

odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: **MATEO BOBAČEK**

BROJ INDEKSA:

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

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$$f'''(t) + 2f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 4, f'(0) = 4, f''(0) = 5.$$

2. Neka je  $C$  krivulja sa parametrizacijom  $\mathbf{r}(t) = \frac{t}{4}\mathbf{i} + (\cos(t) + 5)\mathbf{j} + \sin t\mathbf{k}, t \in [0, 4\pi]$ . Zadano je skalarno

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polje  $f(x, y, z) = 1 + z$ . Izračunaj  $\int_C f ds$ .

3. Izaberi bilo koji romb  $R$  u ravnini i na njemu odredi integral  $\iint_R x + y dx dy$ .

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4. Izračunati volumen tijela omeđenog ravninama  $x = 1, x = -1, y = 1, y = -1, z = 5 + x^2, z = -y^2$ .

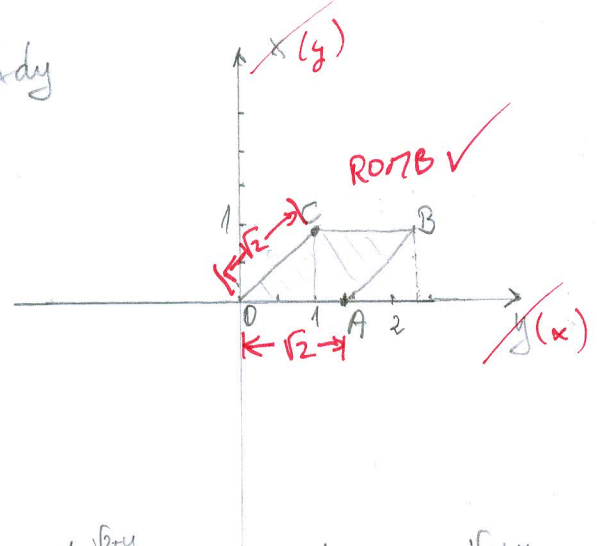
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5. Izračunati  $\iint_S (x^2 + y^2) dS$  ako je  $S$  kružni stožac zadan jednadžbom  $z = \sqrt{x^2 + y^2}$  i  $0 \leq z \leq 5$ .

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Ukupno:  
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3.  $\iint_R x + y dx dy$



$$c^2 = a^2 + b^2$$

$$c^2 = 1 + 1$$

$$c = \sqrt{2}$$

- $O(0,0)$
- $A(1,0)$
- $B(1+\sqrt{2},1)$
- $C(1,1)$

$\overline{CB} \dots y = 1$   
 $\overline{CA} \dots y = 0$

$$(x_2 - x_1)(y - y_1) = (y_2 - y_1)(x - x_1)$$

$$(1 + \sqrt{2} - 1)(y - 0) = (1 - 0)(x - 1 - \sqrt{2})$$

$\overline{AB} \dots y = x - \sqrt{2}$

$O(0,0) \quad C(1,1) \quad \sqrt{2} + y = x$

$$(1 - 0)(y - 0) = (1 - 0)(x - 0)$$

$\overline{OC} \dots y = x$   
 $x = y$

$$\iint_R x + y dx dy = \int_0^1 \int_0^{\sqrt{2}+y} (x+y) dx dy = \int_0^1 \left[ \frac{x^2}{2} + yx \right]_0^{\sqrt{2}+y} dy$$

$$= \int_0^1 \left[ \frac{1}{2}(\sqrt{2}+y)^2 + y(\sqrt{2}+y) - \left( \frac{1}{2}y^2 + y^2 \right) \right] dy$$

$$= \int_0^1 \left[ \frac{1}{2}(2 + 2\sqrt{2}y + y^2) + \sqrt{2}y + y^2 - \frac{1}{2}y^2 - y^2 \right] dy$$

$$= \int_0^1 \left( 1 + \sqrt{2}y + \frac{1}{2}y^2 + \sqrt{2}y + y^2 - \frac{1}{2}y^2 - y^2 \right) dy = \int_0^1 (2\sqrt{2}y + 1) dy =$$

$$= 2\sqrt{2} \frac{y^2}{2} + y \Big|_0^1 = \sqrt{2}y^2 + y \Big|_0^1 = \sqrt{2} + 1$$

④  $x=1$   $x \in [-1, 1]$   
 $x=-1$   $y \in [-1, 1]$   
 $y=1$   
 $y=-1$   $z \in [-y, 5+x^2]$

$$z=5+x^2 \quad z=-y^2 \quad V = \int_{-1}^1 \int_{-1}^1 \int_{-y^2}^{5+x^2} 1 \, dz \, dx \, dy = \int_{-1}^1 \int_{-1}^1 (5+x^2 - (-y^2)) \, dx \, dy$$

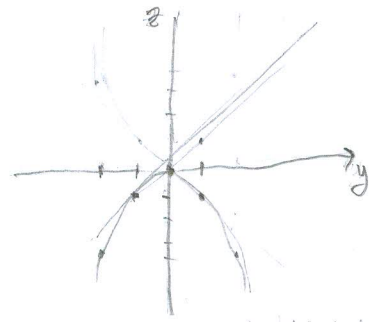
$$= \int_{-1}^1 \int_{-1}^1 (5+x^2 - y^2) \, dx \, dy = \int_{-1}^1 \left[ 5x + \frac{1}{3}x^3 - y^2x \right]_{-1}^1 dy = \int_{-1}^1 \left[ 5 + \frac{1}{3} - y^2 - (-5 - \frac{1}{3} + y^2) \right] dy$$

$$= \int_{-1}^1 \left( \frac{16}{3} - y^2 + \frac{16}{3} - y^2 \right) dy = \int_{-1}^1 (-2y^2 + \frac{32}{3}) dy = -\frac{2}{3}y^3 + \frac{32}{3}y \Big|_{-1}^1$$

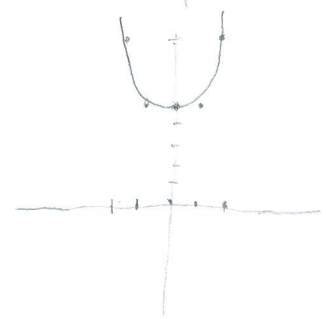
$$= -\frac{2}{3} + \frac{32}{3} - \left( -\frac{2}{3} \cdot (-1)^3 + \frac{32}{3}(-1) \right) = -\frac{2}{3} + \frac{32}{3} - \frac{2}{3} + \frac{32}{3}$$

$$= 20$$

$-y^2$	0	-1	1	-2	2
$z=-y^2$	0	-1	1	-4	-4



$x$	0	1	1	2	2
$z=x^2+5$	5	5	5	6	6



②  $r = \begin{bmatrix} \frac{t}{4} \\ \cos(t)+5 \\ \sin(t) \end{bmatrix} \quad t \in [0, 4\pi]$

$$r'(t) = \begin{bmatrix} \frac{1}{4} \\ -\sin(t) \\ \cos(t) \end{bmatrix}$$

$$\|r'(t)\| = \sqrt{\left(\frac{1}{4}\right)^2 + (-\sin(t))^2 + (\cos(t))^2} = \sqrt{\frac{1}{16} + \sin^2(t) + \cos^2(t)} = \sqrt{\frac{1}{16} + 1} = \sqrt{\frac{17}{16}} = \frac{\sqrt{17}}{4}$$

$$\int_C f \, ds = \int_0^{4\pi} (1 + \sin(t)) \cdot \frac{\sqrt{17}}{4} \, dt = \int_0^{4\pi} \left( \frac{\sqrt{17}}{4} + \frac{\sqrt{17}}{4} \sin(t) \right) dt = \frac{\sqrt{17}}{4}t - \frac{\sqrt{17}}{4} \cos(t) \Big|_0^{4\pi}$$

$$= \frac{\sqrt{17}}{4} 4\pi - \frac{\sqrt{17}}{4} \cos(4\pi) - \left( \frac{\sqrt{17}}{4} \cdot 0 - \frac{\sqrt{17}}{4} \cos(0) \right)$$

$$= \sqrt{17} \pi - \frac{\sqrt{17}}{4} + \frac{\sqrt{17}}{4} = \sqrt{17} \pi$$

