MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ F4

IME I PREZIME: MIHOVIL JORDAN

BROJ INDEKSA: 17-2-0231-2012

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 5 = 0$. *Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem!* Dalje izracunaj: $\overline{\left(\frac{z_1 - z_2}{z_2 + 5}\right)}$ i $\operatorname{Im}\left(\overline{\left(\frac{z_2}{z_1}\right)}\right)$. 4+3+2+6 — 13

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{aligned} x_1 + x_2 - x_3 - 3x_4 + 4x_5 &= 2 \\ 3x_1 + x_2 - x_3 - x_4 &= 2 \\ 9x_1 + x_2 - 2x_3 - x_4 - 2x_5 &= 5 \\ x_1 - x_2 \quad - x_4 + 2x_5 &= 1 \end{aligned}$$

3. Odrediti domenu funkcije $g(x) = \sqrt{x^2 + x - 5} - \arctan\left(2x^2 - x\right)$. 15

4. Odrediti tok funkcije $f(x) = \dfrac{x^2 - 2}{x^2 + 3}$ 20(graf)

5. Odrediti i provjeriti uvrštavanjem: $\lim\limits_{x \to -4} \dfrac{x^2 - 3}{x^2 + 8x + 16} =$ 4+1

6. Odredi derivaciju funkcije $f(x) = \dfrac{2}{\cos^2(5x)}$ 10

7. Odrediti tangentu na funkciju $f(x) = \log_2 x$ tamo gdje je $x = 2$. Nacrtati graf funkcije i nacrtati izračunatu tangentu. 15+3+2

Ukupno: 52

(1) $z^2 - z + 5 = 0$

$z_{1,2} = \dfrac{1 \pm \sqrt{-19}}{2}$ $\quad z_2 = \dfrac{1}{2} + \dfrac{\sqrt{19}}{2} i$

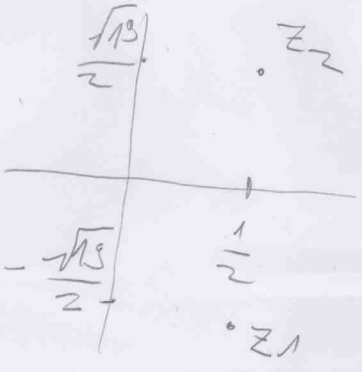
PR:

$\left(\dfrac{1}{2} \pm \dfrac{\sqrt{19}}{2} i\right)^2 - \left(\dfrac{1}{2} \pm \dfrac{\sqrt{19}}{2} i\right) + 5 = 0$

$\dfrac{1}{4} \pm \dfrac{\sqrt{19}}{2} i - \dfrac{19}{4} - \dfrac{1}{2} \mp \dfrac{\sqrt{19}}{2} i + 5 = 0$

$\dfrac{-9}{2} - \dfrac{1}{2} + 5 = 0$

$-5 + 5 = 0$



(1)

$z^2 - z + 5 = 0$

$z_{1,2} = \frac{1 \pm \sqrt{-19}}{2} = \frac{1 \pm i\sqrt{19}}{2}$

PR:

$\left(\frac{1}{2} - \frac{\sqrt{19}}{2} i\right)^2 - \left(\frac{1}{2} - \frac{\sqrt{19}}{2} i\right) + 5 = 0$

$\frac{1}{4} + \frac{\sqrt{19}}{2} i - \frac{19}{4} - \frac{1}{2} + \frac{\sqrt{19}}{2} i + 5 = 0$

$-5 + 5 = 0$

$0 = 0$

$\frac{z_2}{z_1} = \frac{\frac{1 + i\sqrt{19}}{2}}{\frac{1 - i\sqrt{19}}{2}} = \frac{1 + i\sqrt{19}}{1 - i\sqrt{19}} \cdot \frac{1 + i\sqrt{19}}{1 + i\sqrt{19}}$

$= \frac{1 + 2i\sqrt{19} - 19}{1 + 19}$

$= \frac{-18 + 2i\sqrt{19}}{20}$

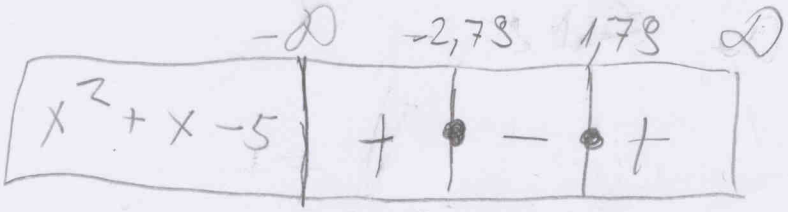
$\overline{\left(\frac{z_2}{z_1}\right)} = \frac{-19 - 2i\sqrt{19}}{20}$

$\operatorname{Im}\left(\overline{\left(\frac{z_2}{z_1}\right)}\right) = -\frac{\sqrt{19}}{10}$ ✓

(3) $x^2+x-5 \geq 0$

$x_{1,2} = \frac{-1 \pm \sqrt{21}}{2}$

$x_1 = -2{,}79 \qquad x_2 = 1{,}79$



$D(g) = \langle -\infty, -2{,}79] \cup [1{,}79, +\infty \rangle$ ✓

(6) $f(x) = \frac{2}{\cos^2(5x)} = 2(\cos(5x))^{-2}$

$f'(x) = -4(\cos(5x))^{-3} \cdot (-\sin(5x)) \cdot 5$

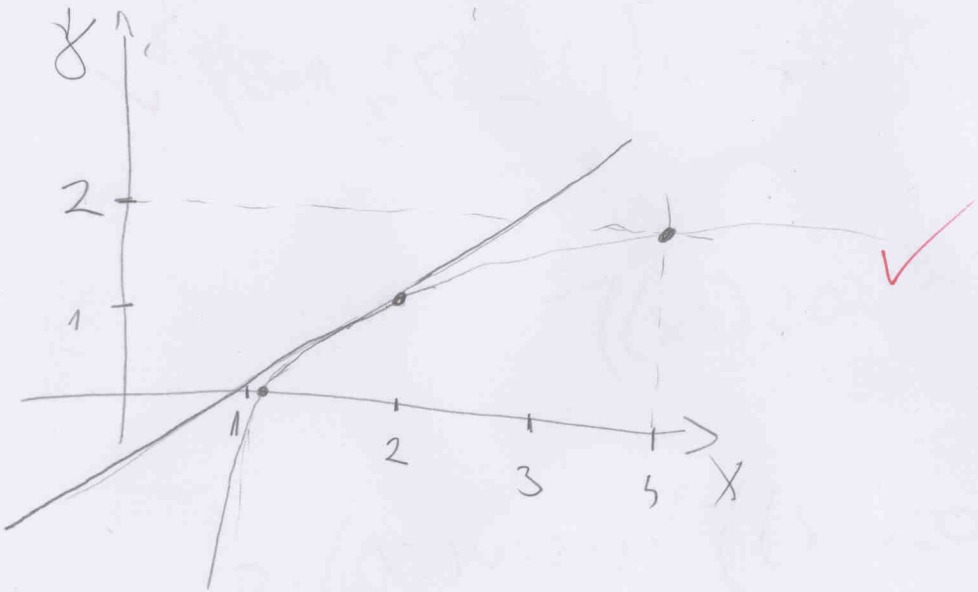
$= \frac{20 \sin(5x)}{\cos^3(5x)}$ ✓

(7) $f(x) = \log_2 x$ $\quad x_0 = 2 \quad y_0 = 1$

$f'(x) = \frac{1}{x \ln 2}$ $\quad f'(x) = \frac{1}{2 \ln 2}$

$t \dots y = \frac{1}{2 \ln 2}(x-2) + 1$

$y = \frac{1}{2 \ln 2} x - \frac{1}{\ln 2} + 1$



✓

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MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{F}4$

IME I PREZIME: IVAN PADOVAN **BROJ INDEKSA:** 0247033119

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 53

9) $\lim_{x \to -4} \dfrac{x^2 - 3}{x^2 + 2x + 16}$

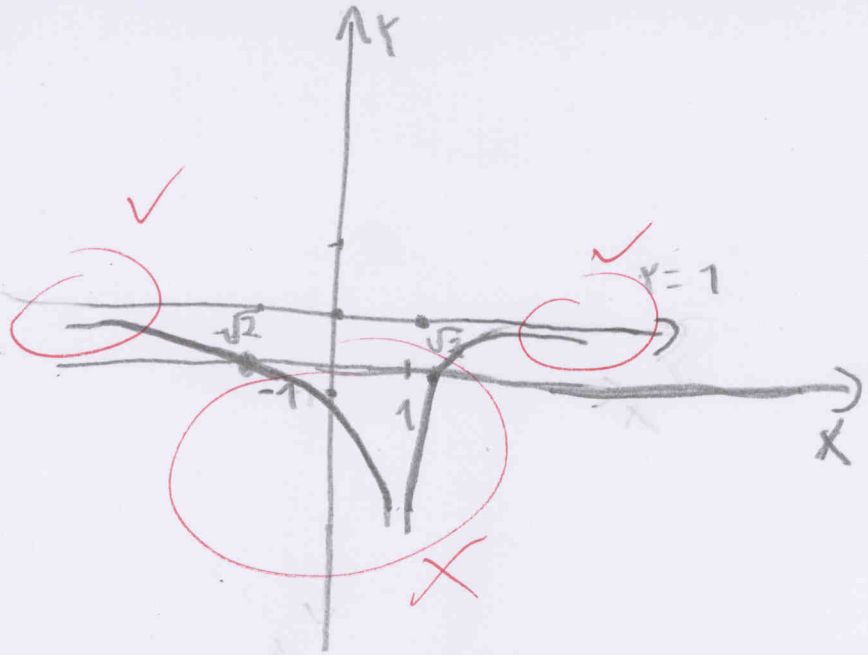
$\lim_{x \to -4} \dfrac{(-4)^2 - 3}{(-4)^2 + 2x + 16}$

$\lim_{x \to -4} \dfrac{16 - 3}{16 + 2x + 16}$

$\lim_{x \to -4} \dfrac{13}{32 + 2x}$ ×

GRAF NAN PAJOUAN

(4)



13

6) KRITIČNE TOČKE

$f'(x) = 0 \qquad 10x = 0 \;/:10 \qquad \left(0, -\frac{2}{3}\right)$

$x = 0$

7) TOČKE ZAKL.

$f''(x) = 0 \qquad -60x^2 + 10 = 0$

$-60x^2 = 90 \;/:(-60)$

$x^2 = \frac{90}{60} = \frac{9}{6} = \frac{3}{2} \;/\sqrt{}$

$x_{1,2} = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{6}}{2}$

$\frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$ //

8) MONOTONOST

	$-\infty$		$-\sqrt{2}$		0		$\sqrt{2}$		$+\infty$
$f'(x)$		+		−		−		+	
$f(x)$		↗		↘		↘		↗	

NEMA MINIMUM NI MAKSIMUM

9) ZAKRIVLJENOST

	$-\infty$		$-\frac{\sqrt{6}}{2}$		0		$\frac{\sqrt{6}}{2}$		$+\infty$
$f'(x)$		−		−		−		−	
$f(x)$		∩		∩		∩		∩	

IVAN TADOVAN

4) NULTOČKE

$x^2 - 2 = 0 \qquad x^2 = 2 \mid \sqrt{}$

$x_{1,2} = \pm \sqrt{2}$

NULTOČKE : $(-\sqrt{2}, 0), (\sqrt{2}, 0)$

5) DERIVACIJE

$$f'(x) = \left(\frac{x^2-2}{x^2+3}\right)' = \frac{2x \cdot (x^2+3) - (x^2-2) \cdot 2x}{(x^2+3)^2}$$

$$= \frac{2x^3 + 6x - 2x^3 + 4x}{(x^2+3)^2} = \frac{10x}{x^4 + 6x^2 + 9}$$

$$f''(x) = \left(\frac{10x}{x^4+6x^2+9}\right)' = \frac{10(x^4+6x^2+9) - 10x(4x^3+12x)}{(x^2+3)^4}$$

$$= \frac{10x^4 + 60x^2 + 90 - 40x^4 - 120x^2}{(x^2+3)^4}$$

$$= \frac{-60x^2 + 90}{(x^2+3)^4}$$

→

6) KRITIČNE TOČKE

$f'(x) = 0 \qquad 10x = 0 \;/:10 \qquad \left(0, -\frac{2}{3}\right)$

$x = 0$

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$-60x^2 = 90 \;/:(-60)$

$x^2 = \frac{90}{60} = \frac{9}{6} = \frac{3}{2} \;/\sqrt{}$

$x_{1,2} = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{6}}{2}$

$\frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$

8) MONOTONOST

	$-\infty$		$-\sqrt{2}$		0		$\sqrt{2}$		$+\infty$
$f'(x)$		$+$		$-$		$-$		$+$	
$f(x)$		↗		↘		↘		↗	

NEMA MINIMUM I MAKSIMUM

9) ZAKRIVLJENOST

	$-\infty$		$-\frac{\sqrt{6}}{2}$		0		$\frac{\sqrt{6}}{2}$		$+\infty$
$f'(x)$		$-$		$-$		$-$		$+$	
$f(x)$		∩		∩		∩		∪	

IVAN TADOVAN

4) NULTOČKE

$x^2 - 2 = 0 \qquad x^2 = 2 \quad / \sqrt{\ }$

$x_{1,2} = \pm \sqrt{2}$

NULTOČKE : $(-\sqrt{2}, 0)$, $(\sqrt{2}, 0)$

5) DERIVACIJE

$$f'(x) = \left(\frac{x^2-2}{x^2+3}\right)' = \frac{2x \cdot (x^2+3) - (x^2-2) \cdot 2x}{(x^2+3)^2}$$

$$= \frac{2x^3 + 6x - 2x^3 + 4x}{(x^2+3)^2} = \frac{10x}{x^4 + 6x^2 + 9}$$

$$f''(x) = \left(\frac{10x}{x^4+6x^2+9}\right)' = \frac{10(x^4+6x^2+9) - 10x(4x^3+12x)}{(x^2+3)^4}$$

$$= \frac{10x^4 + 60x^2 + 90 - 40x^4 - 120x^2}{(x^2+3)^4}$$

$$= \frac{-60x^2 + 90}{(x^2+3)^4}$$

→

④ $f(x) = \frac{x^2-2}{x^2+3}$

1) DOMENA

$x^2+3 \neq 0$ TO VRIJEDI ZA SVAKI $x \in \mathbb{R}$

$D_f = \mathbb{R}$

2) ASIMPTOTE

NEMA V.A.

H.A. $\lim_{x\to\infty} \frac{x^2-2 \,/:x^2}{x^2+3 \,/:x^2} = \lim_{x\to\infty} \frac{1-\frac{2}{x^2}}{1+\frac{3}{x^2}}$

$= \frac{1-0}{1+0} = \frac{1}{1} = 1$ // $\quad y=1$ JE H.A.

KOSE ASIMP. NEMA

3) GLOBALNA SVOJSTVA

$f(-x) = \frac{(-x)^2-2}{(-x)^2+3} = \frac{x^2-2}{x^2+3} = f(x)$

FUNKCIJA JE PARNA

NIJE PERIODIČNA

3)

$$g(x) = \sqrt{x^2+x-5} - \operatorname{arctan}(2x^2 - x)$$

UVJETI

I $x^2+x-5 \geq 0$

$$x_{1,2} = \frac{-1 \pm \sqrt{1-4\cdot 1\cdot(-5)}}{2\cdot 1}$$

$$x_{1,2} = \frac{-1 \pm \sqrt{21}}{2}$$

$$x_1 = \frac{-1+\sqrt{21}}{2}$$

$$x_2 = \frac{-1-\sqrt{21}}{2}$$

II DOMENA FUNKCIJE arctg JE $\mathbb{R}$

$$x^2+x+5 = \left(x - \frac{-1+\sqrt{21}}{2}\right)\left(x - \frac{-1-\sqrt{21}}{2}\right)$$

	$-\infty$	$\frac{-1-\sqrt{21}}{2}$		$\frac{-1+\sqrt{21}}{2}$	$+\infty$
$x - \frac{-1+\sqrt{21}}{2}$		−	−	0	+
$x - \frac{-1-\sqrt{21}}{2}$		−	0	+	+
		+	−	+	

RJEŠENJE: $D_g = \left\langle -\infty, \frac{-1-\sqrt{21}}{2}\right] \cup \left[\frac{-1+\sqrt{21}}{2}, +\infty\right\rangle$

$$x \in \left\langle -\infty, \frac{-1-\sqrt{21}}{2}\right] \cup \left[\frac{-1+\sqrt{21}}{2}, +\infty\right\rangle$$

④ $f(x) = \frac{x^2-2}{x^2+3}$

1) DOMENA

$x^2+3 \neq 0$ TO VRIJEDI ZA SVAKI $x \in \mathbb{R}$

$D_f = \mathbb{R}$

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H.A. $\lim_{x\to\infty} \frac{x^2-2 \,/:x^2}{x^2+3 \,/:x^2} = \lim_{x\to\infty} \frac{1-\frac{2}{x^2}}{1+\frac{3}{x^2}}$

$= \frac{1-0}{1+0} = \frac{1}{1} = 1$ // $y=1$ JE H.A.

KOSE ASIMP. NEMA

3) GLOBALNA SVOJSTVA

$f(-x) = \frac{(-x)^2-2}{(-x)^2+3} = \frac{x^2-2}{x^2+3} = f(x)$

FUNKCIJA JE PARNA

NIJE PERIODIČNA

$x_3 + 6x_4 - 10x_5 = -3$

$x_3 + 6r - 10u = -3$

$x_3 = -3 - 6r - 10u$

$$x = \begin{bmatrix} 0 \\ 2 \\ -3 \\ 0 \\ 0 \end{bmatrix} + r \begin{bmatrix} -1 \\ -2 \\ -6 \\ 1 \\ 0 \end{bmatrix} + u \begin{bmatrix} 2 \\ 4 \\ 10 \\ 0 \\ 1 \end{bmatrix}$$

②

⑥ $f(x) = \dfrac{2}{\cos^2(5x)}$

$$f'(x) = \left(\frac{2}{\cos^2(5x)}\right)'$$

$$= -\frac{2}{\cos^2(5x)} \cdot 2\cos(5x) \cdot (-\sin(5x)) \cdot 5$$

$$= \frac{-2}{\cos(5x)} \cdot 2 \cdot (-\sin(5x)) \cdot 5$$

$$= \frac{20\sin(5x)}{\cos(5x)} = 20\tan(5x)$$

IVAN PADOVAN

①

②

$$x_1 + x_2 - x_3 - 3x_4 + 4x_5 = 2$$
$$3x_1 + x_2 - x_3 - x_4 = 2$$
$$3x_1 + x_2 - 2x_3 - x_4 - 2x_5 = 5$$
$$x_1 - x_2 - x_4 + 2x_5 = 1$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -1 & -3 & 4 & 2 \\ 3 & 1 & -1 & -1 & 0 & 2 \\ 3 & 1 & -2 & -1 & -2 & 5 \\ 1 & -1 & 0 & -1 & 2 & 1 \end{array}\right] \begin{array}{l} \cdot(-3)\cdot I + II \\ \sim \cdot(-3)\cdot I + III \\ IV - I \end{array} \sim \left[\begin{array}{ccccc|c} 1 & 1 & -1 & -3 & 4 & 2 \\ 0 & -2 & 2 & 8 & -12 & -4 \\ 0 & -8 & 7 & 26 & -38 & -13 \\ 0 & -2 & 1 & 2 & -2 & -1 \end{array}\right] \begin{array}{l} \\ \\ \\ \\ \end{array} \; I:(-2)$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 1 & -1 & -3 & 4 & 2 \\ 0 & 1 & -1 & -4 & 6 & 2 \\ 0 & -8 & 7 & 26 & -38 & -13 \\ 0 & -2 & 1 & 2 & -2 & -1 \end{array}\right] \begin{array}{l} I - II \\ 8\cdot II + III \\ 2\cdot II + IV \\ \\ \end{array} \sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & -2 & 0 \\ 0 & 1 & -1 & 4 & 6 & 2 \\ 0 & 0 & -1 & -6 & 10 & 3 \\ 0 & 0 & -1 & -6 & 10 & 3 \end{array}\right] \begin{array}{l} \\ \\ \\ IV - III \end{array} \; I:(-1)$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & -2 & 0 \\ 0 & 1 & -1 & 4 & 6 & 2 \\ 0 & 0 & 1 & 6 & -10 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] III + II \sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & 2 & -4 & 2 \\ 0 & 0 & 1 & 6 & -10 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

2 - PARAMETARSKO RJEŠENJE

$x_4 = \lambda$, $x_5 = \mu$, $\lambda, \mu \in \mathbb{R}$

$x_1 + x_4 - 2x_5 = 0$

$x_1 + \lambda - 2\mu = 0$

$x_1 = -\lambda + 2\mu$

$x_2 - 2x_4 - 4x_5 = 2$

$x_2 + 2\lambda - 4\mu = 2$

$x_2 = -2\lambda + 4\mu + 2$

✓

→