MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓ G3

IME I PREZIME: DRAGAN ASIĆ **BROJ INDEKSA:** 0269078375

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja uvrštavanjem. 12+3
2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{aligned} 4A - 3B + 4C + 3D &= 3 \\ -3A + 4B - 3C - 4D &= 4 \\ -A - B + C + 3D &= 0 \\ 4A + 4B + 4C - 4D &= -4 \end{aligned}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 + 4x + 4} - 4x$. 5+15 8
4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{x+3}{x^2-4}$. 15(graf)
5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^3)$. 2+4+6+8
6. Zadana je funkcija $f(x) = \sqrt{4+3x}$. Kolika je derivacija $f'(2)$? Koji su lokalni ekstremi? 10+5

Ukupno: 13

(2)

$$\begin{aligned} 4A - 3B + 4C + 3D &= 3 \\ -3A + 4B - 3C - 4D &= 4 \\ -A - B + C + 3D &= 0 \\ 4A + 4B + 4C - 4D &= -4 \end{aligned}$$

$$A = \left[\begin{array}{cccc|c} 4 & -3 & 4 & 3 & 3 \\ -3 & 4 & -3 & -4 & 4 \\ -1 & -1 & 1 & 3 & 0 \\ 4 & 4 & 4 & -4 & -4 \end{array}\right] \quad \text{III} \cdot (-1)$$

$$A = \left[\begin{array}{cccc|c} 1 & 1 & -1 & -3 & 0 \\ 3 & 4 & -3 & -4 & 4 \\ 4 & -3 & 4 & 3 & 3 \\ 4 & 4 & 4 & -4 & -4 \end{array}\right] \begin{array}{l} \text{II} - 3 \cdot \text{I} \\ \text{III} - 4 \cdot \text{I} \\ \text{IV} - 4 \cdot \text{I} \end{array}$$

$$A = \left[\begin{array}{cccc|c} 1 & 1 & -1 & -3 & 0 \\ 0 & 1 & 0 & 5 & 4 \\ 0 & -7 & 8 & 15 & 3 \\ 0 & 0 & 8 & 8 & -4 \end{array}\right] \quad \text{III} - \text{IV}$$

$$A = \left[\begin{array}{cccc|c} 1 & 1 & -1 & -3 & 0 \\ 0 & 1 & 0 & 5 & 4 \\ 0 & -7 & 1 & 7 & -1 \\ 0 & 0 & 8 & 8 & -4 \end{array}\right] \quad \text{IV} : 8$$

$$A = \left[\begin{array}{cccc|c} 1 & 1 & -1 & -3 & 0 \\ 0 & 1 & 0 & 5 & 4 \\ 0 & -7 & 1 & 7 & -1 \\ 0 & 0 & 1 & 1 & -1 \end{array}\right] \begin{array}{l} \text{I} - \text{II} \\ \\ \\ \text{III} - (-7) \cdot \text{I} \end{array}$$

$$A=\left[\begin{array}{cccc|c} 1 & 0 & -1 & -2 & -4 \\ 0 & 1 & 0 & 5 & 4 \\ 0 & 0 & 1 & 42 & 27 \\ 0 & 0 & 1 & 11 & -4 \end{array}\right] \quad \text{I}+\text{III}$$

$$A=\left[\begin{array}{cccc|c} A & B & C & D & \\ 1 & 0 & 0 & 34 & 23 \\ 0 & 1 & 0 & 5 & 4 \\ 0 & 0 & 1 & 42 & 27 \\ 0 & 0 & 0 & 31 & -4 \end{array}\right] \qquad D=(t) \quad t\in\mathbb{R}$$

$A+34t=23$

$A=-34t+23$

$B+5t=4$

$B=-5t+4$

$C+42t=27$

$D=\in\mathbb{R}$ ✗

③ $g(x)=\sqrt{x^2+4x+4}-4x$

$x^2+4x+4\geq 0$

$(x+2)^2\geq 0$

$D(g)=\mathbb{R}$ ✓

ASIMPTOTE

VERTIKALNE NEMA

HORIZONTALNA

$$\lim_{x\to\infty}\left(\sqrt{x^2+4x-4}-4x\right)=\lim_{x\to\infty}\left(|x+2|-4x\right)=-\infty$$

NEMA HORIZONTALNE ASIMPTOTA

KOSE

$$\lim_{x\to\infty}\frac{g(x)}{x}=\lim_{x\to\infty}\frac{\sqrt{x^2+4x+4}-4x}{x}:\frac{x}{x}=\lim_{x\to\infty}\frac{\sqrt{1+\frac{4}{x}+\frac{4}{x^2}}}{1}=-3$$

$k=-3$

DRAGAN ASIC

(5) $k = \lim_{x\to\infty} \frac{g(x)}{x} = \lim_{x\to\infty} \frac{\sqrt{x^2+4x+4}-4x}{x}$

$l = \lim_{x\to\infty} (g(x) - kx) = \lim_{x\to\infty} (\sqrt{x^2+4x+4} - 4x + 3x) =$

$= \lim_{x\to\infty} (\sqrt{x^2+4x+4} - x) \cdot \frac{\sqrt{x^2+4x+4} + x}{\sqrt{x^2+4x+4} + x}$

$\lim_{x\to\infty} \frac{4x+4}{\sqrt{x^2+4x+4}+x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{4+\frac{4}{x}}{\sqrt{1+\frac{4}{x}+\frac{4}{x^2}}+1} = 2$

PRAVAC $y = -3x + 2$ JE KOSA ASIMPTOTA

RJEŠENJA

LIJEVA?

(4) $f(x) = \frac{x+3}{x^2-4}$

NULTOČKA $x = -3$

DOMENA $= \mathbb{R} \setminus \{-2, 2\}$

VER. ASIMPTOTE

$\lim_{x\to -2} \frac{x+3}{x^2-4} = \frac{5}{0} = \infty$ $\lim_{x\to 2} \frac{x+3}{x^2-4} = \frac{5}{0} = \infty$

PRAVCI $x = -2$ i $x = 2$ SU VERTIKALNE ASIMPTOTE

HORIZONTALNA ASIMPTOTA

$\lim_{x\to\infty} \frac{x+3}{x^2-4} = 0 \quad -x = 0.5$

JE HORIZONTALNA ASIMP.

SKICA GRAFA?

KOSE NEMA

10-7-81.4
07 05 14
0.0142,27

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IME I PREZIME: KREŠIMIR PIPLICA

BROJ INDEKSA: 0269078329

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Ukupno: 32

⑥ $f(x) = \sqrt{4+3x}$ - derivacija $f'(2)$, lokalni ekstremi;

KREŠIMIR PIPLICA

$$f'(x) = \left(\sqrt{4+3x}\right)' = \frac{1}{2\sqrt{4+3x}} \cdot (4+3x)' = \frac{1}{2\sqrt{4+3x}} (0+3) =$$

$$f'(x) = \frac{3}{2\sqrt{4+3x}}$$

$$f'(2) = \frac{3}{2\sqrt{4+3\cdot 2}} \quad \frac{3}{2\sqrt{4+6}} \quad \frac{3}{2\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{3\sqrt{10}}{\sqrt{10}}$$

$$f'(x) = 0 \quad \frac{3}{2\sqrt{4+3x}} = 0 \mid \cdot 2\sqrt{4+3x}$$

$3 \neq 0$

Nema stacionarnih točaka, temena ni lokalnih ekstrema. ✗

LOK. EKSTREM MOŽE BITI U TOČKI GDJE DERIVACIJA NE POSTOJI

(5) $h(x) = \operatorname{arctan}(x^3)$ - domena, periodičnost, neparnost, druga derivacija

KREŠIMIR PIPLICA

domena $D(f) = \mathbb{R}$ ✓

$h(x) = \operatorname{arctan}(-x)^3 = \operatorname{arctan}(-x)$?

$$h'(x) = -\frac{1}{1+(x^3)^2} \cdot (x^3)' = -\frac{1}{1+x^6} \cdot 3x^2 = \frac{-3x^2}{1+x^6}$$

$$h''(x) = \frac{(-3x^2)'(1+x^6) - (3x^2)(1+x^6)'}{(1+x^6)^2}$$

$$= \frac{-6x^2 - (1+x^6)\cdot 3x^2 \cdot 6x^5}{(1+x^6)^2} = \frac{-6x^2 - 6x^8 + 18x^7}{(1+x^6)^2} =$$

$$= \frac{-6x^2(x^6 + x^5 - 1)}{(1+x^6)^2}$$

$h(-x)\ \operatorname{arctan}(-x)^3 = \operatorname{arctg}(-x^3)$

neparna

$-\operatorname{arctg} x^3$

$D(f) = \mathbb{R}$

H.A.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x+3}{x^2-4} \;\Big|:x^2 = \lim_{x\to\infty} \frac{\frac{x}{x^2}+\frac{3}{x^2}}{\frac{x^2}{x^2}-\frac{4}{x^2}} = \frac{0}{1} = 0$$

K.A. - nema kose asimptote zato što ima horizontalnu H.A. $y=0$

IKREŠIMIR PIPLICA

(4) $f(x) = \frac{x+3}{x^2-4}$

a) domena $x^2-4 \neq 0$

$x^2 \neq 4$

$x \neq \pm 2$

$D(f) = \mathbb{R} \setminus \{-2, 2\}$

b) nultočke

$y = 0 \quad \frac{x+3}{x^2-4} = 0 \mid \cdot x^2-4$

$x+3 = 0$

$x = -3$

$(-3, 0)$

na y-osi $x = 0$

$f(0) = \frac{3}{-4}$

$(0, -\frac{3}{4})$

c) asimptote

V.A. $\lim_{x \to -2^-} \frac{x+3}{x^2-4} = \frac{+\text{broj}}{+0} = +\infty$

$\lim_{x \to -2^+} \frac{x+3}{x^2-4} = \frac{+}{-0} = -\infty$

$x = -2$

$\lim_{x \to 2^-} \frac{x+3}{x^2-4} = \frac{+}{-0} = -\infty$

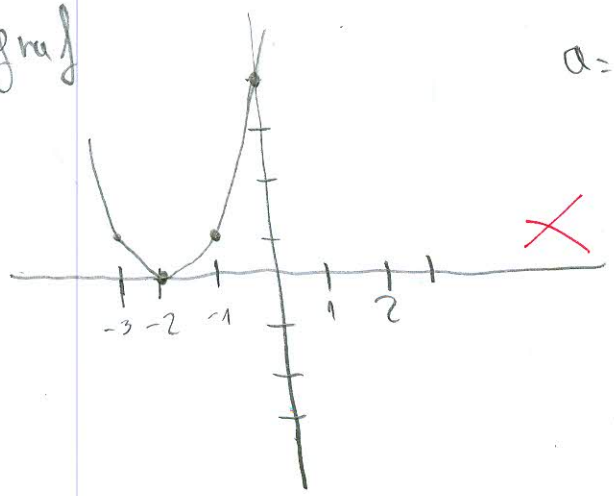
$\lim_{x \to 2^+} \frac{x+3}{x^2-4} = \frac{+}{+0} = +\infty$

$x = 2$

KREŠIMIR PIPLICA

(3) graf

$a = 1 > 0 \cup$



③ kosa asimptota $y = kx + l$

$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\sqrt{x^2+4x+4}+4x}{x} \;|:x = \lim_{x\to\infty} \frac{\sqrt{\frac{x^2}{x^2}+\frac{4x}{x^2}+\frac{4}{x^2}}-\frac{4x}{x}}{\frac{x}{x}}$

$= \frac{\sqrt{1-4}}{1} = \frac{1-4}{1} = -3$

$K.A = y = 3x+2$ ✗

$l = \lim_{x\to\infty} [f(x) - kx]$

$= \lim_{x\to\infty} \left[\sqrt{x^2+4x+4} - 4x + 3x\right] = \lim_{x\to\infty} \left[\sqrt{x^2+4x+4} - x\right] = \frac{\sqrt{x^2+4x+4}+x}{\sqrt{x^2+4x+4}-x}$

$= \lim_{x\to\infty} \frac{x^2+4x+4-x^2}{\sqrt{x^2+4x+4}+x} \;|:x = \lim_{x\to\infty} \frac{\frac{4x}{x}+\frac{4}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{4x}{x^2}+\frac{4x}{x^2}}+\frac{x}{x}} = \frac{4}{2} = 2$

③ $g(x) = \sqrt{x^2+4x+4} - 4x$ - domena, asimptote

$x^2+4x+4 \geq 0$

$x^2+4x+4 = 0$

$(x+2)^2 = 0 \;|\sqrt{}$

$x+2 = 0$

$x_{1,2} = -2$

$D(f) = \mathbb{R}$ ✓

nema horizontalne asimptote

horiz. asimptota

$\lim_{x\to\infty} \sqrt{x^2+4x+4} - 4x \cdot \frac{\sqrt{x^2+4x+4}+4x}{\sqrt{x^2+4x+4}+4x} =$

$\lim_{x\to\infty} \frac{(\sqrt{x^2+4x+4})^2 - (4x)^2}{\sqrt{x^2+4x+4}+4x} = \lim_{x\to\infty} \frac{x^2+4x+4-16x^2}{\sqrt{4x^2+4x+4}+4x} \;/:x^2 =$

$= \lim_{x\to\infty} \frac{-15x^2+4x+4}{\sqrt{4x^2+4x+4}+4x} = \lim_{x\to\infty} \frac{-\frac{15x^2}{x^2}+\frac{4x}{x^2}+\frac{4}{x^2}}{\sqrt{\frac{4x^2}{x^4}+\frac{4x}{x^4}+\frac{4}{x^4}}+\frac{4x}{x^2}}$

$= -\frac{15}{0} = -\infty$

Zato što je domena skup $\mathbb{R}$ nema vertikalne asimptote

KREŠIMIR PIPLICA

② $4A - 3B + 4C + 3D = 3$

$-3A + 4B - 3C - 4D = 4$

$-A - B + C + 3D = 0$

$4A + 4B + 4C - 4D = -4$

$$\left[\begin{array}{cccc|c} 4 & -3 & 4 & 3 & 3 \\ -3 & 4 & -3 & -4 & 4 \\ -1 & -1 & 1 & 3 & 0 \\ 4 & 4 & 4 & -4 & -4 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & -1 & -3 & 0 \\ -3 & 4 & -3 & -4 & 4 \\ 4 & -3 & 4 & 3 & 3 \\ 4 & 4 & 4 & -4 & -4 \end{array}\right] \begin{array}{l} \\ +3\cdot(-4) \\ \\ + \\ + \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & -1 & -3 & 0 \\ 0 & 7 & -6 & -13 & 4 \\ 0 & -7 & 8 & 15 & 3 \\ 0 & 0 & 8 & 8 & -4 \end{array}\right] + \sim \left[\begin{array}{cccc|c} 1 & 1 & -1 & -3 & 0 \\ 0 & 7 & -6 & -13 & 4 \\ 0 & 0 & 2 & 2 & 7 \\ 0 & 0 & 8 & 8 & -4 \end{array}\right] \cdot(-4) \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & -1 & -3 & 0 \\ 0 & 7 & -6 & -13 & 4 \\ 0 & 0 & 2 & 2 & 7 \\ 0 & 0 & 0 & 0 & -32 \end{array}\right] \Rightarrow \quad 0 \neq 32$$

Sustav nema rješenja jer $0 \neq 32$ ✓

KREŠIMIR PIPLICA

① $\frac{|z|}{2+2i} = 3i$

$z = x + yi$

$z = \sqrt{x^2+y^2}$

$y+2 = t$

$x^2t^2 + y^2t^2 = 9x^2 + 18x^2t^2 + 9t^4$

$\frac{\sqrt{x^2+y^2}}{x+yi+2i} = 3i$

$\frac{\sqrt{x^2+y^2}}{x+(y+2)i} = 3i$

$\frac{\sqrt{x^2+y^2}}{x+(y+2)i} \cdot \frac{x-(y+2i)}{x-(y+2i)} = 3i$

$\frac{\sqrt{x^2+y^2}\,[x-(y+2)i]}{x^2+(y+2)^2i} = 3i$

$\frac{x\sqrt{x^2+y^2}}{x^2+(y+2)^2} = 3 \;\Big|\cdot x^2+(y+2)^2$

$-(y+2)\sqrt{x^2+y^2} = 3(x^2+y+2)^2 \;\Big|^2$

$(y+2)^2\cancel{\sqrt{x^2+y^2}^2} = 9\left[x^2+(y+2)^2\right]^2$

$(y+2)^2(x^2+y^2) = 9(x^4+2x^2(y+2)^2+(y+2)^4)$

$x^2(y+2)^2 + y^2(y+2)^2 = 9x^2 + 18x^2(y+2)^2 + 9(y+2)^4$

?

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IME I PREZIME: Petra Delalić **BROJ INDEKSA:** 0269080611

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Ukupno: 0

(1) $\frac{|z|}{z+2i} = 3i$

z_1: $\sqrt{x^2+y^2}$

$\frac{\sqrt{x^2+y^2}}{x-yi+2i} = 3i$ z_2

$z_1 = \sqrt{x^2+y^2} = 3$

$x+y=3$

$z_2 = x+y+2i = 3$

$x+y=1$?

(2)

$$\begin{bmatrix} 4 & -3 & 4 & 3 & | & 3 \\ -3 & 4 & -3 & -4 & | & 4 \\ -1 & -1 & 1 & 3 & | & 0 \\ 4 & 4 & 4 & -4 & | & -4 \end{bmatrix} \cdot 3 \; + \; \cdot 4 \; \sim \; \begin{bmatrix} 4 & -3 & 4 & 3 & | & 3 \\ 0 & 7 & 0 & 7 & | & 25 \\ -1 & -1 & 1 & 3 & | & 0 \\ 4 & 4 & 4 & -4 & | & -4 \end{bmatrix} + \cdot 4 \; \sim \; \begin{bmatrix} 4 & -3 & 4 & 3 & | & 3 \\ 0 & 7 & 0 & 7 & | & 25 \\ 0 & -7 & 8 & 15 & | & 3 \\ 4 & 4 & 4 & -4 & | & -4 \end{bmatrix} -$$

$$\sim \begin{bmatrix} 4 & -3 & 4 & 3 & | & 3 \\ 0 & 7 & 0 & 7 & | & 25 \\ 0 & -7 & 8 & 15 & | & 3 \\ 0 & 7 & 0 & 7 & | & 7 \end{bmatrix} + \sim \begin{bmatrix} 4 & -3 & 4 & 3 & | & 3 \\ 0 & 7 & 0 & 7 & | & 25 \\ 0 & 0 & 8 & 22 & | & 28 \\ 0 & -7 & 0 & 7 & | & 7 \end{bmatrix} + \sim \begin{bmatrix} 4 & -3 & 4 & 3 & | & 3 \\ 0 & 7 & 0 & 7 & | & 25 \\ 0 & 0 & 8 & 22 & | & 28 \\ 0 & 0 & 0 & 14 & | & 32 \end{bmatrix} \Rightarrow$$

$14D = 32 \; / \cdot \frac{1}{14}$

$D = 8$

$C + 3D = 28$

$C + 3 \cdot 8 = 28$

$C = 28 - 24$

$C = 4$

$4B - 3C - 4D = 25$

$4B - (3 \cdot 4) - (4 \cdot 8) = 25$

$4B - 12 - 32 = 25$

$4B - 44 = 25$

$4B = 25 + 44$

$4B = 69 \; / \cdot \frac{1}{4}$

$B = 17.25$

$4A - 3B + 4C + 3D = 3$

$4A - (3 \cdot 17.25) + (4 \cdot 4) + (3 \cdot 8) = 3$

$4A - 51.75 + 16 + 24 = 3$

$4A - 11.75 = 3$

$4A = 3 + 11.75$

$4A = 14.75 \; / \cdot \frac{1}{4}$

$A = 3.6875$

PROVJERA ?

(3) $g(x) = \sqrt{x^2 - 4x + 4} - 4x$

1) DOMENA

$x \neq 0$

$x^2 - 4x + 4 \neq 0$

$(x-2)^2 \neq 0 \; / \sqrt{}$

1) $x - 2 \neq 0$ 2) $x + 2 \neq 0$

$x \neq 2$ $x \neq -2$ ✗

$D_f \dots \langle -\infty, -2 \rangle \cup \langle 2, +\infty \rangle$ ✗

2) ASIMPTOTE

V.A.

$\lim_{x \to -2^-}$

$-2 - 0.0001$

-2.0001

$\sqrt{x^2 - 4x + 4} - 4x = \sqrt{4^+ + 8^+ + 4} + 8^+ = +\infty$

$\lim_{x \to 2^+}$

$2 + 0.0001$

2.0001

$\sqrt{x^2 - 4x + 4} - 4x = \sqrt{4^+ - 8^+ + 4} - 8^+ = \sqrt{0^+} - 8^+ = -\infty$

$x = 2$ slijeva

$x = 2$ s desna

H. A.

$\lim_{x \to \pm\infty} \sqrt{x^2 - 4x + 4} - 4x = \sqrt{(x-2)^2} - 4x = x - 2 - 4x = -2$

$x = -2$

ROSE NEMA ✗

Petra Delalić

6) $h(x) = \operatorname{arctg}(x^3)$

① domena

arctang $[-1,1]$ $\operatorname{arctg} x^3 = \operatorname{arctg} x(x^2)$

$Df \langle -1,1 \rangle$ ✗

$-1 < x(x^2) < 1$

$x(x^2) > -1$

1) $x > -1$

2) $x^2 > -1$ /√

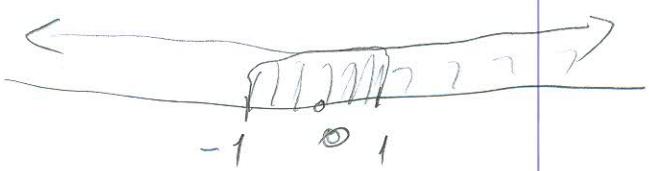
$x = \pm -1$

$x(x^2) < 1$

1) $x < 1$

2) $x^2 < 1$ /√

$x < \pm 1$



② parnost

$h(-x) = \operatorname{arctg}(-x^3)$

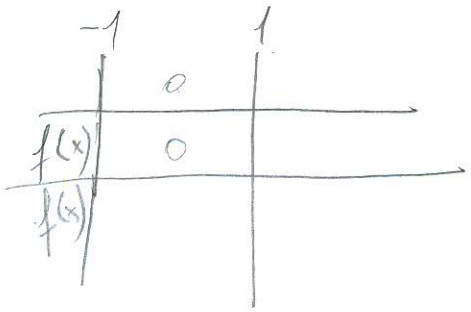
nije parna ZAŠTO ?

③ periodičnost ?

$h(x)' = \text{arctang}' \cdot (x^3)'$

$h(x)' = \frac{1}{x-1} \cdot 3x^2$ ✗

$h(x)' = \frac{3x^2}{x-1}$



$\frac{3-0}{0-1} < \frac{0}{-1} = 0$

④ druga derivacija

$h(x)'' = \frac{(3x^2)' \cdot x-1 + 3x^2 \cdot (x-1)'}{(x-1)^2}$

$= \frac{0-1+3x^2}{x^2-2x+1}$

$= \frac{-1+3x^2}{x^2-2x+1}$

H. A

$\lim_{x \to \pm\infty}$

$$\frac{x+3}{x^2-4} \to \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \frac{(x+3)' \cdot (x^2-4) + (x+3)(x^2-4)'}{(x^2-4)^2} = \frac{1 \cdot (x^2-4) + (x+3)\,x}{(x^2-4)^2}$$

$$= \frac{x^2-4+x^2+3x}{x^4+16}$$

$$\frac{x+3}{(x-2)(x+2)} = \frac{(x+3)' \cdot (x-2)(x+2) + (x-2)(x+2)' \cdot (x+3)}{\left((x-2)(x+2)\right)^2} = \frac{(x-2)(x+2) + x+3}{\left((x-2)(x+2)\right)^2}$$

nema horizontalne

$$\frac{x+3}{(x-2)(x+2)}$$

K A

$y = kx + l$

$$k = \frac{f(x)}{x} = \frac{\frac{x+3}{x^2-4}}{\frac{x}{1}} = \frac{\frac{x+3}{(x-2)(x+2)}}{\frac{x}{1}} \qquad \frac{x+3}{x(x-2)x(x+2)} = \frac{x+3}{x^2-2x+x^2+2x}$$

$$= \frac{x+3}{2x^2}$$

(5) EKSTREMI

$$f(x)' = \frac{(x+3)'}{(x^2-4)'} = (x+3)'(x^2-4) + (x+3)(x^2-4)$$

SKICA GRAFA?

Petra Dekalić

(6) $f(x) = \sqrt{4+3x}$ $\quad f(2) = \sqrt{4+3\cdot 2}$

$f'(2) = ?$ $\quad f(2) = \sqrt{4+6}$

$f'(2) = (\sqrt{10})'$ ✗ $\quad f(2) = \sqrt{10}$

(4) $f(x) = \dfrac{x+3}{x^2-4}$

Df... $\langle -\infty, -2\rangle \cup \langle -2, 2\rangle \cup \langle 2, +\infty\rangle$

1) Domena $x^2 - 4 \neq 0$

$x^2 \neq 4 \mid \sqrt{}$

$x \neq \pm 2$

2) NUL-TOČKE i sjecište sa y osi

$x = 0$ $\quad y = 0$

$f(0) = \dfrac{0+3}{0-4} = \dfrac{3}{-4} = -\dfrac{3}{4}$ $\quad \dfrac{x+3}{x^2-4} = 0$

$\dfrac{x+3}{(x-2)(x+2)} = 0$

3) parnost

$f(-x) = \dfrac{-x+3}{(-x)^2-4} = \dfrac{-x+3}{x-4}$

$x + 3 = 0$

$\boxed{x = 3}$

nit parna nit neparna $(x-2)(x+2) = 0$

$x - 2 = 0$

$\boxed{x = 2}$

$x + 2 = 0$

$\boxed{x = -2}$

4) asimptote

V. A.

$\lim\limits_{x \to -2^-} \dfrac{x+3}{x^2-4} = \dfrac{1^-}{0^-} = -\infty$

$-2 - 0.001$

-2.0001

$\boxed{x = -2}$ s lijeve

$\lim\limits_{x \to -2^+} \dfrac{x+3}{x^2-4} = \dfrac{5^-}{0^-} = -\infty$

$-2 + 0.0001$

-1.9999

$\boxed{x = -2}$ s desne

$\lim\limits_{x \to 2^-} \dfrac{x+3}{x^2-4} = \dfrac{5^-}{0^-} = -\infty$

$2 - 0.0001$

1.9999

$\boxed{x = 2}$ s lijeve

$\lim\limits_{x \to 2^+} \dfrac{x+3}{x^2-4} = \dfrac{5^+}{0^+} = +\infty$

$2 + 0.0001$

2.0001

$\boxed{x = 2}$ s desne

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: MARKO MILOŠ **BROJ INDEKSA:** 17-1-0170-2313

G3

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja uvrštavanjem. 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{aligned} 4A - 3B + 4C + 3D &= 3 \\ -3A + 4B - 3C - 4D &= 4 \\ -A - B + C + 3D &= 0 \\ 4A + 4B + 4C - 4D &= -4 \end{aligned}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 + 4x + 4} - 4x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{x+3}{x^2-4}$. 15(graf)

5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^3)$. 2+4+6+8

6. Zadana je funkcija $f(x) = \sqrt{4 + 3x}$. Kolika je derivacija $f'(2)$? Koji su lokalni ekstremi? 10+5

Ukupno:

(2)

$$\left|\begin{array}{cccc|c} 4 & -3 & 4 & 3 & 3 \\ -3 & 4 & -3 & -4 & 4 \\ -1 & -1 & 1 & 3 & 0 \\ 4 & 4 & 4 & -4 & -4 \end{array}\right| \sim \left|\begin{array}{cccc|c} -1 & -1 & 1 & 3 & 0 \\ -3 & 4 & -3 & -4 & 4 \\ 4 & -3 & 4 & 3 & 3 \\ 4 & 4 & 4 & -4 & -4 \end{array}\right| \cdot(-1) \sim \left|\begin{array}{cccc|c} 1 & 1 & -1 & 3 & 0 \\ -3 & 4 & -3 & -4 & 4 \\ 4 & -3 & 4 & 3 & 3 \\ 4 & 4 & 4 & -4 & -4 \end{array}\right| \begin{array}{l} \cdot 3 \\ + \\ + \\ + \end{array} \begin{array}{l} \cdot(+1) \\ \\ \\ \\ \end{array}$$

MARKO MLAĐ

$$\sim \left|\begin{array}{cccc|c} 1 & 1 & -1 & 3 & 0 \\ 0 & 7 & -6 & 5 & 4 \\ 0 & -7 & 8 & 15 & 3 \\ 0 & 0 & 8 & -16 & -4 \end{array}\right| \cdot 7 \sim \left|\begin{array}{cccc|c} 1 & 1 & -1 & 3 & 0 \\ 0 & 1 & -\frac{6}{7} & \frac{5}{7} & \frac{4}{7} \\ 0 & -7 & 8 & 15 & 3 \\ 0 & 0 & 8 & -16 & -4 \end{array}\right| \begin{array}{l} + \\ \cdot(+1)/\cdot 7 \\ + \\ \\ \end{array}$$

$$\sim \left|\begin{array}{cccc|c} 1 & 0 & -\frac{1}{7} & \frac{16}{7} & -\frac{4}{7} \\ 0 & 1 & -\frac{6}{7} & \frac{5}{7} & \frac{4}{7} \\ 0 & 0 & 2 & 20 & 7 \\ 0 & 0 & 8 & -16 & -4 \end{array}\right| :2 \sim \left|\begin{array}{cccc|c} 1 & 0 & -\frac{1}{7} & \frac{16}{7} & -\frac{4}{7} \\ 0 & 1 & -\frac{6}{7} & \frac{5}{7} & \frac{4}{7} \\ 0 & 0 & 1 & 10 & \frac{7}{2} \\ 0 & 0 & 8 & -16 & -4 \end{array}\right| \begin{array}{l} + \\ + \\ \cdot\frac{1}{7}/\cdot\frac{6}{7}\cdot(-8) \\ \\ \end{array}$$

$$2 \left|\begin{array}{cccc|c} 1 & 0 & 0 & \frac{26}{7} & -\frac{1}{14} \\ 0 & 1 & 0 & \frac{65}{7} & \frac{25}{7} \\ 0 & 0 & 1 & 10 & \frac{7}{2} \\ 0 & 0 & 0 & -96 & -32 \end{array}\right| :(-96) \sim \left|\begin{array}{cccc|c} 1 & 0 & 0 & \frac{26}{7} & -\frac{1}{14} \\ 0 & 1 & 0 & \frac{65}{7} & \frac{25}{7} \\ 0 & 0 & 1 & 10 & \frac{7}{2} \\ 0 & 0 & 0 & 1 & \frac{1}{3} \end{array}\right| \begin{array}{l} + \\ + \\ + \\ \cdot(-\frac{26}{7})/\cdot(-\frac{65}{7})\cdot(-10) \end{array}$$

PROVJERA ?

$$2 \left|\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{55}{42} \\ 0 & 1 & 0 & 0 & \frac{10}{21} \\ 0 & 0 & 1 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & 1 & \frac{1}{3} \end{array}\right|$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ G3

IME I PREZIME: MARKO KARLIĆ BROJ INDEKSA: 17-2-0179-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja uvrštavanjem. 12+3
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5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^3)$. 2+4+6+8
6. Zadana je funkcija $f(x) = \sqrt{4 + 3x}$. Kolika je derivacija $f'(2)$? Koji su lokalni ekstremi? 10+5

Ukupno: 18

1. $\frac{|z|}{2+2i} = 3i$

$\frac{\sqrt{x+yi}}{x+yi+2i} = 3i \quad / \cdot (x+yi+2i)$

$\sqrt{x+yi} = \frac{3i}{x+yi+2i}$

2.)

$$\left|\begin{array}{cccc|c} 4 & -3 & 4 & 3 & 3 \\ -3 & 4 & -3 & -4 & 4 \\ -1 & -1 & 1 & 3 & 0 \\ 4 & 4 & 4 & -4 & -4 \end{array}\right| \quad /:4 \sim \left|\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -1 \\ -3 & 4 & -3 & -4 & 4 \\ -1 & -1 & -1 & 3 & 0 \\ 4 & -3 & 4 & 3 & 3 \end{array}\right| \begin{array}{l} \\ \\ \text{III} + \text{I} \\ \text{IV} + \text{II} \end{array}$$

$$\sim \left|\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -1 \\ -3 & 4 & -3 & -4 & 4 \\ 0 & 0 & 0 & 2 & 3 \\ 1 & 1 & 1 & -1 & 7 \end{array}\right| \quad \text{II} + 3\text{I} \sim \left|\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -1 \\ 0 & 7 & 0 & -7 & 1 \\ 0 & 0 & 0 & 2 & 3 \\ 1 & 1 & 1 & -1 & 7 \end{array}\right| \quad \text{IV} - \text{I}$$

$$\left|\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -1 \\ 0 & 7 & 0 & -7 & 1 \\ 0 & 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 0 & 8 \end{array}\right| \quad /:7 \sim \left|\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -1 \\ 0 & 1 & 0 & -1 & \frac{1}{7} \\ 0 & 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 0 & 8 \end{array}\right| \quad \begin{matrix} \text{I} - \text{II} \\ \\ \text{III} + \text{II} \end{matrix}$$

$$\sim \left|\begin{array}{cccc|c} 1 & 0 & 1 & 0 & -\frac{8}{7} \\ 0 & 1 & 0 & -1 & \frac{1}{7} \\ 0 & 0 & 0 & 1 & \frac{3}{7} \\ 0 & 0 & 0 & 0 & 8 \end{array}\right| \sim \left|\begin{array}{cccc|c} 1 & 0 & 0 & 1 & -\frac{8}{7} \\ 0 & 1 & -1 & 0 & \frac{1}{7} \\ 0 & 0 & 1 & 0 & \frac{3}{7} \\ 0 & 0 & 0 & 0 & 8 \end{array}\right| \quad \text{II} + \text{III}$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & 1 & -\frac{8}{7} \\ 0 & 1 & 0 & 0 & \frac{5}{7} \\ 0 & 0 & 1 & 0 & \frac{3}{7} \\ 0 & 0 & 0 & 0 & 8 \end{array}\right| \rightarrow \text{beskonačno mnogo rješenja}$$

$0 = 8$? ✗

Marko Gurlić

3,) $g(x) = \sqrt{x^2+4x+4} - 4x$

$x_{1,2} = \frac{-4 \pm \sqrt{16-16}}{2} = \frac{-4 \pm 0}{2}$

$x_{1,2} = -\frac{4}{2} = -2$

uvjet korijena

$x^2+4x+4 \geq -2$

$D(g(x)) = \langle -2, +\infty \rangle$ X

V.A. nema

H.A.

$\lim_{x\to+\infty} \sqrt{x^2+4x+4} - 4x \cdot \frac{\sqrt{x^2+4x+4}+4x}{\sqrt{x^2+4x+4}+4x} = \lim_{x\to\pm\infty} \frac{x^2+4x+4+16x^2}{\sqrt{x^2+4x+4}+4x} \,/:x^2 =$

$= \frac{17 + \frac{4x}{x^2} + \frac{4}{x^2}}{\sqrt{1 + \frac{4x}{x^2} + \frac{4}{x^2}} + \frac{4x}{x^2}} = \frac{17}{\sqrt{1}} = \frac{17}{1} = 17$

D.H.A. $y = 17$ X

$\lim_{x\to-\infty} \sqrt{(-x)^2+4\cdot(-x)+4} - 4\cdot(-x) = \lim_{x\to-\infty} \sqrt{x^2-4x+4} + 4x \cdot \frac{\sqrt{x^2-4x+4}-4x}{\sqrt{x^2-4x+4}-4x}$

$= \lim_{x\to-\infty} \frac{x^2-4x+4-16x^2}{\sqrt{x^2-4x+4}-4x} = \lim_{x\to-\infty} \frac{-15x^2-4x+4 \,/:x^2}{\sqrt{x^2-4x+4}-4x \,/:x^2} = \frac{-15}{1} = -15$

L.H.A. $y = -15$ X

K.A. nema

4.) $f(x) = \frac{x+3}{x^2-4}$

1° Domena

$x^2 - 4 \neq 0$

$x^2 \neq 4$

$\boxed{x \neq 2}$

$D(f) = \mathbb{R} \setminus \{2\}$

2° Nultočke

$x + 3 = 0$

$x = -3$

$N(3, 0)$

3° Sjecište s y osi

$f(0) = \frac{0+3}{0^2-4} = -\frac{3}{4}$

$S_y\left(0, -\frac{3}{4}\right)$

4° Ekstremi, rast. pad.

$$f'(x) = \frac{1 \cdot (x^2-4) - (x+3) \cdot 2x}{(x^2-4)^2} = \frac{x^2 - 4 - 2x^2 - 6x}{(x^2-4)^2} = \frac{-x^2 - 6x - 4}{x^4 - 8x^2 + 16}$$

$$x_{1,2} = \frac{-6 \pm \sqrt{36 - 4 \cdot 1 \cdot 4}}{-2} = \frac{-6 \pm \sqrt{20}}{-2}$$

$$x_1 = \frac{-6 + \sqrt{20}}{-2} = 3 - \sqrt{5} \quad (0{,}764)$$

$$x_2 = \frac{-6 - \sqrt{20}}{-2} = 3 + \sqrt{5} \quad (5{,}236)$$

$E\left(3 - \sqrt{5}, -\frac{15 + 13\sqrt{5}}{40}\right)$ (0,8; −1,1)

$E\left(3 + \sqrt{5}, \frac{-15 + 13\sqrt{5}}{40}\right)$ (0,4)

	$-\infty$		$3-\sqrt{5}$		$3+\sqrt{5}$		$+\infty$
$f'(x)$		−		−		−	
$f\ x$		↘		↘		↘	

$f(x) \searrow$ za $x \in \langle -\infty, +\infty \rangle$

5° Asimptote

$\lim_{x \to 2^+} \frac{x+3}{x^2-4} = \frac{2+3}{4-4} = \frac{5}{0^+} = +\infty$

$\lim_{x \to 2^-} \frac{x+3}{x^2-4} = \ldots = \frac{5}{0^-} = -\infty$

$x = 2$ je V. A. obostrano

H. A.: $\lim_{x \to \pm\infty} \frac{x+3}{x^2-4} \ /:x^2 = \frac{0}{1} = 0$

$(-x)^2 = x^2$

$\lim_{x \to -\infty} \frac{-x+3}{x^2-4} \ /:x^2 = \frac{0}{1} = 0$

$\boxed{y = 0 \text{ je H. A.}}$

NASTAVAK ⇒

MARKO KARLIĆ

Kvadrović 4.)

6° (Ne) Parnost

$$f(-x) = \frac{-x+3}{(-x)^2-4} = \frac{-x+3}{x^2-4} = \frac{-(x-3)}{x^2-4}$$

Ni parna ni neparna.

7° Infleksija

$$f''(x) = \frac{(-2x+3)\cdot(x^4-8x^2-16) - (-x^2+3x-4)\cdot(4x^3-16x)}{(x^4-8x^2-16)^2}$$

$$= \frac{-2x^5+16x^3+32x+3x^4-24x^3-48+4x^5-16x^3-12x^4+48x^2-64x}{(x^4-8x^2-16)^2}$$

$$= \frac{2x^5-9x^4+24x^2-32x-48}{(x^4-8x^2-16)^2}$$

Ti nema jer se $f''(x)$ zakomplicirala

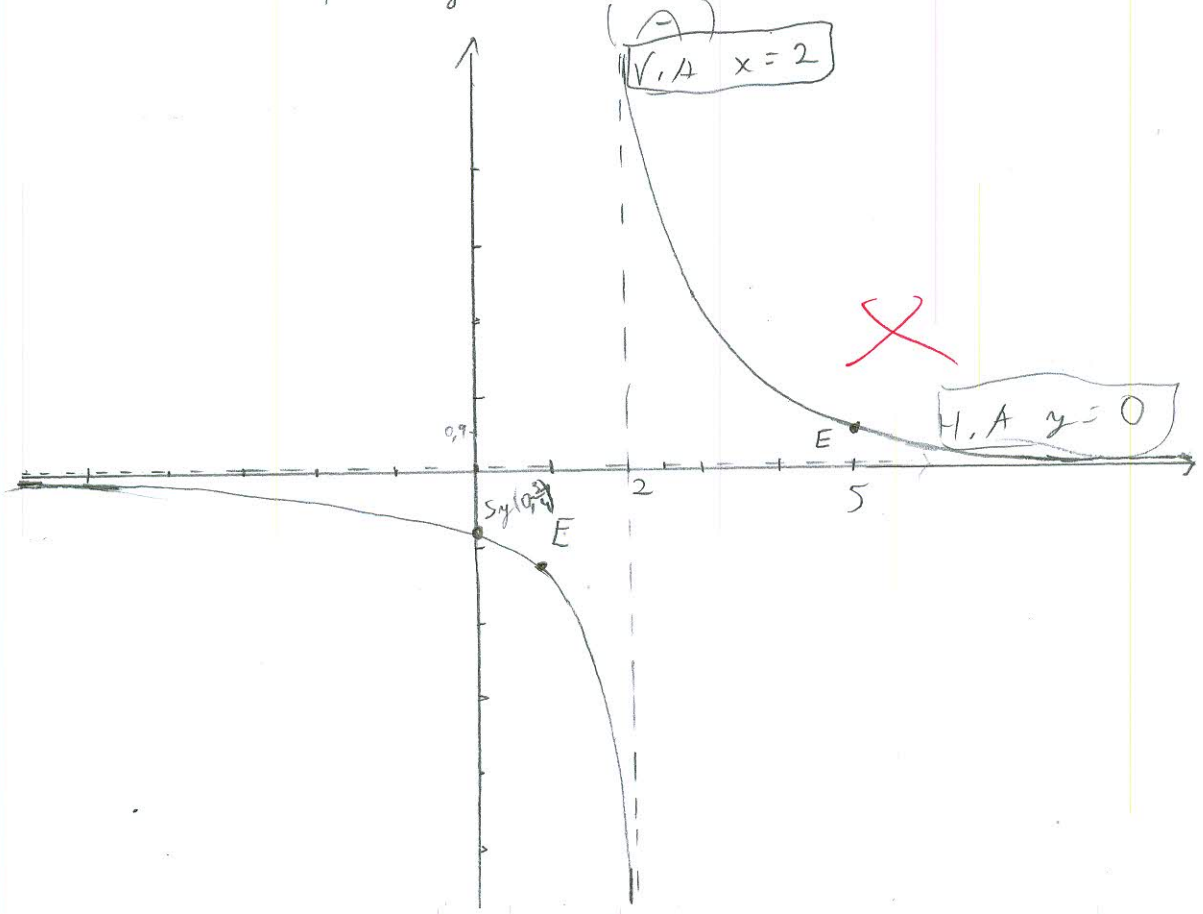
	$-\infty$		2		$+\infty$
$f''(x)$		$-$		$+$	
$f(x)$		$\cap$		$\cup$	

(∀)

$f(x)$ je konveksna za $x \in \langle 2, +\infty \rangle$

$f(x)$ je konkavna za $x \in \langle -\infty, 2 \rangle$

graf.



5.) $h(x) = \arctan(x^3)$ $\quad \frac{1}{1+x^2}$

$$f'(x) = \frac{1}{1+(x^3)^2} \cdot 3x^2 = \frac{3x^2}{1+x^6}$$

$$f''(x) = \frac{6x \cdot (1+x^6) - 3x^2 \cdot 6x^5}{(1+x^6)^2}$$ ✓

$$= \frac{6x + 6x^7 - 18x^7}{(1+x^6)^2} = \frac{-12x^7 + 6x}{x^{12} + 2x^6 + 1}$$

6.)

$$f(x) = \sqrt{4+3x}$$

$$f'(x) = -\frac{1}{2\sqrt{4+3x}} \cdot 3 = \boxed{-\frac{3}{2\sqrt{4+3x}}}$$

$$f'(2) = -\frac{3}{2\sqrt{4+3\cdot 2}} = -\frac{3}{2\sqrt{10}} = \boxed{-\frac{3\sqrt{10}}{20} \approx -0{,}47}$$ ✓

$-3 = 0$ Keine Extrema! ✗