MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ I1

IME I PREZIME: MATHIAS (RIZEL) **BROJ INDEKSA:** 17-2-0329-2013

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^4 - 4 + 2i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 15+3

2. Gaussovom metodom riješi sustav linearnih jednadžbi, a zatim provjeri uvrštavanjem:

$$\begin{aligned} x + 2y - z + u &= -1 \\ 2x + 5y - z + 2u &= -2 \\ 3x - y - 2z + u &= 5 \\ x - y + 3z - 5u &= 6 \end{aligned}$$

Provjeri uvrštavanjem! 16+3

3. Ispitati domenu i sve asimptote funkcije $g(x) = (\sqrt{x^2 + x} - x)$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{x^2-4}{x^2+2}$. 20(graf)

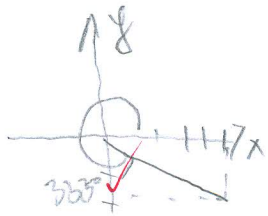
5. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\sin(4x - 2))$. 15

6. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. 8

Ukupno: 59

1. $z^4 - 4 + 2i = 0$

$z^4 = 4 - 2i$



$N = \sqrt{x^2 + y^2} = \sqrt{4^2 + (-2)^2} = \sqrt{20}$

$\operatorname{tg}\varphi = \frac{y}{x} = -\frac{2}{4} = -\frac{1}{2} = -26{,}5^\circ \approx 27^\circ = \frac{3\pi}{20}$

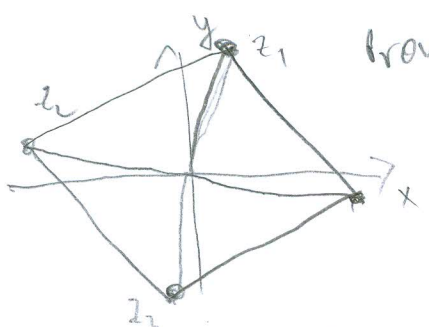
$z^4 = \sqrt{20} \cdot (\cos\frac{3\pi}{20} + i\sin\frac{3\pi}{20})$

$k=0 \Rightarrow z_1 = \sqrt[4]{\sqrt{20}} \cdot (\cos\frac{3\pi}{80} + i\sin\frac{3\pi}{80})$

$k=1 \Rightarrow z_2 = \sqrt[4]{\sqrt{20}} \cdot (\cos\frac{77\pi}{80} + i\sin\frac{77\pi}{80})$

$k=2 \Rightarrow z_3 = \sqrt[4]{\sqrt{20}} \cdot (\cos\frac{117\pi}{80} + i\sin\frac{117\pi}{80})$

$k=3 \Rightarrow z_4 = \sqrt[4]{\sqrt{20}} \cdot (\cos\frac{157\pi}{80} + i\sin\frac{157\pi}{80})$



provjera

$\frac{27\pi}{180} = \frac{3\pi}{60} = \frac{3\pi}{20}$

$\frac{3335}{180} = \frac{1111 - 37\pi}{60} = \frac{37\pi}{20}$

$\frac{37\pi}{20} + \frac{4\pi}{1} = \frac{37\pi + 80\pi}{20} = \frac{117\pi}{20}$

$\frac{37\pi}{20} \quad \frac{37\pi}{80}$

$2 \cdot 3 \cdot \pi$

$\frac{37\pi}{20} \cdot \frac{1}{4}$

$\frac{2\pi}{4} = \frac{37\pi}{20} + \frac{2\pi}{1} = \frac{37\pi + 40}{20}$

$\frac{77\pi}{20} \cdot \frac{4}{1}$

$\frac{117\pi}{20} \cdot \frac{1}{4}$

$\frac{77\pi}{80}$

$\frac{37\pi}{20} + \frac{6\pi}{1} = \frac{37\pi + 120}{20}$

$\frac{117\pi}{80}$

$\frac{157\pi}{20} \cdot \frac{1}{4} = \frac{157\pi}{80}$

MATHIAS GRIBELJ

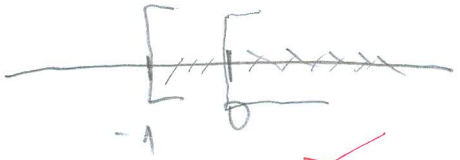
3. $g(x)=\left(\sqrt{x^2+x}-x\right)$ $\quad 1^0\ x^2+x\geq 0$

$x(x+1)\geq 0$

$\boxed{x\geq 0}$

$x+1\geq 0$

$\boxed{x\geq -1}$



$D(g)=\langle -\infty,-1\rangle \cup [1+\infty\rangle$ ✗

$\langle -\infty,-1]\cup[1,+\infty\rangle$ ✗

VERTIKALNE NEMA

$\lim_{x\to\infty}\sqrt{x^2+x}-x=\lim_{x\to\infty}\sqrt{x^2+x}-x\cdot\frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x}=\lim_{x\to\infty}\frac{x^2+x-x^2}{\sqrt{x^2+x}+x}=$

$\lim_{x\to\infty}\frac{x}{\sqrt{x^2+x}+x}:\frac{x}{x}=\lim_{x\to\infty}\frac{1}{\sqrt{1+\frac{1}{x}}+1}=\frac{1}{2}$

$y=\frac{1}{2}$ je H.A. (DESNA)

KOSE NEMA

✗

5. $f(x)=\ln(\sin(4x-2))=\frac{1}{\sin(4x-2)}\cdot\cos(4x-2)\cdot 4=\frac{\cos(4x-2)\cdot 4}{\sin(4x-2)}$ ✓

MATHIAS GRIZELJ

4. $h(x) = \frac{x^2-4}{x^2+2}$

1° $x^2+2 \neq 0$

$x^2 \neq -2$

$x \neq \pm\sqrt{-2} \in \mathbb{R}$

$D(f) = \mathbb{R}$

2° Nultočke

$x^2-4=0$

$(x-2)(x+2)=0$

$x=2 \quad , \quad x=-2$

$T_0(2,0) \; , \; T_0(-2,0)$

3° Ekstremi

$f(x) = \frac{x^2-4}{x^2+2} \quad \frac{u}{v} \; , \; f'(x) = \frac{u' \cdot v - u \cdot v'}{v^2} = \frac{2x \cdot (x^2+2) - (x^2-4 \cdot 2x)}{(x^2+2)^2} =$

$\frac{2x^3+4x-(2x^3-8x)}{(x^2+2)^2} = \frac{2x^3+4x-2x^3+8x}{(x^2+2)^2} = \frac{12x}{(x^2+2)^2}$

f

PARNOST i NEPARNOST

$f(-x) = \frac{(-x)^2-4}{(-x)^2+2} = \frac{x^2-4}{x^2+2}$; $f(x)$ je parna

$f''(x) = \frac{x^2}{(x^2+2)^2} = \frac{u}{v} = \frac{u' \cdot v - u \cdot v'}{v^2} = 2x \cdot ($

4° ASIMPTOTE

$\lim_{x \to \infty} \frac{x^2-4}{x^2+2} \quad / : x^2 = \lim_{x \to \infty} \frac{\frac{x^2}{x^2} - \frac{4}{x^2}}{\frac{x^2}{x^2} + \frac{2}{x^2}} = \frac{1-0}{1+0} = 1 \quad y=1$ je H.A

V. A. NEMA

K. A. NEMA

$$x+2y-z+u=-1$$
$$2x+5y-z+2u=-2$$
$$3x-y-2z+u=5$$
$$x-y+3z+5u=6$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & 5 & 6 \end{bmatrix} \begin{matrix} \\ II-2\cdot I \\ III-3\cdot I \\ IV-I \end{matrix} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -7 & 1 & -2 & 8 \\ 0 & -3 & 4 & 4 & 7 \end{bmatrix} \begin{matrix} I-2\cdot II \\ \\ III+7\cdot II \\ III+3\cdot II \end{matrix} \sim$$

$$\begin{bmatrix} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -2 & 8 \\ 0 & 0 & 7 & 4 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 4 & 1 & 1 \\ 0 & 0 & -6 & 5 & 8 \end{bmatrix} \begin{matrix} \\ \\ :2 \\ :7 \end{matrix} \sim \begin{bmatrix} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & -6 & 5 & 8 \end{bmatrix} \begin{matrix} I+3\cdot III \\ II-III \\ \\ IV+6\cdot III \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 & \frac{4}{4} & \frac{2}{4} \\ 0 & 1 & 0 & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{11}{4} & \frac{14}{4} \end{bmatrix} \cdot\frac{4}{11} \sim$$

$$\begin{bmatrix} 1 & 0 & 0 & 4 & 2 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & \frac{14}{11} \end{bmatrix} \begin{matrix} I-4\cdot IV \\ II+IV \\ III-IV \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & -\frac{34}{11} \\ 0 & 1 & 0 & 0 & \frac{3}{11} \\ 0 & 0 & 1 & 0 & -\frac{3}{11} \\ 0 & 0 & 0 & 1 & \frac{14}{11} \end{bmatrix}$$

$$2-4\cdot\frac{14}{11}=2-\frac{56}{11}=\frac{22-56}{11}=$$
$$-\frac{34}{11}$$

$$-1+\frac{14}{11}=\frac{-11+14}{11}=\frac{3}{11}$$
$$1-\frac{14}{11}=\frac{11-14}{11}=\frac{3}{11}$$

$$-\frac{34}{11}+2\cdot\frac{3}{11}+\frac{3}{11}+\frac{14}{11}=-1$$
$$-\frac{11}{11}=-1$$
$$-1=-1$$

$$2\cdot\left(-\frac{34}{11}\right)+5\cdot\frac{3}{11}+\frac{3}{11}+2\cdot\frac{14}{11}=-2$$
$$-\frac{22}{11}=-2$$
$$-2=-2$$

6. $\begin{vmatrix} 2 & 3 & 0 & -2 \\ 0 & 1 & 4 & -2 \\ 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{vmatrix}$ $\begin{matrix} 0 \\ 1 \\ -2 \\ 4 \end{matrix}$

PRVA DET ✓

$2 \cdot 1 - 0 = 2$

DRUGA DET.

$\begin{matrix} 2 & 3 & 0 & 2 & 3 \\ 0 & 1 & 4 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{matrix}$ $= 12 - 8 = 4$ ✓

DALJE. —

Rang matrice je 4.

ZAŠTO?

4×4

$R = 4.$

$\begin{bmatrix} 1 & 0 & -3 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -2 & 8 \\ 0 & 0 & 7 & 4 & 7 \end{bmatrix} /:8 \begin{bmatrix} 1 & 0 & -3 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & 1 \\ 0 & 0 & 7 & 4 & 7 \end{bmatrix}$ $\begin{matrix} \text{I} + 3 \cdot \text{III} \\ \text{II} - \text{III} \\ \\ \text{IV} - 7 \cdot \text{III} \end{matrix}$ $\sim \begin{bmatrix} 1 & 0 & -3 & \frac{1}{4} & 1 \\ 0 & 1 & 0 & \frac{1}{4} & -1 \\ 0 & 0 & 1 & -\frac{1}{4} & 1 \\ 0 & 0 & 0 & \frac{23}{4} & 0 \end{bmatrix} /: \frac{23}{4}$ $\sim \begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} & 1 \\ 0 & 1 & 0 & \frac{1}{4} & -1 \\ 0 & 0 & 1 & -\frac{1}{4} & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ $\begin{matrix} \text{I} - \frac{1}{4} \cdot \text{IV} \\ \text{II} - \frac{1}{4} \cdot \text{IV} \\ \text{III} + \frac{1}{4} \cdot \text{IV} \end{matrix}$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 1 & 0 & 0 & -\frac{5}{4} \\ 0 & 0 & 1 & 0 & \frac{5}{4} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ ✗

$x + 2y - z + u = -1$

$\frac{3}{4} + \left(2 \cdot \left(-\frac{5}{4}\right)\right) - \frac{5}{4} + 0 = -1$

$\frac{3}{4} - \frac{5}{2} - \frac{5}{4} + 0 = \frac{3 - 10 - 5}{4} = -\frac{12}{4}$

$\frac{3-10}{4} - \frac{5}{4} = -\frac{7}{4} - \frac{5}{4} =$

$-1 - \frac{7}{4} = \frac{-4-7}{4} = -\frac{11}{4}$

MATHIAS GIRIZELJ

3° Ekstremi

$f(x) = \frac{x^2-4}{x^2+2} = \frac{0-4}{0+2} = -\frac{4}{2} = -2 \quad T(0,-2)$ MIN

$12x = 0$

$x = 0$

	$-\infty$		0		$+\infty$
$f'(x)$		$-$	0	$+$	
$f(x)$		↘		↗	

$f'(x) = \frac{12x}{(x^2+2)^2} = \frac{12 \cdot (-10)}{(-10^2+2)^2} = -$

$= \frac{12 \cdot 10}{(10^2+2)^2} = +$

5° KONKAVNOST

	$-\infty$		0		$+\infty$
$f''(x)$		$+$	0	$-$	
$f(x)$		∪		∩	

$\frac{12x}{(x^2+2)^2} = \frac{u' \cdot v - u \cdot v'}{v^2} = \frac{12 \cdot (x^2+2)^2 - (12x \cdot [(2x^2+2) \cdot 2x])}{(x^2+2)^4} =$

$= \frac{12 \cdot (x^2+2)^2 - (48x^3 + 48x^2)}{(x^2+2)^4} =$

$= \frac{12 \cdot (x^2+2)^2 - 48x^3 - 48x^2}{(x^2+2)^4}$

$h(x) = \frac{x^2-4}{x^2+2} = \frac{0-4}{0+2} = -2$

$T(0,-2)$

$f''(x) = \frac{12 \cdot (x^2+2)^2 - 48x^3 - 48x^2}{(x^2+2)^4} =$

$= 1728 + 48000 - 4800$

$= 1728 - 48000$

$12(x^2+2)^2 - 48x^3 - 48x^2 = 0 \quad / : 12$

$(x^2+2)^2 - 4x^3 - 4x^2 = 0$

$x^4 + 4x^2 + 4 - 4x^3 - 4x^2 = 0$

$x^4 - 4x^3 + 4 = 0$

$x^4 - 4x^3 = -4$

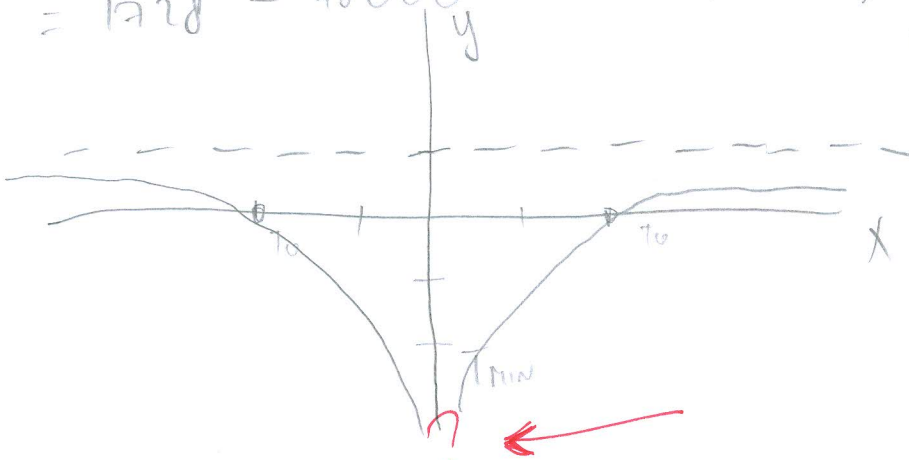
$x^3(x-4) = -4$

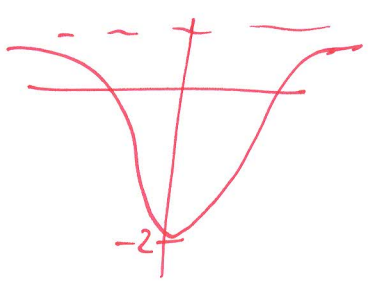
$x^3 = -4$

$x - 4 = -4$

$x = 0$

18





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IME I PREZIME: DINO PETEŠIĆ **BROJ INDEKSA:** 17-2-0314-2013

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^4 - 4 + 2i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 15+3

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6. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. I-III 8

Ukupno:

6. $\begin{bmatrix} 1 & 2 & 0 & -6 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \overset{III-I}{\sim} \begin{bmatrix} 1 & 2 & 0 & -6 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & -1 & 0 & 10 & 0 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \overset{I-2II}{\sim} \begin{bmatrix} 1 & 0 & -8 & -2 & -4 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & -1 & 0 & 10 & 0 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \begin{matrix} III+II \\ IV-II \end{matrix}$

$\sim \begin{bmatrix} 1 & 0 & -8 & -2 & -4 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & 4 & 8 & 1 \\ 0 & 0 & -4 & 0 & 3 \end{bmatrix} \overset{I+2III}{\sim} \begin{bmatrix} 1 & 0 & 0 & 14 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & 4 & 8 & 1 \\ 0 & 0 & -4 & 0 & 3 \end{bmatrix} \overset{II-III}{\sim} \begin{bmatrix} 1 & 0 & 0 & 14 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 4 & 0 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 14 & -2 \\ 0 & 1 & 0 & -10 & 0 \\ 0 & 0 & 4 & 8 & 1 \\ 0 & 0 & -4 & 0 & 3 \end{bmatrix} \overset{IV+III}{\sim}$

$\sim \begin{bmatrix} 1 & 0 & 0 & 14 & -2 \\ 0 & 1 & 0 & -10 & 0 \\ 0 & 0 & 4 & 8 & 1 \\ 0 & 0 & 0 & 8 & 4 \end{bmatrix} \overset{III-IV}{\sim} \begin{bmatrix} 1 & 0 & 0 & 14 & -2 \\ 0 & 1 & 0 & -10 & 0 \\ 0 & 0 & 4 & 0 & -3 \\ 0 & 0 & 0 & 8 & 4 \end{bmatrix} :4 \sim \begin{bmatrix} 1 & 0 & 0 & 14 & -2 \\ 0 & 1 & 0 & -10 & 0 \\ 0 & 0 & 4 & 0 & -3 \\ 0 & 0 & 0 & 2 & 1 \end{bmatrix} \overset{I-7IV}{\sim} \begin{bmatrix} 1 & 0 & 0 & 0 & -9 \\ 0 & 1 & 0 & -10 & 0 \\ 0 & 0 & 4 & 0 & -3 \\ 0 & 0 & 0 & 2 & 1 \end{bmatrix} \overset{II+5IV}{\sim}$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & -9 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 4 & 0 & -3 \\ 0 & 0 & 0 & 2 & 1 \end{bmatrix} \begin{matrix} \\ \\ :4 \\ :2 \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & -9 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & -\frac{3}{4} \\ 0 & 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ Rang: 4 ✓

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IME I PREZIME: ANTE VEDRIĆ **BROJ INDEKSA:** 17- 2-0198 22012

POPUNJAVA NASTAVNIK
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Ukupno: 30

(4) NASTAVAK

$1'(0) = 0$

$\langle -\infty, -2 \rangle$

$\langle -2, +\infty$ INTERVALI

$f'(-3) < 0$ MONOTONOST

$f'(1) > 0$

$1''(x) = 0 \Rightarrow -36x^2 + 24 = 0$

$x^2 = \frac{4}{6} = \frac{2}{3}$

$x = \frac{2\sqrt{3}}{3}$

$1''(0) = 0$

$$1''(x) = \frac{12(x^2+2)^2 - 12x\left(2(x^2+2)2x\right)}{(x^2+2)^4}$$

$$= \frac{12(x^2+2)^2 - 24x(x^2+2)2x}{(x^2+2)^4}$$

$$= \frac{(x^2+2)\left(-48x^2 + 12(x^2+2)\right)}{(x^2+2)^4}$$

ASIMPTOTE

$$\lim_{x\to\infty} \frac{x^2-4}{x^2+2} = \left(\frac{\infty}{\infty}, H\right) = \lim_{x\to\infty} \frac{2x}{2x} = 1 \quad \text{HORIZONTALNA ASIMPTOTA}$$

$$\lim_{x\to-\infty} = \frac{x^2-4}{x^2+2} = 1$$

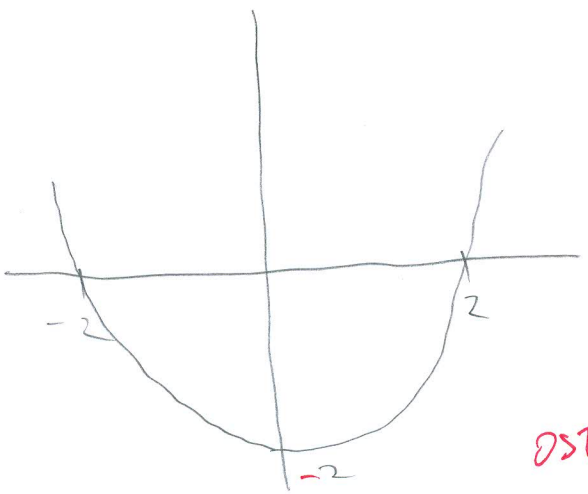
VERTIKALNE NEMA

② $g(x) = \left(\sqrt{x^2 + x} - x\right)$

③ $h(x) = \dfrac{x^2 - 4}{x^2 + 2} \rightarrow x^2 + 2 \neq 0$

Nema vertikalne asimptote

$x^2 \neq -2 \Rightarrow$ kvadrat uvijek pozitivan



7

OSTATAK GRAFA.

1° $\dfrac{x^2 - 4}{x^2 + 2} = 0$

$x^2 = 4 \Rightarrow x_{1,2} = \pm 2$

↑ NULTOČKE

$h'(x) = \dfrac{2x(x^2+2) - (x^2-4)\cdot 2x}{(x^2+2)^2}$

$h'(x) = \dfrac{12x}{(x^2+2)^2} \Rightarrow h'(x) = 0$

$h(0) = -2 \quad (0, -2)$ Stacionarna točka

ANTE VEDRIĆ (NASTAVAK)

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & -4 & -3 \\ 1 & 0 & 0 & 2 & -6 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 0 & -3 \\ 1 & 0 & 0 & 0 & -6 \\ 0 & 1 & 0 & 0 & 4 \end{bmatrix}$$

rang = 4 ✓

(5.)

$f(x) = \ln(\sin(4x-2))$

$f'(x) = \frac{1}{\sin(4x-2)} \cdot (\sin(4x-2))'$

$f'(x) = \frac{1}{\sin(4x-2)} \cdot \cos(4x-2) \cdot (4x-2)'$

$f'(x) = \frac{1}{\sin(4x-2)} \cdot \cos(4x-2) \cdot 4$

$f'(x) = 4 \cdot \frac{\cos(4x-2)}{\sin(4x-2)}$ ✓ , $4x-2 = t$

$f'(x) = 4 \cdot \operatorname{ctg} t$

ANTE VEDRIĆ MAT 1 5. 9. 2014.

①

$x = z^2$

$x^2 - 4 + 2i = 0$

$a = 1$

$b = 0$

$c = -4 + 2i$

$\frac{\pm\sqrt{-(-16+8i)}}{2}$

$= \frac{\pm\sqrt{16-8i}}{2}$

$= \frac{\pm 2\sqrt{4-2i}}{2}$

$x_1 = z^2$

$\sqrt{4-2i} = z^2$

$z = \sqrt[4]{4-2i}$?

$x_2 = z^2$

$-\sqrt{4-2i} = z^2$

$z = -\sqrt[4]{4-2i}$ = ?

②

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 6 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 0 & -3 & 4 & -6 & 7 \end{array}\right) \sim$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 0 & 9 & -2 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 3 & -1 & -2 & 1 & 5 \\ 0 & 9 & -2 & 0 & 1 \end{array}\right) \sim$$

$$\left(\begin{array}{cccc|c} 1 & 3 & 0 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 3 & 1 & 0 & 1 & 5 \\ 0 & 9 & -2 & 0 & 1 \end{array}\right) \quad ?$$

y

$y + z = 0 \Rightarrow y = -z$

$9y - 2z = 1$

$y = \frac{1}{11}$

$-9z - 2z = 1$

$-11z = 1$

$z = -\frac{1}{11}$

$x + \frac{3}{11} + \mu = -1$

$3x + \frac{1}{11} + \mu = 5$

$11x + 3 + 11\mu = -11$

$33x + 1 + 11\mu = 55$

$11x + 11\mu = -14 \Rightarrow 3x + 11\mu = -14$

$33x + 11\mu = 54$

$22x = 68$

$x = \frac{68}{11} = \frac{34}{11}$

$\mu = -\frac{48}{11}$ ✗

⑥

$$\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 0 & 4 & -4 & -3 \\ 1 & 0 & 0 & 2 & -6 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 0 & 0 & 4 \\ 0 & 0 & 4 & -4 & -3 \\ 1 & 0 & 0 & 2 & -6 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 0 & 2 \\ 0 & 0 & 4 & -4 & -3 \\ 0 & 0 & 0 & 2 & -6 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim$$

$$\begin{bmatrix} 0 & 2 & 0 & -2 & 8 \\ 0 & 0 & 4 & -4 & -3 \\ 1 & 0 & 0 & 2 & -6 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 0 & -6 & 0 \\ 0 & 0 & 4 & -4 & -3 \\ 1 & 0 & 0 & 2 & -6 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{I}1$

IME I PREZIME: IVAN BEDAM **BROJ INDEKSA:** 17-1-0130-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^4 - 4 + 2i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 15+3

2. Gaussovom metodom riješi sustav linearnih jednadžbi, a zatim provjeri uvrštavanjem:

$$\begin{aligned} x + 2y - z + u &= -1 \\ 2x + 5y - z + 2u &= -2 \\ 3x - y - 2z + u &= 5 \\ x - y + 3z - 5u &= 6 \end{aligned}$$

Provjeri uvrštavanjem! 16+3

3. Ispitati domenu i sve asimptote funkcije $g(x) = \left(\sqrt{x^2 + x} - x\right)$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{x^2-4}{x^2+2}$. 20(graf)

5. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\sin(4x - 2))$. 15

6. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. 8

Ukupno: 39

3) $f(x) = \sqrt{x^2+x} - x$

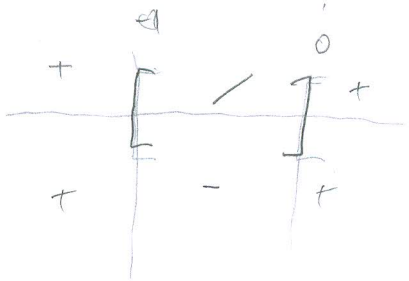
$\sqrt{x^2+x} \geq 0$

$x^2+x \geq 0$

$x(x+1) \geq 0$

$x_1 \geq 0$

$x_2 \geq -1$



$D_f \langle -\infty, -1] \cup [0, +\infty \rangle$ ✓

V.A.

$\lim_{x \to 0^+} \sqrt{0^{+2}+0^+} - 0^+ = \sqrt{0^+ + 0^+} = 0$

$\lim_{x \to -1^+} \sqrt{x^2+x} - x = \lim_{x \to -1^+} \sqrt{1^- - 1^+} + 1^+ = \sqrt{0^-} + 1^+ = 1$ —

$\lim_{x \to -1^-} \sqrt{x^2+x} - x = \lim_{x \to -1^-} \sqrt{1^+ - 1^-} + 1^- = \sqrt{0^+} + 1^- = 1$ —

H.A.

$$\lim_{x \to \infty} \sqrt{x^2+x} - x \left[\infty - \infty\right] = \lim_{x \to \infty} \sqrt{x^2+x} - x \cdot \frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x} = \lim_{x \to \infty} \frac{x^2+x-x^2 \,/:x^2}{\sqrt{x^2+x}+x \,/:x^2}$$

PRVO ODUZETI $x^2 - x^2$ ZATIM DIJELITI BROJNIK I NAZIVNIK SA x^2

$$= \lim_{x \to \infty} \frac{1+\frac{1}{x}-1}{\sqrt{\frac{1}{x^2}+\frac{1}{x^3}}+\frac{1}{x}} = \lim_{x \to \infty} \frac{\frac{1}{x}}{\sqrt{\frac{1}{x^2}+\frac{1}{x^3}}+\frac{1}{x}} = \frac{0}{0} = 0$$

KOSA

$k = \frac{f(x)}{x} \Rightarrow \lim_{x \to \infty} \frac{\sqrt{x^2+x}-x}{x} \cdot \frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x} = \lim_{x \to \infty} \frac{x^2+x-x^2 \,/:x^2}{\sqrt{x^2+x}+x \,/:x^2} = \frac{1+\frac{1}{x}-1}{\sqrt{\frac{1}{x^2}+\frac{1}{x^3}}+1} = \frac{0}{1}$

$l = f(x) - kx \Rightarrow \lim_{x \to \infty} \sqrt{x^2+x} - x - 0 = \frac{0}{0} = 0$

5) $f(x) = \ln(\sin(4x-2))$

$$f'(x) = \frac{1}{\sin(4x-2)} \cdot \sin(4x-2)' = \frac{1}{\sin(4x-2)} \cdot \cos(4x-2) \cdot (4x-2)' = \frac{1}{\sin(4x-2)} \cdot \cos(4x-2) \cdot 4$$

$$= \frac{4\cos(4x-2)}{\sin(4x-2)}$$ ✓

IVAN BERAM

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 6 \end{array}\right) \begin{array}{l} \text{II}-2\text{I} \\ \text{III}-3\text{I} \\ \text{IV}-\text{I} \end{array} \sim \left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -7 & 1 & -2 & 8 \\ 0 & -3 & 4 & -6 & 7 \end{array}\right) \begin{array}{l} \text{I}-2\text{II} \\ \text{III}+7\text{II} \\ \text{IV}+3\text{II} \end{array} \sim \left(\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -2 & 8 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right) \text{II}-\text{IV}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right) \text{IV}-7\text{III} \sim \left(\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & -48 & 0 \end{array}\right) \text{IV}:(-48) \sim \left(\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \begin{array}{l} \text{I}+3\text{III} \\ \text{II}-\text{III} \end{array} \sim$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 7 & 2 \\ 0 & 1 & 0 & -4 & -1 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \begin{array}{l} \text{I}-7\text{IV} \\ \text{II}+4\text{IV} \\ \text{III}-4\text{IV} \end{array} \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \checkmark \quad \begin{pmatrix} x \\ y \\ z \\ u \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$

$x + 2y - z + u = \boxed{-1} \Rightarrow 2 - 2 - 1 + 0 = \boxed{-1}$ ✓

$2x + 5y - z + 2u = \boxed{-2} \Rightarrow 4 - 5 - 1 + 0 = \boxed{-2}$ ✓

$3x - y - 2z + u = \boxed{5} \Rightarrow 6 + 1 - 2 + 0 = \boxed{5}$ ✓

$x - y + 3z - 5u = \boxed{6} \Rightarrow 2 + 1 + 3 - 0 = \boxed{6}$ ✓

IVAN BEDAM

6. $\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \overset{I-2III}{\sim} \begin{bmatrix} 0 & 1 & 0 & -6 & 4 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \overset{I \leftrightarrow III}{\sim} \begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 1 & 0 & -6 & 4 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \overset{III-II}{\overset{IV-II}{\sim}}$

$\begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & -4 & -4 & 3 \\ 0 & 0 & -4 & 4 & 3 \end{bmatrix} \overset{IV+III}{\sim} \begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & -4 & -4 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $R = \{3\}$

hodnost matice je 3 ✗

1. $z^4 - 4 + 2i = 0$

$z^4 = 4 - 2i$

$z = \sqrt[4]{4-2i}$

$w = 4 - 2i$

$|w| = \sqrt{4^2 - 2^2} = 12$ ✗

$\rho = \sqrt[4]{12} = \frac{4}{2} = 2$

$k = 0, 1, 2, 3$

$k = 0$

$z_1 = \sqrt[4]{12} \cdot \left(\cos \frac{2 + 2k\pi}{4} + i \sin \frac{2 + 2k\pi}{4} \right)$

$z_1 = \sqrt[4]{12} \cdot \left(\cos \frac{2+0}{4} + i \sin \frac{2+0}{4} \right)$

$z_1 = \sqrt[4]{12} \left(\cos \frac{1}{2} + i \sin \frac{1}{2} \right)$

$k = 1$

$z_2 = \sqrt[4]{12} \cdot \left(\cos \frac{2 + 2 \cdot 1 \cdot \pi}{4} + i \sin \frac{2 + 2 \cdot 1 \cdot \pi}{4} \right)$

$z_2 = \sqrt[4]{12} \cdot \left(\cos \frac{2 + 2\pi}{4} + i \sin \frac{2 + 2\pi}{4} \right)$

$k = 2$

$z_3 = \sqrt[4]{12} \cdot \left(\cos \frac{2 + 2 \cdot 2 \cdot \pi}{4} + i \sin \frac{2 + 2 \cdot 2 \cdot \pi}{4} \right)$

$z_3 = \sqrt[4]{12} \cdot \left(\cos \frac{2 + 4\pi}{4} + i \sin \frac{2 + 4\pi}{4} \right)$

$k = 3$

$z_3 = \sqrt[4]{12} \cdot \left(\cos \frac{2 + 2 \cdot 3 \cdot \pi}{4} + i \sin \frac{2 + 3 \cdot 2 \cdot \pi}{4} \right)$

$= \sqrt[4]{12} \cdot \left(\cos \frac{2 + 6\pi}{4} + i \sin \frac{2 + 6\pi}{4} \right)$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{I}1$

IME I PREZIME: [illegible] **BROJ INDEKSA:** 0269082198

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^4 - 4 + 2i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 15+3

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Provjeri uvrštavanjem! 16+3

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5. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\sin(4x - 2))$. 15

6. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. 8

Ukupno: 35

$\lim_{x\to+\infty} \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1+1}} = \frac{1}{2}$ ✓

HORIZONTALNA U DESNOJ STRANI

$\lim_{x\to-\infty} y(x) = \lim_{x\to-\infty} \sqrt{x^2+x} - x = \infty = \infty + \infty = \infty$

NEMA HORIZONTALNE U DESNOJ STRANI

$y = kx + l$

$k = \lim_{x\to-\infty} \frac{\sqrt{x^2+x} - x}{x}$

$= \lim_{x\to-\infty} \left(\frac{-1}{x} + \frac{\sqrt{x^2+x}}{x}\right) =$

$= -1 \cdot \lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{1} =$

$= -1 - \sqrt{\lim_{x\to-\infty} \frac{x^2+x}{x^2}}$ (·x²) $=$ $= -1 - \sqrt{\lim_{x\to-\infty} \frac{1+\frac{1}{x}}{x}} =$

$= -1 - 1 = -2$

$k = -2$

KOSA ASIMTOTA ___ ?

$$\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 2 & 3 & 0 & -2 & 1 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 1 & 0 & -10 & 4 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & -10 & 4 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 1 & 0 & 14 & -6 \\ 0 & 1 & 0 & -10 & 4 \\ 0 & 0 & 4 & 8 & -3 \\ 0 & 0 & 0 & 12 & 0 \end{bmatrix}$$

RANG MATRICE JE 4 ✓

5. $f(x) = \ln(\sin(4x-2))$ DOMAĆI ZADATAK

$f'(x) = (\ln(\sin(4x-2)))' = \frac{1}{\sin(4x-2)} \cdot (\sin(4x-2))' =$

$= \frac{1}{\sin(4x-2)} \cdot \cos(4x-2) \cdot (4x-2)' =$

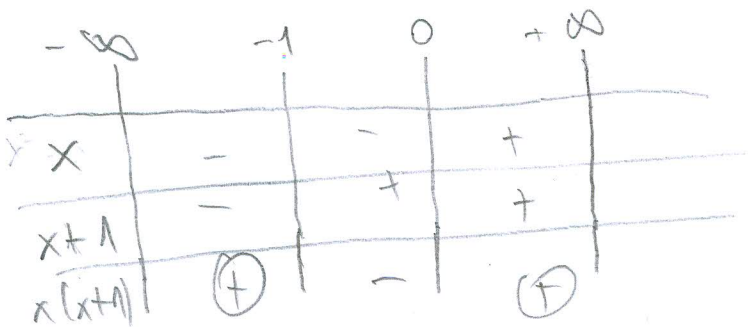
$= \operatorname{ctg}(4x-2) \cdot 4 = 4\operatorname{ctg}(4x-2)$ ✓

3. $g(x) = (\sqrt{x^2+x} - x)$

① DOMENA

$x^2 + x \geq 0$

$x(x+1) \geq 0$



$x \in \langle -\infty, -1] \cup [0, \infty \rangle$ ✓

② ASIMPTOTE

VERTIKALNA U TOČKAMA $x_1 = -1$ $x_2 = 0$

$\lim_{x \to -1} g(x) = \sqrt{(-1)^2 - 1} + 1 = \underline{1}$?

$\lim_{x \to 0} g(x) = \sqrt{0^2 - 0} - 0 = 0$?

HORIZONTALNA U DESNOJ STRANI

$\lim_{x \to +\infty} g(x) = \lim_{x \to \infty} \sqrt{x^2+x} - x =$

$= \lim_{x \to +\infty} (\sqrt{x^2+x} - x) \cdot \frac{\sqrt{x^2-x} + x}{\sqrt{x^2+x} + x} =$

$= \lim_{x \to +\infty} \frac{x^2 + x - x}{\sqrt{x^2+x} + x} = \lim_{x \to +\infty} \frac{x}{\sqrt{x^2+x} + x} : x =$ ⟹ Druga strana nastavak

4. $h(x) = \frac{x^2-4}{x^2+2}$ DOMAĆOJ GOTOVO

① DOMENA FUNKCIJE $\boxed{x \in \mathbb{R}}$

② NULTA TOČKA

$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

$\boxed{x_1 = 2 \quad x_2 = -2}$

③ ASIMPTOTE, HORIZONTALNA VERTIKALNA

$$\lim_{x \to \pm\infty} \frac{x^2-4}{x^2+2} \;/x^2 = \lim_{x \to \pm\infty} \frac{1-\frac{4}{x^2}}{1+\frac{2}{x^2}} = 1$$

HORIZONTALNA LIJEVA I DESNA POLOVI

④ EKSTREMI

$$h'(x) = \left(\frac{x^2-4}{x^2+2}\right) = \frac{2x(x^2+2) - 2x(x^2-4)}{x^4+4x^2+4} =$$

$$= \frac{2x(x^2+2-x^2+4)}{x^4+4x^2+4} = \frac{12x}{x^4+4x^2+4} = \frac{12x}{(x^2+2)^2}$$

$h'(x) = 0$

$12x = 0$

$x = 0$

	$-\infty$		0		$+\infty$
$h'(x)$		$-$		$+$	
$h(x)$		↘		↗	

$x=0$ LOKALNI MINIMUM

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IME I PREZIME: Marko Milolović **BROJ INDEKSA:** 17-2-0146-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^4 - 4 + 2i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 15+3

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Provjeri uvrštavanjem! 16+3

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6. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. 8

Ukupno: 34

⑤ $f(x) = \ln(\sin(4x-2))$

$f'(x) = \frac{1}{\sin(4x-2)} \cdot \cos(4x-2) \cdot 4$

$f'(x) = 4 \cdot \frac{\cos(4x-2)}{\sin(4x-2)}$ // ✓

③ $g(x) = \left(\sqrt{x^2-x} - x\right)$

V.A
$\lim_{x \to 1} \left(\sqrt{x^2+x} - x\right) = \sqrt{1^2+1} - 1 = -1 + \sqrt{2}$
NEMA V.A ✗

H.A
$\lim_{x \to \infty^{\pm}} \left(\sqrt{x^2+x} - x\right) \cdot \frac{\sqrt{x^2+x} - x}{\sqrt{x^2+x} - x}$

$= \frac{x^2+x-x}{\sqrt{x^2+x} - x} = \infty$ ✗

Domena
$x^2 - x \geq 0$
$a = 1$ ∪

$D_f \; x \in \mathbb{R}$ ✗

① $z^4 - 4 + 2i = 0$

$z^4 = 4 - 2i \;/\sqrt{}$

$z = \sqrt[4]{4-2i}$

$|z| = \sqrt{(4^2)+(2^2)}$ ✗

$|z| = 2\sqrt{3}$

$tg\varphi = \frac{4}{-2} =$

MARKO MIL

② $x + 2y - z + u = -1$

$2x + 5y - z + 2u = -2$

$3x - y - 2z + u = 5$

$x - y + 3z - 5u = 6$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 6 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \\ R_4 - R_1 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -7 & 1 & -2 & 8 \\ 0 & -3 & 4 & -6 & 7 \end{array}\right] \begin{array}{l} R_1 - 2R_2 \\ \\ R_3 + 7R_2 \\ R_4 + 3R_2 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -2 & 8 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right] R_3 - R_4 \sim \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right] \begin{array}{l} R_1 + 3R_3 \\ R_2 - R_3 \\ \\ R_4 - 7R_3 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 13 & 2 \\ 0 & 1 & 0 & -4 & -1 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & -34 & 0 \end{array}\right] \cdot(-1/34) \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 13 & 2 \\ 0 & 1 & 0 & -4 & -1 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} R_1 - 13R_4 \\ R_2 + 4R_4 \\ R_3 - 4R_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} x = 2 \\ y = -1 \\ z = 1 \\ u = 0 \end{array}$$

Rješenje:

$x + 2y - z + u = 2 + (2 \cdot (-1)) - 1 + 0 = -1$ ✓

$2x + 5y - z + 2u = 2 \cdot 2 + (5 \cdot (-1)) - 1 + 2 \cdot 0 = -2$ ✓

$3x - y - 2z + u = 3 \cdot 2 + 1 - 2 \cdot (1) + 0 = 5$ ✓

$x - y + 3z - 5u = 2 + 1 + 3 \cdot 1 - 5 \cdot 0 = 6$ ✓

(9) $h(x) = \frac{x^2-4}{x^2+2}$

DOMENA

$x^2+2 \neq 0$

$x^2 \neq -2 \,/\sqrt{}$

$x_{1,2} \neq \pm 2$

$D_f \; \mathbb{R} \setminus \{-2, 2\}$

PRVA DERIVACIJA

$$h'(x) = \frac{2x\cdot(x^2+2)-(x^2-4)\cdot 2x}{(x^2+2)^2} = \frac{2x^3+4x-2x^3+8x}{(x^2+2)^2} = -\frac{4x}{(x^2+2)^2}$$

SKICA GRAFA?

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ $\mathcal{I}1$

IME I PREZIME: LUKA ŠATALIĆ

BROJ INDEKSA: 17-2-0266-2013 0269075417

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^4 - 4 + 2i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* ~~15~~ 13

2. Gaussovom metodom riješi sustav linearnih jednadžbi, a zatim provjeri uvrštavanjem:

$$\begin{aligned} x + 2y - z + u &= -1 \\ 2x + 5y - z + 2u &= -2 \\ 3x - y - 2z + u &= 5 \\ x - y + 3z - 5u &= 6 \end{aligned}$$

Provjeri uvrštavanjem! ~~16~~ 13

3. Ispitati domenu i sve asimptote funkcije $g(x) = (\sqrt{x^2 + x} - x)$. ~~5~~ 15

4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{x^2-4}{x^2+2}$. 20(graf)

5. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\sin(4x - 2))$. 15

6. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. 8

Ukupno: ~~54~~

2. $\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -2 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 6 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -7 & 1 & -2 & 8 \\ 0 & -3 & 4 & -6 & 7 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -2 & 8 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right] \sim$

$\sim \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{6} & 1 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{3}{4} & 2 \\ 0 & 1 & 0 & \frac{1}{4} & -1 \\ 0 & 0 & 1 & -\frac{1}{4} & 1 \\ 0 & 0 & 0 & -\frac{17}{4} & 0 \end{array}\right]$

$x - \frac{3}{4}u = 2$

$y + \frac{1}{4}u = -1$

$z - \frac{1}{4}u = 1$

$-\frac{17}{4}u = 0 \Rightarrow$

$u = 0$

$x = 2$

$y = -1$

$z = 1$

$2 + 2 \cdot (-1) - 1 + 0 = -1$

$2 \cdot 2 + 5 \cdot (-1) - 1 + 2 \cdot 0 = -2$

$3 \cdot 2 - (-1) - 2 \cdot 1 + 0 = 5$

$2 - (-1) + 3 \cdot 1 - 5 \cdot 0 = 6$

⑤ $f(x) = \ln(\sin(4x-2))$

$$f'(x) = \frac{1}{\sin(4x-2)} \cdot \cos(4x-2) \cdot 4$$

$$= \frac{\cos(4x-2)}{\sin(4x-2)} \cdot 4$$

$$= 4 \operatorname{ctg}(4x-2) \checkmark$$

---

① $z^4 - 4 + 2i = 0$

$z^4 = 4 - 2i$

$1^4 = i^4 = (-1)^4 = (-i)^4$

$z_1 = -\sqrt[4]{4-2i} = ?$

$z_2 = -i\sqrt[4]{4-2i} = \ldots ?$

$z_3 = i\sqrt[4]{4-2i} = ?$

$z_4 = \sqrt[4]{4-2i} = \ldots ?$

LUKA SATALIĆ

③ $g(x) = \left(\sqrt{x^2+x} - x\right)$

$x^2 + x \geq 0$

$x(x+1) \geq 0$

$x \in \langle -\infty, -1] \cup [0, +\infty \rangle$

$D_g = \langle -\infty, -1] \cup [0, +\infty \rangle$ ✓

VERTIKALNE:

$\lim_{x \to -1^-} \left(\sqrt{x^2+x} - x\right) = 1$

$\lim_{x \to 0^+} \left(\sqrt{x^2+x} - x\right) = 0$

→ NEMA ASIMPTOTE

HORIZONTALNE I KOSE:

$\lim_{x \to +\infty} \left(\sqrt{x^2+x} - x\right) = \lim_{x \to +\infty} \frac{(x^2+x) - x^2}{\sqrt{x^2+x} + x} = \lim_{x \to +\infty} \frac{x}{\sqrt{x^2+x} + x}$

$= \lim_{x \to +\infty} \frac{1}{\sqrt{1+\frac{1}{x}} + 1} = \frac{1}{1+1} = \frac{1}{2}$

$y = \frac{1}{2}$ je horizontalna asimptota (DESNA)

$\lim_{x \to -\infty} \left(\sqrt{x^2+x} - x\right) = \infty + \infty = \infty$

$$\lim_{x \to -\infty} \frac{\sqrt{x^2+x}-x}{x} = \lim_{x \to -\infty}$$

$$= \lim_{x \to -\infty} \left(-\sqrt{1+\frac{1}{x}} - 1 \right) = -2$$

$$\lim_{x \to -\infty} \left(g(x) + 2x \right) = \lim_{x \to -\infty} \left(\sqrt{x^2+x} + x \right) = \lim_{x \to -\infty} \frac{x}{\sqrt{x^2+x}-x}$$

$$= \lim_{x \to -\infty} \frac{1}{-\sqrt{1+\frac{1}{x}}-1} = -\frac{1}{2}$$

$y = -2x - \frac{1}{2}$ je kosa asimptota ✓