MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ *H2*

IME I PREZIME: Antonio Ivanušić **BROJ INDEKSA:** 17-2-0186-2012

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* — 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2-2}$. — 5+15+5 → 7
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln(4-x^2)$. — 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-2x-(2+1)}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. — 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: — 15

$$\begin{aligned} x+2y-z+u &= 2 \\ 2x+5y-z+2u &= 3 \\ 3x-y-2z+u &= 2 \\ x-y+3z-5u &= 3 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem: $\lim_{x\to 0} \dfrac{|x|}{x}$. — 5

Ukupno: 57

(5)

$$\left[\begin{array}{cccc|c} x+2y & -z & +u & & 2 \\ 2x+5y & -z & +2u & & 3 \\ 3x-y & -2z & +u & & 2 \\ x-y & +3z & -5u & & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 8 & -1 & 0 & 4 \\ 0 & -5 & -1 & -2 & 1 \\ 0 & -3 & -4 & -6 & 5 \end{array}\right] \sim \quad /:(-3)$$

1R · 2R + (-2)
1R · 3R + (-3)
1R · 4R + (-1)

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & -\frac{1}{8} & 0 & \frac{1}{2} \\ 0 & -5 & -1 & -2 & 1 \\ 0 & -3 & -4 & -6 & 5 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & -\frac{1}{8} & 0 & \frac{1}{2} \\ 0 & 0 & \frac{41}{8} & 5 & \frac{11}{2} \\ 0 & 0 & \frac{7}{2} & -6 & \frac{11}{2} \end{array}\right] \cdot \frac{8}{41} \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & -\frac{1}{8} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{40}{41} & \frac{44}{41} \\ 0 & 0 & \frac{7}{2} & -6 & \frac{11}{2} \end{array}\right] \sim$$

2R · 3R + 5
2R · 4R + 3

3R · 4R + (-7/2)

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & -\frac{1}{8} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{40}{41} & \frac{44}{41} \\ 0 & 0 & 0 & -\frac{767}{82} & \frac{197}{82} \end{array}\right] \cdot \left(-\frac{82}{767}\right) \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & -\frac{1}{8} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{40}{41} & \frac{44}{41} \\ 0 & 0 & 0 & 1 & -\frac{197}{767} \end{array}\right]$$

4R · 3R (-40/41)
4R · 1R + (-1)

	$-\infty$		$-2-\sqrt{5}$		$-2+\sqrt{5}$		$+\infty$
$f'(x)$		+	0	−	0	+	
$f(x)$		↗		↘		↗	

NULTOČKE $x^2 - 2x - 3 = 0$

$$x_{1,2} = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-3)}}{2}$$

MAX $\left(-2-\sqrt{5}, -1+\sqrt{5}\right)$

MIN $\left(-2+\sqrt{5}, -1-\sqrt{5}\right)$

$(3, 0)$ $\quad x_1 = 3$

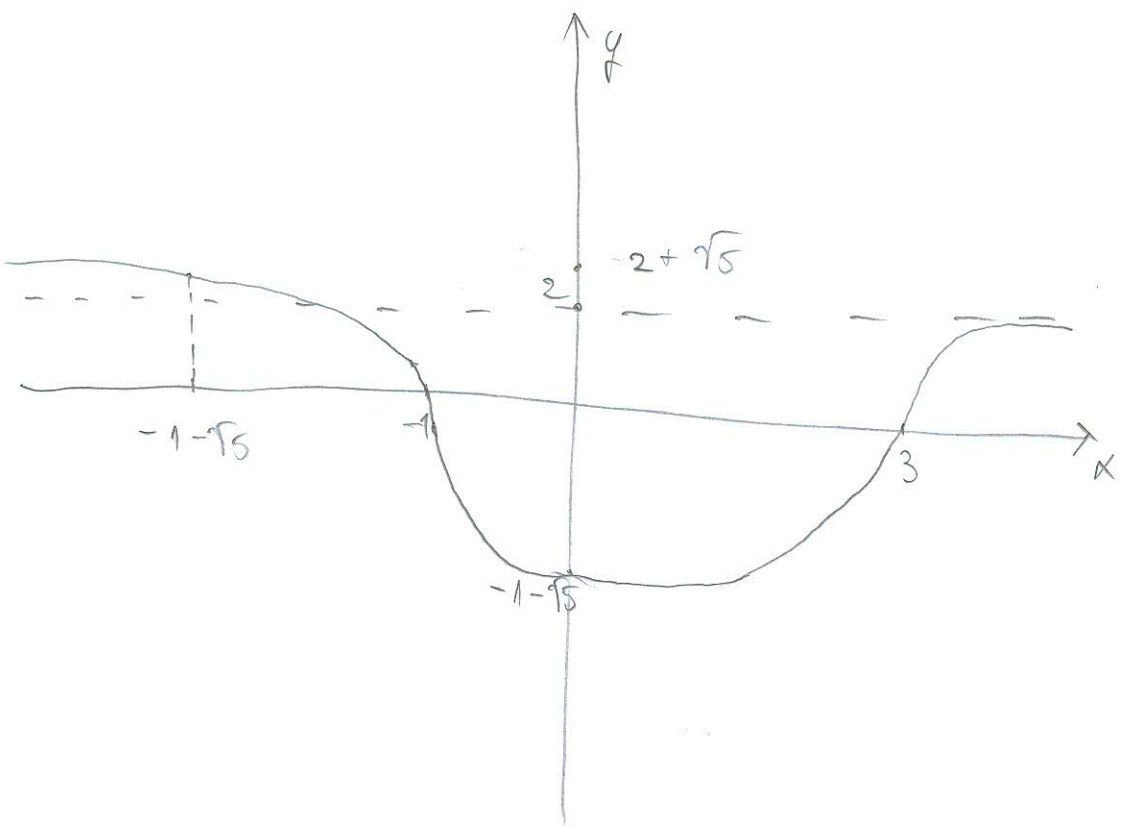
$(-1, 0)$ $\quad x_2 = -1$

HOR. AS

$$\lim_{x \to \pm\infty} \frac{x^2 - 2x - 3}{x^2 + 1} = \lim_{x \to \pm\infty} \frac{2x - 2}{2x} = \left(\frac{\infty}{\infty}\right)' =$$

$$= \lim_{x \to \pm\infty} \frac{2}{2} = 1 \;//$$

$y = 1$ HORIZ. ASIMPTOTA



(4)

$$f(x) = \frac{x^2 - 2x - (2+1)}{x^2+1}$$

DOMENA

$x^2 + 1 \neq 0$

$x^2 \neq 1$

$x \in \mathbb{R}$

$$f'(x) = \frac{(2x-2)(x^2+1) - (x^2-2x-3)\cdot 2x}{(x^2+1)^2} =$$

$$= \frac{(2x-2)(x^2+1) - (x^2-2x-3)\cdot 2x}{(x^2+1)^2}$$

$$= \frac{2x^3 + 2x - 2x^2 - 2 - 2x^2 + 4x^2 + 6x}{(x^2+1)^2}$$

$$= \frac{2x^2 + 8x - 2}{(x^2+1)^2}$$

$$x_{1,2} = \frac{-4 \pm \sqrt{16 - 4\cdot 1 \cdot (-1)}}{2}$$

$$\frac{2x^2 + 8x - 2}{(x^2+1)^2} = 0$$

$$x_1 = -2 + \sqrt{5}$$

$$x_2 = -2 - \sqrt{5}$$

→

$$2x^2 + 8x - 2 = 0$$

$$x^2 + 4x - 1 = 0$$

Antonio Ilanušić

$$\left[\begin{array}{cccc|c} -1 & 2 & -1 & 0 & -1161 \\ 0 & 1 & -\frac{1}{8} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & -\frac{8668}{8437} \\ 0 & 0 & 0 & 1 & -\frac{197}{767} \end{array}\right]$$

$3R \cdot 2R + \frac{1}{8}$

$3R \cdot 1R + 1$

$$-\frac{197}{767} \cdot \frac{44}{41} = -\frac{8668}{8437}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & -1161 \\ 0 & 1 & 0 & 0 & -\frac{394}{767} \\ 0 & 0 & 1 & 0 & -\frac{8668}{8437} \\ 0 & 0 & 0 & 1 & -\frac{197}{767} \end{array}\right]$$

$2R \cdot 1R - 2 \sim$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 594,4 \\ 0 & 1 & 0 & 0 & -\frac{394}{767} \\ 0 & 0 & 1 & 0 & -\frac{8668}{8437} \\ 0 & 0 & 0 & 1 & -\frac{197}{767} \end{array}\right]$$

$594,4 \rightarrow x$

$-593,89 \rightarrow y$

$-1.03 \rightarrow z$

$-0,26 \rightarrow a$

$594,4 + 2 \cdot (-593,89) + 1.03 - 0,26 = 2$

③

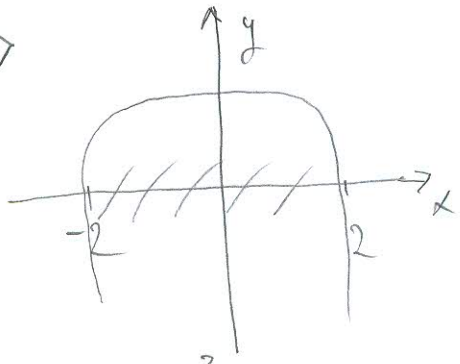
$4 - x^2 > 0$

$4 - x^2 = 0$

$x^2 = 4$

$x = \pm 2$

$x \in \langle -2, 2 \rangle$



$g(-x) = \ln\left(4 - (-x)^2\right) = \ln\left|4 - x^2\right|$

$g(x) = g(-x) \rightarrow$ FUNKCIJA JE PARNA ✓

$g'(x) = \frac{1}{4-x^2} \cdot (-2x) = \frac{-2x}{4-x^2}$

$g'(x) = \frac{-2(4-x^2) + 2x(-2x)}{(4-x^2)^2}$

$= \frac{-8 + 2x^2 - 4x^2}{(4-x^2)^2} = \frac{-2x^2 - 8}{(4-x^2)^2}$ //

$(4-x^2)^2 > 0$

$-2x^2 - 8 < 0$ (ZA BILO KOJI x)

IZ TOGA SLIJEDI DA JE $g''(x) < 0$

PA JE FUNKCIJA KONKAVNA ✓

Antonio Tonusić

②

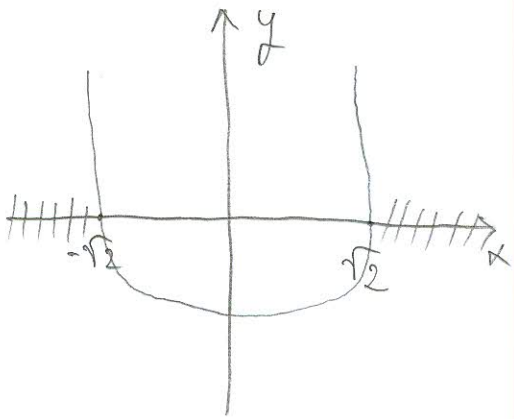
$$f(x) = x - \sqrt{x^2-2}$$

$x^2 - 2 \geq 0$

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$



$x \in \langle -\infty, -\sqrt{2}] \cup [\sqrt{2}, +\infty \rangle$ ✓

$$f'(x) = 1 - \frac{1}{2\sqrt{x^2-2}} \cdot 2x = 1 - \frac{x}{\sqrt{x^2-2}}$$

$$f''(x) = -\frac{\sqrt{x^2-2} - x \cdot \frac{1}{2\sqrt{x^2-2}} \cdot 2x}{\left(\sqrt{x^2-2}\right)^2} \quad ✓ =$$

$$f''(x) = -\frac{\sqrt{x^2-2} - \frac{x}{\sqrt{x^2-2}}}{x^2-2} =$$

$$f''(x) = -\frac{\frac{x^2-2-x^2}{\sqrt{x^2-2}}}{x^2-2} = -\frac{-2}{(x^2-2)^{\frac{3}{2}}} = \frac{2}{(x^2-2)^{\frac{3}{2}}}$$

HORIZONTALNA AS.

$$\lim_{x \to \infty} \left(x - \sqrt{x^2-2}\right) = 0 \quad \text{(DESNA)}$$

L.K.A?

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ $\mathcal{H}2$

IME I PREZIME: GABRIJELA BRKIĆ **BROJ INDEKSA:** 0269069951

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
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Ukupno: 57

(5)

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] \begin{matrix} \\ -2\text{I} \\ -3\text{I} \\ -\text{I} \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & -7 & 1 & -2 & -4 \\ 0 & -3 & 4 & -6 & 1 \end{array}\right] \begin{matrix} -2\text{II} \\ \\ +7\text{II} \\ +3\text{II} \end{matrix}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 4 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 8 & -2 & -11 \\ 0 & 0 & 7 & -6 & -2 \end{array}\right] \begin{matrix} \\ \\ -\text{IV} \\ \\ \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 4 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 & 9 \\ 0 & 0 & 7 & -6 & -2 \end{array}\right] \begin{matrix} +3\text{III} \\ -\text{III} \\ \sim \\ -6\text{III} \end{matrix}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 13 & 31 \\ 0 & 1 & 0 & -4 & -10 \\ 0 & 0 & 1 & 4 & 9 \\ 0 & 0 & 0 & -34 & -65 \end{array}\right] /:(-34) \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 13 & 31 \\ 0 & 1 & 0 & -4 & -10 \\ 0 & 0 & 1 & 4 & 9 \\ 0 & 0 & 0 & 1 & -\frac{65}{34} \end{array}\right] \begin{matrix} -13\text{IV} \\ +4\text{IV} \\ -4\text{IV} \\ \\ \end{matrix}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -\frac{65}{34} \end{array}\right]$$

(5)
$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] \begin{array}{l} \\ II - IV \\ III - 3I \\ IV - I \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & -7 & 1 & -2 & -5 \\ 0 & -3 & 4 & -6 & 1 \end{array}\right] \begin{array}{l} \\ \\ III + 7II \\ IV - 3II \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 8 & -2 & -11 \\ 0 & 0 & 7 & -6 & -2 \end{array}\right] \begin{array}{l} \\ \\ \\ IV - \frac{7}{8} III \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 8 & -2 & -11 \\ 0 & 0 & 0 & -\frac{17}{4} & \frac{61}{8} \end{array}\right]$$

$$-\frac{17}{4} u = \frac{61}{8}$$

$$u = \frac{61}{34}$$

$$8z - 2u = -11$$

$$z = \frac{-11 + 2u}{8} = \frac{435}{272}$$

$$x = 2 + 2y + 2 - u$$

$$x = \frac{271}{272} \quad // \quad \times$$

$$y + z = -1$$

$$y = -1 - z = \frac{163}{272}$$

(6) $$\lim_{x \to 0} \frac{|x|}{x} = \left[\frac{0}{0}\right]^{l'H} = \lim_{x \to 0} \frac{1}{1} = 1 \quad // \quad \times$$

$$\lim_{x \to 0_+} \frac{|x|}{x} = \lim_{x \to 0_+} \frac{x}{x} = 1$$

$$\lim_{x \to 0_-} \frac{|x|}{x} = \lim_{x \to 0_-} \frac{-x}{x} = -1$$

GABRIJELA BRKIĆ

IMATIO 4 RJEŠENJA

① $z^4 - (4-i)^2 = 0$

$(x+yi)^4 = (4-i)^2 \quad / \sqrt{\ }$

$(x+yi)^2 = \pm(4-i)$

$x^2 + 2xyi - y^2 = 4 - i$

$x^2 - y^2 = 4 \qquad y = \sqrt{4-y^2}$

$2xy = -1$

$2\sqrt{4-y^2} \cdot y = -1 \quad /^2$

$2y^2(4-y^2) = 1$

$8y^2 - 2y^4 - 1 = 0$

$y^2 = t$

$8t - 2t^2 - 1 = 0$

$t = \frac{-8 \pm \sqrt{64-8}}{-4} = \frac{-8 \pm 2\sqrt{14}}{-4}$

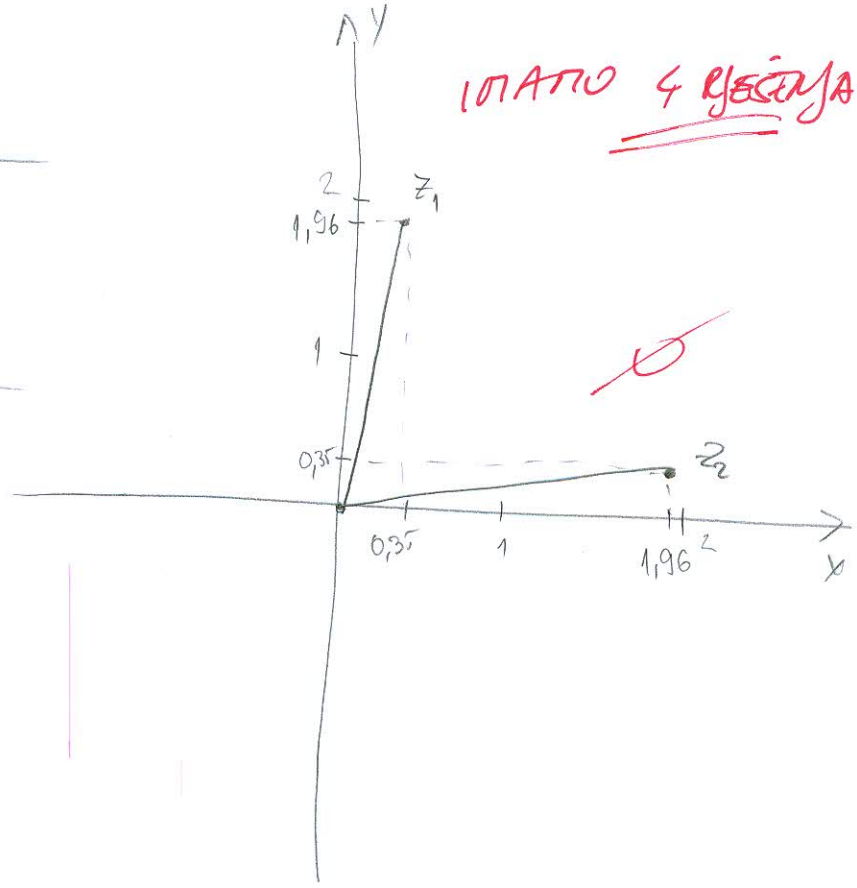
$t_1 = 2 + \frac{\sqrt{14}}{2}$

$t_2 = 2 - \frac{\sqrt{14}}{2}$

$y^2 = t$

$y_1 = \sqrt{t} = \sqrt{2 + \frac{\sqrt{14}}{2}}$

$y_1 = \sqrt{2 - \frac{\sqrt{14}}{2}}$



$x_1 = 0,35 \qquad y_1 = 1,96$

$x_2 = 1,967 \qquad y_2 = 0,35$

$z_1 = 0,35 + 1,96i$

$z_2 = 1,967 + 0,35i$

② $f(x) = x - \sqrt{x^2-2}$

$f'(x) = 1 - \frac{1}{2\sqrt{x^2-2}} \cdot 2x$

$= 1 - \frac{x}{\sqrt{x^2-2}}$

$f''(x) = 0 - \frac{1 \cdot \sqrt{x^2-2} - x \cdot \frac{1}{2\sqrt{x^2-2}} \cdot 2x}{x^2-2}$ ✓

$= \frac{-\sqrt{x^2-2} + \frac{x^2}{\sqrt{x^2-2}}}{x^2-2}$

$= \frac{\frac{x^2 - x^2 + 2}{\sqrt{x^2-2}}}{x^2-2} = \frac{2}{(x^2-2)\sqrt{x^2-2}}$ //

---

④ $h(x) = \frac{x^2 - 2x - (2+1)}{x^2+1}$

$h(x) = \frac{x^2-2x-3}{x^2+1}$

$x^2 - 2x - 3 = x^2 + 1$

$-2x = 4$

$x = -2$

DOMENA

$x^2 + 1 \neq 0$

$x^2 \neq -1$ $\forall x \in \mathbb{R}$

DOMENA : $\mathbb{R}$

N. T

$y^2 - 2x - 2 = 0$

$x_{1,2} = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2}$

$x_1 = 4$ $x_2 = -1$

ASIMPTOTE

V. A NEMA

H. A

$\lim_{x \to \infty} \frac{x^2-2x-3}{x^2+1} = \lim_{x \to \infty} \frac{x^2\left(1 - \frac{3}{x} - \frac{3}{x}\right)}{x^2\left(1 + \frac{3}{x}\right)^2} = 1$

$y = 1$ H. A ✓✓

GABRIJELA BRKIĆ

② $f(x) = x - \sqrt{x^2-2}$

DOMENA

$x^2 - 2 \geq 0$

$x^2 \geq 2$

$x \in \{(-\infty, -\sqrt{2}] \cup [\sqrt{2}, +\infty)\}$ ✓

ASIMPTOTE

V. A

$\lim_{x \to -\sqrt{2}} x - \sqrt{x^2-2} = -\sqrt{2} - \sqrt{2-2} = -\sqrt{2}$

$\lim_{x \to \sqrt{2}} x - \sqrt{x^2-2} = \sqrt{2} - 0 \Rightarrow$ NEMA

NEMA VERTIKALNIH ASIMPTOTA

$\lim_{x \to \pm\infty} x - \sqrt{x^2-2} = \lim_{x \to \pm\infty} (x - \sqrt{x^2-2}) \cdot \frac{(x + \sqrt{x^2-2})}{x + \sqrt{x^2-2}}$

$\lim_{x \to \infty} \frac{x^2 - x^2 + 2}{x + \sqrt{x^2-2}} = \frac{2}{x + \sqrt{x^2-2}} = \left[\frac{2}{\infty}\right] = 0$

HORIZONTALNA ASIMPTOTA $\boxed{y = 0}$ (DESNA)

LIJEVA KOSA ?

→

GABRIJELA BRKIĆ

④ EKSTREMI

$$f'(x) = \frac{(2x-2)(x^2+1) - (x^2-2x-3)(2x)}{(x^2+1)^2}$$

$$= \frac{2x^3 + 2x - 2x^2 - 2 - 2x^3 + 4x^2 + 6x}{(x^2+1)^2}$$

$$= \frac{2x^2 + 8x - 2}{(x^2+1)^2} = 0$$

$2x^2 + 8x - 2 = 0 \quad / : 2$

$x^2 - 4x - 1 = 0$

$x = \frac{-4 \pm \sqrt{16+4}}{2}$

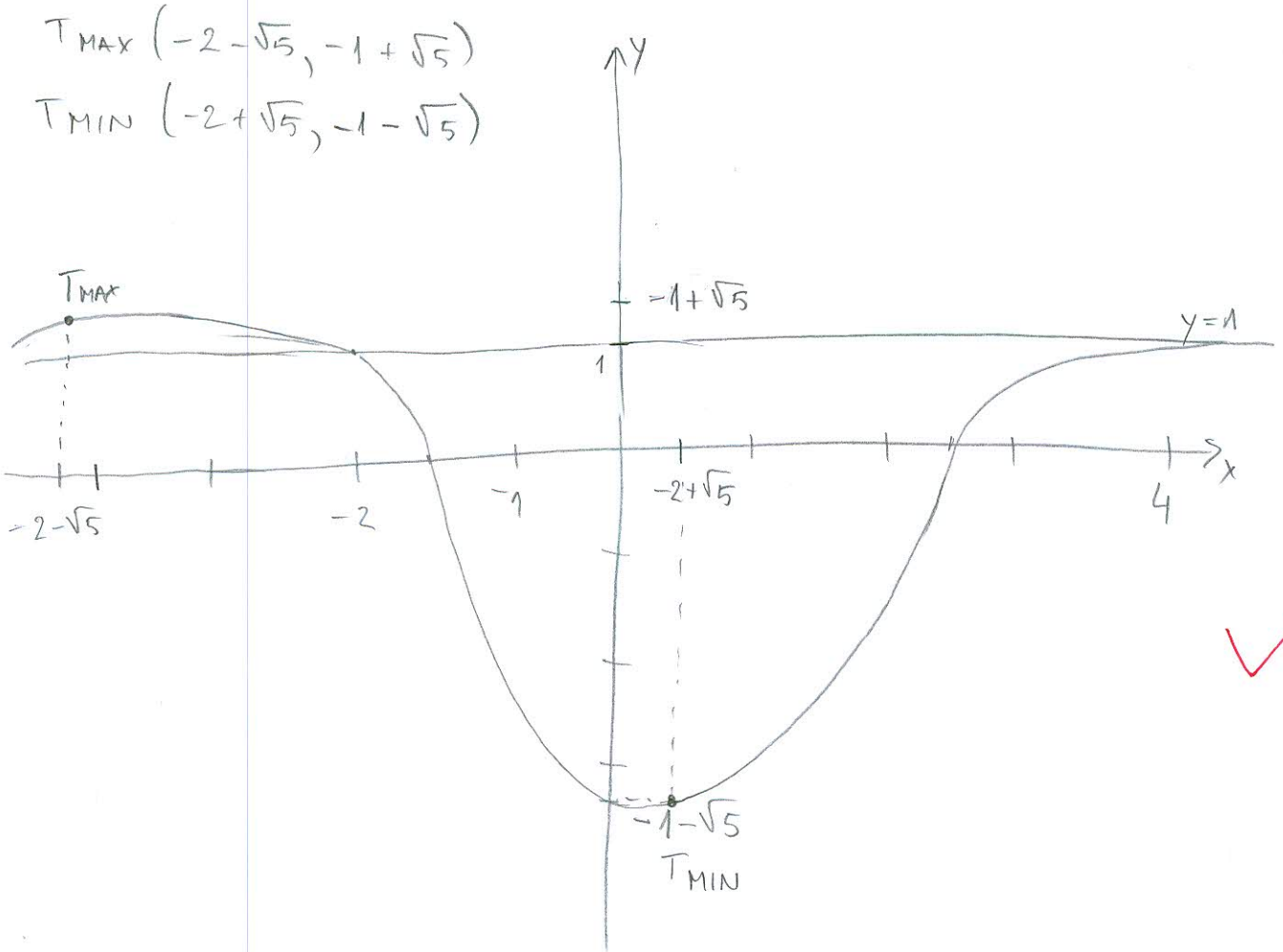
$x_{1,2} = -2 \pm \frac{2\sqrt{5}}{2}$

$x_{1,2} = -2 \pm \sqrt{5}$

	$-\infty$		$-2-\sqrt{5}$		$-2+\sqrt{5}$		$+\infty$
f'		+		+		+	
f		↗		↘		↗	

$T_{MAX}(-2-\sqrt{5}, -1+\sqrt{5})$

$T_{MIN}(-2+\sqrt{5}, -1-\sqrt{5})$



(3) $g(x) = \ln(4-x^2)$

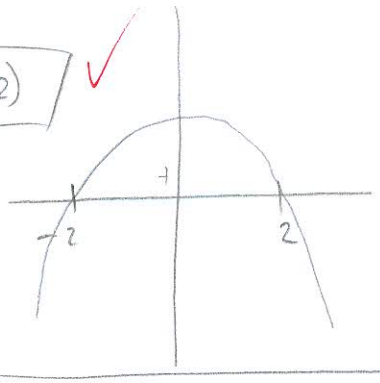
DOMENA

$4-x^2 > 0$

$x^2 = 4$

$x = \pm 2$

$x \in (-2, 2)$ ✓



$g(-x) = \ln(4-(-x)^2) = \ln(4-x^2)$

$g(-x) = g(x) \Rightarrow$ FUNKCIJA JE PARNA ✓

$g'(x) = \frac{1}{4-x^2}(-2x)$

$= \frac{-2x}{4-x^2}$

$g''(x) = \frac{-2(4-x^2)+2x(-2x)}{(4-x^2)^2} = \frac{-8+2x^2-4x^2}{(4-x^2)^2}$

$= \frac{-2x^2-8}{(4-x^2)^2} = 0$ $\quad g'' = 0$ NEMA TOČAKA INFLEKSIJE

$-2x^2-8=0$

$-2x^2=8$

$x^2 \neq -4$

FUNKCIJA JE KONKAVNA NA CIJELOM PODRUČJU DEF. ✓

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IME I PREZIME: JOSIP MATEŠIĆ

BROJ INDEKSA: 0269075368

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6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to 0}\dfrac{|x|}{x}$. — 5

Ukupno: (5)

(4.) $h(x) = \dfrac{x^2-2x-(2+1)}{x^2+1}$

$x \in \mathbb{R}$

$x^2+1 \neq 0$

$x^2 \neq -1 \,/\sqrt{}$

$x \neq \pm\sqrt{-1}$

V.A. ne postoji!

H.A. $\lim\limits_{x\to\infty}\dfrac{x^2-2x-3 \,/x^2}{x^2+1 \,/x^2} = \lim\limits_{x\to\infty}\dfrac{\frac{x^2}{x^2}-\frac{2x}{x^2}-\frac{3}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}} = \lim\limits_{x\to\infty}\dfrac{1}{1} = 1$

$y = 1$

K.A. ne postoji!

$\dfrac{x^2-2x-3}{x^2+1} = 0 \,/\, x^2+1$

$x^2-2x-3=0$

$x_{1,2} = \dfrac{-b\pm\sqrt{b^2-4ac}}{a^2}$

$x_{1,2} = \dfrac{2\pm\sqrt{(-2)^2-4\cdot 1\cdot(-3)}}{1}$

$x_{1,2} = \dfrac{2\pm 4}{1}$

$x_1 = \dfrac{2+4}{1} = 6$

$x_2 = \dfrac{2-4}{1} = -2$

NT(6,0) NT(0,-3)
NT(-2,0)

$$h(x) = \frac{x^2-2x-3}{x^2+1}$$

$$h(x)' = \frac{2x-2(x^2+1)-x^2-2x-3\cdot(2x)}{(x^2+1)^2}$$

$$h(x)' = \frac{2x^3+2x-2x^2-2-2x^3+4x^2+6x}{(x^2+1)^2}$$

$$h(x)' = \frac{2x^2+8x-2}{(x^2+1)^2}$$

$$\frac{2x^2+8x-2}{(x^2+1)^2} = 0 \quad / \cdot (x^2+1)^2$$

$$2x^2+8x-2=0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{a^2}$$

$$x_{1,2} = \frac{-8 \pm \sqrt{8^2-4\cdot 2\cdot(-2)}}{4}$$

$$x_{1,2} = \frac{-8 \pm 4\sqrt{5}}{4}$$

$$x_1 = \frac{-8+4\sqrt{5}}{4} = -2+\sqrt{5} \approx 0{,}24$$

$$x_2 = \frac{-8-4\sqrt{5}}{4} = -2-\sqrt{5} \approx -4{,}24$$

$m(0{,}24, -3{,}24)$

$M(-4{,}24, 1{,}24)$

	$-\infty$		$-4{,}24$		$-3{,}24$		$+\infty$
$h(x)'$		$+$	0	$-$	0	$+$	
$h(x)$		↗		↘		↗	

SKICA GRAFA!

JOSIP MATEŠIĆ

② $f(x)=x-\sqrt{x^2-2}$ $\quad x\in\langle-\infty,-\sqrt{2}\rangle\cup\langle\sqrt{2},+\infty\rangle$ ✗

$D_f=\langle-\infty,-\sqrt{2}]\ [\sqrt{2},+\infty\rangle$

$x^2-2>0$

$x^2>2\ /\sqrt{\ }$

$x>\pm\sqrt{2}$

V.A. $\lim\limits_{x\to\sqrt{2}} x-\sqrt{x^2-2}$

$\lim\limits_{x\to-\sqrt{2}} -\sqrt{2}-\sqrt{(-\sqrt{2})^2-2}=-\sqrt{2}$

$x=\sqrt{2}$

V.A $\lim\limits_{x\to\sqrt{2}} \sqrt{2}-\sqrt{(\sqrt{2})^2-2}=\sqrt{2}$ ✗

H.A. $\lim\limits_{x\to\infty} x-\sqrt{x^2-2}\ :x^2$

$\lim\limits_{x\to\infty} \frac{x}{x^2}-\sqrt{\frac{x^2}{x^2}-\frac{2}{x^2}}=\lim\limits_{x\to\infty} 1$

$y=1$

k.A. ne postoji! ✗

③ $g(x)=\ln(4-x^2)$ $\quad x\in\langle-\infty,-2]\cup[2,+\infty\rangle$ ✗

$-x^2+4\geq 0$

$-x^2\geq -4\ /\cdot(-1)$

$x^2\geq 4\ /\sqrt{\ }$

$x_{1,2}\geq\pm 2$

$g(x)=\ln(4-x^2)$

$g(x)=\ln(4-(-x)^2)$

$g(-x)=\ln(4-x^2)$

Funkcija je PARNA! ✓

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ G3

IME I PREZIME: IVANA FURLAN **BROJ INDEKSA:** 17-2-0173-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja uvrštavanjem. 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{aligned} 4A - 3B + 4C + 3D &= 3 \\ -3A + 4B - 3C - 4D &= 4 \\ -A - B + C + 3D &= 0 \\ 4A + 4B + 4C - 4D &= -4 \end{aligned}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 + 4x + 4} - 4x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{x+3}{x^2-4}$. 15(graf)

5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^3)$. 2+4+6+8

6. Zadana je funkcija $f(x) = \sqrt{4 + 3x}$. Kolika je derivacija $f'(2)$? Koji su lokalni ekstremi? 10+5

Ukupno: 0

2. $\left[\begin{array}{cccc|c} 4 & -3 & 4 & 3 & 3 \\ -3 & 4 & -3 & -4 & 4 \\ -1 & -1 & 1 & 3 & 0 \\ 4 & 4 & 4 & -4 & -4 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 7 \\ -3 & 4 & -3 & -4 & 4 \\ -1 & -1 & 1 & 3 & 0 \\ 4 & 4 & 4 & -4 & -4 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 7 \\ 0 & 7 & 0 & -7 & 25 \\ 0 & 0 & 2 & 2 & 7 \\ 0 & 0 & 0 & & \end{array}\right]$

5. $h(x) = \arctan(x^3)$

$$h'(x) = \frac{x^3}{1+x^2} \cdot 3x^2$$

$$h''(x) = \left(\frac{x^3}{1+x^2}\right)' \cdot 3x^2 + \frac{x^3}{1+x^2} \cdot (3x^2)'$$

$$h''(x) = \frac{x^{3'} \cdot (1+x^2) - x^3 \cdot (1+x^2)'}{(1+x^2)^2}$$

$$= \frac{3x^2 \cdot (1+x^2) - x^3 \cdot 2x}{(1+x^2)^2}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ G3

IME I PREZIME: DUŠEVIĆ KRISTIAN **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja uvrštavanjem. 12+3

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5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^3)$. 2+4+6+8

6. Zadana je funkcija $f(x) = \sqrt{4 + 3x}$. Kolika je derivacija $f'(2)$? Koji su lokalni ekstremi? 10+5

Ukupno: 20

6. $f(x) = \sqrt{4+3x}$

$f'(2) = \sqrt{4+6}$ ✗

$f'(2) = \sqrt{10} = f'(2) = \frac{1}{2\sqrt{10}}$ ✗

4. $f(x) = \frac{x+3}{x^2-4}$

1° DOMENA

$x^2 - 4 \neq 0$

$(x+2)(x-2) \neq 0$ $\quad Df = \langle -\infty, -2 \rangle \cup \langle -2, 2 \rangle \cup \langle 2, +\infty \rangle$

$x \neq -2$

$x \neq +2$

2° NULTOČKE

$x+3 = 0$ $\quad N_1(-3, 0)$

$x = -3$

3° EKSTREMI

$f'(x) = \frac{x+3}{x^2-4} = \frac{x^2-4-(x+3)(2x)}{x^4-8x^2+16} = \frac{x^2-4-2x^2-6}{x^4-8x^2+16}$

$f'(x) = \frac{-x^2-10}{x^4-8x^2+16}$ - - - + + +

$-x^2 - 10 = 0$

NEMA EKSTREMA

KRISTIAN DUJEVIĆ

3. $g(x) = \sqrt{x^2+4x+4} - 4x$

$x^2+4x+4 \geq 0$

$x_{12} \geq \frac{-4 \pm \sqrt{16-16}}{2}$

$x_{12} \geq \frac{-4}{2}$

$x \geq -2$

$D_f : \mathbb{R}$ ✓

1° V.A.

$\lim_{x \to -2^+} \sqrt{x^2+4x+4} - 4x = \lim_{x \to -2^+} \sqrt{(-2)^2-8+4} - 8^+ = \sqrt{4-8+4} - 8^+ = -8^+$

$\lim_{x \to -2^-} \sqrt{x^2+4x+4} - 4x = \lim_{x \to -2^-} \sqrt{(-2)^2-8^-+4} - 8^- = \sqrt{4^+-8^-+4} - 8^- = -8$

NIJE VERTIKALNA ASIMPTOTA

2° H.A

$\lim_{x \to \pm\infty} \sqrt{x^2+4x+4} - 4x \overset{\infty}{=} \lim_{x \to \pm\infty} \sqrt{1-0-0} - 0 = 1$

$y = 1$ → JE HORIZONTALNA ASIMPTOTA ✗

NEMA KOSE ASIMPTOTE ✗

1. $\frac{|z|}{z+2i} = 3i$

$\frac{\sqrt{x^2+y^2}}{x+yi+2i} = 3i \cdot (x+yi+2i)$

$\sqrt{x^2+y^2} = 3i(x+yi+2i)$

$\sqrt{x^2+y^2} = 3xi - 3y - 6 \,/^2$

$x^2+y^2 = -9x^2 + 9y^2 + 36$

$10x^2 - 9y^2 = 36$

$-y^2 = 0$

$y = 0$ → $10x^2 = 36$

$x^2 = 3.6$

$x = 1.89$

$z = 1.89 + 0i$ ✗

KRISTIAN DOJČIĆ

4. 4° ASIMPTOTE

1° V.A

$\lim_{x \to 2^+} \frac{x+3}{x^2-4} = \frac{2^+ + 3}{4^+ - 4} = \frac{5^+}{0^-} = -\infty$

$\lim_{x \to 2^-} \frac{x+3}{x^2-4} = \frac{5^-}{0^-} = +\infty$

$\lim_{x \to -2^+} \frac{x+3}{x^2-4} = \frac{1^+}{0^-} = -\infty$

$\lim_{x \to -2^-} \frac{x+3}{x^2-4} = \frac{1^-}{0^+} = +\infty$

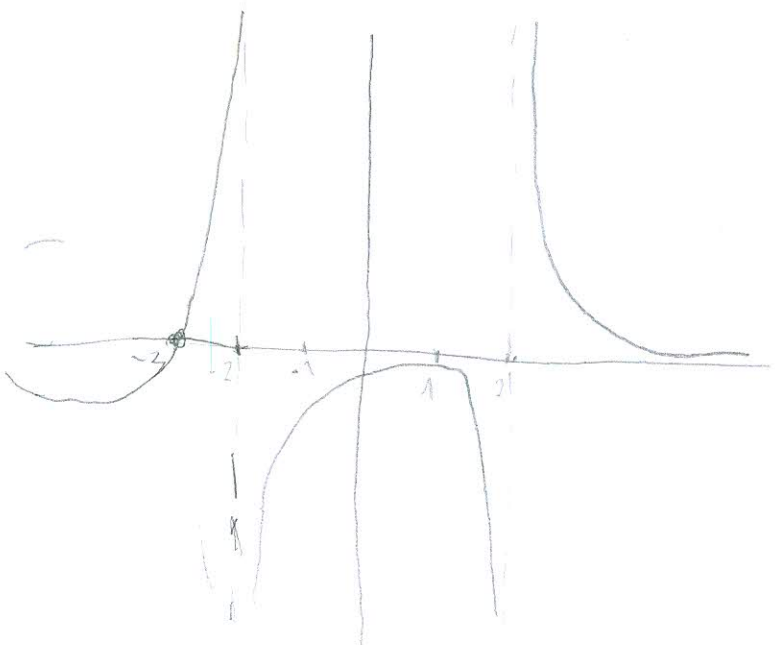
-2 i 2 su VERTIKALNE ASIMPTOTE

2° H.A

$\lim_{x \to \pm\infty} \frac{x+3}{x^2-4} \; / :x^2 = \frac{0}{1} = 0$

$y = 0 \Rightarrow$ JE HORIZONTALNA ASIMTOTA

NEMA KOSE



✓

KRISTIAN DUJEVIĆ

$$\begin{bmatrix} 4 & -3 & 4 & 3 \\ -3 & 4 & -3 & -4 \\ -1 & -1 & 1 & 3 \\ 4 & 4 & 4 & -4 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 0 \\ -4 \end{bmatrix} \leftarrow \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} -1 & 0 & 6 & 1 & \frac{3}{2} \\ 0 & 1 & -6 & -1 & 4 \\ 0 & 0 & -10 & -6 & 7 \\ 0 & 0 & 2 & 2 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} -1 & -1 & 1 & 3 & 0 \\ -3 & 4 & -3 & -4 & 4 \\ 4 & -3 & 4 & 3 & 3 \\ 4 & 4 & 4 & -4 & -4 \end{array}\right] :4$$

$$\left[\begin{array}{cccc|c} -1 & -1 & 1 & 3 & 0 \\ -3 & 4 & -3 & -4 & 4 \\ 4 & -3 & 4 & 3 & 3 \\ 4 & 4 & 4 & -4 & -4 \end{array}\right] \begin{array}{l} II-3I \\ III+4I \\ IV+I \end{array}$$

$$\begin{matrix} A \\ B \\ C \\ D \end{matrix} \begin{bmatrix} 0 \\ -\frac{3}{2} \\ -1 \\ \frac{1}{2} \end{bmatrix} \times$$

PROVJERA

$4 \cdot 0 - 3 \cdot \left(-\frac{3}{2}\right) + 4 \cdot (-1) + 3 \cdot \frac{1}{2} = 2 \neq 3$

$$\begin{matrix} A & B & C & D \end{matrix}$$

$$\left[\begin{array}{cccc|c} -1 & -1 & 1 & 3 & 0 \\ 0 & 1 & -6 & -16 & 4 \\ 0 & -7 & 9 & 15 & 3 \\ 0 & 0 & 2 & 2 & -1 \end{array}\right] II+II$$

$$\left[\begin{array}{cccc|c} -1 & -1 & 1 & 3 & 0 \\ 0 & 1 & -6 & -16 & 4 \\ 0 & -6 & 2 & -1 & 7 \\ 0 & 0 & 2 & 2 & -1 \end{array}\right] I-\frac{1}{2}IV$$

$$\left[\begin{array}{cccc|c} -1 & -1 & 0 & 2 & -\frac{1}{2} \\ 0 & 1 & -6 & -1 & 4 \\ 0 & -6 & 2 & -1 & 7 \\ 0 & 0 & 2 & 2 & -1 \end{array}\right] I+II$$

$$\left[\begin{array}{cccc|c} -1 & 0 & -6 & 1 & \frac{7}{2} \\ 0 & 1 & -6 & -1 & 4 \\ 0 & -6 & 2 & -1 & 7 \\ 0 & 0 & 2 & 2 & -1 \end{array}\right] III+6II$$

$$\left[\begin{array}{cccc|c} -1 & 0 & -6 & 1 & \frac{7}{2} \\ 0 & 1 & -6 & -1 & 4 \\ 0 & 0 & -10 & -6 & 7 \\ 0 & 0 & 2 & 2 & -1 \end{array}\right] I+3II$$

$-2(+2D) = -1 \Rightarrow C = \frac{-1-2D}{2}$

$-10C - 6D = 7$

$-10 \cdot \frac{-1-2D}{2} - 6D = 7$

$5 + 10D - 6D = 7$

$4D = 2$

$D = \frac{1}{2}$

$C = \frac{-1-2 \cdot \frac{1}{2}}{2}$

$C = -1$

$-A + 7D = \frac{7}{2}$

$A = 0$

$B - 6C - 1D = 4$

$B + 6 - \frac{1}{2} = 4$

$B = 4,5 - 6$

$B = -1,5$

$B = -\frac{3}{2}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ G3

IME I PREZIME: MARINO ĆOSIĆ **BROJ INDEKSA:** 0269080599

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja uvrštavanjem. 12+3
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5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^3)$. 2+4+6+8
6. Zadana je funkcija $f(x) = \sqrt{4 + 3x}$. Kolika je derivacija $f'(2)$? Koji su lokalni ekstremi? 10+5

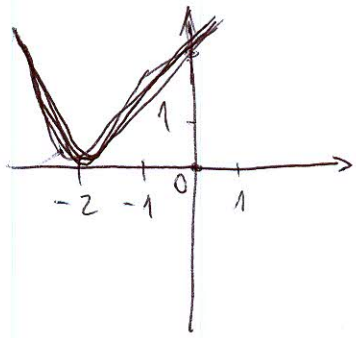
Ukupno: 10

(3) $x^2+4x+4 \geq 0$

$D_f = R$

$(x+2)^2 \geq 0$

$x = -2$



$D_f = R$ ✓

(6) $f(x) = (4x+3x)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(4+3x)^{\frac{1}{2}-1} \cdot 3 = \frac{3}{2}(4+3x)^{-\frac{1}{2}}$

$f'(2) = \frac{3}{2}(4+3\cdot 2)^{-\frac{1}{2}}$

$f'(2) = \frac{3}{2}(4+6)^{-\frac{1}{2}} = \frac{3}{2}10^{-\frac{1}{2}} = \frac{3}{2}\cdot\frac{1}{\sqrt{10}} =$ ✓

$= \frac{3}{2}\cdot\frac{1}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = \frac{3\sqrt{10}}{2\cdot 10} = \frac{3\sqrt{10}}{20}$

① $\frac{|z|}{z+2i} = 3i$

$z = x + iy \quad |z| = x^2 + y^2$

$\frac{x^2+y^2}{x+iy+2i} = 3i$

$\frac{x^2+y^2}{x+(2+y)i} = 3i$

$\frac{x^2+y^2}{x+(2+y)i} \cdot \frac{x-(2+y)i}{x-(2+y)i} = 3i$

$\frac{x(x^2+y^2) - (x^2+y^2)(2+y)i}{x^2+(2+y)^2}$

$x(x^2+y^2) = 0$

$x = 0$

$\frac{-(x^2+y^2)(2+y)}{x^2+(2+y)^2} = 3$

$-2y^2 + y^3 = 3(2+y)^2$

$-2y^2 + y^3 = 3(4+4y+y^2)$

$-2y^2 - y^3 = 12 + 12y + 3y^2$

$z = ?$

② $\begin{vmatrix} 4 & -3 & 4 & 3 \\ -3 & 4 & 3 & -4 \\ -1 & -1 & 1 & 3 \\ 4 & 4 & 4 & -4 \end{vmatrix} = \begin{vmatrix} 4 & -3 & 0 & 0 \\ -3 & 4 & 0 & 0 \\ -1 & -1 & -2 & 2 \\ 4 & 4 & 0 & 0 \end{vmatrix} = 4 \begin{vmatrix} 4 & 0 & 0 \\ -1 & -2 & 2 \\ 4 & 0 & 0 \end{vmatrix} + 3 \begin{vmatrix} -3 & 0 & 0 \\ -1 & -2 & 2 \\ 4 & 0 & 0 \end{vmatrix}$

$= 4 \cdot 4 \begin{vmatrix} -2 & 2 \\ 0 & 0 \end{vmatrix} = 16 \cdot 0 + (-9) \cdot 0 = 0$?

nema kompleksnog rješenja

MARINO ĐOSIĆ

④ $f(x)=\frac{x+3}{x^2-4}$

$x+3=0$ $\quad x^2-4\neq 0$

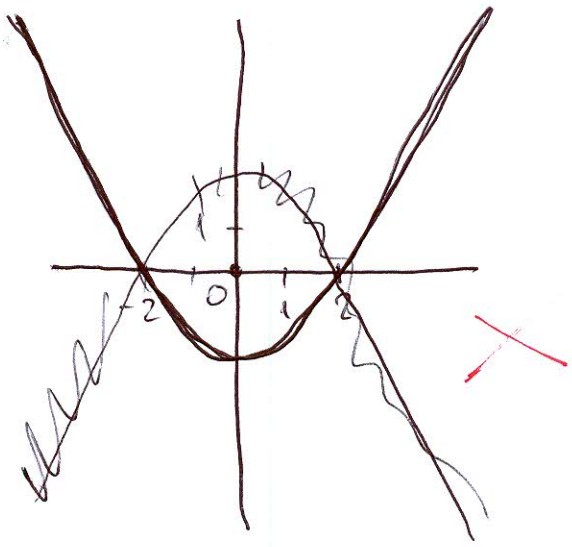
$x=-3$ $\quad x\neq \pm 2$

	$-\infty$	-3	-2	2	$+\infty$
$x+3$	$-$	0 $+$	$+$	$+$	
$x-2$	$-$	$-$	$-$	0 $+$	
$x+2$	$-$	$-$	0 $+$	$+$	
	$+$	$-$	$-$	$+$	

?

~~$x\in\langle -\infty, -3\rangle$~~

$x\in\langle -\infty, -2\rangle \cup \langle 2, +\infty\rangle$



TOK FUNKCIJE NE ZNATE!!!

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ g3

IME I PREZIME: Marin Dušević **BROJ INDEKSA:** 0269081811

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja uvrštavanjem. — 12+3
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5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^3)$. — 2+4+6+8
6. Zadana je funkcija $f(x) = \sqrt{4 + 3x}$. Kolika je derivacija $f'(2)$? Koji su lokalni ekstremi? — 10+5

Ukupno: 70

5

$h(x) = \arctan(x^3)$

$D(h) = R$ ✓

NIJE PERIODIČNA ✓

$h(-x) = \arctan((-x)^3) = -\arctan(x^3)$

NEPARNA JE ✓

$h'(x) = \frac{3x^2}{1+x^6}$ ✓

$h''(x) = \frac{6x(1+x^6) - 3x^2 \cdot 6x^5}{(1+x^6)^2}$ ✓

$= \frac{6x - 12x^7}{(1+x^6)^2}$

(6) $f(x) = \sqrt{4+3x}$

$4+3x \geq 0$

$3x \geq -4$

$x \geq -\frac{4}{3}$

$D(f) = \left[-\frac{4}{3}, +\infty\right>$

$f'(x) = \frac{3}{2\sqrt{4+3x}}$

$f'(2) = \frac{3}{2\sqrt{10}} = \frac{3\sqrt{10}}{20}$

$f'(x) > 0$

$f\left(-\frac{4}{3}\right) = 0$

FUNKCIJA IMA MINIMUM $\left(-\frac{4}{3}, 0\right)$

$= \sqrt{x^2+4x+4} - 4x$

$4x + 4 \geq 0$

$)^2 \geq 0 \rightarrow D(g) = R$ ✓

VERTIKALNIH ASIMPTOTA

$\lim_{x \to -\infty} \left(\sqrt{x^2+4x+4} - 4x\right) = +\infty$

∞ $\frac{\sqrt{x^2+4x+4} - 4x}{x} = \lim_{x \to -\infty} \left(-\sqrt{1+\frac{4}{x}+\frac{4}{x^2}} - 4\right)$

5

$-\infty$ $\left(\sqrt{x^2+4x+4} + x\right) \cdot \frac{\sqrt{x^2+4x+4} - x}{\sqrt{x^2+4x+4} - x}$

$\lim_{x \to -\infty}$ $\frac{x^2+4x+4-x^2}{\sqrt{x^2+4x+4} - x}$ $: x$ $: x$

$\lim_{x \to -\infty}$ $\frac{4+\frac{4}{x}}{-\sqrt{1+\frac{4}{x}+\frac{4}{x^2}} - 1} = -2$

LKA: $y = -5x - 2$ ✓

$\lim_{x \to +\infty}$ $\left(\sqrt{x^2+4x+4} - 4x\right) \cdot \frac{\sqrt{x^2+4x+4} + 4x}{\sqrt{x^2+4x+4} + 4x}$

$\lim_{x \to +\infty}$ $\frac{x^2+4x+4-16x^2}{\sqrt{x^2+4x+4} + 4x}$ $/ : x^2$ $/ : x^2$

$\rightarrow$

$$\frac{-15+\frac{4}{x}+\frac{4}{x^2}}{\sqrt{\frac{1}{x^2}+\frac{4}{x^3}+\frac{4}{x^4}}+4x} = \frac{-15}{0} = \infty$$

$\to +\infty$

$$\lim_{x\to+\infty} \frac{\sqrt{x^2+4x+4}-4x}{x} = \lim_{x\to+\infty}\left(\sqrt{1+\frac{4}{x}+\frac{4}{x^2}}-4\right)$$

$\to +\infty$

$= -3$

$$\lim_{x\to+\infty}\left(\sqrt{x^2+4x+4}-x\right)\cdot\frac{\sqrt{x^2+4x+4}+x}{\sqrt{x^2+4x+4}+x}$$

$\to +\infty$

$$\lim_{x\to+\infty} \frac{x^2+4x+4-x^2}{\sqrt{x^2+4x+4}+x} \;\Big|\; :x \;\Big|\; :x = \lim_{x\to+\infty} \frac{4+\frac{4}{x}}{\sqrt{1+\frac{4}{x}+\frac{4}{x^2}}+1}$$

$\to +\infty$

$= 2$

DKA: $y = -3x+2$ ✓

$z = x + yi$

$|z| = \sqrt{x^2+y^2}$

$\frac{|z|}{+2i} = 3i$

$\frac{\sqrt{x^2+y^2}}{yi+2i} = 3i \Rightarrow \sqrt{x^2+y^2} = 3xi - 3y - 6$

$\sqrt{x^2+y^2} = -3y - 6$

$3x = 0 \rightarrow x = 0$

$|y| = -3y - 6$

$4y = -6$ $\quad$ $2y = -6$

$y = -\frac{3}{2}$ $\quad$ $y = -3$

$z = -3i$

PROVJERA:

$\frac{\sqrt{(-3)^2}}{-3i+2i} = 3i \rightarrow \frac{3}{-i} = 3i \;/\cdot(-i)$

$3 = -3i^2$

$3 = 3$ ✓

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ G3

IME I PREZIME: MARKO VUKELIĆ

BROJ INDEKSA: 0268080097

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja uvrštavanjem. 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{aligned} 4A - 3B + 4C + 3D &= 3 \\ -3A + 4B - 3C - 4D &= 4 \\ -A - B + C + 3D &= 0 \\ 4A + 4B + 4C - 4D &= -4 \end{aligned}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 + 4x + 4} - 4x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{x+3}{x^2-4}$. 15(graf)

5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^3)$. 2+4+6+8

6. Zadana je funkcija $f(x) = \sqrt{4 + 3x}$. Kolika je derivacija $f'(2)$? Koji su lokalni ekstremi? 10+5

Ukupno: 35

5. $h(x) = \arctan(x^3)$

$D(h) = \mathbb{R}$ ✓ FUNKCIJA NIJE PERIODIČKA ✓

$h(-x) = \arctan\left((-x)^3\right) = -\arctan(x^3)$

FUNKCIJA JE NEPARNA ✓

$h'(x) = \frac{1}{1 + x^6} \cdot 3x^2 = \frac{3x^2}{1 + x^6}$ ✓

$h''(x) = \frac{6x(1 + x^6) - 3x^2 \cdot 6x^5}{(1 + x^6)^2}$ ✓

$= \frac{6x + 6x^7 - 18x^7}{(1 + x^6)^2}$

$= \frac{6x - 12x^7}{(1 + x^6)^2}$

6. $f(x) = \sqrt{4+3x}$

$4+3x \geq 0$

$3x \geq -4$

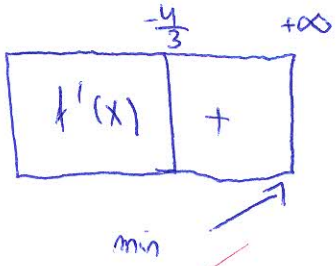
$x \geq -\frac{4}{3}$

$D(f) = \left[-\frac{4}{3}, +\infty\right)$

$f'(x) = \frac{1}{2}(4+3x)^{-\frac{1}{2}} \cdot 3$

$= \frac{3}{2\sqrt{4+3x}}$ ✓

$f'(2) = \frac{3}{2\sqrt{10}}$ ✓



$f\left(-\frac{4}{3}\right) \Rightarrow 0$ min $\left(-\frac{4}{3}, 0\right)$ ✓

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ g3

IME I PREZIME: STIPE BOŽIĆ **BROJ INDEKSA:** 17-1-0134-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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6. Zadana je funkcija $f(x) = \sqrt{4 + 3x}$. Kolika je derivacija $f'(2)$? Koji su lokalni ekstremi? 10+5

Ukupno: 0

2.

$$\left|\begin{array}{cccc|c} 4 & -3 & 4 & 3 & 3 \\ -3 & 4 & -3 & -4 & 4 \\ -1 & -1 & 1 & 3 & 0 \\ 4 & 4 & 4 & -4 & -4 \end{array}\right| \begin{array}{l} \\ \\ \\ +\downarrow \end{array} \sim \left|\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 7 \\ -3 & 4 & -3 & -4 & 4 \\ -1 & -1 & 1 & 3 & 0 \\ 4 & 4 & 4 & -4 & -4 \end{array}\right|$$