MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: MATEO BOBAČEK **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + 2f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 4,\ f'(0) = 4,\ f''(0) = 5\,.$$

2. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t) = \frac{t}{4}\mathbf{i} + (\cos(t) + 5)\,\mathbf{j} + \sin t\mathbf{k}$, $t \in [0, 4\pi]$. Zadano je skalarno polje $f(x, y, z) = 1 + z$. Izračunaj $\int_C f\mathrm{d}s$. 20

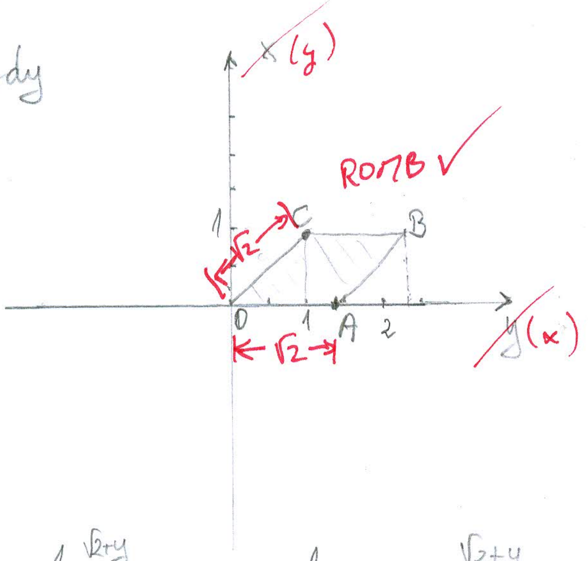
3. Izaberi bilo koji romb R u ravnini i na njemu odredi integral $\iint_R x + y\,\mathrm{d}x\mathrm{d}y$. 20

4. Izračunati volumen tijela omeđenog ravninama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 5 + x^2$, $z = -y^2$. ✓ ~~20~~ 15

5. Izračunati $\iint_S (x^2 + y^2)\mathrm{d}S$ ako je S kružni stožac zadan jednadžbom $z = \sqrt{x^2 + y^2}$ i $0 \leq z \leq 5$. ~~20~~

Ukupno: 75

③ $\iint_R x+y\,dx\,dy$



$c^2 = a^2 + b^2$

$c^2 = 1 + 1$

$c = \sqrt{2}$

$O(0,0)$

$A(\sqrt{2},0)$

$B(1+\sqrt{2},1)$

$C(1,1)$

$\overline{CB} \ldots y = 1$

$\overline{OA} \ldots y = 0$

$(x_2 - x_1)(y - y_1) = (y_2 - y_1)(x - x_1)$

$(1 + \sqrt{2} - \sqrt{2})(y - 0) = (1 - 0)(x - \sqrt{2})$

$\overline{AB} \ldots y = x - \sqrt{2}$

$O(0,0) \quad C(1,1) \qquad \sqrt{2} + y = x$

$(1-0)(y-0) = (1-0)(x-0)$

$\overline{OC} \ldots y = x$

$x = y$

$$\iint_R x+y\,dx\,dy = \int_0^1\int_y^{\sqrt{2}+y}(x+y)\,dx\,dy = \int_0^1 \frac{x^2}{2} + yx \Big|_y^{\sqrt{2}+y} dy \quad ✓$$

$$= \int_0^1 \left[\frac{1}{2}(\sqrt{2}+y)^2 + y(\sqrt{2}+y) - \left(\frac{1}{2}y^2 + y^2\right)\right] dy$$

$$= \int_0^1 \left[\frac{1}{2}(2 + 2\sqrt{2}y + y^2) + \sqrt{2}y + y^2 - \frac{1}{2}y^2 - y^2\right] dy$$

$$= \int_0^1 \left(1 + \sqrt{2}y + \cancel{\frac{1}{2}y^2} + \sqrt{2}y \cancel{+y^2} \cancel{-\frac{1}{2}y^2} \cancel{-y^2}\right) dy = \int_0^1 (2\sqrt{2}y + 1)\,dy =$$

$$= \cancel{2}\sqrt{2}\,\frac{y^2}{\cancel{2}} + y \Big|_0^1 = \sqrt{2}y^2 + y \Big|_0^1 = \sqrt{2} + 1$$

④ $x=1$

$x=-1$

$y=1$

$y=-1$

$x\in[-1,1]$

$y\in[-1,1]$

$z\in[-y,5+x^2]$

$z=5+x^2$

$z=-y^2$

$$V=\int_{-1}^{1}\int_{-1}^{1}\int_{-y^2}^{5+x^2} 1\,dz\,dx\,dy \checkmark = \checkmark \int_{-1}^{1}\int_{-1}^{1} z\Big|_{-y^2}^{5+x^2} dx\,dy \qquad \underline{15}$$

$$=\int_{-1}^{1}\int_{-1}^{1}(5+x^2-y^2)\,dx\,dy = \int_{-1}^{1} 5x+\frac{1}{3}x^3-y^2x\Big|_{-1}^{1} dy = \int_{-1}^{1}\left[5+\frac{1}{3}-y^2-\left(-5-\frac{1}{3}+y^2\right)\right]dy$$

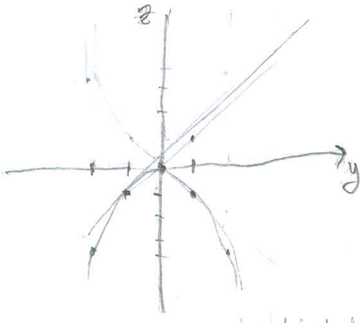
$\times$ $\times$

$$=\int_{-1}^{1}\left(\frac{16}{3}-y^2+\frac{16}{3}-y^2\right)dy = \int_{-1}^{1}\left(-2y^2+\frac{32}{3}\right)dy = -\frac{2}{3}y^3+\frac{32}{3}y\Big|_{-1}^{1}$$

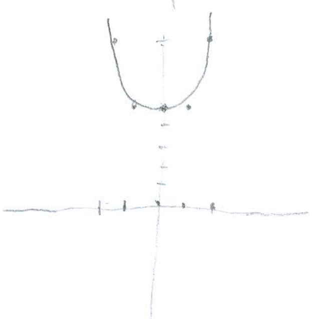
$$=-\frac{2}{3}+\frac{32}{3}-\left(-\frac{2}{3}\cdot(-1)^3+\frac{32}{3}(-1)\right) = -\frac{2}{3}+\frac{32}{3}-\frac{2}{3}+\frac{32}{3}$$

$$=20$$

$-y^2$	0	-1	1	-2	2
$z=-y^2$	0	-1	-1	-4	-4



x	0	1	-1	2	-2
$z=x^2+5$	5	5	5	9	9



② $$r=\begin{bmatrix}\frac{t}{4}\\ \cos(t)+5\\ \sin(t)\end{bmatrix} \qquad t\in[0,4\pi]$$

$$r'(t)=\begin{bmatrix}\frac{1}{4}\\ -\sin(t)\\ \cos t\end{bmatrix}$$

$$\|r'(t)\|=\sqrt{\left(\frac{1}{4}\right)^2+(-\sin(t))^2+(\cos(t))^2} = \sqrt{\frac{1}{16}+\sin^2(t)+\cos^2(t)} = \sqrt{\frac{1}{16}+1} = \sqrt{\frac{17}{16}} = \frac{\sqrt{17}}{4}$$

$$\int_C f\,ds = \int_0^{4\pi}(1+\sin(t))\cdot\frac{\sqrt{17}}{4}\,dt = \int_0^{4\pi}\left(\frac{\sqrt{17}}{4}+\frac{\sqrt{17}}{4}\sin(t)\right)dt = \frac{\sqrt{17}}{4}t-\frac{\sqrt{17}}{4}\cos(t)\Big|_0^{4\pi}$$

$$=\frac{\sqrt{17}}{4}4\pi-\frac{\sqrt{17}}{4}\cos(4\pi)-\left(\frac{\sqrt{17}}{4}\cdot 0-\frac{\sqrt{17}}{4}\cos(0)\right)$$

$$=\sqrt{17}\pi-\frac{\sqrt{17}}{4}+\frac{\sqrt{17}}{4}=\sqrt{17}\pi \quad \checkmark$$

ATEO BOBACEK

$f'''(t) + 2f''(t) + f'(t) + 2f(t) = t \qquad f(0)=4,\ f'(0)=4,\ f''(0)=5$

$s^3F(s) - s^2f(0) - sf'(0) - f''(0) + 2(s^2F(s) - sf(0) - f'(0)) + sF(s) - f(0) + 2F(s) = \frac{1}{s^2}$

$s^3F(s) - 4s^2 - 4s - 5 + 2(s^2F(s) - 4s - 4) + sF(s) - 4 + 2F(s) = \frac{1}{s^2}$

$s^3F(s) - 4s^2 - 4s - 5 + \underline{2s^2F(s)} - 8s - 8 + \underline{sF(s)} - 4 + \underline{2F(s)} = \frac{1}{s^2}$

$s^3 + s + 2s^2 + 2$
$= s(s^2+1) + 2(s^2+1)$
$= (s+2)(s^2+1)$

$F(s)(s^3 + 2s^2 + s + 2) - 4s^2 - 4s - 8s - 17 = \frac{1}{s^2}$

$F(s)(s^3 + 2s^2 + s + 2) = \frac{1}{s^2} + 4s^2 + 12s + 17$

$F(s)(s^3 + 2s^2 + s + 2) = \frac{1 + 4s^4 + 12s^3 + 17s^2}{s^2} \quad / \cdot \frac{1}{(s^3+2s^2+s+2)}$

$F(s) = \frac{4s^4 + 12s^3 + 17s^2 + 1}{s^2(s^3 + 2s^2 + s + 2)} = \frac{4s^4 + 12s^3 + 17s^2 + 1}{s^2(s+2)(s^2+1)} =$

$\downarrow s=-2 \qquad \downarrow \cancel{s^2=-1}$

$\frac{4s^4 + 12s^3 + 17s^2 + 1}{s^2(s+2)(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2} + \frac{Ds+E}{s^2+1} \quad / \cdot s^2(s+2)(s^2+1)$

$4s^4 + 12s^3 + 17s^2 + 1 = As(s^3 + 2s^2 + s + 2) + B(s^3 + 2s^2 + s + 2) + Cs^2(s^2+1) + (s^3 + 2s^2)(Ds + E)$

$4s^4 + 12s^3 + 17s^2 + 1 = As^4 + 2As^3 + As^2 + 2As + Bs^3 + 2Bs^2 + Bs + 2B + Cs^4 + Cs^2 + Ds^4 + Es^3 + 2Ds^3 + 2Es^2$

$4s^4 + 12s^3 + 17s^2 + 1 = s^4(A + C + D) + s^3(2A + B + E + 2D) + s^2(A + 2B + C + 2E) + s(2A + B) + 2B$

$2B = 1 \ /:2$
$\boxed{B = \frac{1}{2}}$

$2A + B = 0$
$2A + \frac{1}{2} = 0$
$2A = -\frac{1}{2}$
$\boxed{A = -\frac{1}{4}}$

$2A + B + E + 2D = 12$
$-\frac{1}{2} + \frac{1}{2} + E + 2D = 12$
$2D = 12 - E$
$D = 6 - \frac{1}{2}E$

$A + 2B + C + 2E = 17$
$-\frac{1}{4} + 1 - \frac{7}{4} + \frac{1}{2}E + 2E = 17$
$-1 + \frac{5}{2}E = 17$
$\frac{5}{2}E = 18$
$\boxed{E = \frac{36}{5}}$

77
-37
40

$A + C + D = 4$
$-\frac{1}{4} + C + 6 - \frac{1}{2}E = 4$
$C = 4 + \frac{1}{4} - 6 + \frac{1}{2}E$
$C = -\frac{7}{4} + \frac{1}{2}E$

$A + C + D = 4$
$-\frac{1}{4} + C + \frac{12}{5} = 4$
$\boxed{C = \frac{37}{20}}$

$2A + B + E + 2D = 12$
$-\frac{1}{2} + \frac{1}{2} + \frac{36}{5} + 2D = 12$
$2D = \frac{24}{5} \ /:2$
$\boxed{D = \frac{12}{5}}$

$\frac{37}{10} - \frac{37}{10} + \frac{74}{5} - \frac{74}{5} = 0$

$\frac{24}{5} + \frac{36}{5} - \frac{24}{5} - \frac{36}{5} = 0$

$F(s) = \frac{-\frac{1}{4}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{\frac{37}{20}}{s+2} + \frac{\frac{12}{5}s + \frac{36}{5}}{s^2+1}$

$F(s) = \frac{-\frac{1}{4}}{s} + \frac{1}{2}\cdot\frac{1}{s^2} + \frac{37}{20}\cdot\frac{1}{s+2} + \frac{12}{5}\cdot\frac{s}{s^2+1^2} + \frac{36}{5}\,\frac{1}{s^2+1^2}$

$f(t) = -\frac{1}{4} + \frac{1}{2}t + \frac{37}{20}\cdot e^{-2t} + \frac{12}{5}\cos(t) + \frac{36}{5}\sin(t)$

$f(0) = -\frac{1}{4} + \frac{37}{20} + \frac{12}{5} = \frac{-5+37+48}{20} = 4$ ✓

$f'(t) = \frac{1}{2} - \frac{37}{10}e^{-2t} - \frac{12}{5}\sin t + \frac{36}{5}\cos t$

$f'(0) = \frac{1}{2} - \frac{37}{10} + \frac{36}{5} = \frac{5-37+72}{10} = 4$

$f''(t) = \frac{37}{5}e^{-2t} - \frac{12}{5}\cos t - \frac{36}{5}\sin t$

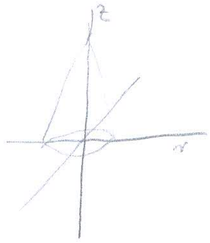
$f''(0) = \frac{37}{5} - \frac{12}{5} = \frac{25}{5} = 5$ ✓

$f'''(t) = -\frac{74}{5}e^{-2t} + \frac{12}{5}\sin t - \frac{36}{5}\cos t$

$f''' + 2f'' + f' + 2f = t + 0\cdot e^{-2t} + 0\cdot\cos t + 0\cdot\sin t = t$ ✓

MATEO BOBAČEK

⑤ $z=\sqrt{x^2+y^2}$ $0\leq z\leq 5$ $\iint\limits_S (x^2+y^2)ds$



$z^2=x^2+y^2$

$x^2+y^2=r^2$ $r\in[0,5]$

$z^2=r^2/\sqrt{\ }$

$z=r$

$z=5$

$r=5$

0

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: ANTE RAŽOV **BROJ INDEKSA:** 0269082684

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$f'''(t) + 2f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 4,\ f'(0) = 4,\ f''(0) = 5\,.$$

2. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t) = \frac{t}{4}\mathbf{i} + (\cos(t) + 5)\,\mathbf{j} + \sin t\mathbf{k}$, $t \in [0, 4\pi]$. Zadano je skalarno polje $f(x, y, z) = 1 + z$. Izračunaj $\int_C f\mathrm{d}s$. ~~20~~

3. Izaberi bilo koji romb R u ravnini i na njemu odredi integral $\iint_R x + y\,\mathrm{d}x\mathrm{d}y$. ~~20~~

4. Izračunati volumen tijela omeđenog ravninama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 5 + x^2$, $z = -y^2$. ~~20~~

5. Izračunati $\iint_S (x^2 + y^2)\mathrm{d}S$ ako je S kružni stožac zadan jednadžbom $z = \sqrt{x^2 + y^2}$ i $0 \le z \le 5$. 20

Ukupno: ~~0~~

② $r(t) = \frac{t}{4} i + (\cos(t)+5) j + \sin t k, \quad t \in (0, 4\pi) \quad f(x,y,z) = 1+z$

$r \begin{bmatrix} \frac{t}{4} \\ \cos t + 5 \\ \sin t \end{bmatrix} \rightarrow t = (0, 4\pi) \qquad \|r'(t)\| \sqrt{\left(\frac{1}{4}\right)^2 + \sin^2 t + \cos^2 t} = \sqrt{\frac{1}{16} + 0 + 0} = \frac{1}{4}$

$r' \begin{bmatrix} \frac{1}{4} \\ -\sin t \\ \cos t \end{bmatrix}$

$Dz \int_{\Gamma} (1+z)\,dz \quad \int_0^{4\pi} \|r'(t)\|(1+z)\,dt = \int_0^{4\pi} \underline{\left(\frac{1}{4} + \frac{1}{4} z\right)\Big|_z = \frac{1}{4} z + \frac{1}{4}\frac{z^2}{2}\Big|_0^{4\pi}} = \pi + \frac{1}{4}\cdot\frac{16\pi}{2} =$ ×

$= \pi + 2\pi = 3\pi$

③ $\iint x + y\,dx\,dy \left| \begin{matrix} x\,dx = t \\ y\,dy = u \end{matrix} \right| = \underline{\int_0^1 t + u} = \frac{t^2}{2} + \frac{u^2}{2}\Big|_0^1 = \frac{1}{2}x^2 + \frac{1}{2}y^2\Big|_0^1 = \frac{1}{2} + \frac{1}{2} = 1$ ×

④ V = ?. RAVNINE $\quad x = 1 \quad x = -1 \quad y = 1 \quad y = -1 \quad z = 5 + x^2 \quad z = -y^2$

y 2 r

$V = \iiint r\,dz\,dx\,dy = \underline{\int_0^{2\pi}\int_{-1}^{1}\int_{5+\cos\varphi}^{-\sin^2\varphi} r\,dr\,dx\,dy}$ ×

$x = 5 + \cos\varphi$

$y = -\sin^2\varphi$

→ OKRENI →

$$V=\int_0^{2\pi}\int_{-1}^{1}\int_{5+\cos\varphi}^{-\sin^2\varphi} r\,dz\,dr\,d\varphi=\int_0^{2\pi}\int_{-1}^{1} r\left(\left(-\sin^2\varphi\right)-\left(5+\cos\varphi\right)\right)dr\,d\varphi=\int_0^{2\pi}\int_{-1}^{1}\left(r\sin^2\varphi-5r-r\cos\varphi\right)dr\,d\varphi=$$

$$V=\int_0^{2\pi}\left(-\frac{r^2}{2}\cos^2\varphi-5\frac{r^2}{2}+r\sin\varphi\right)\Big|_{-1}^{1}d\varphi=\int_0^{2\pi}\left(-\frac{1}{2}\cos^2\varphi-\frac{5}{2}+\sin\varphi\right)-\left(-\frac{1}{2}\cos^2\varphi-\frac{5}{2}-\sin\varphi\right)=$$

$$V=\int_0^{2\pi}\left(\cancel{-\frac{1}{2}\cos^2\varphi}\cancel{-\frac{5}{2}}+\sin\varphi\cancel{+\frac{1}{2}\cos^2\varphi}\cancel{+\frac{5}{2}}+\sin\varphi\right)d\varphi=2\pi$$

~~⑤ $\iint_S (x^2+y^2)\,dS$ S → [illegible] $z=\sqrt{x^2+y^2}$; $0\leq z\leq 5$~~

① NASTAVAK S DRUGOG PAPIRA ZADATAK BR. ①

$$F(s)=\frac{1}{4}\frac{1}{s}+\frac{1}{2}\frac{1}{s^2}+\frac{7}{4}\frac{1}{s+2}+\frac{2s}{s^2+1}+7\frac{1}{s^2+1}$$

$$f(t)=\frac{1}{4}+\frac{1}{2}t+\frac{7}{4}e^{-t}+2\cos t+7\sin t \quad \times$$

$$f(0)=\frac{1}{4}+\frac{7}{4}+2=4 \checkmark$$

$$f'(t)=\frac{1}{2}-\frac{7}{4}e^{t}-2\sin t+7\cos t$$

$$f'(0)=\frac{1}{2}-\frac{7}{4}+7=5 \quad \times$$

03.09.14.   ANTE KAZOV   Br. Ind. 0269082684   MATEMATIKA 3

① $f'''(t) + 2f''(t) + f'(t) + 2f(t) = t$ $f(0)=4$ $f'(0)=4$ $f''(0)=5$

$$s^3F(s) - s^2\overset{=4}{f(0)} - s\overset{=4}{f'(0)} - \overset{=5}{f''(0)} + 2\left(s^2F(s) - s\overset{=4}{f(0)} - \overset{=4}{f'(0)}\right) + sF(s) - \overset{=4}{f(0)} + 2F(s) = \frac{1}{s^2}$$

$$s^3F(s) - 4s^2 - 4s - 5 + 2s^2F(s) - 8s - 8 + sF(s) - 4 + 2F(s) = \frac{1}{s^2}$$

$$F(s)(s^3 + 2s^2 + s + 2) - 4s^2 - 4s - 5 - 8s - 8 - 4 = \frac{1}{s^2}$$

$$F(s)(s^3 + 2s^2 + s + 2) - 4s^2 - 12s - 17 = \frac{1}{s^2}$$

$$F(s)(s^3 + 2s^2 + s + 2) = 4s^2 + 12s + 17 + \frac{1}{s^2} = \frac{4s^4 + 12s^3 + 17s^2 + 1}{s^2} \Big/ \cdot s^3 + 2s^2 + s + 2)$$

$$F(s) = \frac{4s^4 + 12s^3 + 17s^2 + 1}{s^2(s^3 + 2s^2 + s + 2)} = \frac{4s^4 + 12s^3 + 17s^2 + 1}{s^2(s+2)(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2} + \frac{Ds+E}{s^2+1} \Big/ \cdot s^2(s+2)(s^2+1)$$

$$4s^4 + 12s^3 + 17s^2 + 1 = (As^2 + 2As)(s^2+1) + (Bs + 2B)(s^2+1) + Cs^2(s^2+1) + Ds^4 + 2Ds^3 + Es^3 + 2Es^2$$

$$4s^4 + 12s^3 + 17s^2 + 1 = \underline{As^4} + \underline{As^2} + \underline{2As^3} + \underline{2As} + \underline{Bs^3} + \underline{Bs} + \underline{2Bs^2} + 2B + \underline{Cs^4} + \underline{Cs^2} + \underline{Ds^4} + \underline{2Ds^3} + \underline{Es^3} + \underline{2Es^2}$$

$A + C + D = 4$

$\frac{1}{4} + \frac{7}{4} + D = 4$

$D = 4 - 2$

$\boxed{D = 2}$

$\frac{1}{4} + 2 \cdot \frac{1}{2} + \frac{7}{4} + 2E = 17$

$\frac{1}{4} + 1 + \frac{7}{4} + 2E = 17$

$2E = 17 - 3$

$\boxed{E = 7}$

$2A + B + 2D + E = 12$

$2 \cdot \frac{1}{4} + \frac{1}{2} + 2D + E = 12$

$2D + E = 11$

$E = 11 - 2D$

$A + 2B + C + 2E = 17$

$\frac{1}{4} + 1 + C + 22 - \cancel{4D} = 17$

$C - 4D = 17 - 22 - 1 - \frac{1}{4}$

$C - 4D = -6\frac{1}{4}$

$C + D = 4 - \frac{1}{4} \Big/ \cdot 4$

$4C + 4D = 16 - 1$

$4C + \cancel{4D} = 15$

$C - \cancel{4D} = -6\frac{1}{4}$

$5C = 15 - \frac{25}{4}$

$5C = \frac{60-25}{4} = \frac{35}{4}$

$\boxed{C = \frac{\frac{35}{4}}{\frac{5}{1}} = \frac{7}{4}}$

$2A + B = 1$

$2A = 1 - \frac{1}{2}$

$2A = \frac{1}{2}$

$2B = 1$

$\boxed{A = \frac{1}{4}}$ $\boxed{B = \frac{1}{2}}$

$A = \frac{\frac{1}{2}}{2} = \frac{1}{4}$

→ RJEŠENJE NA PRVOM PAPIRU

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: MARIN GVOZDEN

BROJ INDEKSA: 17-2-0137-2011.

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~
$$f'''(t) + 2\,f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 4,\ f'(0) = 4,\ f''(0) = 5\,.$$
2. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t) = \frac{t}{4}\mathbf{i} + (\cos(t) + 5)\,\mathbf{j} + \sin t\mathbf{k}$, $t \in [0, 4\pi]$. Zadano je skalarno polje $f(x, y, z) = 1 + z$. Izračunaj $\int_C f\mathrm{d}s$. 20
3. Izaberi bilo koji romb R u ravnini i na njemu odredi integral $\iint_R x + y\,\mathrm{d}x\mathrm{d}y$. ~~20~~
4. Izračunati volumen tijela omeđenog ravninama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 5 + x^2$, $z = -y^2$. ~~20~~
5. Izračunati $\iint_S (x^2 + y^2)\mathrm{d}S$ ako je S kružni stožac zadan jednadžbom $z = \sqrt{x^2 + y^2}$ i $0 \le z \le 5$. ~~20~~

Ukupno:
20

2.

$$r(t) = \begin{pmatrix} \frac{t}{4} \\ \cos(t)+5 \\ \sin t \end{pmatrix} \qquad r'(t) = \begin{pmatrix} \frac{1}{4} \\ -\sin t \\ \cos t \end{pmatrix} \qquad t \in [0, 4\pi]$$

$$\|r'(t)\| = \sqrt{\left(\frac{1}{4}\right)^2 + (-\sin t)^2 + (\cos t)^2}$$

$$= \sqrt{\frac{1}{16} + (\sin^2 t + \cos^2 t)} = \sqrt{\frac{17}{16}} = \frac{\sqrt{17}}{4}$$

$$\int f\,ds = \int_0^{4\pi} \frac{\sqrt{17}}{4}(1 + \sin t)\,dt = \int_0^{4\pi} \left(\frac{\sqrt{17}}{4} + \frac{\sqrt{17}}{4}\sin t\right) dt =$$

$$= \left(\frac{\sqrt{17}}{4}t - \frac{\sqrt{17}}{4}\cos t\right)\Big|_0^{4\pi} = \left(\frac{\sqrt{17}}{\cancel{4}}\cancel{4}\pi - \frac{\sqrt{17}}{4}\right) -$$

$$\left(0 - \frac{\sqrt{17}}{4}\right) = \sqrt{17}\pi - \frac{\sqrt{17}}{4} + \frac{\sqrt{17}}{4} = \underline{\underline{\sqrt{17}\pi}}$$ ✓

1.

$$f'''(t)+2f''(t)+f'(t)+2f(t)=t$$

$f(0)=4$

$f'(0)=4$

$f''(0)=5$

$$s^3F(s)-s^2f(0)-sf'(0)-f''(0)+2\left(s^2F(s)-sf(0)-f'(0)\right)+$$

$$+sF(s)-f(0)+\frac{1}{2}F\left(\frac{s}{2}\right)=\frac{1}{s^2}$$ DALJE?

5* NASTAVAK

$$\int_0^{2\pi}\int_0^5\left(25\cos^2\varphi+25\sin^2\varphi\right)\cdot\left(5-r^2\right)=\int_0^{2\pi}\int_0^5\left(125\cos^2\varphi+125\sin^2\varphi-\right.$$

$$\left.25r^2\cos^2\varphi-25r^2\sin^2\varphi\right)dr\,d\varphi=\int_0^{2\pi}\left(125r\cos^2\varphi+125r\sin^2\varphi\right.$$

$$\left.-\frac{25}{3}r^3\cos^2\varphi-\frac{25}{3}r^3\sin^2\varphi\right)\Big|_0^5 d\varphi=\int_0^{2\pi}\left(625\cos^2\varphi+625\sin^2\varphi\right.$$

$$\left.-\frac{3125}{3}\cos\varphi-\frac{3125}{3}\sin^2\varphi\right)\cancel{d\varphi}=\int_0^{2\pi}\left(\frac{625}{2}(1+\cos2\varphi)+\frac{625}{2}(1-\cos2\varphi)\right.$$

$$\left.-\frac{3125}{6}(1+\cos2\varphi)-\frac{3125}{6}(1-\cos2\varphi)\right)=0$$ //

MARIN GVOZDEN

4. $x=1 \quad y=1 \quad z=5+x^2$

$x=-1 \quad y=-1$

$z=-y^2 \Longrightarrow -(-1)^2=-1$

$$\int_{-1}^{1}\int_{-1}^{1}\int_{\textcircled{-1}\times}^{5+x^2} 1\,dz\,dx\,dy = \int_{-1}^{1}\int_{-1}^{1} z\Big|_{-1}^{5+x^2} dx\,dy = \int_{-1}^{1}\int_{-1}^{1}(5+x^2+1)\,dx\,dy =$$

$$\int_{-1}^{1}\int_{-1}^{1}(6+x^2)\,dx\,dy = \int_{-1}^{1}\left(6x+\frac{1}{3}x^3\right)\Big|_{-1}^{1} dy = \int_{-1}^{1}\left(6+\frac{1}{3}\right)-\left(-6-\frac{1}{3}\right)dy$$

$$\int_{-1}^{1}\frac{38}{3}\,dy = \frac{38}{3}y\Big|_{-1}^{1} = \left(\frac{38}{3}+\frac{38}{3}\right) = \frac{76}{3}\,//$$

5.

$z=\sqrt{x^2+y^2} \qquad z\in[r^2,5] \qquad x=r\cos\varphi$

$z^2=x^2+y^2 \qquad r\in[0,5] \qquad y=r\sin\varphi$

$z^2=r^2 \qquad \varphi\in[0,2\pi] \qquad z=z$

$r=5$

$dx\,dy\,dz = r\,dr\,dz\,d\varphi$

$$\int_{0}^{2\pi}\int_{0}^{5}\int_{r^2}^{5}(r^2\cos^2\varphi+r^2\sin^2\varphi)\,dz\,dr\,d\varphi \times = \int_{0}^{2\pi}\int_{0}^{5}(r^2\cos^2\varphi+r^2\sin^2\varphi)\cdot z\Big|_{r^2}^{5}$$

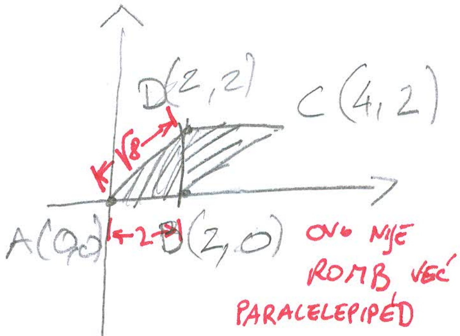
$$\int_{0}^{2\pi}\int_{0}^{5}(r^2\cos^2\varphi+r^2\sin^2\varphi)\cdot(5-r^2) = \int_{0}^{2\pi}\int_{0}^{5}(5r^2\cos^2\varphi+5r^2\sin^2\varphi)-(r^4\cos^2\varphi+r^4\sin^2\varphi)$$

~~$\int_{0}^{2\pi}\left(\frac{5}{3}r^3\cos^2\varphi+\frac{5}{3}r^3\sin^2\varphi\right)-\frac{1}{5}r^5\cos^2\varphi-\frac{1}{5}r^5\sin^2\varphi\Big|_{0}^{5}$~~

~~$\int_{0}^{2\pi}\left(\frac{625}{3}\cos^2\varphi+\frac{625}{3}\sin^2\varphi\right)-625\cos^2\varphi-625\sin^2\varphi$~~

MARIN GVERDSAN

3. $(\overset{4}{x_2}-\overset{2}{x_1})(y-\overset{2}{y_1})=(\overset{2}{y_2}-\overset{2}{y_1})(x-\overset{2}{x_1})$



OVO NIJE ROMB VEĆ PARALELEPIPED

$\iint_{2} x+y\,dx\,dy$

$\overline{AB}$.... $2\cdot y=0\cdot x$

$\underline{y=0}$

$\overline{BC}$.... $2y=2\cdot(x-2)$

$2y=2x-4 \quad /:2$

$\underline{y=x-2}$

$\overline{AD}$.... $2y=2x$

$\underline{y=x}$

$\overline{DC}$.... $2\cdot(y-2)=0\cdot(x-2)$

$2y-4=0$

$2y=4$

$\underline{y=2}$

INT 1.

$$\int_0^2\int_0^x (x+y)\,dy\,dx=\int_0^2\left(xy+\frac{1}{2}y^2\right)\Big|_0^x=\int_0^2\left(x\cdot x+\frac{1}{2}x^2\right)dx=$$

$$=\int_0^2\left(x^2+\frac{1}{2}x^2\right)=\int_0^2\frac{3}{2}x^2\,dx=\frac{1\cancel{3}}{2\cancel{6}}x^3\Big|_0^2=\frac{1}{2}\cdot 8=\underline{4}$$

INT 2.

$$\int_2^4\int_{x-2}^2 (x+y)\,dy\,dx=\int_2^4\left(xy+\frac{1}{2}y^2\right)\Big|_{x-2}^2dx=\int_2^4\left(x\cdot 2+\frac{1}{2}\cdot 4\right)-\left(x\cdot(x-2)+\frac{1}{2}(x-2)^2\right)$$

$$=\int_2^4(\cancel{2x}+2)-x^2-\cancel{2x}+\frac{1}{2}(x^2-4x+4)=\int_2^4 2-x^2+\frac{x^2}{2}-2x+2 \quad \boxed{\frac{16}{3}}$$

$$=\int_2^4\left(-\frac{x^2}{2}-2x+4\right)dx=\left(-\frac{x^3}{6}-x^2+4x\right)\Big|_2^4=\left(-\frac{32}{3}-\cancel{16}+\cancel{16}\right)-\left(-\frac{4\cancel{8}}{\cancel{6}3}-4+8\right)$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: Josip Šimičev **BROJ INDEKSA:** 17-1-OPI-2011

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$f'''(t) + 2\,f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 4,\ f'(0) = 4,\ f''(0) = 5\,.$$

2. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t) = \frac{t}{4}\mathbf{i} + (\cos(t) + 5)\,\mathbf{j} + \sin t\mathbf{k}$, $t \in [0, 4\pi]$. Zadano je skalarno polje $f(x, y, z) = 1 + z$. Izračunaj $\int_C f\mathrm{d}s$. ~~20~~

3. Izaberi bilo koji romb R u ravnini i na njemu odredi integral $\iint_R x + y\,\mathrm{d}x\mathrm{d}y$. ~~20~~

4. Izračunati volumen tijela omeđenog ravninama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 5 + x^2$, $z = -y^2$. 20

5. Izračunati $\iint_S (x^2 + y^2)\mathrm{d}S$ ako je S kružni stožac zadan jednadžbom $z = \sqrt{x^2 + y^2}$ i $0 \le z \le 5$. ~~20~~

Ukupno:
20

4.
$x=1 \quad x=-1$
$y=1 \quad y=-1$
$z=5+x^2$
$z=-y^2$

$$V=\int_{-1}^{1}\int_{-1}^{1}\int_{-y^2}^{5+x^2} 1\,dz\,dy\,dx \ ✓$$

$$=\int_{-1}^{1}\int_{-1}^{1} z\Big|_{-y^2}^{5+x^2} dy\,dx = \int_{-1}^{1}\int_{-1}^{1} 5+x^2+y^2\,dy\,dx$$

$$=\int_{-1}^{1} 5y+yx^2+\frac{y^3}{3}\Big|_{-1}^{1} dx$$

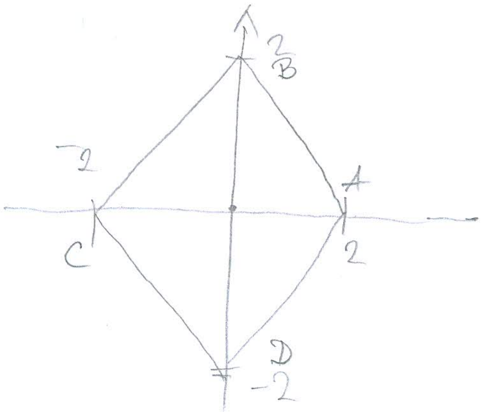
$$=\int_{-1}^{1} 5+x^2+\frac{1}{3}-\left(-5-x^2-\frac{1}{3}\right)dx = \int_{-1}^{1} 5+x^2+\frac{1}{3}+5+x^2+\frac{1}{3}\,dx$$

$$=\int_{-1}^{1} 10+2x^2+\frac{2}{3}\,dx = \int_{-1}^{1} \frac{32}{3}+2x^2\,dx = \frac{32}{3}x+\frac{2}{3}x^3\Big|_{-1}^{1}$$

$$=\left(\frac{32}{3}+\frac{2}{3}\right)-\left(-\frac{32}{3}\right)-\frac{2}{3} = \frac{32}{3}+\frac{2}{3}+\frac{32}{3}+\frac{2}{3} = \frac{68}{3} \ ✓$$

NAŽALOST, UNATOČ MNOGO UČINJENOG, U 4 ZADATKA SE POTKRALA MANJA POGREŠKA, KOJA IH JE UPROPASTILA.

3. $\iint\limits_{D} (x+y)\,dxdy$



A(2,0) C(-2,0)
B(0,2) D(0,-2)

$\overline{AB}$ A(2,0) B(0,2)

$(0-2)(y-0)=(2-0)(x-2)$

$-2y=2x-4$

$y=-x+2$

$\overline{BC}$ (0,2) C(-2,0)

$(-2-0)(y-2)=(0-2)(x-0)$

$-2y+4=-2x$

$-2y=-2x-4$

$y=+x\,\textcircled{+4}$

×

$\overline{AD}$ A(2,0) D(0,-2)

$(0-2)(y-0)=(-2-0)(x-2)$

$-2y=-2x+4$

$y=x-2$

$\overline{CD}$ C(-2,0) D(0,-2)

$(0+2)(y-0)=(-2-0)(x+2)$

$2y=-2x-4$

$y=-x-2$

$$\int_{-2}^{0}\int_{-x-2}^{x+4} 1\,dydx+\int_{0}^{2}\int_{x-2}^{-x+2} 1\,dy\,dx=\int_{-2}^{0} y\Big|_{-x-2}^{x+4} dx+\int_{0}^{2} y\Big|_{x-2}^{-x+2} dx$$

$$=\int_{-2}^{0}(x+4)-(-x-2)\,dx+\int_{0}^{2}(-x+2)-(x-2)\,dx=\int_{-2}^{0}2x+6\,dx+\int_{0}^{2}-2x+4\,dx$$

$$==\frac{\cancel{2}x^2}{\cancel{2}}+6x\Big|_{-2}^{0}-x^2+4x\Big|_{0}^{2}=4-12-4+8=-4$$ ×

RJEŠENJE = 0

1. $f'''(t)+2f''(t)+f'(t)+2f(t)=t \quad f(0)=4 \quad f'(0)=4 \quad f''(0)=5$

$$s^3F(s)-s^2f(0)-sf'(0)-f''(0)+2sF(s)-2sf(0)-2f'(0)+sF(s)-f(0)+2f(s)=\frac{1}{s^2}$$

$$s^3F(s)-4s^2-4s-5+2sF(s)-8s-8+sF(s)-4+2f(s)=\frac{1}{s^2}$$

$$F(s)=(s^3+2s+s+2)=\frac{1}{s^2}+4s^2+4s+8s+17$$

$$=\frac{4s^4+12s^3+17s^2+1}{s^2} \quad /:(s^3+2s+s+2)$$

$(s+2)(s^2+1)$

$$=\frac{4s^4+12s^3+17s^2+1}{s^2(s+2)(s^2+1)} \checkmark$$

$$\frac{A}{s^2}+\frac{B}{s}+\frac{C}{(s+2)}+\frac{Ds+E}{(s^2+1)} \quad /\cdot s^2(s+2)(s^2+1)$$

$$A\cdot(s^3+2s+s+2)+Bs(s^3+2s+s+2)+C(s^4+s^2)+(Ds+E)(s^3+2s^2)$$

$$=As^3+2As+As+2A+Bs^4+2Bs^2+Bs^2+2Bs+Cs^4+Cs^2+Ds^4+2Ds^3+Es^3+2Es^2$$

$$=As^3+3As+2A+Bs^4+3Bs^2+2Bs+Cs^4+Cs^2+Ds^4+2Ds^3+Es^3+2Es^2$$

$4s^4+12s^3+17s^2+1=$

$A=\frac{1}{2}$

$B+C+D=4$

$A+2D+E=12$

$3B+C+2E=17$

$2B+3A=0$

$2A=1$

$A=\frac{1}{2}$

$C+D=4-B=4+\frac{3}{4}=\frac{19}{4}$

$2D+E=12-A=12-\frac{1}{2}=\frac{23}{2}$

$\underline{C+2E=17-3B=17+\frac{9}{4}=\frac{77}{4}}$

$2B=-3A$

$B=-\frac{3}{2}A=-\frac{3}{2}\cdot\frac{1}{2}=-\frac{3}{4}$

$E=\frac{23}{2}-2D$

$C+2E=C+23-4D=\frac{77}{4}/:4$

$C=\frac{19}{4}-D$;

$C+D=\frac{19}{4}$

~~RJEŠENJE~~ ?

IZA ↓

$D=\frac{19}{4}-C=\frac{19}{4}-\frac{91}{12}=\boxed{-\frac{17}{6}}$

$2E=\frac{77}{4}-C$

$=\frac{77}{4}-\frac{91}{12}=\frac{35}{6}$

$E=\boxed{\frac{35}{6}}$

$C=\boxed{\frac{91}{12}}$

$\left.\begin{aligned}\frac{1}{4}C+\frac{23}{4}-D&=\frac{77}{16}\\ C+D&=\frac{19}{4}\end{aligned}\right\}-$

$-\frac{3}{4}C+\frac{23}{4}=\frac{1}{16}$

$-\frac{3}{4}C=\frac{1}{16}-\frac{23}{4}=-\frac{91}{16}/:-\frac{3}{4}$

Josip Šimičev

2. $r(t) = \frac{t}{4} i + \cos(t) + 5 j + \sin t\, k \qquad t \in [0, 4\pi] \qquad f(x,y,z) = 1 + z \quad \int_c f ds.$

$$r(t) = \begin{pmatrix} \frac{t}{4} \\ \cos t + 5 \\ \sin t \end{pmatrix} = r'(t) = \begin{pmatrix} \frac{1}{4} \\ -\sin t \\ \cos t \end{pmatrix}$$

$$\|r'(t)\| = \sqrt{\left(\frac{1}{4}\right)^2 + (-\sin t)^2 + (\cos t)^2} = \sqrt{\frac{1}{16} + \underbrace{\sin^2 t + \cos^2 t}_{1}} = \sqrt{\frac{1}{16} + 1} = \frac{1}{4} \quad \frac{\sqrt{17}}{4} \quad X$$

$$\int_0^{4\pi} (1 + \sin t) \cdot \frac{1}{4} dt = \int_0^{4\pi} 1 + \frac{1}{4} \sin t \, dt = 1\ell - \frac{1}{4} \cos \ell \Big|_0^{4\pi} \qquad X$$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

---

1. NASTAVAK

$A = \frac{1}{2} \quad B = -\frac{3}{4} \quad C = \frac{91}{12} \quad D = -\frac{17}{6} \quad E = \frac{35}{6}$

$$F(s) = \frac{1}{2} s^2 - \frac{3}{4} \frac{1}{s} + \frac{91}{12} \frac{1}{s+2} - \frac{17}{6} \frac{s}{(s^2+1)} + \frac{35}{6} \frac{1}{(s^2+1)}$$

$$f(t) = \frac{1}{2} t - \frac{3}{4} + \frac{91}{12} \cdot e^{-2t} - \frac{17}{6} \cdot \cos t + \frac{35}{6} \cdot \sin(t) \quad X$$

$f(0) = 4$

$$f(0) = -\frac{3}{4} + \frac{91}{12} - \frac{17}{6} = \frac{-9+91-34}{12} = \frac{48}{12} = 4 \ ✓$$

$$f'(t) = \frac{1}{2} - \frac{91}{6} e^{-2t} + \frac{17}{6} \sin t + \frac{35}{6} \cos t$$

$$f'(0) = \frac{1}{2} - \frac{91}{6} + \frac{35}{6} = \frac{3 - 91 + 35}{6} = -\frac{53}{6} \neq 4$$

$$\begin{array}{r} 91 \\ -38 \\ \hline 53 : 6 = 8\frac{5}{6} \end{array}$$

Josip Šimičev

$$\iint\limits_S (x^2+y^2)\, ds$$

$z=\sqrt{x^2+y^2} \quad 0 \leq z \leq 5$ $\qquad x^2+y^2=r^2 \quad r=z$

$z^2=x^2+y^2=r^2$ $\qquad r \in [0,5]$

$r^2=z^2=5^2=25 \Rightarrow r=\sqrt{25} = 5$ $\qquad \varphi \in [0,2\pi]$

$z^2=x^2+y^2$

$$\frac{\partial z}{\partial x}=\frac{2x}{2z} \qquad \frac{\partial z}{\partial y}=\frac{2y}{2z}$$

$$=\sqrt{1+\left(\frac{2x}{2z}\right)^2+\left(\frac{2y}{2z}\right)^2}=\sqrt{1+\frac{4x^2}{4z^2}+\frac{4y^2}{4z^2}}=\sqrt{1+\frac{4r^2}{4z^2}}=\sqrt{\frac{4z^2+4z^2}{4z^2}}=\sqrt{\frac{8z^2}{4z^2}}=\sqrt{2} \ ✓$$

NEDOSTAJE FAKTOR r ZBOG PRELASKA U POLARNE KOORDINATE

$$\int_0^{2\pi}\int_0^{5} \underbrace{(r\sin\varphi)^2+(r\cos\varphi)^2\cdot\sqrt{2}}_{} \; \overset{\times}{} dr\, d\varphi$$

$$=\int_0^{2\pi}\int_0^{5} r^2\sin^2\varphi + r^2\cos^2\varphi \, dr\, d\varphi = \int_0^{2\pi}\int_0^{5} \sqrt{2}\, r^2\sin^2\varphi + \sqrt{2}\, r^2\cos^2\varphi \, dr\, d\varphi$$

$$=\int_0^{2\pi} \sqrt{2}\cdot\frac{r^3}{3}\sin^2\varphi + \sqrt{2}\,\frac{r^3}{3}\cos^2\varphi \Big|_0^5 \, d\varphi$$

$$=\int_0^{2\pi} \sqrt{2}\cdot\frac{125}{3}\sin^2\varphi + \sqrt{2}\cdot\frac{125}{3}\cos^2\varphi \, d\varphi$$

$$=\int_0^{2\pi} \sqrt{2}\cdot\frac{125}{3}\underbrace{(\sin^2\varphi+\cos^2\varphi)}_{1} \, d\varphi$$

$$=\sqrt{2}\cdot\frac{125}{3}\,\varphi\Big|_0^{2\pi} = \sqrt{2}\cdot\frac{125}{3}2\pi - \sqrt{2}\cdot\frac{125}{3}\cdot 0 = \frac{\sqrt{2}\cdot 250\pi}{3} = \frac{250\pi\sqrt{2}}{3}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: DARIAN RADMAN **BROJ INDEKSA:** 57635 -2009.

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$f'''(t) + 2\,f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 4,\ f'(0) = 4,\ f''(0) = 5\,.$$

2. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t) = \frac{t}{4}\mathbf{i} + (\cos(t) + 5)\,\mathbf{j} + \sin t\mathbf{k}$, $t \in [0, 4\pi]$. Zadano je skalarno polje $f\,(x, y, z) = 1 + z$. Izračunaj $\int_C f \mathrm{d}s$. 20

3. Izaberi bilo koji romb R u ravnini i na njemu odredi integral $\iint_R x + y\,\mathrm{d}x\mathrm{d}y\,.$ 20

4. Izračunati volumen tijela omeđenog ravninama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 5 + x^2$, $z = -y^2$. 20

5. Izračunati $\iint_S (x^2 + y^2)\mathrm{d}S$ ako je S kružni stožac zadan jednadžbom $z = \sqrt{x^2 + y^2}$ i $0 \le z \le 5$. 20

Ukupno: 60

① $f'''(t) + 2f''(t) + f'(t) + 2f(t) = t$

$f(0) = 4,\ f'(0) = 4,\ f''(0) = 5$

$s^3F(s) - s^2f(0) - sf'(0) - f''(0) + 2s^2F(s) - 2sf(0) - 2f'(0)$

$+ sF(s) - f(0) + 2F(s) = \frac{1}{s^2}$

$s^3F(s) - 4s^2 - 4s - 5 + 2s^2F(s) - 8s - 8 + sF(s) - 4 + 2F(s) = \frac{1}{s^2}$

$F(s)\cdot(s^3 + 2s^2 + s + 2) = \frac{1}{s^2} + 4s^2 + 12s + 17$

$F(s)\cdot(s^3 + 2s^2 + s + 2) = \frac{4s^4 + 12s^3 + 17s^2 + 1}{s^2}$

$F(s) = \frac{4s^4 + 12s^3 + 17s^2 + 1}{s^2\cdot(s+2)(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2} + \frac{Ds + E}{s^2 + 1}$

$4s^4 + 12s^3 + 17s^2 + 1 = (As^2 + 2As)(s^2+1) + (Bs + 2B)(s^2+1) + Cs^2(s^2+1)$

$+ (Ds + E)(s^3 + 2s^2)$

$4s^4 + 12s^3 + 17s^2 + 1 = As^4 + As^2 + 2As^3 + 2As + Bs^3 + Bs + 2Bs^2 + 2B$

$+ Cs^4 + Cs^2 + Ds^4 + 2Ds^3 + Es^3 + 2Es^2$

$4 = A + C + D$

$+12 = 2A + B + 2D + E$

$+17 = A + 2B + C + 2E$

$0 = 2A + B$

$1 = 2B \Rightarrow B = \frac{1}{2} \Rightarrow A = -\frac{1}{4}$

$\frac{17}{4} = C + D \quad (1)$

$\begin{cases} +12 = 2D + E \quad (2) \quad / \cdot (-2) \\ \frac{65}{4} = C + 2E \end{cases}$

$-24 = -4D - 2E$ $\quad / +$

$\frac{65}{4} = C + 2E$

$-\frac{31}{4} = C - 4D$

$\frac{17}{4} = C + D \quad / \cdot 4$

$-\frac{31}{4} = C - 4D$

$17 = 4C + 4D$ $\quad / +$

$-\frac{31}{4} = C - 4D$

$\frac{37}{4} = 5C \Rightarrow C = \frac{37}{20}$

$D = \frac{12}{5} \Rightarrow E = \frac{36}{5}$

$$F(s) = \frac{-\frac{1}{4}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{\frac{37}{20}}{s+2} + \frac{\frac{12}{5}s + \frac{36}{5}}{s^2+1}$$

$$f(t) = -\frac{1}{4} + \frac{1}{2}t + \frac{37}{20}e^{-2t} + \frac{12}{5}\cos(t) + \frac{36}{5}\sin(t)$$ ×

$f(0) = -\frac{1}{4} + \frac{37}{20} + \frac{12}{5} = \frac{-5+37+24}{20}$

$= \frac{56}{20} \neq 4$

② $r(t) = \frac{t}{4}\vec{i} + (\cos t + 5)\vec{j} + \sin t\,\vec{k}, \quad t \in [0, 4\pi]$

$f(x,y,z) = 1 + z \qquad \int_C f\,ds = ?$

$f_{\circ r}(t) = 1 + \sin t$

$\|r'(t)\| = \sqrt{\left(\frac{1}{4}\right)^2 + (-\sin t)^2 + (\cos t)^2}$

$\|r'(t)\| = \sqrt{\frac{1}{16} + 1} = \sqrt{\frac{17}{16}} = \frac{\sqrt{17}}{4}$

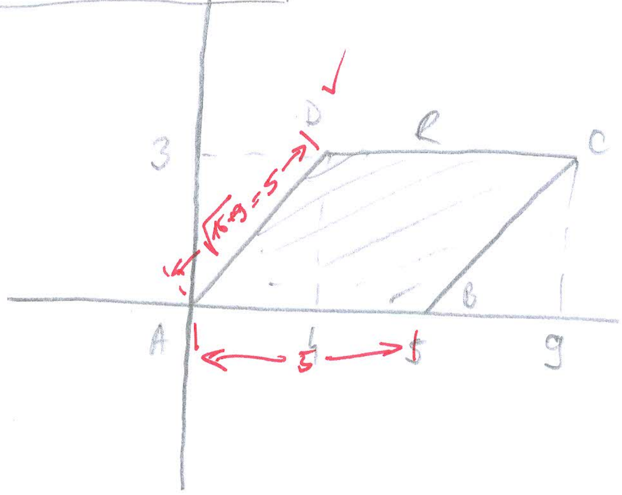
$$\int_C f\,ds = \int_a^b (f_{\circ r}) \cdot \|r'\|\,dt$$

$$= \int_0^{4\pi} (1 + \sin t) \cdot \frac{\sqrt{17}}{4}\,dt = \frac{\sqrt{17}}{4}\int_0^{4\pi} 1 + \sin t\,dt$$ ✓

$$= \frac{\sqrt{17}}{4}\left(t - \cos t \Big|_0^{4\pi}\right) = \frac{\sqrt{17}}{4}(4\pi - 1 - (0 - 1)) = \frac{\sqrt{17}}{4} \cdot (4\pi - 1 + 1) = \underline{\sqrt{17}\,\pi}$$ ✓

DARIAN RADMAN

③ A(0,0)
B(5,0)
C(9,3)
D(4,3)



DOISTA JE ROMB! ✓

$$\iint_R (x+y)\,dx\,dy$$

$$= \int_{y_1=0}^{y_2=3} \int_{x_1=\frac{4}{3}y}^{x_2=\frac{4}{3}y+5} x+y \; dx\,dy$$ ✓

20

$$= \int_{y_1=0}^{y_2=3} \frac{x^2}{2}+yx \Bigg|_{x_1=\frac{4}{3}y}^{x_2=\frac{4}{3}y+5} dy$$

$$= \int_0^3 \frac{16}{8}y^2+\frac{20}{3}y+\frac{25}{2}+\frac{4}{3}y^2+5y-\frac{16}{18}y^2-\frac{4}{3}y^2\,dy$$

$$= \int_0^3 \frac{10}{9}y^2+\frac{35}{3}y+\frac{25}{2}\,dy = \frac{10y^3}{27}+\frac{35y^2}{6}+\frac{25}{2}y\Bigg|_0^3$$

$$= 10+\frac{105}{2}+\frac{75}{2} = 100$$ ✓

$y_{AD} \ldots y=\frac{3}{4}x \nearrow x=\frac{4}{3}y$

$y_{BC} \ldots y=-\frac{15}{4}+\frac{3}{4}x$

$\swarrow$

$\frac{3}{4}x = y+\frac{15}{4} \;/\cdot\frac{4}{3}$

$x=\frac{4}{3}y+5$

④ $x=1$

$x=-1$

$y=1$

$y=-1$

$z=5+x^2$

$z=-y^2$

$$V=\int_{-1}^{1}\int_{-1}^{1}\int_{-y^2}^{5+x^2} dz\,dy\,dx$$ ✓

$$V=\int_{-1}^{1}\int_{-1}^{1} 5+x^2+y^2\,dy\,dx$$

$$V=\int_{-1}^{1} 5y+x^2y+\frac{y^3}{3}\Big|_{-1}^{1}\,dx$$

$$V=\int_{-1}^{1} 5+x^2+\frac{1}{3}-\left(-5-x^2-\frac{1}{3}\right)dx$$

$$V=\int_{-1}^{1} 10+2x^2+\frac{2}{3}\,dx=\int_{-1}^{1}\left(\frac{32}{3}+2x^2\right)dx$$

$$V=\frac{32}{3}x+\frac{2}{3}x^3\Big|_{-1}^{1}=\frac{32}{3}+\frac{2}{3}+\frac{32}{3}+\frac{2}{3}$$

$$V=\frac{68}{3}$$ ✓

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: RIKARDO PEROVIĆ **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~
$$f'''(t) + 2\,f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 4,\ f'(0) = 4,\ f''(0) = 5\,.$$
2. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t) = \frac{t}{4}\mathbf{i} + (\cos(t) + 5)\,\mathbf{j} + \sin t\mathbf{k}$, $t \in [0, 4\pi]$. Zadano je skalarno polje $f\,(x, y, z) = 1 + z$. Izračunaj $\displaystyle\int_C f\mathrm{d}s$. ~~20~~
3. Izaberi bilo koji romb R u ravnini i na njemu odredi integral $\iint_R x + y\,\mathrm{d}x\mathrm{d}y\,.$ 20
4. Izračunati volumen tijela omeđenog ravninama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 5 + x^2$, $z = -y^2$. 20
5. Izračunati $\iint_S (x^2 + y^2)\mathrm{d}S$ ako je S kružni stožac zadan jednadžbom $z = \sqrt{x^2 + y^2}$ i $0 \leq z \leq 5$. 20

Ukupno: 0

2.)

$$\|r\| = \begin{bmatrix} \frac{t}{4} \\ \cos t + 5 \\ \sin t \end{bmatrix} \quad \|r'\| = \begin{bmatrix} \frac{1}{4} \\ -\sin t \\ \cos t \end{bmatrix} \qquad t \in [0, 4\pi]$$

$$f(x, y, z) = 1 + \sin t$$

$$\|r'\| = \sqrt{\left(\tfrac{1}{4}\right)^2 + (-\sin t)^2 + (\cos t)^2} = \sqrt{\tfrac{1}{16} + \sin^2 t + \cos^2 t}$$ ✓

$$= \sqrt{\tfrac{1}{16} + 1} = \sqrt{\tfrac{17}{16}} = \frac{\sqrt{17}}{4}$$ ✓

$$f = \int_0^{4\pi} \underbrace{\frac{\sqrt{17}}{4} + (1+\sin t)}\ dr\,dt = \int_0^{4\pi} \frac{\sqrt{17}}{4}\,dr + \int_0^{4\pi} (1+\sin t)\,dt =$$ ✗

$$= \frac{\sqrt{17}}{4}\, r \Big|_0^{4\pi} + (1 - \cos t)\Big|_0^{4\pi} = \frac{\sqrt{17}}{4} \cdot 4\pi + \left[\cancel{1} - \cancel{1} - (\cancel{1} - \cancel{1})\right]$$

$$= \pi\sqrt{17}$$

RIKARDO PEROVIĆ MAT. III

$f(0)=4$

$f'(0)=4$

$f''(0)=5$

1.) $f'''(t)+2f''(t)+f'(t)+2f(t)=t$

$s^3F(s)-s^2f(0)-sf'(0)-f''(0)+2s^2F(s)-2sf(0)-2f'(0)+sF(s)-f(0)+$
$2F(s)=\frac{1}{s^2}$

$\underline{s^3F(s)}-4s^2-4s-5+\underline{2s^2F(s)}-8-8+\underline{sF(s)}-4+\underline{2F(s)}=\frac{1}{s^2}$

$F(s)(s^3+2s^2+s+2)=\frac{1}{s^2}+4s^2-4s+21$

$Fs(s^3+2s^2+s+2)=\frac{1+4s^4-4s^3+21s}{s^2}\quad /:(s^3+2s^2+s+2)$

$$Fs=\frac{\frac{4s^4-4s^3+21s+1}{s^2}}{s^3+2s^2+s+2}=\frac{4s^4-4s^3+21s+1}{s^2\underbrace{(s^3+2s^2+s+2)}_{\underbrace{[s^2(s+2)+s+2]}_{(s+2)(s^2+1)}}}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: Romana Novosel **BROJ INDEKSA:** 17-2-0097-2011

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + 2\,f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 4,\ f'(0) = 4,\ f''(0) = 5\,.$$

2. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t) = \frac{t}{4}\mathbf{i} + (\cos(t) + 5)\,\mathbf{j} + \sin t\mathbf{k}$, $t \in [0, 4\pi]$. Zadano je skalarno polje $f\,(x, y, z) = 1 + z$. Izračunaj $\int\limits_C f\mathrm{d}s$. 20

3. Izaberi bilo koji romb R u ravnini i na njemu odredi integral $\iint\limits_R x + y\,\mathrm{d}x\mathrm{d}y\,.$ 20

4. Izračunati volumen tijela omeđenog ravninama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 5 + x^2$, $z = -y^2$. 20

5. Izračunati $\iint_S (x^2 + y^2)\mathrm{d}S$ ako je S kružni stožac zadan jednadžbom $z = \sqrt{x^2 + y^2}$ i $0 \leq z \leq 5$. 20

Ukupno: 0

①

$$f'(t) = sF(s) - f(0) \Rightarrow f'(0) = 4$$

$$f''(t) = s^2F(s) - s\,f(0) - f'(0) \Rightarrow f''(0) = 5$$

$$f'''(t) = s^3F(s) - s^2 f(0) - s\,f'(0) - f''(0)$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: IVAN DONAT GRŽAN

BROJ INDEKSA: 57648-2009

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + 2\,f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 4,\ f'(0) = 4,\ f''(0) = 5\,.$$

2. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t) = \frac{t}{4}\mathbf{i} + (\cos(t) + 5)\,\mathbf{j} + \sin t\mathbf{k}$, $t \in [0, 4\pi]$. Zadano je skalarno polje $f\,(x, y, z) = 1 + z$. Izračunaj $\int\limits_C f\mathrm{d}s$. 20

3. Izaberi bilo koji romb R u ravnini i na njemu odredi integral $\iint\limits_R x + y\,\mathrm{d}x\mathrm{d}y\,.$ 20

4. Izračunati volumen tijela omeđenog ravninama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 5 + x^2$, $z = -y^2$. 20

5. Izračunati $\iint_S (x^2 + y^2)\mathrm{d}S$ ako je S kružni stožac zadan jednadžbom $z = \sqrt{x^2 + y^2}$ i $0 \leq z \leq 5$. 20

Ukupno: 0

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: JOSIP HROMIN **BROJ INDEKSA:** 57125

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + 2\,f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 4,\ f'(0) = 4,\ f''(0) = 5\,.$$

2. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t) = \frac{t}{4}\mathbf{i} + (\cos(t) + 5)\,\mathbf{j} + \sin t\mathbf{k}$, $t \in [0, 4\pi]$. Zadano je skalarno polje $f\,(x, y, z) = 1 + z$. Izračunaj $\int\limits_C f\mathrm{d}s$. 20

3. Izaberi bilo koji romb R u ravnini i na njemu odredi integral $\iint\limits_R x + y\,\mathrm{d}x\mathrm{d}y\,.$ 20

4. Izračunati volumen tijela omeđenog ravninama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 5 + x^2$, $z = -y^2$. 20

5. Izračunati $\iint_S (x^2 + y^2)\mathrm{d}S$ ako je S kružni stožac zadan jednadžbom $z = \sqrt{x^2 + y^2}$ i $0 \le z \le 5$. 20

Ukupno:

0