\frac{df}{dx} = (\sin x)' \cdot \cos y + \sin x \cdot \underbrace{(\cos y)'}_{=0}$

$= \cos x \cdot \cos y$

$\frac{d^2f}{dx^2} = (\cos x)' \cos y + \cos x \cdot \underbrace{(\cos y)'}_{=0}$

$= -\sin x \cdot \cos y$

$\frac{df}{dy} = \underbrace{(\sin x)'}_{=0} \cos y + \sin x (\cos y)'$

$= 0 - \sin x \sin y$

$\frac{d^2f}{dy^2} = (-\sin x)' \cdot \sin y + - \sin x (\sin y)'$

$= - \sin x \cdot \cos y$

$\frac{df}{dy} = 0$

$\cos x \cdot \cos y = 0$

$\cancel{0}$

1° $\cos x = 0$

x

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xoo	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: MARIO IVANAC **BROJ INDEKSA:** 17-1-0096-2011	

1. Pronaći opće rješenje ODJ $y'' - y' = x^2$ i provjeriti dobiveno rješenje. 10+5
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \sin(x) \cdot \cos(y)$ na kvadratu $x \in [0, 2\pi]$, $y \in [0, 2\pi]$. 15
3. Pronaći ravninu koja dira graf funkcije $f(x,y) = xy - \ln(xy)$ povučenu u točki $(4, 1, z_0)$ tog grafa. 15
4. $\int_0^2 x \sin x^2 \, dx = ?$ ~~20~~
5. $\int_0^1 \frac{2x}{x^2 - x - 2} = ?$ ~~15~~
6. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$. ~~20~~

Ukupno: 15

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
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Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
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3) $F(x,y) = xy - \ln(xy)$ $T(4, 1, z_0)$ $x = 4$, $y = 1$

$z_0 = 4 \cdot 1 - \ln(4 \cdot 1)$

$z_0 = 4 - \ln 4$

$z_0 = 2.613705639$

$\frac{\partial f}{\partial x} = f_x = y - \frac{1}{x} = 1 - \frac{1}{4} = \boxed{\frac{3}{4}}$ $\frac{\partial f}{\partial y} = f_y = x - \frac{1}{y} = 4 - \frac{1}{1} = \boxed{4}$

$z - z_0 = f_x(x - x_1) + f_y(y - y_1)$

$z - 2.613705639 = \frac{3}{4}(x - 4) + 4(y - 1)$

$z - 2.613705639 = \frac{3}{4}x - 3 + 4y - 4$

$z = \frac{3}{4}x - 3 + 4y - 4 + 2.613705639$

$z = \frac{3}{4}x + 4y - 4.386294361$ $_{//}$

✓

5) $\int_0^1 \frac{2x}{x^2-x-2} = \begin{bmatrix} x^2-x-2=t \\ 2x-1\,dx=dt \end{bmatrix}$ ... $dx = \frac{dt}{2x}$

$(x^2-x-2) = x(x-1)(x-2)$

4) $\boxed{\int_0^2 \sin x^2 \, x\,dx}$

$\int_0^2 \boxed{x \sin x^2 \, dx} = \begin{bmatrix} x^2=t \\ 2x\,dx=dt \\ x\,dx=\frac{dt}{2} \end{bmatrix} = \int_0^2 \sin t \frac{dt}{2} = \frac{1}{2}\int_0^2 \sin t \, dt = -\frac{1}{2}\cos t \Big|_0^2$ GRANICE INTEGRACIJE!!!

$= -\frac{1}{2}\cos x^2 \Big|_0^2 = \left(-\frac{1}{2}\cos 2^2\right) - \left(-\frac{1}{2}\cos 0^2\right) = -0.498782025 + \frac{1}{2}$

$= 1.217975 \cdot 10^{-3}$

6) $x+y^2=6 \Rightarrow y+x^2=6 \Rightarrow y=-x^2+6$

$x+y+1=0 \Rightarrow y=-x-1$

$y=-x^2+6$

$y=-x-1$

$-x^2+6=-x-1$

$-x^2+6+x+1=0$

$-x^2+x+7=0$

$x_{1,2} = \frac{-1 \pm \sqrt{1-4\cdot(-1)\cdot 7}}{2\cdot(-1)}$

$x_{1,2} = \frac{-1 \pm \sqrt{29}}{-2}$

$x_1 = \frac{-1+\sqrt{29}}{2} \approx 4.385$

$x_2 = \frac{-1-\sqrt{29}}{2} \approx -6.385$

$P = \int_{-6.385}^{4.385} \left((-x^2+6) - (-x-1)\right) dx$

$P = \int_{-6.385}^{4.385} (-x^2+6+x+1)\, dx$

$P = -\frac{x^3}{3} + 6x + \frac{x^2}{2} + x \Big|_{-6.385}^{-6.385}$

$P = 12.20385696 + 62.45748471$

$P = 74.66134167$

MARIO IVANAC 17-1-0096-2011

5) $\int_0^1 \frac{2x}{x^2-x-2} \Rightarrow \frac{x(x-1)(x-2)}{}$ X

$x=0$, $x=1$, $x=2$

A B C

$I = \frac{2x}{x(x-1)(x-2)}$ X $= \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2} \quad / \cdot x(x-1)(x-2)$

$I = 2x = A(x-1)(x-2) + B\,x(x-2) + C\,x(x-1)$

$\boxed{x=0}$ $2 \cdot 0 = A(0-1)(0-2) + \cancel{B}^{\to 0} + \cancel{C}^{\to 0}$

$0 = 2A$

$\boxed{A=0}$

$\boxed{x=1}$ $2 \cdot 1 = \cancel{A}^{\to 0} + B\,1(1-2) + \cancel{C}^{\to 0}$

$2 = B-1$

$\boxed{B=-2}$

$\boxed{x=2}$ $2 \cdot 2 = \cancel{A}^{\to 0} + \cancel{B}^{\to 0} + C\,2(2-1)$

$4 = C2$

$\boxed{C=2}$

$I = 0\cancel{\int \frac{dx}{x}}^{\to 0} - 2\int \frac{dx}{x-1} + 4\int \frac{dx}{x-2}$ X

$= 0 - 2\ln|x-1| + 4\ln|x-2| \Big|_0^1$

$= (0 - 2\ln|1-1| + 4\ln|1-2|) - (0 - 2\ln|0-1| + 4\ln|0-2|)$

$= (0 - 2\ln 0 + 4\ln|-1|) - (0 - 2\ln|-1| + 4\ln|-2|)$

$= (0 - \infty + 0) - (0 - 0 + 2{,}772588722) = -2{,}772588722$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xoo

IME I PREZIME: NIKOLA MILETIĆ **BROJ INDEKSA:** 17-2-0265-2013

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Pronaći opće rješenje ODJ $y'' - y' = x^2$ i provjeriti dobiveno rješenje. 10+5
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Ukupno:

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x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
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Tablica nekih integrala

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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
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(5) $$\int_0^1 \frac{2x}{x^2-x-2} = \frac{-2}{(x+1)} + \frac{4}{(x-2)}\Bigg|_0^1 = \left[\left(\frac{-2}{1+1} + \frac{4}{1-2}\right) - \left(\frac{-2}{0+1} + \frac{4}{0-2}\right)\right] = \frac{1}{3} - 0$$

$$= \frac{1}{3} = 0.\overline{3}$$

∅

$$\frac{2x}{x^2-x-2} = \frac{A}{(x+1)} + \frac{B}{(x-2)} \Big/ \cdot (x+1)(x-2)$$

PROVJERA NUMERIČKOM INTEGRACIJOM BI VAS SPASILA...

$x^2 - x - 2 = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{1 \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$x_{1,2} = \frac{1 \pm \sqrt{9}}{2}$$

$$x_1 = \frac{1-3}{2} = -1$$

$$x_2 = \frac{1+3}{2} = 2$$

$x^2 - x - 2 = (x+1)(x+2)$

$2x = A(x-2) + B(x+1)$

$2x = Ax + 2A + Bx + B$

$A + B = 2$

$2A + B = 0 \Rightarrow B = -2A$

$= -2 \cdot (-2)$

$= 4$

$A - 2\cancel{A} = 2$

$-A = 2 / \cdot (-1)$

$A = -2$

⑥ $x+y^2=6 \Rightarrow x=-y^2+6$

$x+y+1=0 \Rightarrow x=-y-1$

x	-2	-1	0	1	2
$-y^2+6$	2	5	6	5	6

x	-2	-1	0	1	2
$-y-1$	1	0	-1	-2	-3

$-y^2+6=-y-1$

$-y^2+6+y+1=0$

$-y^2+y+7=0 \,/\cdot(-1)$

$y^2-y-7=0$

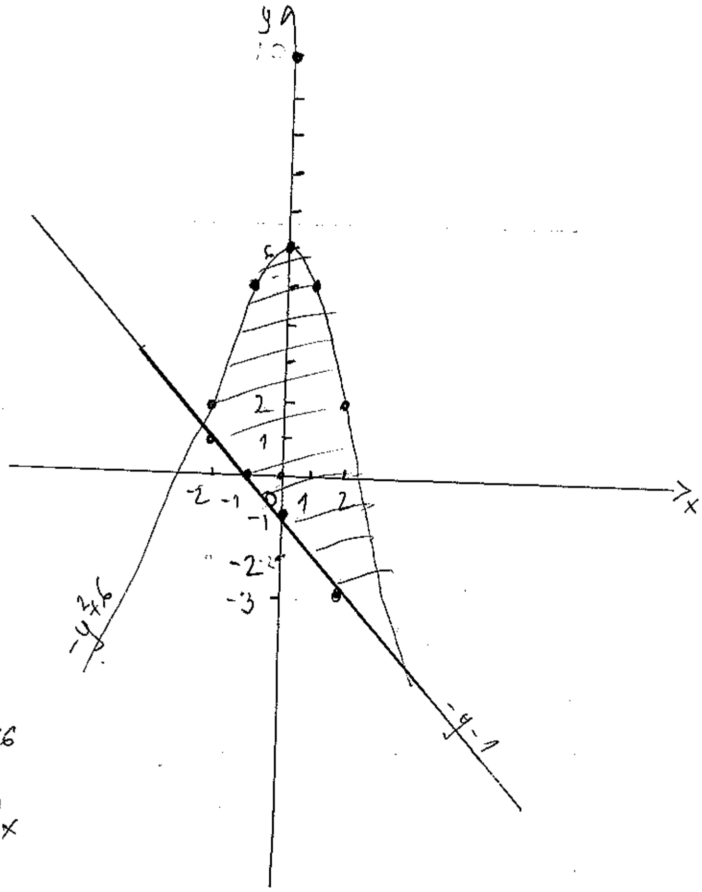
$y_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$y_{1,2}=\frac{1\pm\sqrt{1-4\cdot1\cdot(-7)}}{2}$

$y_{1,2}=\frac{1\pm\sqrt{29}}{2}$

$y_1=\frac{1-\sqrt{29}}{2}=-2.1926$

$y_2=\frac{1+\sqrt{29}}{2}=3.1926$



$$\int_{-2.1926}^{3.1926}(y^2-y-7)\,dx=\int_{-2.1926}^{3.1926}y^2\,dx-\int_{-2.1926}^{3.1926}y\,dx-\int_{-2.1926}^{3.1926}7\,dx$$

$$=\frac{y^3}{3}-\frac{y^2}{2}-7x\Big|_{-2.1926}^{3.1926}=\left[\frac{(3.1926)^3}{3}-\frac{(3.1926)^2}{2}-7\cdot(3.1926)\right]$$

$$-\left[\frac{(-2.1926)^3}{2}-\frac{(-2.1926)^2}{2}-7\cdot(-2.1926)\right]$$

$=-16.5975-9.4308$ $\varnothing$

$=(-)\,26.0283$

POVRŠINA NEGATIVNA ?

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** 11. 9. 2014. xoo	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: Matija Miočić **BROJ INDEKSA:** 17-1-0110-2012	

1. Pronaći opće rješenje ODJ $y'' - y' = x^2$ i provjeriti dobiveno rješenje. 10+5
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$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

(6) $x + y^2 = 6 \qquad x + y + 1 = 0$

$x = 6 - y^2 \qquad x = -1 - y$

(4) $\int\limits_0^2 x \sin x^2 dx = \int\limits_0^2 x^2 \sin x \, dx = \left[\begin{matrix} x^2 = u /' & \sin x \, dx = dv /\int \\ 2x = du & -\cos x = v \end{matrix} \right]$ 0

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xoo

IME I PREZIME: LUKA GULAN **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Pronaći opće rješenje ODJ $y'' - y' = x^2$ i provjeriti dobiveno rješenje. 10+5
2. Odrediti lokalne ekstreme funkcije $f(x, y) = \sin(x) \cdot \cos(y)$ na kvadratu $x \in [0, 2\pi]$, $y \in [0, 2\pi]$. 15
3. Pronaći ravninu koja dira graf funkcije $f(x, y) = xy - \ln(xy)$ povučenu u točki $(4, 1, z_0)$ tog grafa. 15
4. $\displaystyle\int_0^2 x \sin x^2 \, dx = ?$ 20
5. $\int_0^1 \dfrac{2x}{x^2 - x - 2} = ?$ 15
6. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$. 20

Ukupno: 0

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x \, dx = -\ln \lvert\cos x\rvert + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln \lvert x\rvert + C$	$\int \cot x \, dx = \ln \lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
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MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: [illegible] **BROJ INDEKSA:** 17-2-023 0-20

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Pronaći opće rješenje ODJ $y'' - y' = x^2$ i provjeriti dobiveno rješenje. 10+5
2. Odrediti lokalne ekstreme funkcije $f(x, y) = \sin(x) \cdot \cos(y)$ na kvadratu $x \in [0, 2\pi]$, $y \in [0, 2\pi]$. 15
3. Pronaći ravninu koja dira graf funkcije $f(x, y) = xy - \ln(xy)$ povučenu u točki $(4, 1, z_0)$ tog grafa. 15
4. $\displaystyle\int_0^2 x \sin x^2 \, dx = ?$ 20
5. $\displaystyle\int_0^1 \frac{2x}{x^2 - x - 2} = ?$ 15
6. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$. 20

Ukupno: 0

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\| \frac{a+x}{a-x} \right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln \|\cos x\| + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\| \frac{x-a}{x+a} \right\| + C$
$\int \frac{dx}{x} = \ln \|x\| + C$	$\int \cot x \, dx = \ln \|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\| x + \sqrt{x^2 \pm a^2} \right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
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6

$\cancel{x + y^2 = 6}$

$\cancel{y^2 = 6 - x}$

$\cancel{y = -x - 1}$

$\cancel{6 - x = x + 1}$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xoo

POPUNJAVA NASTAVNIK

IME I PREZIME: DINO PAVELA

BROJ INDEKSA: 17-2-0195-2012

	Broj ↓ bodova
1. Pronaći opće rješenje ODJ $y'' - y' = x^2$ i provjeriti dobiveno rješenje.	10+5
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \sin(x) \cdot \cos(y)$ na kvadratu $x \in [0, 2\pi]$, $y \in [0, 2\pi]$.	15
3. Pronaći ravninu koja dira graf funkcije $f(x,y) = xy - \ln(xy)$ povučenu u točki $(4, 1, z_0)$ tog grafa.	15
4. $\int\limits_0^2 x \sin x^2 \, dx = ?$	20
5. $\int_0^1 \frac{2x}{x^2 - x - 2} = ?$	15
6. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$.	20
Ukupno:	$\varnothing$

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
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$\sin x$	$\cos x$
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
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4. $\int\limits_0^2 x \sin x^2 \, dx$

$\int\limits_0^2 x \cos x^2 \, dx$

5) $\int_0^1 \frac{2x}{x^2-x-2}$

$\int_0^1 \frac{2x}{x^2-x-2}$

1. $y'' - y' = x^2$

$z = y' \Rightarrow y'' = z'$

$z' - z = x$

$y_0 = e^{-\int f(x)\,dx}$

6. $x+y^2=6$

$x+y+1=0$