MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OXO

POPUNJAVA NASTAVNIK

IME I PREZIME: MARINO ZUBČIĆ **BROJ INDEKSA:** 17-2-0216-2012

	Broj ↓ bodova
1. Riješiti diferencijalnu jednadžbu: $4y'' - y = x \sin x$ i na kraju provjeriti rješenje.	~~20~~
2. Riješi diferencijalnu jednadžbu $x^2 y y' = 1 - x^2$ uz rubni uvjet $y(1) = 1$.	15
3. Obrazloži diferencijabilnost funkcije $f(x, y) = e^y + 2xy + \frac{1}{y}$ na njenoj domeni.	10
4. $\int\limits_0^{\pi} (x \cos x + e^{1-3x})\, dx = ?$	20
5. $\int\limits_0^{2} \frac{x^2\, dx}{1+x} = ?$	15
6. Četverokut $ABCD$ je određen točkama $A(0,0)$, $B(3,0)$, $C(3,3)$ i $D(2,1)$. Integriranjem mu pronađi površinu.	20
Ukupno:	70

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln \lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln \lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

(4) $\int\limits_0^{\pi} (x\cos x + e^{1-3x})\, dx = I$

$\int\limits_0^{\pi} x\cos x\, dx + \int\limits_0^{\pi} e^{1-3x}\, dx \Rightarrow$

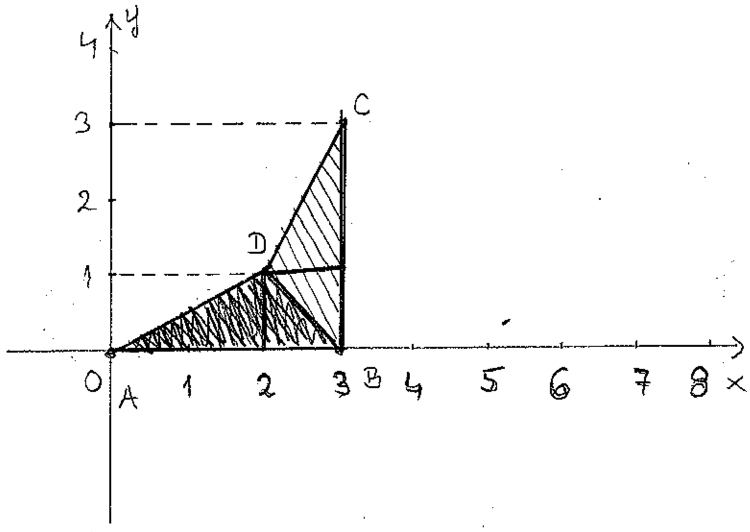
(1) $x\cos x\, dx = \left| \begin{matrix} u = x & \cos x\, dx = dv \\ du = dx & v = +\sin x \end{matrix} \right| = x\sin x - \int \sin x\, dx$

$= (x \sin x + \cos x)\Big|_0^{\pi} = (\pi \sin \pi + \cos\pi) - (0 \sin 0 + \cos 0) = -1 - 1 = -2$ ✓

(2) $\int e^{1-3x} dx = \left| \begin{matrix} t = 1 - 3x \\ dt = -3dx \end{matrix} \right| = \int -\frac{1}{3} e^t dt = -\frac{1}{3} e^t = -\frac{1}{3} e^{1-3x} \Big|_0^{\pi}$

$= -\frac{1}{3}[e^{1-3x} - e^{1-0}] = -\frac{1}{3}[e^{1-3\pi} - e] = \frac{1}{3} e [1 - e^{-3\pi}]$

(1)+(2) $\underline{I} = -2 + \frac{1}{3} e [1 - e^{-3\pi}]$ ✓

⑥ 

$\Rightarrow$ NA DRUGOM PAPIRU

② $\int_0^2 \frac{x^2 dx}{1+x} = \begin{bmatrix} 1+x=t \\ dx = dt \end{bmatrix} = \int_0^2 \frac{(t-1)^2}{t} dt = \int_0^2 \frac{t^2-2t+1}{t} dt$

$= \int_0^2 \left(t - 2 + \frac{1}{t}\right) dt = \frac{t^2}{2} - 2t + \ln t = \left(\frac{(1+x)^2}{2} - 2(1+x) + \ln(1+x)\right)\Big|_0^2$

$= \left(\frac{9}{2} - 6 + \ln(3)\right) - \left(\frac{1}{2} - 2 + \ln(1)\right) = 4.5 - 6 + \ln(3) - 0.5 + 2 + 0$

$= \ln(3) \approx 1.098612289$ ✓

② $x^2 y y' = 1 - x^2 \quad / : x^2$ $\qquad y(1) = 1$

$y y' = \frac{1-x^2}{x^2}$

$y y' = \frac{1}{x^2} - 1 \quad / \int$

$\int y\, dy = \int \left(\frac{1}{x^2} - 1\right) dx$

$\frac{y^2}{2} = -\frac{1}{x} + x + c \quad \Rightarrow \quad \frac{1}{2} = -\frac{1}{1} + 1 + c$

$c = \frac{5}{2}$

RJEŠENJE: $\frac{y^2}{2} = -\frac{1}{x} + x + \frac{5}{2}$ ✓

⑥ 2 TROKUTA $\Rightarrow \Delta ABD$; ΔBDC

MARINO ZUBČIĆ

$$P(ABD) = \frac{1}{2}\left|0(0-1) + 3(1-0) + 2(0-0)\right|$$

$$= \frac{1}{2}|3| = \frac{3}{2}$$

$$P(BDC) = \frac{1}{2}\left|3(3-1) + 3(1-0) + 2(0-3)\right|$$

$$= \frac{1}{2}|3\cdot 2 + 3 - 6| = \frac{1}{2}|3| = \frac{3}{2}$$

$$P(ABCD) = \frac{3}{2} + \frac{3}{2} = \frac{6}{2} = \underline{\underline{3}} \checkmark$$

① $4y'' - y = x\sin x$

HOMOGENO

$$4y'' - y = 0 \qquad y'' = r^2 e^{rx}$$

$$4r^2 e^{rx} - e^{rx} = 0 \qquad y' = re^{rx} \qquad a = 0$$

$$4r^2 - 1 = 0 \qquad y = e^{rx} \qquad b = -1$$

$$4r^2 = 1$$

$$r^2 = \frac{1}{4} \Rightarrow r = \pm\frac{1}{2}$$

$$y_{1h} = e^{\frac{1}{2}x}, \quad y_{2h} = e^{-\frac{1}{2}x}$$

$$\boxed{y_H = c_1 e^{\frac{1}{2}x} + c_2 e^{-\frac{1}{2}x}, \quad c_1, c_2 \in \mathbb{R}}$$

PARTICULARNO

$$f(x) = x\sin x = x e^{ax} \sin x \Rightarrow \begin{matrix} a = 0 \\ b = 1 \end{matrix}$$

DALJE ___

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OXO

POPUNJAVA NASTAVNIK

IME I PREZIME: Ivan Klanac **BROJ INDEKSA:** 17-2-0098-2011

	Broj ↓ bodova
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Ukupno:	35

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e^x	e^x
$a^x\ (a > 0)$	$a^x \ln a$
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
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⑤ $\int\limits_0^2 \frac{x^2 dx}{1+x} dx = \int\limits_0^2 \frac{x^2-1+1}{x+1} dx = \int\limits_0^2 \frac{(x-1)(x+1)+1}{x+1} =$

$f(x) = \frac{x^2 dx}{1+x}$ $1+x \neq 0$ $x=-1$

određeni integral nije nepravi

$= \int\limits_0^2 (x-1)\, dx + \left| \int\limits_0^2 \frac{1}{x+1} dx \right| \rightarrow \left| \begin{matrix} x+1=t \\ dx=dt \end{matrix} \right| \begin{matrix} x-1=t \\ dx=dt \end{matrix}$

$D(f) \in \mathbb{R}$

$[0,2] \in \mathbb{R}$

$= \int\limits_0^2 (x-1)\, dx + \int\limits_0^2 \frac{dt}{t}$

$= \int\limits_0^2 t\, dt + \int\limits_0^2 \frac{dt}{t}$

$= \frac{t^2}{2}\Big|_0^2 + \ln|t|\Big|_0^2$

$= \frac{x-1^2}{2}\Big|_0^2 + \ln|x+1|\Big|_0^2$

$= \frac{2-1^2}{2} - \frac{0-1^2}{2} + \ln|2+1| - \ln|0-1|$

$= \underbrace{\frac{1}{2} - \frac{1}{2}}_{0} + \ln|3| - \underbrace{\ln|-1|}_{0}$

$= \ln 3 = 1{,}09$ ✓

Ivan Klanac

(4) $\int_0^{\pi} (x\cos x + e^{1-3x})\,dx$

$$= \underbrace{\int_0^{\pi} x\cos x\,dx}_{I_1} + \underbrace{\int_0^{\pi} e^{1-3x}\,dx}_{I_2} = I$$

$$I_1 = \int_0^{\pi} x\cos x\,dx = \begin{bmatrix} u = x & dv = \cos x \\ du = dx & v = \sin x \end{bmatrix}$$

$$= x\sin x\Big|_0^{\pi} - \int_0^{\pi} \sin x\,dx = x\sin x\Big|_0^{\pi} + \cos x$$

$$= (\underbrace{\pi\cdot\sin\pi}_{0} - \underbrace{0\cdot\sin 0}_{0}) + (\underbrace{\cos\pi}_{-1} - \underbrace{\cos 0}_{-1}) = -1-1 = -2 \checkmark$$

$$I_2 = \int_0^{\pi} e^{1-3x}\,dx = -\frac{1}{3}\cdot e\cdot e^{-3}\Big|_0^{\pi} = \frac{1}{3}e(e^{-3\pi}-1)$$

$$I_2 = \int_0^{\pi} e^{+1-3x}\,dx$$

$$= \left[-\frac{1}{3}e^{1-3x}\right]_0^{\pi}$$

$$= -\frac{1}{3}e^{1-3\pi} + \frac{1}{3}e^{1}$$

$$= -\frac{1}{3}e\cdot e^{-3\pi} + \frac{1}{3}e^{1}$$

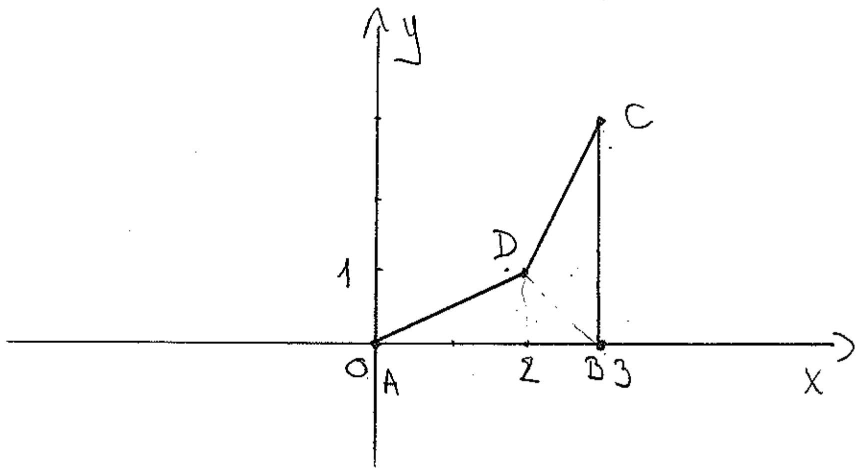
$$= \frac{1}{3}e\left(1 - e^{-3\pi}\right)$$

$$I = I_1 + I_2$$

$$I = \frac{e}{3}(1-e^{-3\pi}) - 2 \checkmark$$

Ivan Klanac
17-2-0098-2011

⑥ $A(0,0)$ $B(3,0)$ $C(3,3)$ $D(2,1)$



$A(\overset{x_1}{0},\overset{y_1}{0})$ $B(\overset{x_2}{3},\overset{y_2}{0})$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(3-0)(y-0)=(0-0)(x-0)$

$3(y-0)=0(x-0)$

$3y=x \ /:3$

$AB\ldots y=\frac{x}{3}$

$C(\overset{x_1}{3},\overset{y_1}{3})$ $D(\overset{x_2}{2},\overset{y_2}{1})$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(2-3)(y-3)=(1-3)(x-3)$

$-1(y-3)=-2(x-3)$

$-y+3=-2x+6$

$-y=-2x+6-3$

$-y=-2x+3 \ /\cdot(-1)$

$CD\ldots y=2x-3$

$B(\overset{x_1}{3},\overset{y_1}{0})$ $C(\overset{x_2}{3},\overset{y_2}{3})$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(3-3)(y-0)=(3-0)(x-3)$

$0(y-0)=3(x-3)$

$BC\ldots y=3x-9$

$D(\overset{x_1}{2},\overset{y_1}{1})$ $A(\overset{x_2}{0},\overset{y_2}{0})$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(0-2)(y-1)=(0-1)(x-2)$

$-2(y-1)=-1(x-2)$

$-2y+2=-x+2$

$-2y=-x+2-2$

$-2y=-x \ /:(-2)$

$DA\ldots y=\frac{x}{2}$

$\Rightarrow$

Ivan Klarac

$$P=\int_0^3 \frac{x}{2}-\frac{x}{3} \;+\int_2^3 2x-3-(3x-9)$$

$$P=\int_0^3 \frac{1}{6}x\,dx \;+\int_2^3 2x-3-3x+9$$

$$P=\frac{1}{6}\int_0^3 x\,dx \;+\int_2^3 -x+6$$

$$P=\frac{1}{6}\cdot\frac{x^2}{2}\Big|_0^3 \;+\left(-\frac{x^2}{2}+6x\right)\Big|_2^3$$

$$P=\frac{1}{6}\cdot\frac{3^2}{2}-\left(\frac{1}{6}\cdot\frac{0^2}{2}\right)+\left(-\frac{3^2}{2}+6\cdot 3-\left(-\frac{2^2}{2}+6\cdot 2\right)\right)$$

$$P=\frac{1}{6}\cdot\frac{3^2}{2}-\frac{1}{6}\cdot\frac{0^2}{2}-\frac{3^2}{2}+6\cdot 3+\frac{2^2}{2}-6\cdot 2$$

$$P=\frac{3}{4}-0-\frac{9}{2}+18+2-12$$

$$P=4.25 \;//$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** oxo

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: [redacted] **BROJ INDEKSA:** [redacted]

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Ukupno: 30

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln \lvert x \rvert + C$	$\int \cot x \, dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
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$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} \left[x\sqrt{a^2-x^2} + a^2 \arcsin \left(\frac{x}{a}\right)\right] + C$	

5.

$$\int\limits_0^2 \frac{x^2 dx}{1+x} \quad \left[\begin{array}{lll} 1+x=t & za \ x=2 & t=3 \\ dx=dt & za \ x=0 & t=1 \\ x=t-1 & & \end{array}\right] \quad \int\limits_1^3 \frac{(t-1)^2}{t} dt = \int\limits_1^3 \left(t-2+\frac{1}{t}\right) dt$$

$$= \left(\frac{t^2}{2} - 2t + \ln|t|\right)\Big|_1^3 = \left(\frac{3^2}{2} - 6 + \ln|3|\right) - \left(\frac{1}{2} - 2 + \ln|1|\right) =$$

$$= \frac{9}{2} - 6 + \ln 3 - \frac{1}{2} + 2 - \ln 1 = \ln 3 \checkmark$$

$$4. \int\limits_0^\pi \left(x\cos x + e^{1-3x}\right) dx \quad \left[\begin{array}{ll} x\cos x & dv = \cos x \\ u = x & v = \int \cos x \, dx \\ du = dx & v = \sin x \end{array}\right]$$

$$uv - \int v du = x \cdot \sin x - \int \sin x dx = x\sin x - \cos x$$

$$\left.\left(x\sin x-\cos x \pm \frac{1}{3}e^{1-3x}\right)\right|_0^{\pi} = \pi\sin\pi+\cos\pi+\frac{1}{3}e^{1-3\pi}-\cos 0-\frac{1}{3}e=$$

$$=-1+\frac{1}{3}e^{-1-3\pi}-1-\frac{1}{3}e=\frac{1}{3}e^{1-3\pi}-\frac{1}{3}e \quad \times \quad \emptyset$$

2. $x^2yy'=1-x^2$

$$ydy=\left(\frac{1}{x^2}-1\right)dx$$

$$\frac{y^2}{2}=-\frac{1}{x}-x+C$$

$$y=\sqrt{-\frac{2}{x}-2x+2C}$$

$$y(1) \to \sqrt{-2-2+2C}=1$$

$$2C-4=1$$

$$C=\frac{5}{2}$$

$$y_p=\sqrt{5-\frac{2}{x}-2x} \quad \checkmark$$

1. $4y''-y=x\sin x$

$$4r^2-1=0$$

$$r_{1,2}=\frac{0\pm\sqrt{0+16}}{2} \begin{cases} \frac{4}{2}=2 \\ -\frac{4}{2}=-2 \end{cases}$$

$$y_H=C_1e^{r_1x}+C_2xe^{r_2x}=C_1e^{2x}+C_2\,e^{-2x}$$

$$y_p=x\sin x$$

$$x\sin x=e^{\alpha x}\left(P_m(x)\cos(\beta x)+Q_m(x)\sin(\beta x)\right)$$

$$\alpha=0 \qquad \beta=1$$

$$\alpha+\beta i=k$$

$$0+i=k$$

$$k=0$$

$$y_p=x^0e^{0\cdot x}\left((A+Bx)\cos+(B+Cx)\sin\right)$$

DALJE . . .

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POPUNJAVA NASTAVNIK

Broj ↓ bodova

IME I PREZIME: ANGELO KOSOVIĆ **BROJ INDEKSA:** 17-2-0264-2013

1. Riješiti diferencijalnu jednadžbu: $4y'' - y = x \sin x$ i na kraju provjeriti rješenje. 20
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3. Obrazloži diferencijabilnost funkcije $f(x, y) = e^y + 2xy + \frac{1}{y}$ na njenoj domeni. ~~10~~
4. $\displaystyle\int_0^{\pi} \left(x \cos x + \mathrm{e}^{1-3x}\right) \mathrm{d}x = ?$ ~~20~~ ~~70~~
5. $\displaystyle\int_0^{2} \frac{x^2 \, \mathrm{d}x}{1 + x} = ?$ ~~15~~
6. Četverokut $ABCD$ je određen točkama $A(0,0)$, $B(3,0)$, $C(3,3)$ i $D(2,1)$. Integriranjem mu pronađi površinu. 20

Ukupno: ~~30~~

20

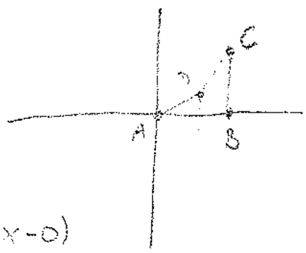
Kosov

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha \, \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, \mathrm{d}x = -\ln \lvert \cos x \rvert + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln \lvert x \rvert + C$	$\int \cot x \, \mathrm{d}x = \ln \lvert \sin x \rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int \mathrm{e}^x \, \mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, \mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, \mathrm{d}x = \frac{1}{2} \left[x \sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, \mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2} \, \mathrm{d}x = \frac{1}{2} \left[x \sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a}\right)\right] + C$	

6) $A(0,0)$ $B(3,0)$ $C(3,3)$ $D(2,1)$



$(x_2 - x_1)(y - y_1) = (y_2 - y_1)(x - x_1)$

$(3-0)(y-0) = (0-0)(x-0)$

$3y = 0$

$\overline{AB} \ldots \ y = 0$

$(3-3)(y-0) = (3-0)(x-3)$

$0 = 3x - 9$

$3x = 9 \ /:3$

$\overline{BC} \ldots \ x = 3$

$(2-3)(y-3) = (1-3)(x-3)$

$-y + 3 = -2x + 6$

$-y = -2x + 6 - 3$

$-y = -2x + 3 \ /\cdot(-1)$

$\overline{CD} \ldots \ y = 2x - 3$

$(2-0)(y-0) = (1-0)(x-0)$

$2y = x$

$\overline{AD} \ldots \ y = \frac{1}{2}x$

$P = \int_0^2 \left(\frac{1}{2}x - (0)\right)dx + \int_2^3 \left(2x - 3 - (0)\right)dx$

$P = \frac{1}{2} \frac{x^2}{2} \Big|_0^2 + x^2 - 3x \Big|_2^3$

$P = \frac{x^2}{4} \Big|_0^2 + x^2 - 3x \Big|_2^3$

$P = 1 + 9 - 9 - (4 - 6)$

$P = 1 + 0 + 2$

$P = 1 + 2$

$P = 3$

3) $f(x,y) = e^y + 2xy + \frac{1}{y}$

$Df = \mathbb{R}^2$

$Df = \{(x,y) \in \mathbb{R}^2\}$ $\cancel{\emptyset}$

$\left| \begin{matrix} u = x & dv = \cos x\,dx \\ du = dx & v = \sin x \end{matrix} \right|$ $\quad$ $\begin{matrix} 1-3x = t \\ -3dx = dt \\ dx = -\frac{1}{3}dt \end{matrix}$

4) $\int_0^{\pi} (x\cos x + e^{1-3x})\,dx$

$-x\sin x - \cos x - \frac{1}{3} e^{1-3x} \Big|_0^{\pi}$

$-\pi \cdot \sin\pi - \cos\pi - \frac{1}{3} e^{1-3\pi} - (0 - \cos 0 - \frac{1}{3} e^1)$

$0 + 1 - \frac{1}{3} e^{4\pi} - (-1 - \frac{1}{3} e^1)$

$= 1 - \frac{1}{3} e^{\pi} + 1 + \frac{1}{3} e^1$

$\int (x\cos x + e^{1-3x})\,dx = \int x\cos x\,dx + \int e^{1-3x}\,dx$

$= -x\sin x + \int \sin x\,dx - \frac{1}{3}\int e^t\,dt$

$= -x\sin x - \cos x - \frac{1}{3} e^{1-3x} + C$

$\cancel{10}$ $\times$

PROVJERA NUMERIČKOM INTEGRACIJOM BI VAS SPASILA...

$(-x\sin x - \cos x)' = \cancel{-\sin x} \underline{- x\cos x} \cancel{+ \sin x}$

5)

$\int_0^2 \frac{x^2\,dx}{1+x} =$

$\frac{x^3}{3} \ln|1+x| \Big|_0^2$

$\frac{2^3}{3} \ln|3| - (0 \ln|1|)$

$\frac{8}{3} \ln|3| - 0 = \frac{8}{3} \ln|3|$

$\int \frac{x^2\,dx}{1+x} = \left| \begin{matrix} 1+x = t \\ dx = dt \end{matrix} \right| = \underline{\int x^2 \cdot \frac{dt}{t} = \frac{x^3}{3} \ln|1+x| + C}$ $\times$

$1+x \neq 0$
$x \neq -1$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OXO

POPUNJAVA NASTAVNIK

IME I PREZIME: TOMISLAV BULIĆ **BROJ INDEKSA:** 17-2-0271-2013

	Broj ↓ bodova
1. Riješiti diferencijalnu jednadžbu: $4y'' - y = x \sin x$ i na kraju provjeriti rješenje.	20
2. Riješi diferencijalnu jednadžbu $x^2 y y' = 1 - x^2$ uz rubni uvjet $y(1) = 1$.	~~15~~
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4. $\int_0^\pi \left(x \cos x + e^{1-3x}\right) dx = ?$	~~20~~
5. $\int_0^2 \frac{x^2 \, dx}{1+x} = ?$	15
6. Četverokut $ABCD$ je određen točkama $A(0,0)$, $B(3,0)$, $C(3,3)$ i $D(2,1)$. Integriranjem mu pronađi površinu.	~~20~~
Ukupno:	Ø

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_a x \ (a > 0)$	$\frac{1}{x \ln a}$
e^x	e^x
$a^x \ (a > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
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Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
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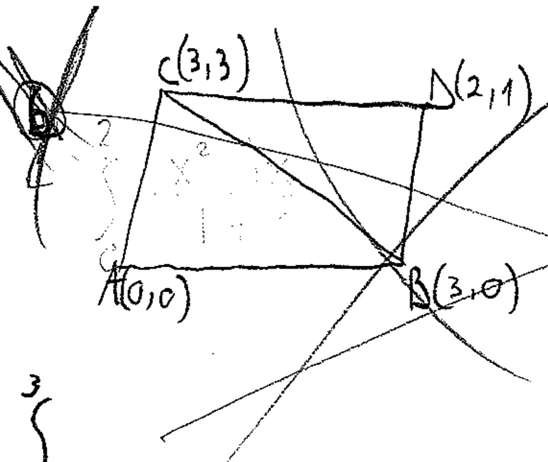

$\int_0^3$

① ΔABC

$A(\overset{x_1}{0}, \overset{y_1}{0})$

$B(\overset{x_2}{3}, \overset{y_2}{0})$

$C(3,3)$

$\overline{AB}$

$(y - y_1)(x_2 - x_1) = (y_2 - y_1)(x - x_1)$

$= (y - 0)(3 - 0) = (0 - 0)(x - 0)$

$= \underline{3y = 0}$

$\overline{AC}$

$(y - 0)(3 - 0) = (3 - 0)(x - 0)$

$= 3y = 3x \Rightarrow \underline{3y - 3x = 0}$

$\overline{BC}$

$(y - 0)(3 - 3) = (3 - 0)(x - 3)$

$0 = 3x - 9$

$\underline{3x - 9 = 0}$

→

② $x^2 y y' = 1 - x^2$

$y \cdot \frac{dy}{dx} = \frac{1-x^2}{x^2}\, dx$

$\frac{1}{2}y^2 = \int \frac{dx}{x^2} - \int \frac{x^2}{x^2} dx$

$\frac{1}{2}y^2 = \int x^{-2} dx - \int dx$

$\frac{1}{2}y^2 = -\frac{1}{x} - x + C$

$\frac{1}{2}y^2 = \frac{-1-x^2}{x} \Big| \cdot 2$

$y^2 = 2\left(\frac{-1-x^2}{x}\right)$

$y = \sqrt{2\left(\frac{-1-x^2}{x}\right)}$

$y(1) = \sqrt{2\left(\frac{-1-1}{1}\right)}$

$\cancel{y(1) = \sqrt{-4}}$

④ $\int_0^{\pi} x\cos x + e^{1-3x} dx = \int_0^{\pi} x\cos x\, dx + \int_0^{\pi} e^{1-3x} dx$

$= \left| \begin{matrix} u = x & dv = \cos x \\ du = dx & v = -\sin x \end{matrix} \right| = -x\sin x + \int_0^{\pi} \sin x\, dx + \int_0^{\pi} e^{1-3x} dx$

$v = \sin x$

$= (-x\sin x + \cos x)\Big|_0^{\pi} + \int_0^{\pi} e^{1-3x} dx = \left| \begin{matrix} 1+3x = t \Rightarrow t_1 = 1 \\ -3dx = dt \quad t_2 = 1-3\pi \\ dx = -\frac{1}{3}dt \end{matrix} \right.$

$(-x\sin x + \cos x)\Big|_0^{\pi} - \frac{1}{3}\int_1^{1-3\pi} e^t dt =$

$(-x\sin x + \cos x)\Big|_0^{\pi} - \frac{1}{3}e^{1-3x}\Big|_1^{1-3\pi} + C =$

$1 - (-\pi \cdot 0 - 1) - \frac{1}{3}\left(e^{1-3} - e^{1-3+9\pi}\right) + C$

$= 2 - \frac{1}{3}\left(e^{-2} - e^{-2+9\pi}\right) + C$

$= 2 - \frac{1}{3e^2} + \frac{1}{3e^{2-9\pi}} + C$

⑥ $A(0,0)$

$B(3,0)$

$C(3,3)$

$D(2,1)$

$P = P_1 + P_2$

$P_1 = \iint_D dx\,dy - \int_0^2 dx \int_0^1 dy = \int_0^2 dx(1-0) = \int_0^2 dx = 2$

$\int_D dx\,dy = \int_2^3 dx \int_0^3 dy = \int_2^3 dx(3-0) = 3\int_2^3 dx = 3(3-2) = 3$

$P = 2+3 = 5$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OXO

POPUNJAVA NASTAVNIK

Broj ↓ bodova

IME I PREZIME: MARIN MATEK

BROJ INDEKSA: 17-1-0111-12

1. Riješiti diferencijalnu jednadžbu: $4y'' - y = x\sin x$ i na kraju provjeriti rješenje.	~~20~~
2. Riješi diferencijalnu jednadžbu $x^2yy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$.	~~15~~
3. Obrazloži diferencijabilnost funkcije $f(x,y) = e^y + 2xy + \frac{1}{y}$ na njenoj domeni.	10
4. $\int\limits_0^\pi \left(x\cos x + e^{1-3x}\right)\,dx = ?$	~~20~~
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6. Četverokut $ABCD$ je određen točkama $A(0,0)$, $B(3,0)$, $C(3,3)$ i $D(2,1)$. Integriranjem mu pronađi površinu.	~~20~~
Ukupno:	Ø

f	$\frac{df}{dx}$
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$\ln x$	$\frac{1}{x}$
$\log_a x\ (a > 0)$	$\frac{1}{x\ln a}$
e^x	e^x
$a^x\ (a > 0)$	$a^x \ln a$
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$\tan x$	$\frac{1}{\cos^2 x}$
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$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\,dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x\,dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

1. $4y'' - y = x \sin x$

$4r^2 - 1 = 0$

$r^2 = \frac{1}{4}$

$r_1 = \frac{1}{2}$

$r_2 = -\frac{1}{2}$

$y_H = C_1 e^{\frac{1}{2}x} + C_2 e^{-\frac{1}{2}x}$

-7

$x \sin x = e^{\alpha x}(P_m(x)\cos(\beta x) + Q_n(x)\sin(\beta x))$

$\alpha = 0$, $\beta = 1$ $\alpha + \beta i = 0 \neq r_1 \neq r_2 \Rightarrow \boxed{k = 0}$

$x \sin x = (P_m(x)\cos x + Q_n(x)\sin(x))$

$P_m(x) =$ $Q_n(x) = \sin x$

$m = 1$ $n = 1$

$N = 1$

$y_p = x^k e^{\alpha x}(S_N(x)\cos(\beta x) + T_N(x)\sin(\beta x))$

$y_p = ((Ax+B)\cdot\cos x + (Cx+D)\cdot\sin x)$

$y_p = Ax\cos x + B\cos x + Cx\sin x + D\sin x$

$y' = A\cos x - Ax\sin x - B\sin x + C\sin x + Cx\cos x + D\cos x$

$y'' = -A\sin x - A\sin x - Ax\cos x - B\cos x + C\cos x - Cx\sin x - D\sin x$

$4y'' - y = x\sin x$

$4(-2A\sin x - Ax\cos x - B\cos x + C\cos x - Cx\sin x - D\sin x) - (Ax\cos x$
$+ B\cos x + Cx\sin x + D\sin x) = x\sin x$

$= -8A\sin x - 4Ax\cos x - 4B\cos x + 4C\cos x - 4Cx\sin x - 4D\sin x - Ax\cos x$
$- B\cos x - Cx\sin x - D\sin x = x\sin x$

$-5C = 1$; $-4A = 0$ $-4B + 4C - D = 0$ $-8A - 4D - D = 0$

$C = -\frac{1}{5}$ $A = 0$ $-5B = -4C$ $-8A - 5D = 0$

$B = \frac{4}{5}C$ $-5D = 8A$

$D = -\frac{8A}{5}$

$B = -\frac{4}{25}$

$D = 0$

$Y_p = -\frac{4}{25}\cos x - \frac{1}{5}x\sin x$

$y = y_H + y_p$

$y = C_1 e^{\frac{1}{2}x} + C_2 e^{-\frac{1}{2}x} - \frac{4}{25}\cos x - \frac{1}{5}x\sin x$ X

Ø

PROVJERA?

$y' = \frac{1}{2}C_1 e^{\frac{1}{2}x} - \frac{1}{2}C_2 e^{-\frac{1}{2}x} + \frac{4}{25}\sin x - \frac{1}{5}\sin x - \frac{1}{5}x\cos x$

$y'' = \frac{1}{4}C_1 e^{\frac{1}{2}x} + \frac{1}{4}C_2 e^{-\frac{1}{2}x} - \frac{1}{25}\cos x - \frac{1}{5}\cos x + \frac{1}{5}x\sin x$

$4y'' - y = \underbrace{0 + 0 + x\sin x - \frac{24}{25}\cos x + \frac{4}{25}\cos x}$

$\neq x\sin x$

2. $x^2 y y' = 1 - x^2 \quad / : x^2$ $\quad y(1) = 1$

$y y' = \frac{1}{x^2} - 1$

$y\frac{dy}{dx} = \frac{1}{x^2} - 1 \quad / \cdot dx$

$y\,dy = (\frac{1}{x^2} - 1)\,dx$

$\int y\,dy = \int \frac{1}{x^2}\,dx - \int dx$

$\frac{y^2}{2} = -\frac{1}{x} - x + C_1 \quad / : 2$

$y^2 = -\frac{1}{2x} - \frac{x}{2} + \frac{C}{2}$ X Ø

$1^2 = -\frac{1}{2} - \frac{1}{2} + \frac{C}{2}$

$1 = -\frac{1}{4} + \frac{C}{2}$

$\frac{C}{2} = 1 + \frac{1}{4}$

$C = \frac{10}{4}$ //

MAR IN MATEK

5. $\int_0^2 \frac{x^2}{1+x}dx = \frac{x^2}{2} + x - \ln|x+1| \Big|_0^2$

$= \left(\frac{2^2}{2} + 2 - \ln|3|\right) - \left(\frac{0^2}{2} + 0 - \ln|1|\right) =$

$= 2,90$

$\frac{x^2}{1+x}dx = \quad \int \frac{x^2+1-1}{1+x}dx = \cancel{\int \frac{x^2+1}{1+x}dx} - \int \frac{1}{1+x}dx = \left[\begin{array}{l} 1+x=t \\ dx=dt \end{array}\right]$

$= \int x + \int dx - \int \frac{dt}{t}$

$= \frac{x^2}{2} + x - \ln|x+1|$ ~~6~~

$x^2 : x+1 = x - 1 + \frac{1}{x+1}$

$\underline{-x^2 \pm x}$

$-x$

$\underline{\mp x \mp 1}$

1

MATEK
MARIN

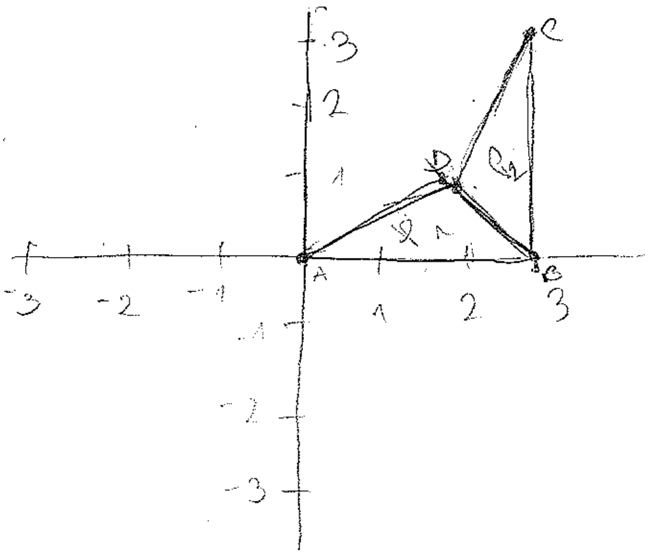
6. ABCD

$A(0, 0)$

$B(3, 0)$

$C(3, 3)$

$D(2, 1)$



ABD

$A(\overset{x_1}{0}, \overset{y_1}{0})$ $B(\overset{x_2}{3}, \overset{y_2}{0})$ $B(\overset{x_1}{3}, \overset{y_1}{0})$ $D(\overset{x_2}{2}, \overset{y_2}{1})$ $A(\overset{x_1}{0}, \overset{y_1}{0})$ $D(\overset{x_2}{2}, \overset{y_2}{1})$

$(x_2 - x_1)(y - y_1) = (y_2 - y_1)(x - x_1)$

$3y = 0$ $-y = x - 3$ $2y = x$

AB... $y = 0$ BD.. $y = -x + 3$ AD... $y = \frac{x}{2}$

DBC

$D(2, 1)$
$B(3, 0)$

DB.... $-x + 3$

$B(\overset{x_1}{3}, \overset{y_1}{0})$ $C(\overset{x_2}{3}, \overset{y_2}{3})$ $(x_2 - x_1)(y - y_1) = (y_2 - y_1)(x - x_1)$

$D(\overset{x_1}{2}, \overset{y_1}{1})$
$C(3, 3)$

$0 = 3x - 9$

$3x = 9$

BC... $x = 3$

$y - 1 = 2x - 4$

DC...$y = 2x - 3$

4. $\int_0^{\pi}(x\cos x + e^{1-3x})dx$

$= \int x\cos x^{*_1} + \int e^{1-3x}dx^{*_2}$

$= \frac{\sin x}{x} - \frac{1}{3}\int e^t dt$

$= -\sin x - \frac{1}{3}e^{1-3x}\Big|_0^{\pi}$

$= 0,906$ //

$*_2 = e^{1-3x} = \Big| \begin{array}{l} 1-3x = t \\ -3\,dx = dt \\ dx = \frac{dt}{3} \end{array}$

$*_1 \int x\cos x\,dx = \Big| \begin{array}{l} x = t \\ dx = dt \end{array}$

$= \int \cos t\,dt = -\sin x$

$P = P_1 + P_2$

$P = \int_0^3 (\frac{x}{2} - 0) + \int_2^3 (2x - 3 - (-x+3))$

$P = \int_0^3 \frac{x}{2} + \int_2^3 2x - 3 + x - 3$

$P = \int_0^3 \frac{x}{2}dx + \int_2^3 3x\,dx - \int_2^3 6\,dx$

$P = \frac{1}{2}\int x\,dx + 3\int x\,dx - 6\int dx = \frac{x^2}{4}\Big|_0^3 + \frac{3x^2}{2} - 6x\Big|_2^3$

$(\frac{9}{4}) + (\frac{27}{2} - 18 - (6 - 12)) = \frac{9}{4} + \frac{27}{2} - 18 + 6$

$= \frac{15}{4}$ ~~11~~

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OXO

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: SANDRO GRDOVIĆ BROJ INDEKSA: 17-2-0213-2012

1. Riješiti diferencijalnu jednadžbu: $4y'' - y = x \sin x$ i na kraju provjeriti rješenje. 20
2. Riješi diferencijalnu jednadžbu $x^2 y y' = 1 - x^2$ uz rubni uvjet $y(1) = 1$. 15
3. Obrazloži diferencijabilnost funkcije $f(x, y) = e^y + 2xy + \frac{1}{y}$ na njenoj domeni. 10
4. $\int_0^{\pi} \left(x \cos x + \mathrm{e}^{1-3x}\right) \mathrm{d}x = ?$ 20
5. $\int_0^{2} \frac{x^2 \, \mathrm{d}x}{1 + x} = ?$ ~~15~~
6. Četverokut $ABCD$ je određen točkama $A(0,0)$, $B(3,0)$, $C(3,3)$ i $D(2,1)$. Integriranjem mu pronađi površinu. ~~20~~

Ukupno: Ø

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, \mathrm{d}x = -\ln \lvert \cos x \rvert + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln \lvert x \rvert + C$	$\int \cot x \, \mathrm{d}x = \ln \lvert \sin x \rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int \mathrm{e}^x \mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, \mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, \mathrm{d}x = \frac{1}{2} \left[x \sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, \mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2} \, \mathrm{d}x = \frac{1}{2} \left[x \sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a}\right)\right] + C$	

1. $4y'' - y = x \sin x$

$5. \int_0^2 \frac{x^2 \, dx}{1+x} = x^2 \cdot \frac{dx}{1+x}$ $\left[\begin{array}{l} u = x^2 \\ du = 2x\,dx \end{array} \right.$

$dv = \frac{1}{1-x} \, dv$

$v = \ln|1-x| = c$

$x^2 \cdot \ln(1+x) \Big|_0^2 - \int_0^2 \ln(1+x) \; 2x\,dx =$

$x^2 \cdot \ln(1+x) \Big|_0^2 - \ln(1+x)2x \Big|_0^2$

$4 \cancel{\ln 3} - 4\ln 1 - \cancel{4\ln 3} + 0$

$-4\ln 1 = -4 \cdot 0 = 0$

$4. \int_0^{\pi} \left(x\cos x + e^{1-3x} \right) dx = \int_0^{\pi} x \cos dx = \int_0^{\pi} e^{1-3x}$

$\int e^{1-3x} \, dx = e^{1-3x} \left[\begin{array}{l} t = 1-3x \\ dt = -3dx \; , \quad dx = -\frac{dt}{3} \end{array} \right]$

$\int e^t \cdot -\frac{dt}{3} = -\frac{1}{3} \int e^t \, dt = \frac{1}{3} e^t + t$

$= \frac{1}{3} e^{1-3t} \; \ldots =$?

$\emptyset$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OXO

POPUNJAVA NASTAVNIK

IME I PREZIME: JOSIP MARIĆ

BROJ INDEKSA: 17-2-0227-2012

	Broj ↓ bodova
1. Riješiti diferencijalnu jednadžbu: $4y'' - y = x \sin x$ i na kraju provjeriti rješenje.	20
2. Riješi diferencijalnu jednadžbu $x^2 y y' = 1 - x^2$ uz rubni uvjet $y(1) = 1$.	15
3. Obrazloži diferencijabilnost funkcije $f(x,y) = e^y + 2xy + \frac{1}{y}$ na njenoj domeni.	10
4. $\int_0^{\pi} (x \cos x + e^{1-3x})\, dx = ?$	20
5. $\int_0^{2} \frac{x^2\, dx}{1+x} = ?$	15
6. Četverokut $ABCD$ je određen točkama $A(0,0)$, $B(3,0)$, $C(3,3)$ i $D(2,1)$. Integriranjem mu pronađi površinu.	20
Ukupno:	Ø

f	$\frac{df}{dx}$
x^{α} $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_a x$ $(a > 0)$	$\frac{1}{x \ln a}$
e^x	e^x
a^x $(a > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

1. $4y'' - y = x \sin x$

$x^2 y y' = 1 - x^2 \quad \{y(1) = 1\}$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OXO

IME I PREZIME: TONI STOŠIĆ **BROJ INDEKSA:** 57817-2009

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $4y'' - y = x \sin x$ i na kraju provjeriti rješenje. ~~20~~
2. Riješi diferencijalnu jednadžbu $x^2 y y' = 1 - x^2$ uz rubni uvjet $y(1) = 1$. 15
3. Obrazloži diferencijabilnost funkcije $f(x, y) = e^y + 2xy + \frac{1}{y}$ na njenoj domeni. ~~10~~
4. $\displaystyle\int_0^{\pi} \left(x \cos x + e^{1-3x}\right) \mathrm{d}x = ?$ ~~20~~
5. $\displaystyle\int_0^{2} \frac{x^2 \, \mathrm{d}x}{1+x} = ?$ ~~15~~
6. Četverokut $ABCD$ je određen točkama $A(0,0)$, $B(3,0)$, $C(3,3)$ i $D(2,1)$. Integriranjem mu pronađi površinu. 20

Ukupno: Ø

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_a x \ (a > 0)$	$\frac{1}{x \ln a}$
e^x	e^x
$a^x \ (a > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
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Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, \mathrm{d}x = -\ln \lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln \lvert x \rvert + C$	$\int \cot x \, \mathrm{d}x = \ln \lvert \sin x \rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, \mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, \mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, \mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2} \, \mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

5.

$$\int_0^2 \frac{x^2 \, dx}{1+x}$$

$$\int \frac{x^2 \, dx}{1+x}$$

Ø

3. $f(x,y) = e^y + 2xy + \frac{1}{y}$

$\partial_x f = 2y$

$\partial_{xx} f = 0$

$\partial_{xy} f = 2$

$\partial_y f = e^y + 2x + 1$

$\partial_{yy} f = e^y$

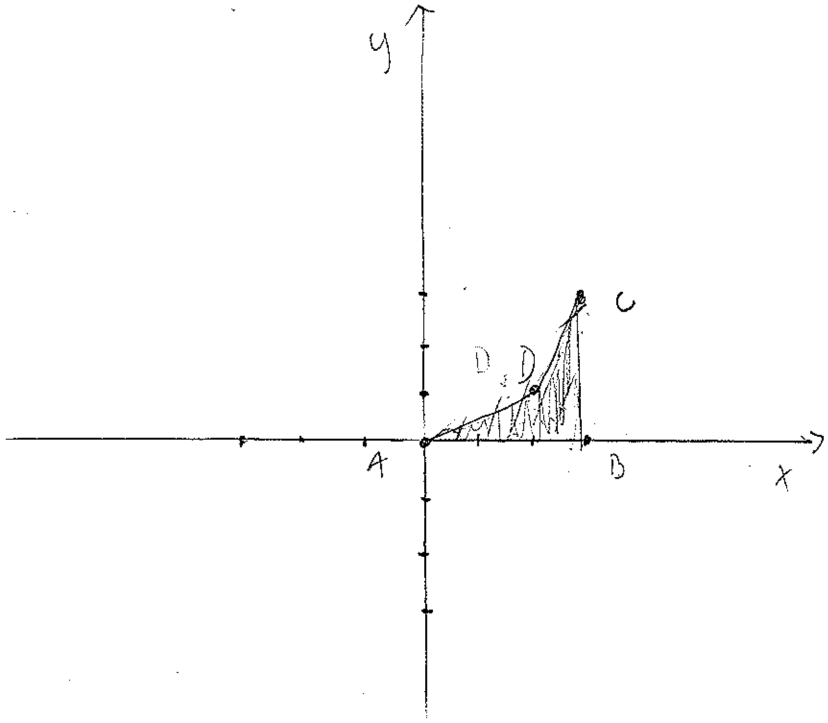
STACIONARNE TOČKE:

$\partial_x f = 2y$

$\partial_y f = e^y + 2x + 1$

$\emptyset$

1.



$\frac{1}{4}$

$A(0,0)$ $B(3,0)$

$x_1\,y_1$ $x_2\,y_2$

$$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$$

$$(3-0)(y-0)=(0-0)(x-0)$$

$$3y=0$$

$$y=0$$

$B(3,0)$ $C(3,3)$

$x_1\,y_1$ $x_2\,y_2$

$$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$$

$$(3-3)(y-0)=(3-3)(x-3)$$

$$0=0$$

4. $\int_0^{\pi} \left(x \cos x + e^{1-3x}\right) dx =$

$$\int \left(x \cos x + e^{1-3x}\right) dx = \int x \cos x \, dx \int e^{1-3x} \, dx =$$

$$\int x \cos x \, dx = \left[\begin{array}{l} u = x \,,\ dv = \cos x \, dx \\ du = 1 \,,\ v = \int \cos x \\ \qquad\qquad v = -\sin x \, dx \end{array} \right.$$