MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** oxx

POPUNJAVA NASTAVNIK

IME I PREZIME: Šimun Zdrilić

BROJ INDEKSA: 17-2-0297-2013

	Broj ↓ bodova
1. Riješiti $y'' - y = -x + 1$ i odrediti posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 1$, $y' = 0$.	20
2. Provjeriti da funkcija $f(x) = x\sin(2x) + 1$ zadovoljava diferencijalnu jednadžbu $y'' + 4y = 4\cos(2x) + 4$ i početne uvjete $y(0) = 1$ i $y'(0) = 0$.	10
3. Skicirati razinske krivulje, na skici označiti smjer rasta i diskutirati diferencijabilnost funkcije $f(x,y) = \frac{1}{x^2+y^2}$.	10+5+5
4. Numeričkom integracijom odrediti vrijednost $\int_0^2 \sin(x^2)\,dx$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$)	20
5. $\int\limits_0^4 \frac{x\,dx}{x^2+2x-3} = ?$	~~15~~
6. Integriranjem izračunati površinu trokuta zadanog točkama $A(-1,0)$, $B(0,1)$, $C(1,-1)$.	~~15~~
Ukupno:	Ø

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
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$\cos x$	$-\sin x$
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Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
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$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

6) $A(-1,0), B(0,1) C(1,-1)$

$AB: (y-y_1)(x_2-x_1) = (x-x_1)(y_2-y_1)$

$= (y-0)(0+1) = (x+1)(1)$

$= \quad \underline{y = x+1}$

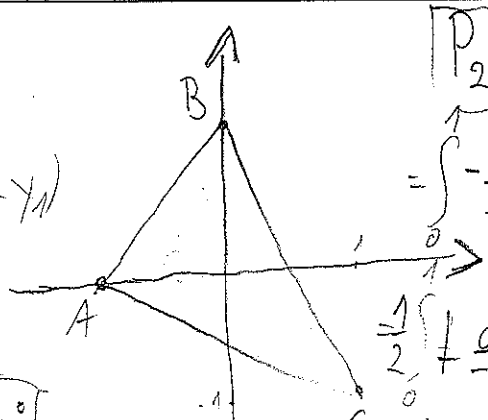
$BC = (y-0)(1) = (x-0)(-1-1)$

$= \quad y = -2x$

$AC = (y-0)(2) = (x+1)(-1)$

$= \quad 2y = -x-1$

$y = \frac{-x-1}{2}$



$\boxed{P_1}: \int_{-1}^{0} x+1-\left(\frac{-x-1}{2}\right)dx = \int_{-1}^{0} x+1+\frac{x}{2}+\frac{1}{2}dx = \int_{-1}^{0}\frac{3x}{2}+\frac{3}{2}dx = \frac{3}{2}\int_{-1}^{0} x+1\,dx =$

$= \begin{bmatrix} x+1 = t \\ dt = dx \end{bmatrix} \frac{3}{2}\int t\,dt = \frac{3}{2}\left|\frac{t^2}{2}\right.= \frac{3}{2}\left|_{-1}^{0}\frac{(x+1)^2}{2}\right. = \frac{1}{2} - 0 = \frac{1}{2}\cdot\frac{3}{2} = \frac{3}{4}$

$\boxed{P = \frac{3}{4} + \frac{3}{2} = \frac{9}{4}}$

$\boxed{P_2}: \int_0^1 -2x-\left(\frac{-x-1}{2}\right)dx = \int -2x+\frac{x+1}{2}dx$

$= \int_0^1 -\frac{3}{2}x+\frac{1}{2}dx = \frac{1}{2}\int_0^1 -3x+1\,dx = \begin{bmatrix} t = -3x+1 \\ dt = -3dx \\ dx = \frac{dt}{-3} \end{bmatrix}$

$= \frac{1}{2}\int_0^1 t\frac{dt}{-3} = \frac{1}{2}\left|_0^1 \frac{t^2}{2}\frac{dt}{-3}\right. = \frac{1}{2}\left|_0^1 \frac{(-3x+1)^2}{2}\right. dx$

$C = \frac{1}{2}(4-1) = \frac{3}{2}$

$$3)\ \int_0^4 \frac{x\,dx}{x^2+2x-3} = \int_0^4 \frac{x\,dx}{(x-1)(x+3)} = \int_0^4 \frac{\frac{1}{4}}{(x-1)} + \frac{\frac{3}{4}}{(x+3)}dx = \frac{1}{4}\int_0^4 \frac{dx}{x-1} + \frac{3}{4}\int_0^4 \frac{dx}{x+3}$$

0 !!

nepravi integral ✓

$x^2+2x-3=0$

$x_{1,2} = \frac{-2 \pm \sqrt{16}}{2}$

$x_1 = 1$

$x_2 = -3$

$x_1 = \lim_{b \to 1^-} b$

$$= \frac{1}{4}\int_0^{x_1} \frac{dx}{(x-1)} + \frac{1}{4}\int_{x_2}^4 \frac{dx}{(x-1)} + \frac{3}{4}\int_0^4 \frac{dx}{x+3} = \frac{1}{4}\int_0^{x_1} \frac{du}{u} + \frac{1}{4}\int_{x_2}^4 \frac{dt}{t} + \frac{3}{4}\int_0^4 \frac{dv}{v}$$

$x_2 = \lim_{b \to 1^+} b$

$\left[\begin{array}{l} t = u = x-1 \\ dt = du = dx \end{array}\right]$ $\left[\begin{array}{l} v = x+3 \\ dv = dx \end{array}\right]$

$$\frac{x}{(x-1)(x+3)} = \frac{A}{(x-1)} + \frac{B}{(x+3)} \Big/ \cdot (x-1)(x+3)$$

$x = Ax + 3A + Bx - B$

$1 = A + B$

$0 = 3A - B$

$1 = 4A$

$A = \frac{1}{4}$

$B = \frac{3}{4}$

$$= \frac{1}{4}\Big|_0^{x_1} \ln|x-1| + \frac{1}{4}\Big|_{x_2}^4 \ln|x-1| + \frac{3}{4}\Big|_0^4 \ln|x+3|$$

$$= \frac{1}{4}\left[\ln\left|\lim_{b\to 1^-} b - 1\right| - \ln|0-1|\right] + \frac{1}{4}\left[\ln 3 - \lim_{b\to 1^+} b\right] + \frac{3}{4}[\ln 7 - \ln 3]$$

$$= \frac{1}{4}\left(\ln|0^-| - \ln 1\right] + \frac{1}{4}\left[\ln 2^-\right] + 0{,}847$$

$$= \frac{1}{4}\left(\ln 0^+ - \ln 1\right) + \frac{1}{4}\left(\ln 2^-\right) + 0{,}847$$

$$= \underline{\frac{1}{4}\left(\ln 0^+\right)} \cdot \frac{1}{4}\left(\ln 2^-\right) + 0{,}847$$

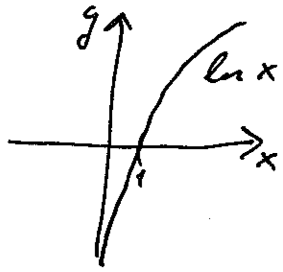
$= ?$

TREBALO JE DOVRŠITI :

$$\lim_{b\to 1_-} \ln|x-1| = \lim_{y\to 0_+} \ln y = \text{"}\ln 0_+\text{"} = -\infty$$

$$\lim_{b\to 1_+} \ln|x-1| = \ldots = -\infty$$

REZULTAT $= -\infty + \infty = N/P$



MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** oxx

IME I PREZIME: ANTON RAŠIĆ **BROJ INDEKSA:** 17-2-0084-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 15

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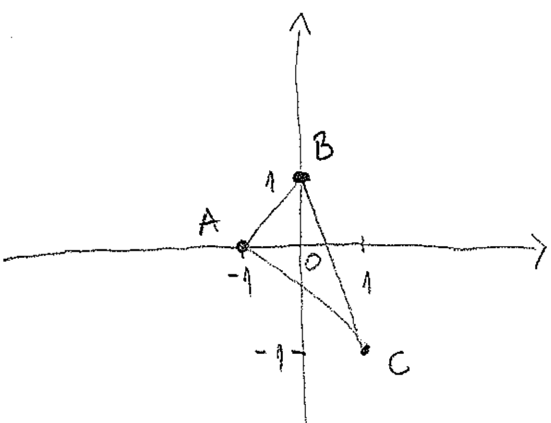
(6) $A(-1,0)$, $B(0,1)$ $C(1,-1)$



$(y-y_1)(x_2-x_1) = (y_2-y_1)(x-x_1)$

$AB = (y-0)(0+1) = (1-0)(x+1)$

$y = x+1$

BC $(y-1)(1-0) = (-1-1)(x-0)$

$y-1 = -2x$

$y = -2x+1$

$AC = (y-0)(1+1) = (-1-0)(x+1)$

$2y = -1(x+1)$

$2y = -x-1 \quad /:2$

$y = -\frac{1}{2}x - \frac{1}{2}$

$$P_1 = \int_{-1}^{0} (x+1) - \left(-\frac{1}{2}x - \frac{1}{2}\right) dx = \int_{-1}^{0} \left(x + 1 + \frac{1}{2}x + \frac{1}{2}\right) dx = \int_{-1}^{0} \left(\frac{x+2x}{2}\right) + \left(\frac{1}{2} + \frac{1}{1}\right) = \int_{-1}^{0} \frac{3}{2}x + \frac{1+2}{2}$$

$$= \int_{-1}^{0} \left(\frac{3}{2}x + \frac{3}{2}\right) dx = \frac{3}{2}\int_{-1}^{0} x\,dx + \frac{3}{2}\int dx = \frac{3}{2} \cdot \frac{x^2}{2}\Big|_{-1}^{0} + \frac{3}{2} \cdot x\Big|_{-1}^{0}$$

$$= \frac{3}{2} \cdot \left(\frac{0^2}{2} - \frac{(-1)^2}{2}\right) + \frac{3}{2} \cdot (0+1) = \frac{3}{2} \cdot \left(-\frac{1}{2}\right) + \frac{3}{2}$$

$$= -\frac{3}{4} + \frac{3}{2} = \frac{-3+6}{4} = \frac{3}{4}$$

$$P_2 = \int_{0}^{1} (-2x+1) - \left(-\frac{1}{2}x - \frac{1}{2}\right) = \int_{0}^{1} \left(-2x + 1 + \frac{1}{2}x + \frac{1}{2}\right) dx = \int_{0}^{1} \left(\frac{-4x + x}{2} + \frac{2+1}{2}\right) dx$$

$$= \int_{0}^{1} \left(-\frac{3}{2}x + \frac{3}{2}\right) dx = -\frac{3}{2}\int_{0}^{1} x\,dx + \frac{3}{2}\int_{0}^{1} dx = -\frac{3}{2} \cdot \frac{x^2}{2}\Big|_{0}^{1} + \frac{3}{2} \cdot x\Big|_{0}^{1}$$

$$= -\frac{3}{2}\left(\frac{1}{2} - \frac{0}{2}\right) + \frac{3}{2}(1-0) = -\frac{3}{2} \cdot \frac{1}{2} + \frac{3}{2} = -\frac{3}{4} + \frac{3}{2} = \frac{-3+6}{4} = \frac{3}{4}$$

$$P = P_1 + P_2 = \frac{3}{4} + \frac{3}{4} = \frac{3+3}{4} = \frac{\cancel{6}^{3}}{\cancel{4}_{2}} = \frac{3}{2}$$ ✓

$$-\frac{2x}{1} + \frac{x}{2} = \frac{-4x + x}{2} = -\frac{3x}{2}$$

$$\frac{1}{2} + \frac{1}{1} = \frac{1+2}{2} = \frac{3}{2}$$

$$-\frac{3}{4} + \frac{3}{2} = \frac{-3+6}{4} = \frac{3}{4}$$

$$-\frac{3}{4} + \frac{3}{2} = \frac{-3+6}{4} = \frac{3}{4}$$

$$\frac{3}{4} + \frac{3}{4} = \frac{6}{4} = \frac{3}{2}$$

$$\frac{x}{2} + \frac{x}{1} = \frac{x+2x}{2} = \frac{3x}{2}$$

$$\frac{1}{2} + \frac{1}{1} = \frac{1+2}{2} = \frac{3}{2}$$

ANTON RAŠIĆ $\boxed{\text{OXX}}$

(5) $\int_0^4 \frac{x\,dx}{x^2+2x-3} = \int_0^4 \frac{\cancel{x}}{x^{\cancel{2}}}dx + \frac{\cancel{x}}{2\cancel{x}}dx - \frac{1}{3}x\,dx = \int_0^4 \frac{1}{x}dx + \frac{1}{2}dx - \frac{1}{3}x\,dx$

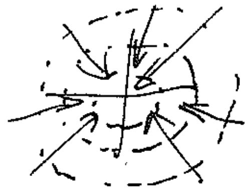
$= \ln|x| \Big|_0^4 + \frac{1}{2}\cdot x\Big|_0^4 - \frac{1}{3}\cdot\frac{x^2}{2}\Big|_0^4 = \ln 4 + \frac{1}{2}\cdot 4 - \frac{1}{3}\cdot\left(\frac{4^2}{2}\right)$

$= \ln 4 + 2 - \frac{1}{3}\cdot 8 = \ln 4 + 2 - \frac{8}{3} = 1{,}39 + 2 - \frac{8}{3} = \frac{4{,}16+6-8}{3}$

$= \frac{2{,}16}{3} = 0{,}72$ X

NISTE PROVJERILI DOMENU, TJ. EVENTUALNI SINGULARITET U INTERVALU INTEGRACIJE.

(3) $f(x,y) = \frac{1}{x^2+y^2}$

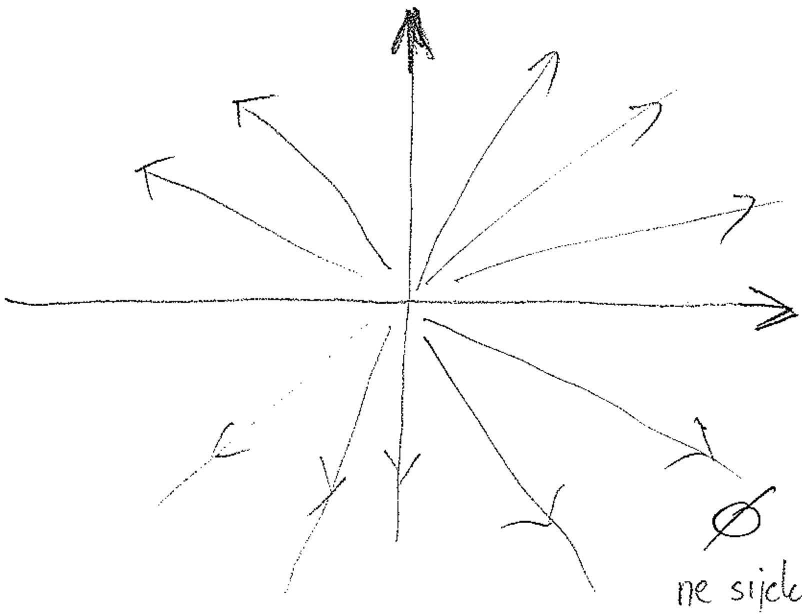


$\frac{1}{x^2+y^2} = 0$

$x^2+y^2 \neq 0$

$D_f = \langle -\infty, 0\rangle \cup \langle 0, +\infty\rangle$

$D_f = \mathbb{R}^2 \setminus \{(0,0)\}$



$\emptyset$ ne sijeku se u nuli.

VIDIMO IZ DOMENA
DIFERENCIJABILNOST NA DOMENI
JER JE KOMPOZICIJA ELEMENTARNIH FUNKCIJA.

(4) $\int_0^2 \sin(x^2)\,dx$

1) $y'' - y = -x + 1$

$x = 0$

$y = 1$

$y' = 0$

$y'' - y = -x + 1$

$y'' - y + x - 1 = 0$

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IME I PREZIME: ANTE STANIŠIĆ **BROJ INDEKSA:** 17-1-0066-2011	

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$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
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5) $\displaystyle\int_0^4 \frac{x\,dx}{x^2+2x-3}$

$1 \notin D_f \quad 1 \in [0,4]$

$x = 1$ je V.A.

ČINI SE DA SE INTEGRAL NEPRAVI !!! ...

$X_{12} = \frac{-2 \pm \sqrt{4+12}}{2} = \frac{-2 \pm \sqrt{16}}{2} = \frac{-2 \pm 4}{2}$

$X_1 = \frac{-2-4}{2} = \frac{-6}{2} = -3$ A

$X_2 = \frac{-2+4}{2} = \frac{2}{2} = 1$ B

$a(x - x_1)(x - x_2)$

$\underbrace{(x+3)}_{A}\underbrace{(x-1)}_{B}$

$I = \frac{x}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1} \quad /(x+3)(x-1)$

$x = A(x-1) + B(x+3)$

$x_1 = -3 \qquad x_2 = 1$

$-3 = -4A \qquad 1 = 4B$

$A = \frac{3}{4} \qquad B = \frac{1}{4}$

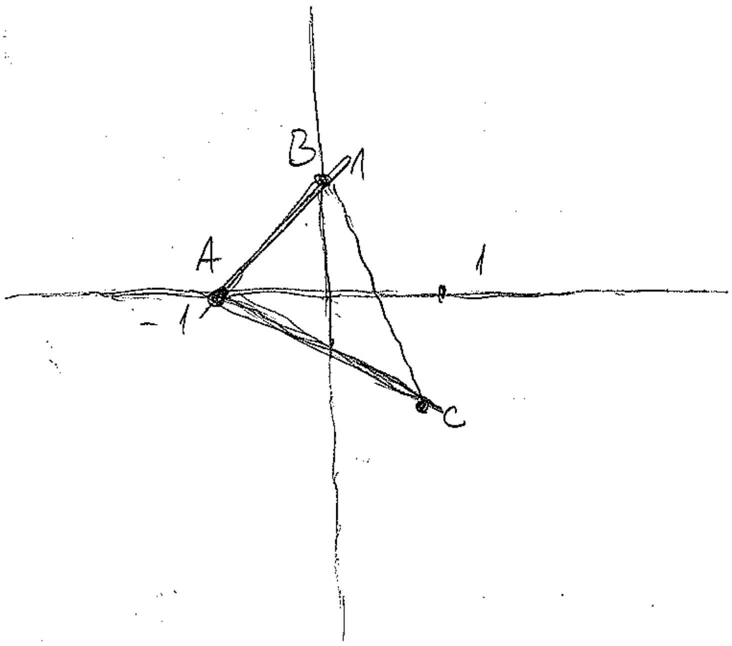
$I = \frac{3}{4}\int_0^4 \frac{dx}{x+3} + \frac{1}{4}\int_0^4 \frac{dx}{x-1}$

$I = \frac{3}{4}\ln|x+3| + \frac{1}{4}\ln|x-1| \Big|_0^4$

$I = \frac{3}{4}\ln|7| + \frac{1}{4}\ln|3| - \left(\frac{3}{4}\ln|3| + \frac{1}{4}\ln|-1|\right)$

$I = \frac{3}{4}\ln(7) + \frac{1}{4}\ln 3 - \frac{3}{4}\ln 3 - \frac{1}{4}\ln(-1)$

$A(-1,0) \quad B(0,1) \quad C(1,-1)$



$A(\underset{x_1}{-1}, \underset{y_1}{0}) \quad C(\underset{x_2}{1}, \underset{y_2}{-1})$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y = \frac{-1-0}{1+1}(x+1)$$

$$y = \frac{-1}{2}(x+1)$$

$$y = -\frac{1}{2}x - \frac{1}{2}$$

$B(\underset{x_1}{0}, \underset{y_1}{1}) \quad C(\underset{x_2}{1}, \underset{y_2}{-1})$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 1 = \frac{-1-1}{1-0}(x-0)$$

$$y - 1 = -2x$$

$$\underline{y = -2x + 1}$$

AB

$A(\underset{x_1}{-1}, \underset{y_1}{0}) \quad B(\underset{x_2}{0}, \underset{y_2}{1})$

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y = \frac{1-0}{0+1}(x+1)$$

$$\underline{y = x + 1}$$

$$P_1 = \int_{-1}^{0}\left((x+1) - \left(-\frac{1}{2}x - \frac{1}{2}\right)\right)dx = \int_{-1}^{0}\left(x + 1 + \frac{1}{2}x + \frac{1}{2}\right)dx$$

$$= \int_{-1}^{0}\left(\frac{3}{2}x + \frac{3}{2}\right)dx = \frac{3}{2}\cdot\frac{x^2}{2} + \frac{3}{2}x \Big|_{-1}^{0}$$

$$= \frac{3x^2}{4} + \frac{3}{2}x \Big|_{-1}^{0} = \left(\cancel{\frac{3\cdot 0^2}{4}} + \cancel{\frac{3}{2}\cdot 0}\right) - \left(\frac{3\cdot 1}{4} + \frac{3}{2}\cdot(-1)\right) = -\left(-\frac{3}{4}\right) = \boxed{\frac{3}{4}}\ P_1$$

$$P_2 = \int_{0}^{1}\left((-2x+1) - \left(-\frac{1}{2}x - \frac{1}{2}\right)\right)dx = \int_{0}^{1}\left(-2x + 1 + \frac{1}{2}x + \frac{1}{2}\right)dx = \int_{0}^{1}\left(-\frac{3}{2}x + \frac{3}{2}\right)dx$$

$$P = -\frac{3x^2}{4} + \frac{3}{2}x \Big|_{0}^{1} = \left(-\frac{3}{4} + \frac{3}{2}\right) - \left(\cancel{-\frac{3\cdot 0}{4}} + \cancel{\frac{3}{2}\cdot 0}\right) = \frac{3}{4}$$

$$P = P_1 + P_2 \Rightarrow \boxed{P = \frac{3}{2}} \ \checkmark$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** oxx	POPUNJAVA NASTAVNIK
IME I PREZIME: MIHOVIL PEDISIĆ BROJ INDEKSA:	Broj ↓ bodova

1. Riješiti $y'' - y = -x + 1$ i odrediti posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 1$, $y' = 0$. 20
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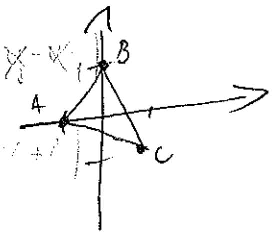
5. $\displaystyle\int_0^4 \frac{x\,dx}{x^2+2x-3} = ?$ ~~15~~
6. Integriranjem izračunati površinu trokuta zadanog točkama $A(-1, 0)$, $B(0, 1)$, $C(1, -1)$. 15

Ukupno: 15

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\,dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\|x + \sqrt{x^2\pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

6. $A(-1,0)$
$B(0,1)$
$C(1,-1)$



$AB \ldots\ (y-0)(0-(-1)) = (x+1)(1-0)$

$y = x+1$

$BC \ldots\ (y-1)(1-0) = (x-0)(-1-1)$

$y-1 = -2x$

$y = -2x+1$

$AC \ldots\ (y-0)(2) = (x+1)(-1-0)$

$2y = -x-1$

$y = \frac{-x-1}{2}$

$P = P_1 + P_2$

$P_1 = \int_{-1}^{0} x+1-\left(\frac{-x-1}{2}\right) = \int_{-1}^{0} x+1+\frac{x}{2}+\frac{1}{2}$

$= \left|\frac{x^2}{2} + x + \frac{x^2}{4} + \frac{x}{2}\right|_{-1}^{0}$

$= 0 - \left(\frac{1}{2} - 1 + \frac{1}{4} - \frac{1}{2}\right) = \frac{3}{4}$

$P_2 = \int_0^1 -2x+1+\frac{x}{2}+\frac{1}{2} =$

$\left| -x^2 + x + \frac{x^2}{4} + \frac{x}{2}\right|_0^1 = \left(-1+1+\frac{1}{4}+\frac{1}{2}\right) - 0 = \frac{1+2}{4} = \frac{3}{4}$

$\boxed{P = P_1 + P_2 = \frac{3}{4} + \frac{3}{4} = \frac{6}{4} \quad P = \frac{3}{2}}$

5. $$\int_0^4 \frac{x\,dx}{x^2+2x-3} = \left\{ \begin{aligned} t &= x^2+2x-3 \;/' \\ \frac{dt}{2} &= (2x+2)\,dx \;/:2 \end{aligned} \right\} = \int_0^4 \frac{x+1-1}{x^2+2x-3}\,dx =$$

$$\frac{dt}{2} = (x+1)\,dx$$

$$= \int_0^4 \frac{x+1}{x^2+2x-3}\,dx - \underbrace{\int_0^4 \frac{dx}{x^2+2x+3}}_{\#} = \frac{1}{2}\int_0^4 \frac{dt}{t} - \int_0^4$$

$$= \left| \frac{1}{2}\left(\ln(x^2+2x-3)\right) - \frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| \right|_0^4 = \left(\frac{1}{2}\ln|4^2+8-3| - \frac{1}{4}\ln\left|\frac{4-1}{4+3}\right|\right) - \left(\frac{1}{2}\ln|-3| - \frac{1}{4}\ln\left|\frac{1}{3}\right|\right)$$

OVO JE NEPRAVI INTEGRAL

$$= 1{,}734 - 0{,}824 = 0{,}91$$

$$\# \Rightarrow \int \frac{dx}{x^2+2x-3} = \int \frac{dx}{x^2+2x+1-4} = \int \frac{dx}{(x+1)^2-2^2} = \left\{ \begin{aligned} t &= x+1 \;/' \\ dt &= dx \end{aligned} \right\} =$$

$$\int \frac{dt}{t^2-2^2} = \frac{1}{4}\ln\left|\frac{t-2}{t+2}\right|$$

$$= \frac{1}{4}\ln\left|\frac{x-1}{x+3}\right|$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** oxx

IME I PREZIME: FILIP MEŠTROVIĆ **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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5. $\displaystyle\int_0^4 \frac{x\,dx}{x^2+2x-3} = ?$ ~~15~~
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Ukupno: ∅

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
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$(x-3)(x+1) = x^2 + x - 3x - 3$

5. $\displaystyle\int_0^4 \frac{x\,dx}{x^2+2x-3}$ $\qquad \frac{x}{x^2+2x-3} = \frac{A}{(x-3)} + \frac{B}{(x+1)} \;/\cdot(x^2+2x-3)$

$x_{1,2} = \frac{-2 \pm \sqrt{4+12}}{2}$

$x_1 = 3 \quad x_2 = -1$

$x = Ax + A + Bx - 3B$

$1 = A + B \qquad 1 = 4B$

$0 = A - 3B \qquad B = \frac{1}{4} \qquad A = \frac{3}{4}$

$A = 3B$

X=1 JE V.A. UNUTAR INTERVALA INTEGRACIJE [0,4] OVO JE NEPRAVI INTEGRAL!!!

$$\int \frac{x\,dx}{x^2+2x-3} = \frac{3}{4}\ln\lvert x-3\rvert + \frac{1}{4}\ln\lvert x+1\rvert + C$$

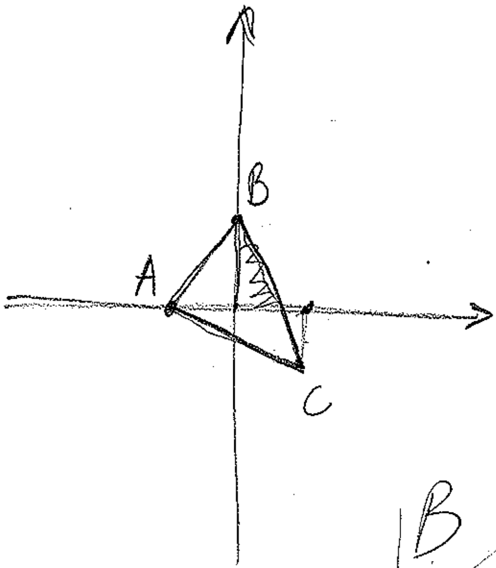
$$\int_0^4 \frac{x\,dx}{x^2+2x-3} = \left(\frac{3}{4}\underbrace{\ln\lvert 1\rvert}_{0} + \frac{1}{4}\ln\lvert 5\rvert\right) - \left(\frac{3}{4}\ln\lvert -3\rvert + \frac{1}{4}\underbrace{\ln(1)}_{0}\right)$$

$$= \frac{1}{4}\ln\lvert 5\rvert - \frac{3}{4}\ln\lvert 3\rvert$$

$0{,}402359 + 0{,}8239592 = \underline{-0{,}4216}$

$A(-1,0)$
$B(0,1)$
$C(1,-1)$

6.

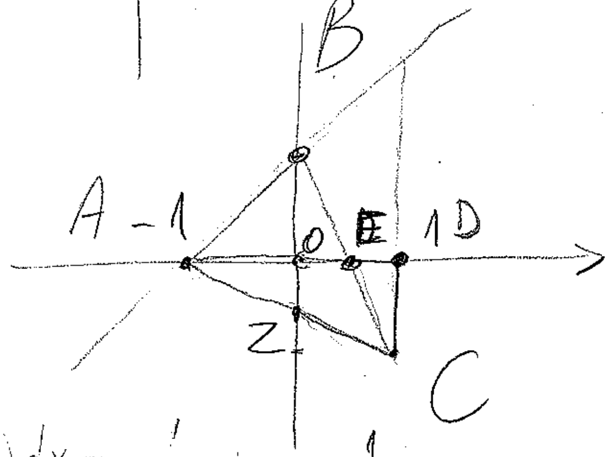


$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$\overline{BC}=y-1=\frac{-1-1}{1-0}(x-0)$$

$$y-1=-2x \qquad -2x+1=0$$

$$y=-2x+1 \qquad y(0)=1 \qquad x=\frac{1}{2}$$



$$AB \quad y-0=\frac{1-0}{0+1}(x+1)$$

$$y=x+1$$

$$AC=y-0=\frac{-1-0}{1+1}(x+1)$$

$$y=-\frac{1}{2}x-\frac{1}{2}$$

$$P=\cancel{BOE}+CDOZ-CED+ZOA+A\cancel{BO}$$

$$\int_0^{1/2}(2x+1)\,dx+\int_{-1}^{0}(x+1)\,dx+\int_1^0\left(-\frac{1}{2}x-\frac{1}{2}-(-2x+1)\right)dx+\int_0^{-1}\left(-\frac{1}{2}x-\frac{1}{2}\right)dx=$$

$$=(x^2+x)\Big|_0^{1/2}+\left(\frac{x^2}{2}+x\right)\Big|_{-1}^{0}+\left(\frac{3}{4}x^2-\frac{3}{2}x\right)\Big|_1^0+\left(-\frac{x^2}{4}-\frac{1}{2}x\right)\Big|_0^{-1}$$

$$=\left(\frac{3}{4}\right)+\left(-\frac{1}{2}+1\right)+\left(0-\frac{3}{4}+\frac{3}{2}\right)+\left(\left(-\frac{1}{4}+\frac{1}{2}\right)-0\right)=$$

$$=\frac{3}{4}+\frac{2}{4}+\frac{3}{4}+\frac{3}{4}=\boxed{\frac{11}{4}}$$

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POPUNJAVA NASTAVNIK

IME I PREZIME: MAHADIĆ FRANE **BROJ INDEKSA:** 17-1-0077-2011

Broj ↓ bodova

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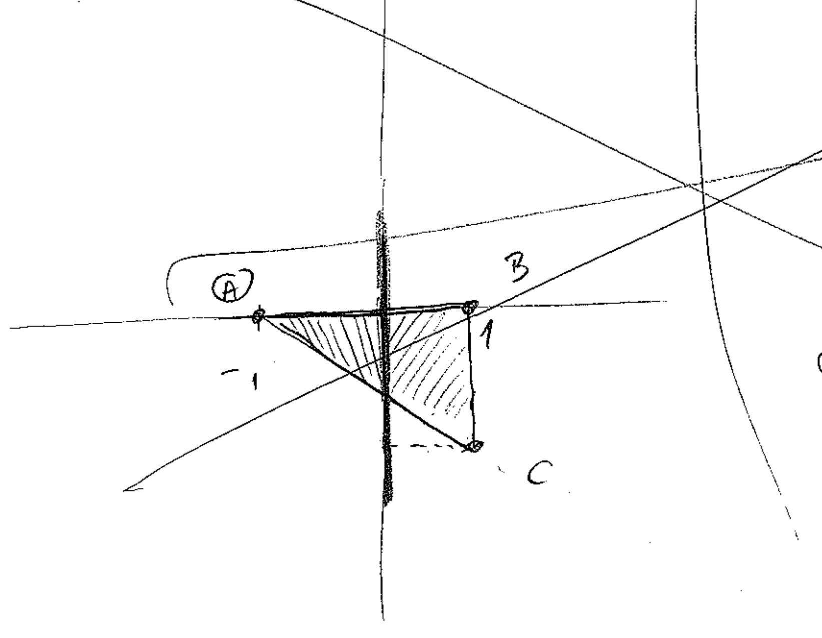
6) $A(-1,0)$, $B(0,1)$ $C(1,-1)$



x_1 y_1 x_2 y_2

(AB) $A(-1,0)$ $B(0,1)$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 0 = \frac{1-0}{0+1}(x+1)$

$y - 0 = 1(x+1)$

$y - 0 = x + 1$

$y = x + 1$ (AB)

$(-1,0,1)$

AC

$A(-1,0) \quad C(1,-1)$

$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-0 = \frac{-1-0}{1+1}(x+1)$

$y-0 = -\frac{1}{2}(x+1)$

$y-0 = -\frac{1}{2}x - \frac{1}{2}$

$y = -\frac{1}{2}x - \frac{1}{2}$

BC

$B(0,1) \quad C(1,-1)$

$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-1 = \frac{-1-1}{1-0}(x-0)$

$y-1 = -\frac{2}{1}(x-0)$

$y-1 = -2(x-0)$

$y-1 = -2x+0$

$y-1 = -2x$

$y = -2x+1$

AB $= y = x+1$

AC $y = -\frac{1}{2}x - \frac{1}{2}$

BC $y = -2x+1$

$P_1 = \int_{-1}^{0} AB - AC$

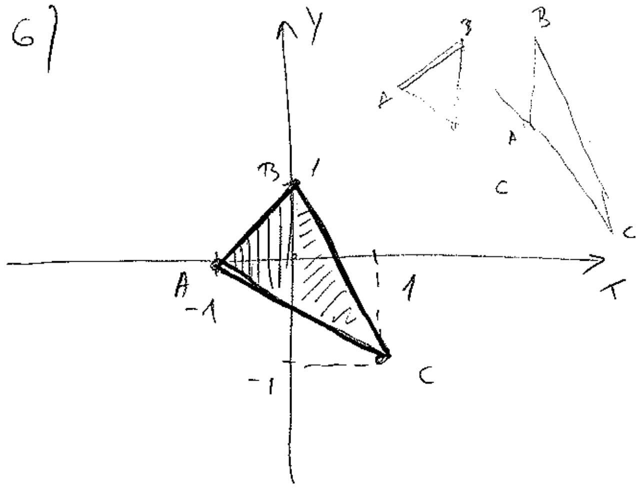
$P_1 = \int_{-1}^{0} (x+1) - \left(-\frac{1}{2}x - \frac{1}{2}\right) dx = \int_{-1}^{0} \left(x+1+\frac{1}{2}x+\frac{1}{2}\right) dx$

$P_1 = \int_{-1}^{0} \frac{3}{2}x + \frac{3}{2} = \frac{3}{2}\cdot\frac{x^2}{2} + \frac{3}{2}x \Big|_{-1}^{0} = \frac{3x^2}{4} + \frac{3}{2}x \Big|_{-1}^{0} =$

$P_1 = \left(\frac{3\cdot 0^2}{4} + \frac{3}{2}\cdot 0\right) - \left(\frac{3\cdot(-1^2)}{4} + \frac{3}{2}\cdot(-1)\right) = 0 - \left(-\frac{3}{4}\right) = P_1 = \frac{3}{4}$

$P_1 = 0 \left(-\frac{3}{4}\right) = \frac{3}{4}$

6)



MAHADIĆ FRANE

$A(-1,0)\quad B(0,1)$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-0=\frac{1-0}{0+1}(x+1)$$

$$y-0=x+1$$

$$y=x+1 \longrightarrow AB$$

$A(-1,0)\quad C(1,-1)$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-0=\frac{-1-0}{1+1}(x+1)$$

$$y=-\frac{1}{2}x-\frac{1}{2} \Rightarrow AC$$

$B(0,1)\quad C(1,-1)$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-1=\frac{-1-0}{1+1}(x-0)$$

$$y-1=-2(x-0)$$

$$y=-2x+1 \Rightarrow BC$$

AB - AC

$$\int_{-1}^{0}(x+1)-\left(-\frac{1}{2}x-\frac{1}{2}\right)dx=\int_{-1}^{0}\left(x+1+\frac{1}{2}x+\frac{1}{2}\right)dx=\int_{-1}^{0}\frac{3}{2}x+\frac{3}{2}$$

$$=\frac{3\cdot x^2}{4}+\frac{3}{2}x\Big|_{-1}^{0}=\left(\frac{3\cdot 0^2}{4}+\frac{3}{2}\cdot 0\right)-\left(\frac{3\cdot(-1^2)}{4}+\frac{3}{2}\cdot(-1)\right)=0-\left(-\frac{9}{4}\right)$$

$$P_1=\frac{9}{4}$$

$\int_0^1 BC-AC$

$$P_2=\int_0^1(-2x+1)-\left(-\frac{1}{2}x-\frac{1}{2}\right)dx=\int_0^1\left(-2x+1+\frac{1}{2}x+\frac{1}{2}\right)dx$$

$$=\int_0^1-\frac{3}{2}x+\frac{3}{2}\,dx=-\frac{3}{2}\cdot\frac{x^2}{2}+\frac{3}{2}x\Big|_0^1=-\frac{3x^2}{4}+\frac{3}{2}x\Big|_0^1=$$

$$=\left(\frac{-3\cdot 1^2}{4}+\frac{3}{2}\cdot 1\right)-\left(-\frac{3\cdot 0^2}{4}+\frac{3}{2}\cdot 0\right)$$

$$=\frac{3}{4}+0=\frac{3}{4}$$

$$P=P_1+P_2$$

$$P=\frac{9}{4}+\frac{3}{4}=3$$

5) $$\int_0^4 \frac{x\,dx}{x^2+2x-3} =$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** oxx **IME I PREZIME:** Radovčić Rikardo **BROJ INDEKSA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

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5. $\displaystyle\int_0^4 \frac{x\,dx}{x^2+2x-3} = ?$ ~~15~~
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Ukupno: $\emptyset$

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_a x\ (a > 0)$	$\frac{1}{x\ln a}$
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Tablica nekih integrala		
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
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(5)

$$\underline{I} = \int_0^4 \frac{x\,dx}{x^2+2x-3} \qquad \left[\begin{array}{l} x^2+2x-3=t \\ (2x+2)\,dx = dt \quad /:2 \\ (x+1)\,dx = \frac{dt}{2} \end{array}\right] \qquad \emptyset$$

NEPRAVI INTEGRAL !!!

$$= \int_0^4 \frac{x+1-1}{x^2+2x-3}\,dx \stackrel{\times}{=} \int_0^4 \frac{x+1}{x^2+2x-3}\,dx - \int_0^4 \frac{dx}{x^2+2x-3}$$

$$= \int_{-3}^{21} \frac{dt}{t} - \underline{I_1} = \frac{1}{2}\cdot \ln(t)\Big|_{-3}^{21} - \underline{I_1}$$

$$\underline{I} = \frac{1}{2}\left(\ln 21 - \ln 3\right) = I_1$$

$$= \int_0^4 \frac{x\,dx}{x^2+2x-3} = \left[x^2+2x-3 = (x+1)^2-1-3 = (x+1)^2-4 \right]$$

$$= \int_0^4 \frac{dx}{(x+1)^2-4} = \left| \begin{array}{ll} x+1=t & u^2=4 \\ dx=dt & u=2 \end{array} \right] = \int_1^5 \frac{dt}{t^2-u^2}$$

$$= \frac{1}{2u} \ln\left|\frac{t-u}{t+u}\right| = \frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| \Bigg|_1^5$$

$$\underline{I_1} = \frac{1}{4}\ln\left|\frac{5-1}{5+3}\right| - \frac{1}{4}\ln\left(\frac{1-1}{1+3}\right)$$

$$I_1 = \frac{1}{4}\ln\left|\frac{2}{8}\right| - \frac{1}{4}\ln|0|$$

Radovčić Rikardo

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** oxx

IME I PREZIME: VALENTINO ŠARE **BROJ INDEKSA:** 17-2-0149-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti $y'' - y = -x + 1$ i odrediti posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 1$, $y' = 0$. 20
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x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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5.

$$\int_0^4 \frac{x\,dx}{x^2+2x-3} = \int_0^4 \frac{x\,dx}{x^2+2x-1+(-2)} = \int_0^4 \frac{x}{x^2}dx + \int_0^4 \frac{x}{2x}dx - \int_0^4 x\,dx$$

NEPRAVI INTEGRAL !!!

$$\int_0^4 \frac{x}{x^2}dx + \int_0^4 \frac{x}{2x}dx - \int_0^4 \frac{x}{1}dx$$
$$= \int \frac{1}{x}dx + \frac{1}{2}$$

$$= \int_0^4 \frac{x}{\frac{x^3}{3}} + \frac{1}{2}\int_0^4 \frac{x}{\frac{x^2}{2}} - \int_0^4 x$$

$$= \int_0^4 \frac{3x}{x^3} + \frac{1}{2}\int_0^4 \frac{2x}{x^2} - \int_0^4 x$$

$$= \int_0^4 \frac{3\cdot 4}{4^3} + \frac{1}{2}\cdot\frac{2\cdot 4}{4^2} - 4 - 0$$

$$= -\frac{57}{16} = -3{,}5625$$

$f(x) = x\sin(2x)$

$y'' + 4y = 4\cos(2x) + 4$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. PIŠITE DVOSTRANO! — oxx

IME I PREZIME: STJEPAN VOJTA — BROJ INDEKSA: 17-2-0187-2012

POPUNJAVA NASTAVNIK — Broj ↓ bodova

1. Riješiti $y'' - y = -x + 1$ i odrediti posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 1$, $y' = 0$. — ~~20~~ Ø
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1. $y'' - y = -x + 1$ $\qquad x=0,\ y=1,\ y=0$

$x^2 - x = 0$

$x_{1,2} = \frac{1 \pm \sqrt{1-0}}{2}$

$\boxed{x_1 = 1}$

$\boxed{x_2 = 0}$

$y = C_1 x^1 + C_2 x^0$

$y = C_1 x + C_2$

Ø

$Y = e^x(Ax+B)$ $\qquad \boxed{Y=0}$

$Y = Ax + B$

$Ax + B = -x + 1$ $\qquad \boxed{B=1}$

$Ax = -1$

$\boxed{A=-1}$

$y' = 2x - 1$

$y'' = 2$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OXX	POPUNJAVA NASTAVNIK
IME I PREZIME: Josip Kovaček **BROJ INDEKSA:**	Broj ↓ bodova

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