

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **DINO DOKOZA**

BROJ INDEKSA: **1219036348**

H2

- Riješiti jednačbu: $z^4 - (4 - i)^2 = 0$. Prikaži rješenja u kompleksnoj ravnini!
- Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2 - 2}$.
- Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln(4 - x^2)$.
- Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \frac{x^2 - 2x - (2 + 1)}{x^2 + 1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira.
- Gaussovom metodom riješiti matični sustav i obavezno provjeri rješenje:

$$\begin{aligned} x + 2y - z + u &= 2 \\ 2x + 5y - z + 2u &= 3 \\ 3x - y - 2z + u &= 2 \\ x - y + 3z - 5u &= 3 \end{aligned}$$

- Izračunati i provjeriti uvrštavanjem: $\lim_{x \rightarrow 0} \frac{|x|}{x}$.

6

12+3

5+15+5

5+5+10

20(graf)

15

5

Ukupno:

43

$$5. \begin{bmatrix} 0 & 2 & 0 & 0 & 2 \\ 2 & 5 & 0 & 2 & 3 \\ 3 & 0 & -2 & 0 & 2 \\ 0 & 0 & 3 & -5 & 3 \end{bmatrix} \xrightarrow{1:2} \sim \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 2 & 5 & 0 & 2 & 3 \\ 3 & 0 & -2 & 0 & 2 \\ 0 & 0 & 3 & -5 & 3 \end{bmatrix} \xrightarrow{-2R_1} \begin{bmatrix} 3 & -2 & 0 & 0 & 0 \\ 0 & 3 & -5 & 2 & 1 \\ 0 & 0 & 3 & -5 & 3 \\ 0 & 0 & 3 & -5 & 3 \end{bmatrix} \xrightarrow{-4R_2} \begin{bmatrix} 3 & -2 & 0 & 0 & 0 \\ 0 & 3 & -5 & 2 & 1 \\ 0 & 0 & 3 & -5 & 3 \\ 0 & 0 & 3 & -5 & 3 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 3 & -2 & 0 & 0 & 0 \\ 0 & 0 & 3 & -5 & 3 \\ 0 & 3 & -5 & 2 & 1 \\ 0 & 0 & 3 & -5 & 3 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 3 & -2 & 0 & 0 & 0 \\ 0 & 3 & -5 & 2 & 1 \\ 0 & 0 & 3 & -5 & 3 \\ 0 & 0 & 3 & -5 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -5 & -2 & -2 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 3 & -5 & 3 \\ 2 & 5 & 0 & 2 & 3 \end{bmatrix} \xrightarrow{-2 \cdot 1R} \begin{bmatrix} 1 & -5 & -2 & -2 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 3 & -5 & 3 \\ 0 & 15 & 4 & 6 & 2 \end{bmatrix} \xrightarrow{-15 \cdot 2R} \begin{bmatrix} 1 & -5 & -2 & -2 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 3 & -5 & 3 \\ 0 & 0 & 4 & 6 & -13 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -5 & -2 & -2 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -7 & -11 & 16 \\ 0 & 0 & 4 & 6 & -13 \end{bmatrix} \xrightarrow{+5 \cdot 2R} \begin{bmatrix} 1 & 0 & -2 & -2 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -7 & -11 & 16 \\ 0 & 0 & 4 & 6 & -13 \end{bmatrix} \xrightarrow{(-1)} \begin{bmatrix} 1 & 0 & -2 & -2 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 7 & 11 & -16 \\ 0 & 0 & 4 & 6 & -13 \end{bmatrix} \xrightarrow{-4 \cdot 3R} \begin{bmatrix} 1 & 0 & -2 & -2 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 7 & 11 & -16 \\ 0 & 0 & 0 & -38 & 51 \end{bmatrix} \xrightarrow{1 \cdot -38}$$

$$\begin{bmatrix} 1 & 0 & -2 & -2 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 7 & 11 & -16 \\ 0 & 0 & 0 & -38 & 51 \end{bmatrix}$$

?

$$\textcircled{1} z^4 - (4-i)^2 = 0$$

$$z^4 = (4-i)^2$$

$$z^4 = 16 - 8i + i^2$$

$$z^4 = 16 - 8i - 1$$

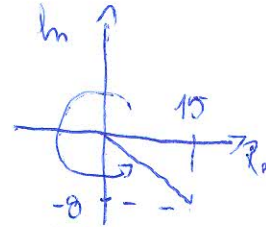
$$z^4 = 15 - 8i$$

$$w = 15 - 8i$$

$$x = 15$$

$$y = -8$$

$$|w| = \sqrt{15^2 + 8^2} = 17 \checkmark$$



$$\varphi = 2\pi - 0,4899573263$$

$$\varphi = 5,793227981 \checkmark$$

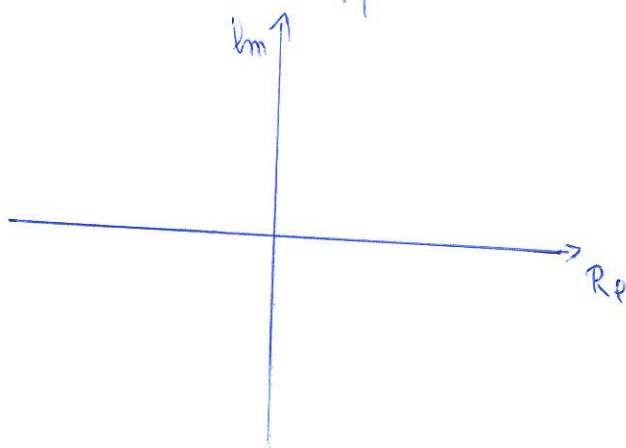
$$w_0 = \sqrt[4]{17} \left(\cos \frac{5,793227981}{4} + i \sin \frac{5,793227981}{4} \right)$$

$$= \sqrt[4]{17} (0,9996805353 + i 0,02527503381)$$

$$w_1 = \sqrt[4]{17} \left(\cos \left(\frac{5,793227981 + 2\pi}{4} \right) + i \sin \left(\frac{5,793227981 + 2\pi}{4} \right) \right)$$

$$w_2 = \sqrt[4]{17} \left(\cos \left(\frac{5,793227981 + 4\pi}{4} \right) + i \sin \left(\frac{5,793227981 + 4\pi}{4} \right) \right)$$

$$w_3 = \sqrt[4]{17} \left(\cos \left(\frac{5,793227981 + 6\pi}{4} \right) + i \sin \left(\frac{5,793227981 + 6\pi}{4} \right) \right)$$



! 2 PĂCUNATI

$$= \lim_{x \rightarrow \infty} (-x - \sqrt{x^2 - 2}) \cdot \frac{-x + \sqrt{x^2 - 2}}{-x + \sqrt{x^2 - 2}}$$

$$= \lim_{x \rightarrow \infty} \frac{x^2 - (x^2 - 2)}{-x + \sqrt{x^2 - 2}}$$

$$= \lim_{x \rightarrow \infty} \frac{2}{-x + \sqrt{x^2 - 2}} = \frac{2}{\infty} = 0$$

$y = 0$ je horizontalna asimptota ✓ (DESNA) NIJE LIJEVA

3) nema drugu asimptotu jer postoji horizontalna X

POSTOJI LIJEVA K.A.

DERIVACIJE

$$f(x) = x - \sqrt{x^2 - 2} = x - (x^2 - 2)^{\frac{1}{2}}$$

$$f'(x) = 1 - \frac{1}{2}(x^2 - 2)^{-\frac{1}{2}} \cdot (x^2 - 2)'$$

$$= 1 - \frac{1}{2}(x^2 - 2)^{-\frac{1}{2}} \cdot 2x = 1 - (x^2 - 2)^{-\frac{1}{2}} \cdot x$$

$$= 1 - \frac{x}{\sqrt{x^2 - 2}}$$

$$f''(x) = \frac{1}{2}(x^2 - 2)^{-\frac{3}{2}} \cdot 2x \cdot x - (x^2 - 2)^{-\frac{1}{2}} \quad \checkmark$$

$$= \frac{x^2}{\sqrt{(x^2 - 2)^3}} - \frac{1}{\sqrt{x^2 - 2}}$$

$$(2) f(x) = x - \sqrt{x^2 - 2}$$

DINO DOKOZA

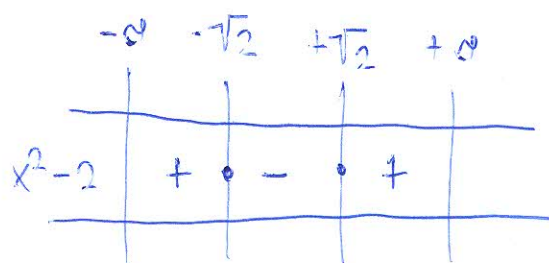
DOMENA

$$x^2 - 2 > 0$$

$$x^2 - 2 = 0$$

$$x^2 = 2 / \sqrt{\quad}$$

$$x = \pm \sqrt{2}$$



$$D_f: \langle -\infty, -\sqrt{2} \rangle \cup [\sqrt{2}, +\infty) \checkmark$$

ASIMPTOTE

1.) V. A

$$\lim_{x \rightarrow -\sqrt{2}} (x - \sqrt{x^2 - 2}) = -\sqrt{2}$$

niży V.A.

$$\lim_{x \rightarrow +\sqrt{2}} (x - \sqrt{x^2 - 2}) = \sqrt{2}$$

niży V.A.

2) H. A

$$\lim_{x \rightarrow -\infty} (x - \sqrt{x^2 - 2}) = \lim_{x \rightarrow \infty} (-x - \sqrt{(-x)^2 - 2})$$

$$= \lim_{x \rightarrow \infty} (-x - \sqrt{x^2 - 2})$$

$$= \lim_{x \rightarrow \infty} (-x - \sqrt{x^2 - 2})$$

$$3. \quad g(x) = \ln(4-x^2)$$

DOMENA

$$4-x^2 > 0$$

$$4-x^2 = 0$$

$$-x^2 = -4$$

$$x^2 = 4$$

$$x = \pm 2$$

	$-\infty$	-2	2	$+\infty$
$4-x^2$		-	+	-

$$D_g: (-2, 2) \quad \checkmark$$

PARNOST

$$g(x) = \ln(4-x^2)$$

$$g(-x) = \ln(4-(-x)^2)$$

$$= \ln(4-x^2)$$

$$g(x) = g(-x) \quad \text{funkcija je parna} \quad \checkmark$$

ZAKRIVLJENOST

$$g'(x) = \frac{1}{4-x^2} \cdot (-2x) = -\frac{2x}{4-x^2}$$

$$g''(x) = \frac{2(4-x^2) - 2x(-2x)}{(4-x^2)^2} = \frac{8-2x^2+4x^2}{(4-x^2)^2}$$

$$= -\frac{8+2x^2}{(4-x^2)^2}$$

$$g''(x) = - \frac{8 + 2x^2}{(4 - x^2)^2}$$

DINO

DUKOZA

$$g''(x) = 0$$

$$- \frac{8 + 2x^2}{(4 - x^2)^2} = 0$$

$$\Rightarrow 8 + 2x^2 = 0 \quad | : 2$$

$$4 + x^2 = 0$$

$$x^2 = -4$$

$$x = \pm \sqrt{-4}$$

memer realitas ajengnya

