

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

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H2

- Riješiti jednačbu:  $z^4 - (4 - i)^2 = 0$ . Prikaži rješenja u kompleksnoj ravnini!
- Odrediti domenu, sve asimptote i drugu derivaciju funkcije  $f(x) = x - \sqrt{x^2 - 2}$ .
- Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije  $g(x) = \ln(4 - x^2)$ .
- Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije  $h(x) = \frac{x^2 - 2x - (2+1)}{x^2 + 1}$ . Ne treba ispitivati zakrivljenost jer se izraz komplicira.
- Gaussovom metodom riješiti matricni sustav i obavezno provjeri rješenje:

12+3  
5+15+5  
5+5+10  
20(graf)  
15

$$\begin{aligned} x + 2y - z + u &= 2 \\ 2x + 5y - z + 2u &= 3 \\ 3x - y - 2z + u &= 2 \\ x - y + 3z - 5u &= 3 \end{aligned}$$

- Izračunati i provjeriti uvrštavanjem:  $\lim_{x \rightarrow 0} \frac{|x|}{x}$ .

5

Ukupno:

*57*

⑤

$$\begin{bmatrix} x + 2y - z + u & | & 2 \\ 2x + 5y - z + 2u & | & 3 \\ 3x - y - 2z + u & | & 2 \\ x - y + 3z - 5u & | & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & | & 2 \\ 2 & 5 & -1 & 2 & | & 3 \\ 3 & -1 & -2 & 1 & | & 2 \\ 1 & -1 & 3 & -5 & | & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & | & 2 \\ 0 & 8 & -1 & 0 & | & 4 \\ 0 & -5 & -1 & -2 & | & 1 \\ 0 & -3 & -4 & -6 & | & 15 \end{bmatrix}$$

$1R \cdot 2R + (-2)$   
 $1R \cdot 3R + (-2)$   
 $1R \cdot 4R + (-1)$

$$\begin{bmatrix} 1 & 2 & -1 & 1 & | & 2 \\ 0 & 8 & -1 & 0 & | & 4 \\ 0 & -5 & -1 & -2 & | & 1 \\ 0 & -3 & -4 & -6 & | & 15 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & | & 2 \\ 0 & 1 & -\frac{1}{8} & 0 & | & \frac{1}{2} \\ 0 & 0 & \frac{41}{8} & 5 & | & \frac{11}{2} \\ 0 & 0 & \frac{7}{2} & -6 & | & \frac{11}{2} \end{bmatrix} \xrightarrow{\cdot \frac{8}{41}}$$

$2R \cdot 3R + 5$   
 $2R \cdot 4R + 3$

$$\begin{bmatrix} 1 & 2 & -1 & 1 & | & 2 \\ 0 & 1 & -\frac{1}{8} & 0 & | & \frac{1}{2} \\ 0 & 0 & 1 & \frac{40}{41} & | & \frac{44}{41} \\ 0 & 0 & 0 & -\frac{797}{82} & | & \frac{197}{82} \end{bmatrix} \xrightarrow{\cdot (-\frac{82}{797})}$$

$3R \cdot 4R + (-\frac{7}{2})$   
 $4R \cdot 3R (-\frac{40}{41})$   
 $4R \cdot 1R + (-1)$

$$\begin{bmatrix} 1 & 2 & -1 & 1 & | & 2 \\ 0 & 1 & -\frac{1}{8} & 0 & | & \frac{1}{2} \\ 0 & 0 & 1 & \frac{40}{41} & | & \frac{44}{41} \\ 0 & 0 & 0 & 1 & | & -\frac{197}{797} \end{bmatrix}$$

$-\infty$        $-2-\sqrt{5}$        $-2+\sqrt{5}$        $+\infty$

$h'(x)$	+	-	+	
$h(x)$	↗	↘	↗	

MULTIPLICARE  $x^2 - 2x - 3 = 0$

$$x_{1,2} = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-3)}}{2}$$

$$\begin{cases} \text{MAX} (-2 - \sqrt{5}, -1 + \sqrt{5}) \\ \text{MIN} (-2 + \sqrt{5}, -1 - \sqrt{5}) \end{cases}$$

$$(3, 0)$$

$$x_1 = 3$$

$$(-1, 0)$$

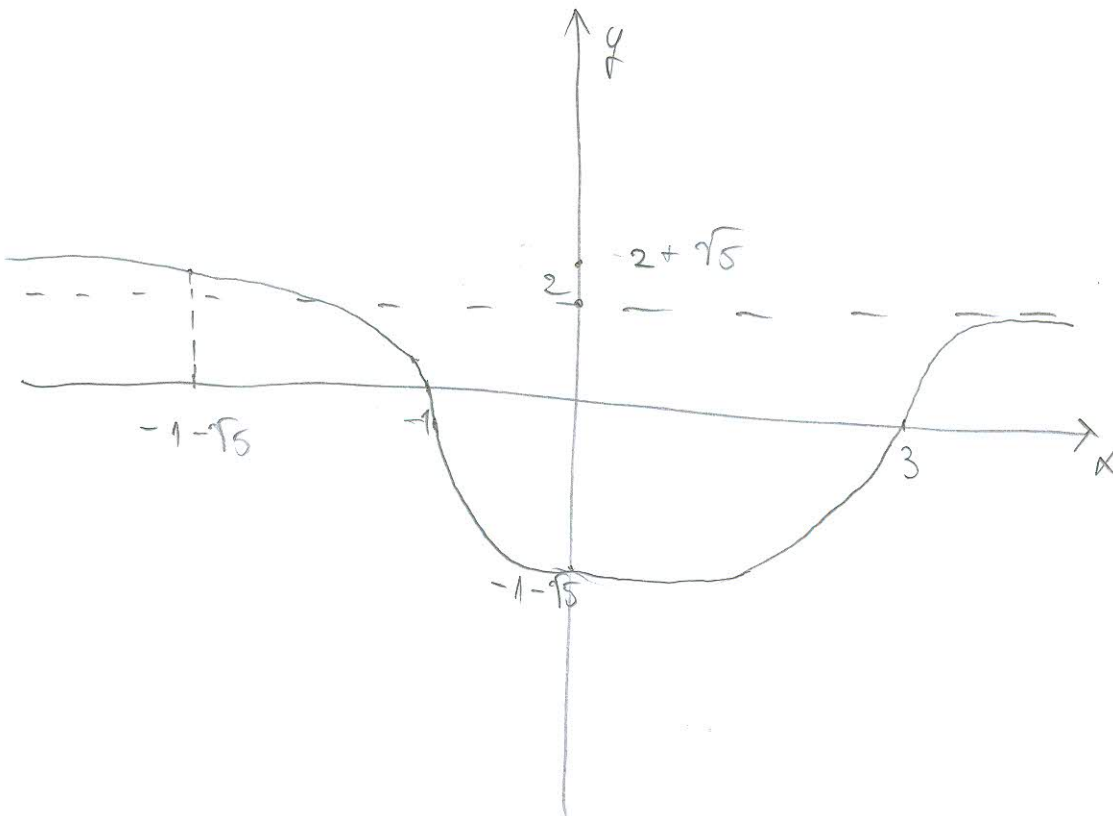
$$x_2 = -1$$

HOR. AS

$$\lim_{x \rightarrow \pm\infty} \frac{x^2 - 2x - 3}{x^2 + 1} = \lim_{x \rightarrow \pm\infty} \frac{2x - 2}{2x} = \left(\frac{\infty}{\infty}\right)' =$$

$$= \lim_{x \rightarrow \pm\infty} \frac{2}{2} = 1 //$$

$y = 1$       HORIZ. ASIMPTOTA



$$9) \quad h(x) = \frac{x^2 - 2x - (2+1)}{x^2 + 1}$$

DOMENNA

$$x^2 + 1 \neq 0$$

$$x^2 \neq -1$$

$$x \in \mathbb{R}$$

$$h'(x) = \frac{(2x-2)(x^2+1) - (x^2-2x-3) \cdot 2x}{(x^2+1)^2} =$$

$$= \frac{(2x-2)(x^2+1) - (x^2-2x-3) \cdot 2x}{(x^2+1)^2}$$

$$= \frac{2x^3 + 2x - 2x^2 - 2 - 2x^2 + 4x^2 + 6x}{(x^2+1)^2}$$

$$= \frac{2x^2 + 8x - 2}{(x^2+1)^2}$$

$$x_{1,2} = \frac{-4 \pm \sqrt{16 - 4 \cdot 1 \cdot (-1)}}{2}$$

$$\frac{2x^2 + 8x - 2}{(x^2+1)^2} = 0$$

$$x_1 = -2 + \sqrt{5}$$

$$x_2 = -2 - \sqrt{5} \rightarrow$$

$$2x^2 + 8x - 2 = 0$$

$$x^2 + 4x - 1 = 0$$

Antonio Ivanušić

$$\left[ \begin{array}{cccc|c} -1 & 2 & -1 & 0 & -1161 \\ 0 & 1 & -\frac{1}{8} & 0 & \frac{1}{2} \\ 0 & 0 & 7 & 0 & \frac{8668}{8437} \\ 0 & 0 & 0 & 7 & \frac{197}{767} \end{array} \right]$$

$$3R \cdot 2R + \frac{1}{8}$$

$$3R \cdot 1R + 1$$

$$-\frac{197}{767} \cdot \frac{44}{41} = -\frac{8668}{8437}$$

$$\left[ \begin{array}{cccc|c} 1 & 2 & 0 & 0 & -1161 \\ 0 & 1 & 0 & 0 & \frac{394}{767} \\ 0 & 0 & 1 & 0 & \frac{8668}{8437} \\ 0 & 0 & 0 & 7 & \frac{197}{767} \end{array} \right]$$

$$2R \cdot 1R - 2 \sim$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 594,4 \\ 0 & 1 & 0 & 0 & \frac{394}{767} \\ 0 & 0 & 1 & 0 & \frac{8668}{8437} \\ 0 & 0 & 0 & 1 & \frac{197}{767} \end{array} \right]$$

$$594,4 \rightarrow x$$

$$-593,89 \rightarrow y$$

$$-1,03 \rightarrow z$$

$$-0,26 \rightarrow w$$

$$594,4 + 2 \cdot (-593,89) + 1,03 - 0,26 = 2$$

③

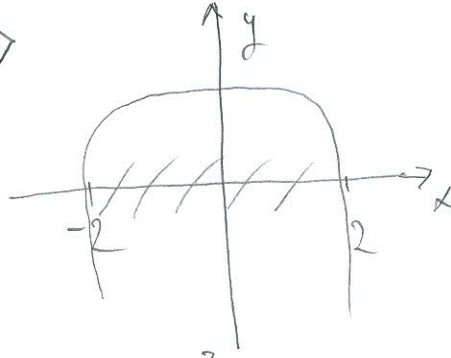
$$4 - x^2 > 0$$

$$4 - x^2 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$$x \in \langle -2, 2 \rangle$$



$$g(-x) = \ln(4 - (-x)^2) = \ln(4 - x^2)$$

$g(x) = g(-x) \rightarrow$  FUNKCIJA JE PARNA ✓

$$g'(x) = \frac{1}{4-x^2} \cdot (-2x) = \frac{-2x}{4-x^2}$$

$$g''(x) = \frac{-2(4-x^2) + 2x(-2x)}{(4-x^2)^2}$$

$$= \frac{-8 + 2x^2 - 4x^2}{(4-x^2)^2} = \frac{-2x^2 - 8}{(4-x^2)^2} //$$

$$(4-x^2)^2 > 0$$

$$-2x^2 - 8 < 0 \quad (\text{ZA BIKO KOJI } x)$$

IZ TOGA SLEDI DA JE  $g''(x) < 0$

PA JE FUNKCIJA KONKAVNA ✓

②

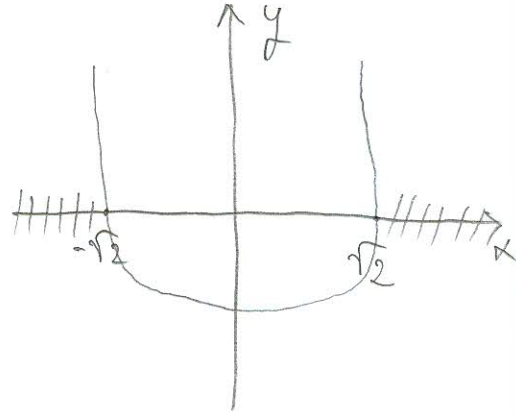
$$f(x) = x - \sqrt{x^2 - 2}$$

$$x^2 - 2 \geq 0$$

$$x^2 - 2 = 0$$

$$x^2 = 2$$

$$x = \pm \sqrt{2}$$



$$x \in \langle -\infty, -\sqrt{2} \rangle \cup \langle \sqrt{2}, +\infty \rangle \quad \checkmark$$

$$f'(x) = 1 - \frac{1}{2\sqrt{x^2-2}} \cdot 2x = 1 - \frac{x}{\sqrt{x^2-2}}$$

$$f''(x) = \frac{-\sqrt{x^2-2} - x \cdot \frac{1}{2\sqrt{x^2-2}} \cdot 2x}{(\sqrt{x^2-2})^2} \quad \checkmark =$$

$$f''(x) = \frac{-\sqrt{x^2-2} - \frac{x^2}{\sqrt{x^2-2}}}{2}$$

$$f''(x) = \frac{\frac{x^2-2-x^2}{\sqrt{x^2-2}}}{x^2-2} = \frac{-2}{(x^2-2)^{\frac{3}{2}}} = \frac{2}{(x^2-2)^{\frac{3}{2}}} \quad \checkmark$$

HORIZONTALNA AS.

$$\lim_{x \rightarrow \infty} (x - \sqrt{x^2 - 2}) = 0 \quad (\text{DESNA})$$

L.K.A.?

