MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

xoo

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POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Pronaći opće rješenje ODJ $y'' - y' = x^2$ i provjeriti dobiveno rješenje. 10+5
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \sin(x) \cdot \cos(y)$ na kvadratu $x \in [0, 2\pi]$, $y \in [0, 2\pi]$. 15
3. Pronaći ravninu koja dira graf funkcije $f(x,y) = xy - \ln(xy)$ povučenu u točki $(4, 1, z_0)$ tog grafa. 15
4. $\int_0^2 x \sin x^2 \, dx = ?$ 20
5. $\int_0^1 \frac{2x}{x^2 - x - 2} = ?$ 15
6. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y = 0$. 20

Ukupno: 20

f	$\frac{df}{dx}$
$x^\alpha \, (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \, (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \, (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$

$\int x^\alpha \, dx = \frac{x^{\alpha+1}}{\alpha+1}, \; \alpha \neq -1$

$\int \frac{dx}{x} = \ln|x| + C$

$\int e^x \, dx = e^x + C$

$\int a^x \, dx = \frac{a^x}{\ln a} + C$

$\int \sin x \, dx = -\cos x + C$

$\int \cos x \, dx = \sin x + C$

$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$

$\int \tan x \, dx = -\ln|\cos x| + C$

$\int \cot x \, dx = \ln|\sin x| + C$

$\int \frac{dx}{\cos^2 x} = \tan x + C$

$\int \frac{dx}{\sin^2 x} = -\cot x + C$

$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$

$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right| + C$

$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C$

$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$

$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$

(4.) $\int_0^2 x \sin x^2 dx =$

$x^2 = u$

$2x\,dx = du$

$x\,dx = \frac{1}{2} du$

$\int_0^2 \sin(x^2)\, x\,dx = \int_0^2 \sin(u) \frac{1}{2} du = \frac{1}{2} \int_0^2 \sin(u)\,du =$

$= -\frac{1}{2} \cos(u) \Big|_0^2 = -\frac{1}{2} \cos(x^2) \Big|_0^2 =$

$= -\frac{1}{2} \cos(4) + \frac{1}{2} \cos(0)$

$= 0,8268$ ✓

IVAN [illegible] GRIAN

$\int_0^{\frac{\pi}{2}} \sin x \cdot 2\sin x^2 \, dx =$

$\sin x^2 = u$

$2\sin x = du$

(5.) $\int_0^1 \frac{2x}{x^2 - x - 2} =$

$= |d(x^2 - x - 2) = (2x - 1)\,dx|$

$= \int_0^1 \frac{2x - 1 + 1}{x^2 - x + 2} dx$

$= \int_0^1 \frac{2x - 1}{x^2 - x} dx + \int_0^1 \frac{dx}{x^2 - x - 2}$

$= \frac{1}{x^2 - x + 2} = \frac{1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$

$= \frac{A(x-2) + B(x+1)}{(x+1)(x-2)} = \frac{Ax - 2A + Bx + B}{(x+1)(x-2)}$

$x_1 = 0$

$y_1 = 0$

$1 = -2A + B$

$1 = Ax + Bx$

$B = 1 + 2A$

$Ax = 1 - Bx$