

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

IME I PREZIME: **FRANKO ŠIMUČINA**

BROJ INDEKSA: **17-1-0222-2013**

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

F4

1. Neka su  $z_1$  i  $z_2$  rješenja kvadratne jednadžbe  $z^2 - z + 5 = 0$ . Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem! Dalje izračunaj:  $\left(\frac{z_1 - z_2}{z_2 + 5}\right)$  i  $\text{Im}\left(\frac{z_2}{z_1}\right)$ .

4+3+2+6

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

10+5

$$x_1 + x_2 - x_3 - 3x_4 + 4x_5 = 2$$

$$3x_1 + x_2 - x_3 - x_4 = 2$$

$$9x_1 + x_2 - 2x_3 - x_4 - 2x_5 = 5$$

$$x_1 - x_2 - x_4 + 2x_5 = 1$$

3. Odrediti domenu funkcije  $g(x) = \sqrt{x^2 + x - 5} - \arctan(2x^2 - x)$ .

15

4. Odrediti tok funkcije  $f(x) = \frac{x^2 - 2}{x^2 + 3}$

20(graf)

5. Odrediti i provjeriti uvrštavanjem:  $\lim_{x \rightarrow -4} \frac{x^2 - 3}{x^2 + 8x + 16} =$

4+1

6. Odredi derivaciju funkcije  $f(x) = \frac{2}{\cos^2(5x)}$

10

7. Odrediti tangentu na funkciju  $f(x) = \log_2 x$  tamo gdje je  $x = 2$ . Nacrtati graf funkcije i nacrtati izračunatu tangentu.

15+3+2

Ukupno:

**65**

$$g) f(x) = \frac{x^2 - 2}{x^2 + 3}$$

~~skiciraj~~

$$x^2 + 3 \neq 0$$

$$x^2 \neq -3$$

$$D(f) = \mathbb{R}$$

$$x^2 - 2 = 0$$

$$x = \pm \sqrt{2} \text{ NULTOČKE}$$

$$f(0) = -\frac{2}{3} \rightarrow S\left(0, -\frac{2}{3}\right)$$

$$\lim_{x \rightarrow \infty} \frac{x^2 - 2}{x^2 + 3} \stackrel{/:x^2}{=} \lim_{x \rightarrow \infty} \frac{1 - \frac{2}{x^2}}{1 + \frac{3}{x^2}} = 1$$

HIA...  $y=1$  NEMA VERTIKALNE NI  
KOSE ASIMPTOTE

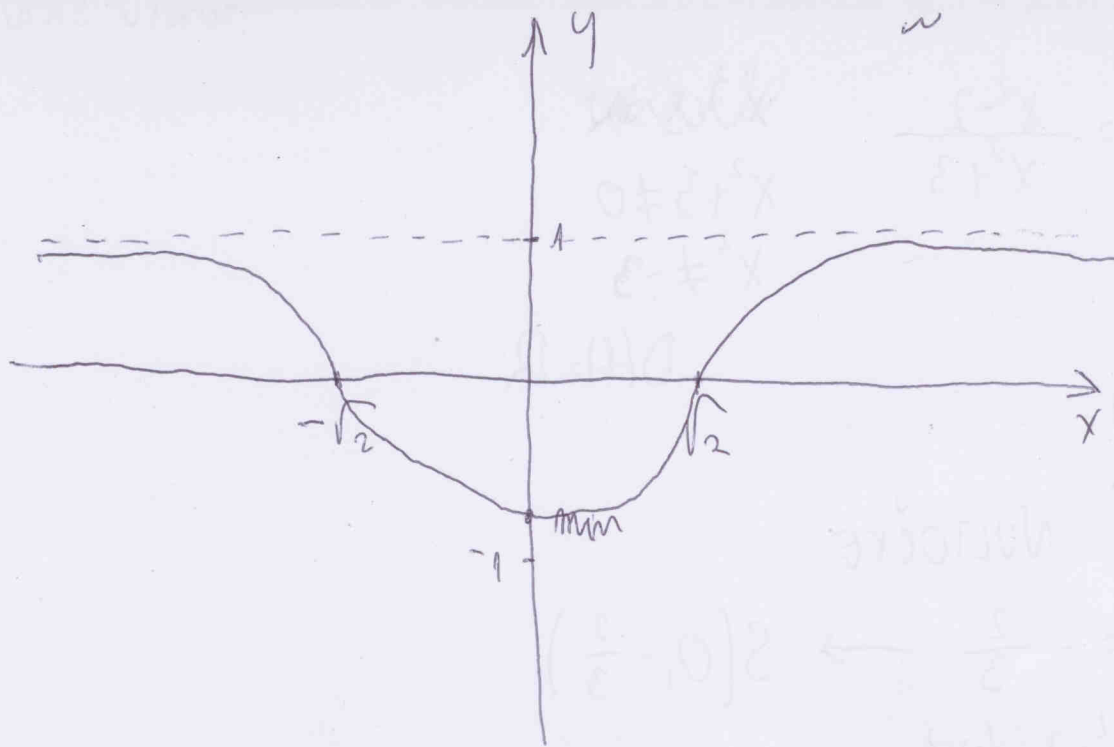
$$f'(x) = \frac{2x(x^2 + 3) - 2x(x^2 - 2)}{(x^2 + 3)^2}$$

$$= \frac{2x^3 + 6x - 2x^3 + 4x}{(x^2 + 3)^2} = \frac{10x}{(x^2 + 3)^2}$$

$$f'(x) = 0 \rightarrow x = 0$$

	$-\infty$	$0$	$+\infty$
$f'(x)$		-	+

$$\min\left(0, -\frac{2}{3}\right)$$



$(\frac{1}{2}, 0)$  min



$$3) g(x) = \sqrt{x^2 + x - 5} \quad \text{or } (2x^2 - x)$$

FRANKO ŠIMURINA

$$x^2 + x - 5 \geq 0 \quad x_1 = -2.79$$

$$x_{1,2} = \frac{-1 \pm \sqrt{21}}{2} \quad x_2 = 1.79$$

	$-\infty$	$-2.79$	$1.79$	$+\infty$
$x^2 + x - 5$		+	-	+

$$D(g) = \langle -\infty, -2.79 \rangle \cup [1.79, +\infty)$$

$$6) f(x) = \frac{2}{\cos^2(5x)}$$

$$f'(x) = \frac{-2 \cdot 2 \cos(5x) \cdot (-\sin(5x)) \cdot 5}{\cos^4(5x)}$$

$$= \frac{20 \sin(5x)}{\cos^3(5x)} \quad \checkmark$$

$$5) \lim_{x \rightarrow -4} \frac{x^2 - 3}{x^2 + 8x + 16} = \frac{13}{0} = \infty \quad \times$$

~~lim~~  
x → -4

$$\frac{13}{0} \neq \infty$$

$$\frac{13}{0_+} = +\infty$$

$$\frac{13}{0_-} = -\infty$$

7)

$$f(x) = \log_2 x$$

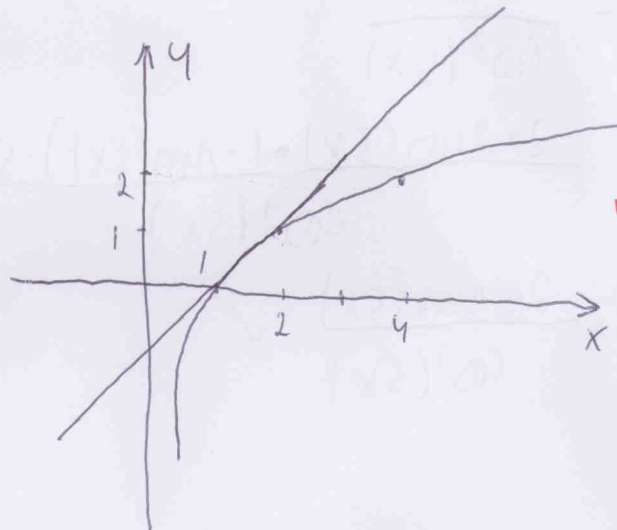
$$x=2 \rightarrow f(2)=1$$

$$f'(x) = \frac{1}{x \ln 2}$$

$$f'(2) = 0.72$$

$$t... y = 0.72(x-2) + 1$$

$$y = 0.72x - 0.44$$



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POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME:

BROJ INDEKSA:

STIPE BREULJ

17-1-0192-2013

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15+3+2

Ukupno:

80

$$2. \begin{bmatrix} 1 & 1 & -1 & -3 & 4 & | & 2 \\ 3 & 1 & -1 & -1 & 0 & | & 2 \\ 9 & 1 & -2 & -1 & -2 & | & 5 \\ 1 & -1 & 0 & -1 & 2 & | & 1 \end{bmatrix} \begin{array}{l} \text{II} - 3\text{I} \\ \text{III} - 9\text{I} \\ \text{IV} - \text{I} \end{array} \sim \begin{bmatrix} 1 & 1 & -1 & -3 & 4 & | & 2 \\ 0 & -2 & 2 & 8 & -12 & | & -4 \\ 0 & -8 & 7 & 26 & -38 & | & -13 \\ 0 & -2 & 1 & 2 & -2 & | & -1 \end{bmatrix} \begin{array}{l} :(-2) \\ \sim \end{array}$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & | & 2 \\ 0 & 1 & -1 & -4 & 6 & | & 2 \\ 0 & -8 & 7 & 26 & -38 & | & -13 \\ 0 & -2 & 1 & 2 & -2 & | & -1 \end{bmatrix} \begin{array}{l} \text{I} - \text{II} \\ \text{III} + 8\text{II} \\ \text{IV} + 2\text{II} \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 1 & -2 & | & 0 \\ 0 & 1 & -1 & -4 & 6 & | & 2 \\ 0 & 0 & -1 & -6 & 10 & | & 3 \\ 0 & 0 & -1 & -6 & 10 & | & 3 \end{bmatrix} \begin{array}{l} \sim \\ :(-1) \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -2 & | & 0 \\ 0 & 1 & -1 & -4 & 6 & | & 2 \\ 0 & 0 & 1 & 6 & -10 & | & -3 \\ 0 & 0 & -1 & -6 & 10 & | & 3 \end{bmatrix} \begin{array}{l} \sim \\ \text{IV} + \text{III} \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 1 & -2 & | & 0 \\ 0 & 1 & -1 & -4 & 6 & | & 2 \\ 0 & 0 & 1 & 6 & -10 & | & -3 \\ 0 & 0 & 0 & 0 & 0 & | & 0 \end{bmatrix} \begin{array}{l} \sim \\ \text{II} + \text{III} \end{array}$$

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ 1 & 0 & 0 & 1 & -2 & | & 0 \\ 0 & 1 & 0 & 2 & -4 & | & -1 \\ 0 & 0 & 1 & 6 & -10 & | & -3 \end{bmatrix}$$

$$x_4 \rightarrow t, t \in \mathbb{R}$$

$$x_5 \rightarrow s, s \in \mathbb{R}$$

$$x_1 + x_4 - 2x_5 = 0$$

$$x_1 + t - 2s = 0$$

$$x_1 = 2s - t$$

$$x_2 + 2x_4 - 4x_5 = -1$$

$$x_2 + 2t - 4s = -1$$

$$x_2 = 4s - 2t - 1$$

$$x_3 + 6x_4 - 10x_5 = -3$$

$$x_3 + 6t - 10s = -3$$

$$x_3 = 10s - 6t - 3$$

$$x_1 = 2s - t$$

$$x_2 = 4s - 2t - 1$$

$$x_3 = 10s - 6t - 3$$

$$x_4 = t, t \in \mathbb{R}$$

$$x_5 = s, s \in \mathbb{R}$$

6.  $f(x) = \frac{2}{\cos^2(5x)}$

$$f'(x) = \frac{2' \cdot \cos^2(5x) - 2 \cdot (\cos^2(5x))'}{(\cos^2(5x))^2}$$

$$= \frac{-2 \cdot 2 \cos 5x \cdot (-\sin 5x) \cdot 5}{(\cos^2(5x))^2}$$

$$= \frac{-20 \cos 5x \cdot (-\sin 5x)}{(\cos^2(5x))^2} = \frac{20 \sin(5x)}{\cos^3(5x)}$$

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ 1 & 0 & 0 & 1 & -2 \mid 0 \\ 0 & 1 & 0 & 2 & -4 \mid -1 \\ 0 & 0 & 1 & 6 & -10 \mid -3 \end{bmatrix}$$

$$x_4 \rightarrow t, t \in \mathbb{R}$$

$$x_5 \rightarrow s, s \in \mathbb{R}$$

$$x_1 + x_4 - 2x_5 = 0$$

$$x_1 + t - 2s = 0$$

$$x_1 = 2s - t$$

$$x_2 + 2x_4 - 4x_5 = -1$$

$$x_2 + 2t - 4s = -1$$

$$x_2 = 4s - 2t - 1$$

$$x_3 + 6x_4 - 10x_5 = -3$$

$$x_3 + 6t - 10s = -3$$

$$x_3 = 10s - 6t - 3$$

$$x_1 = 2s - t$$

$$x_2 = 4s - 2t - 1$$

$$x_3 = 10s - 6t - 3$$

$$x_4 = t, t \in \mathbb{R}$$

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6.  $f(x) = \frac{2}{\cos^2(5x)}$

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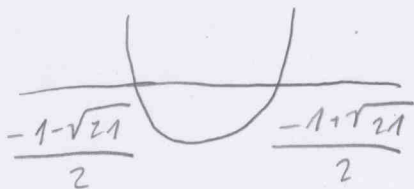
$$= \frac{-20 \cos 5x \cdot (-\sin 5x)}{(\cos^2(5x))^2} = \frac{20 \sin(5x)}{\cos^3(5x)}$$



$$3. g(x) = \sqrt{x^2 + x - 5} - \arctan(2x^2 - x)$$

$$x^2 + x - 5 \geq 5$$

$$x_{1,2} = \frac{-1 \pm \sqrt{21}}{2}$$



$$D_f = \left\langle -\infty, \frac{-1 - \sqrt{21}}{2} \right] \cup \left[ \frac{-1 + \sqrt{21}}{2}, +\infty \right) \quad \checkmark$$

$$4. f(x) = \frac{x^2 - 2}{x^2 + 3} = \frac{x^2 + 3 - 5}{x^2 + 3} = 1 - \frac{5}{x^2 + 3}$$

MULTOCENE -  $x_1 = -\sqrt{2}$ ,  $x_2 = \sqrt{2}$

DOMENA -  $\mathbb{R}$

$f(-x) = f(x)$  - PARNA FUNKCIJA

ASIMPTOTE:

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \left( 1 - \frac{5}{x^2 + 3} \right) = 1 \quad \gamma = 1 - \text{HORIZONTALNA ASIMPTOTA}$$

VERTIKALNA - NEMA JE

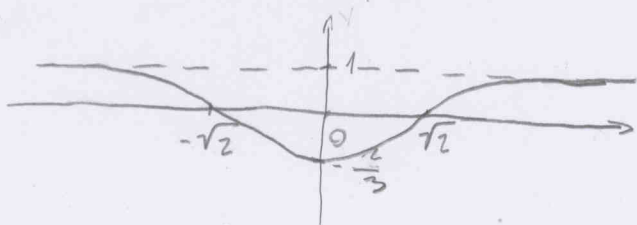
$$\lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{x^2 - 2}{x(x^2 + 3)} = 0 - \text{KOSE NBMA}$$

$$f'(x) = \frac{10x}{(x^2 + 3)^2}$$

$$f(x) = 0 \Leftrightarrow x = 0$$

	$-\infty$	$0$	$+\infty$
$f'$	-	+	
$f$	$\searrow$	$\nearrow$	

LOKALNI MINIMUM U TOČCI  $(0, -\frac{2}{3})$



$$5. \lim_{x \rightarrow 4} \frac{x^2 - 3}{x^2 + 8x + 16} = \frac{\lim_{x \rightarrow 4} (x^2 - 3)}{\lim_{x \rightarrow 4} (x^2 + 8x + 16)} = \frac{13}{0} = \infty$$

PROVERBA:  $x = 4.1 \Rightarrow \frac{x^2 - 3}{x^2 + 8x + 16} = -19.81$

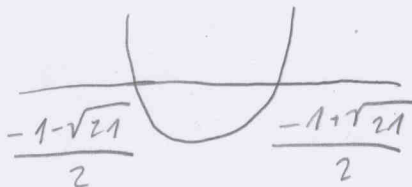
$$x = 4.01 \Rightarrow \frac{x^2 - 3}{x^2 + 8x + 16} = -19.0801$$

$$x = 4.01 \Rightarrow \frac{x^2 - 3}{x^2 + 8x + 16} = 19.0801$$

$$3. g(x) = \sqrt{x^2 + x - 5} - \arctan(2x^2 - x)$$

$$x^2 + x - 5 \geq 5$$

$$x_{1,2} = \frac{-1 \pm \sqrt{21}}{2}$$



$$D_f = \left\langle -\infty, \frac{-1 - \sqrt{21}}{2} \right] \cup \left[ \frac{-1 + \sqrt{21}}{2}, +\infty \right) \quad \checkmark$$

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DOMENA -  $\mathbb{R}$

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$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \left( 1 - \frac{5}{x^2 + 3} \right) = 1 \quad \gamma = 1 - \text{HORIZONTALNA ASIMPTOTA}$$

VERTIKALNA - NEMA JE

$$\lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{x^2 - 2}{x(x^2 + 3)} = 0 - \text{KOSE NBMA}$$

$$f(x) = \frac{10x}{(x^2 + 3)^2}$$

$$f(x) = 0 \Leftrightarrow x = 0$$

	$-\infty$	$0$	$+\infty$
$f'$	-	+	
$f$	$\searrow$	$\nearrow$	

$$7. f(x) = \log_2 x$$

$$f'(x) = \frac{1}{x \ln 2} \quad f'(2) = \frac{1}{2 \ln 2}$$

$f(2) = 1$  TANGENTA PROLAZI TOČKOM  $(2, 1)$

IMA KOEFICIJENT SMJERA  $k = \frac{1}{2 \ln 2} = \frac{1}{2}$

JEDNAČINA TANGENTE

$$\log_2 x - 1 = \frac{1}{2 \ln 2} (x - 2)$$

$$= \frac{1}{2 \ln 2} x - 1 - \frac{1}{\ln 2}$$

$$1 - \frac{1}{\ln 2} \approx -0.44$$

