MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ ε5

IME I PREZIME: Ante Podišić **BROJ INDEKSA:** 17-1-0114-2017

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 8 + 6i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2 + 6}$ 10
3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2 + 2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote) 20(graf)
4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

   (a) $\lim_{x\to 0}\left(\frac{\sqrt{6+x}-\sqrt{6}}{x}\right) =$ 7+2

   (b) $\lim_{n\to\infty}\left(\frac{x^2+8}{x^2}\right) =$ 4+2

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcrcl} 4x & - & y & + & z & + & 2u & = & 14 \\ 2x & + & y & & & - & 3u & = & 2 \\ x & - & y & + & 2z & + & u & = & 3 \\ 2x & + & y & + & z & - & 4u & = & 0 \end{array}$$

6. Računanjem ranga provjeri da li je matrica ima puni rang: $\mathbf{B} = \begin{bmatrix} -0.15 & 0 & -1 & 0.25 \\ 0.2 & 0 & 0 & 0 \\ 0.3 & 1 & 2 & -0.5 \\ -0.15 & 0 & 0 & 0.25 \end{bmatrix}$. 5

7. Odrediti tangentu na funkciju $f(x) = \log_8 x$ tamo gdje je $x = 8$. Nacrtati graf funkcije i nacrtati izračunatu tangentu. 7+3

Ukupno: 44

(1.) $z^3 - 8 + 6i = 0$

$z^3 = 8 - 6i$

$z = \sqrt[3]{8 - 6i}$

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{64 + 36}$

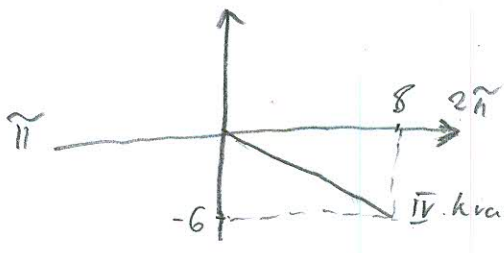
$r = 10$ ✓

$\operatorname{tg}\varphi = \frac{y}{x}$

$\operatorname{tg}\varphi = \frac{-6}{8} = -\frac{3}{4}$

$\varphi = 2\pi - 0{,}643$

$\varphi = 5{,}64$ ✓



$z = \sqrt[3]{10} \cdot \left(\cos\frac{5{,}64 + 2k\pi}{3} + i\sin\frac{5{,}64 + 2k\pi}{3}\right)$

$k = 0, 1, 2$

z_1 za $k = 0$

$z_1 = 2{,}15 \cdot \left(\cos\frac{5{,}64}{3} + i\sin\frac{5{,}64}{3}\right) = 2{,}15 \cdot (-0{,}304 + 0{,}952\,i)$

$z_1 = -0{,}65 + 2{,}04\,i$

z_2 za $k=1$

$$z_2 = 2{,}15 \cdot \left(\cos \frac{5{,}64 + 2\pi}{3} + i \sin \frac{5{,}64 + 2\pi}{3}\right)$$

$$z_2 = 2{,}15 \cdot (-0{,}673 - 0{,}735\,i)$$

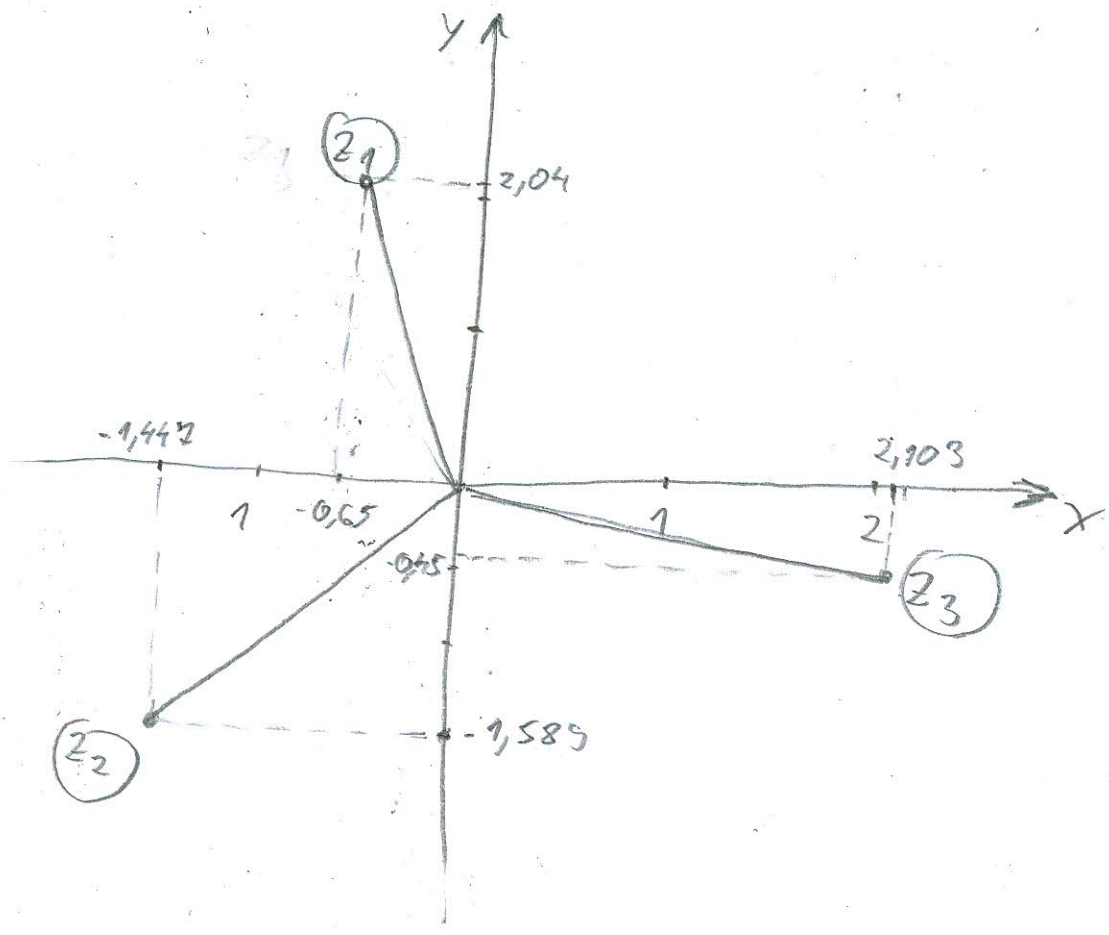
$$\boxed{z_2 = -1{,}447 - 1{,}589\,i}$$

z_3 za $k=2$

$$z_3 = 2{,}15 \cdot \left(\cos \frac{5{,}64 + 4\pi}{3} + i \sin \frac{5{,}64 + 4\pi}{3}\right)$$

$$\boxed{z_3 = 2{,}15 \cdot (0{,}978 - 0{,}213\,i)} \Rightarrow \boxed{z_3 = 2{,}103 - 0{,}458\,i}$$

Rješenja u kompleksnoj ravnini



(5.) Derivacija

$f(x) = (x^2+6)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(x^2+6)^{-\frac{1}{2}} \cdot 2x$

$f'(x) = \frac{2x}{2\sqrt{x^2+6}}$

$f'(x) = 0$

$2x = 0 \Rightarrow \boxed{x = 0}$

$f''(x) = \frac{2 \cdot 2\sqrt{x^2+6} - 2x \cdot 2 \cdot (x^2+6)^{\frac{1}{2}} \cdot 2x}{4 \cdot (x^2+6)}$

$f''(0) = \frac{4 \cdot \sqrt{6}}{24} = 0,40 > 0 \Rightarrow m(0, \sqrt{6})$

minimum u $m(0, \sqrt{6})$ ✓

Anto Podišić 17-1-0114-2012 5. IX. 2014.

(2.) $g(x)=\sqrt{x^2+6}$ - parabola okrenuta prema gore!

(1.) Domena

$x^2+6 \geq 0$

$x \geq \sqrt{-6}$ - $D_f(x)$; $x \in \mathbb{R}$

(2.) Asimptote

V. A - nema

H. A

$$\lim_{x\to\infty} \sqrt{x^2+6} \cdot \frac{\sqrt{x^2+6}}{\sqrt{x^2+6}} = \lim_{x\to\infty} \frac{x^2+6 \;/:x^2}{\sqrt{x^2+6}\;/:x^2} = \left[\frac{1}{\infty}\right] = 0 //$$

- nema H. A

K. A

$y = kx + l$

$$k = \lim_{x\to\infty} \frac{\sqrt{x^2+6}\;/:x}{x\;/:x} = \lim_{x\to\infty} \frac{\sqrt{1}}{1} \Rightarrow \boxed{k = \pm 1}$$

$$l = \lim_{x\to\infty} \left(\sqrt{x^2+6} - 1x\right) = \lim_{x\to\infty} \sqrt{x^2+6} - x \cdot \frac{\sqrt{x^2+6}+x}{\sqrt{x^2+6}+x}$$

$$\lim_{x\to\infty} \frac{x^2+6-x^2\;/:x}{\sqrt{x^2+6}+x\;/:x} = \left[\frac{0}{\sqrt{1}+1}\right] = 0 //$$

$y = x$, $y = -x$ - 2 kose asimptote

(3.) Nultočke

$\sqrt{x^2+6} = 0$

$x^2+6 = 0$

$x = \sqrt{-6}$ - Nema nultočaka!

Ante Podišić
5. IX. 2014

3.) $h(x)=\sqrt{x^2+2x} \;/^2 \Rightarrow y^2=x^2+2x \Rightarrow$

1.) Domena

$x^2+2x \geq 0$

$x(x+2)\geq 0 \;\begin{cases} x\geq 0 \\ x\geq -2 \end{cases}$

	$-\infty$		-2		0		$+\infty$
x		−		−	0	+	
$x+2$		−	0	+		+	
		(+)		−	0	(+)	

$D_f(x): x\in\langle -\infty, -2] \cup [0, +\infty\rangle$

2.) Asimptote

V. A - nema !!

H. A

$\lim\limits_{x\to\infty} \sqrt{x^2+2x} \;/\cdot \frac{\sqrt{x^2+2x}}{\sqrt{x^2+2x}} = \lim\limits_{x\to\infty} \frac{x^2+2x \;/:x^2}{\sqrt{x^2+2x}\;/:x^2} = \left[\frac{1}{0}\right] = \infty$

$\Rightarrow$ nema H. A

K. A

$y=kx+l$

$k=\lim\limits_{x\to\infty}\frac{f(x)}{x}=\lim\limits_{x\to\infty}\frac{\sqrt{x^2+2x}\;/:x}{x\;/:x}=\left[\frac{\sqrt{1}}{1}\right] \Rightarrow \boxed{k=\pm 1}$

$l=\lim\limits_{x\to\infty}(f(x)-kx)=\lim\limits_{x\to\infty}\sqrt{x^2+2x}-x \;/\cdot \frac{\sqrt{x^2+2x}+x}{\sqrt{x^2+2x}+x}$

$\lim\limits_{x\to\infty}\frac{x^2+2x-x^2\;/:x}{\sqrt{x^2+2x}+x\;/:x}=\lim\limits_{x\to\infty}\frac{\frac{2x}{x}=2}{\sqrt{\frac{x^2}{x^2}+\frac{2x}{x^2}}+\frac{x}{x}}=\left[\frac{2}{2}\right]=1$

$\frac{x^2}{x^2}=1 \quad \frac{2x}{x^2}=0 \quad \frac{x}{x}=1$

Kosa asimptota

$y=x+1$

$y=-x-1$

✓