MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO

IME I PREZIME: MARINO ZUBČIĆ **BROJ INDEKSA:** 17-2-0216-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \cos(2x)$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2} - 2$. 15
3. Izračunati tangencijalnu ravninu na graf funkcije $f(x,y) = \frac{y^2}{x+1}$ u točki $T(1,3,z_0)$. 15
4. $\int\limits_0^1 xe^x \, dx = ?$ 20
5. $\int\limits_0^2 \frac{3x}{x^2-2x+1} = ?$ 15
6. $\int\limits_2^{+\infty} \frac{dx}{1-x^2} = ?$ ~~20~~

Ukupno: 20

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

④ $\int\limits_0^1 xe^x dx = xe^x \Big|_0^1 - \int\limits_0^1 e^x dx = (e-0) - e^x \Big|_0^1 =$

$= e - (e-1) = 1$ ✓

⑤ $\int\limits_0^2 \frac{3x}{x^2-2x+1} = \int\limits_0^2 \frac{3x}{x^2-2x+1} dx = \int\limits_0^2 \frac{3x}{(x-1)^2} dx = \left| \begin{array}{l} t = x-1 \\ dt = dx \\ x = t+1 \end{array} \right|$ 0

NEPRAVI INTEGRAL

$= \int\limits_0^2 \frac{3t+3}{t^2} dt = \int\limits_0^2 \frac{3t}{t^2} dt + \int\limits_0^2 \frac{3}{t^2} dt =$

$= 3\int\limits_0^2 \frac{1}{t} dt + 3\int\limits_0^2 \frac{1}{t^2} dt = 3\ln|t| \Big|_0^2 + 3\left(-\frac{1}{t}\right)\Big|_0^2$

$= 3\ln|x-1| \Big|_0^2 + 3\left(\frac{1}{1-x}\right)\Big|_0^2$ →

$$= 3\left(\cancel{\ln|1|} - \ln|-1|\right) + 3\left(\frac{1}{-1} - \frac{1}{1}\right) = -6$$

$$⑥ \int_2^{+\infty} \frac{dx}{1-x^2} = \frac{1}{1-x^2}\,dx = \frac{1}{2}\int_2^{+\infty} \frac{1}{1-x} + \frac{1}{2}\int_2^{+\infty}\frac{1}{1+x} = -\frac{1}{2}\ln|1-x|\Big|_2^{+\infty} + \frac{1}{2}\ln|x+1|\Big|_2^{+\infty}$$

$\hookrightarrow (1-x)(1+x)$

$$\frac{A}{1-x} + \frac{B}{1+x} = \frac{1}{1-x^2} \Big/ \cdot (1-x^2) \qquad = +\infty$$

$$A(1+x) + B(1-x) = 1$$

$$A + Ax + B - Bx = 1$$

$$A + B = 1$$

$$A - B = 0 \Rightarrow A = \frac{1}{2} \quad B = \frac{1}{2}$$

$$① \; y'' + 2y' + 5y = \cos(2x)$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO

IME I PREZIME: ANGELO KOSOVIĆ **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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6. $\int\limits_2^{+\infty} \frac{\mathrm{d}x}{1-x^2} = ?$ 20

Ukupno: 20

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
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α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
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$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x \,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4. $\int\limits_0^1 x e^x dx =$

$\int x e^x dx = \begin{bmatrix} u = x & dv = e^x dx \\ du = dx & v = e^x \end{bmatrix} = x e^x - \int e^x dx = x e^x - e^x + C$

$= x e^x - e^x \Big/_0^1 = (1 \cdot e^1 - e^1) - (0 \cdot e^0 - e^0) = 0 - (-1) = 1$ ✓

5. $\int\limits_0^2 \frac{3x}{x^2-2x+1}$

$x = \frac{1}{2}(2x-2)+1$

$x^2 - 2x + 1 = 0$

$x = \frac{2 \pm \sqrt{4-4}}{2}$

$x = \frac{2 \pm 0}{2}$

$x = 1$

$0 \to 1 \leftarrow 2$

$\int \frac{3x}{x^2-2x+1} dx = 3\int \frac{x\,dx}{x^2-2x+1} = \begin{bmatrix} t = x^2-2x+1 \\ dt = (2x-2)dx \end{bmatrix}$

$= 3\int \frac{dt}{t} \cdot \frac{1}{2}\int \frac{(2x-2)dx}{x^2-2x+1} = 3\ln|t| \cdot \frac{1}{2}\int \frac{(2x-2)dx}{(x^2-2x+1)}$

$= 3\ln|t| \cdot \frac{1}{2}\int \frac{(2x-2)dx}{(x-1)^2} = 3\ln|t| \cdot \frac{1}{2}\int \frac{2x\,dx}{(x-1)^2} \cdot \left(-2\int \frac{dx}{(\underbrace{x-1}_{v})^2}\right.$

$= 3\ln|t| \cdot \frac{1}{2}\int \frac{2x\,dx}{(x-1)^2} \cdot \left(-2\ln|v^2|\right) = 3\ln|t| \cdot \int \frac{x\,dx}{(x-1)^2} \cdot \left(-2\ln|v^2|\right)$

0

6) 5. $\int_{2}^{+\infty} \frac{dx}{1-x^2} = \begin{bmatrix} 1-x=t \\ -dx=dt \end{bmatrix} = -\int \frac{dt}{t^2} = -\ln|(1-x)^2|$ $\quad x^2=1$, $x=\pm 1$

$= \lim_{a\to 1} -\ln|1-x^2| - \lim_{b\to -1} -\ln|(1-x^2)|$?

$= \lim_{a\to 1} -\ln|0| - \lim_{b\to -1} -\ln|0|$

$= -(-\infty) - (-(-\infty)) = \infty - \infty = n/p$

2. $\frac{1}{1+x^2+y^2} - 2$

$\partial_x f = -2x$ $\quad \partial_y f = 0$ $\quad T(0,0)$ $\quad A<0$

$-2x = 0 \,/:(-2)$ $\quad \Delta > 0$

$\partial_{xx} f = -2$ $\quad x=0$ $\quad A = \partial_{xx} f(T) = -2$

$\partial_y f = 0$

$\partial_{xy} f = 0$ $\quad -2y = 0$ $\quad \Delta = \begin{vmatrix} \partial_{xx} f & \partial_{xy} f \\ \partial_{yx} f & \partial_{yy} f \end{vmatrix}$ $\quad f(0,0) = \frac{1}{1+0+0} - 2 = \frac{1}{1} - 2 = 1-2 = -1$

$y=0$

$\partial_y f = -2y$ $\quad \Delta \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix} = 4$ $\quad f_{max} = -1$ u $T(0,0)$

$\partial_{yy} f = -2$

3. $f_{(x,y)} = \frac{y^2}{x+1}$ $\quad T(1,3,z_0)$

$z_0 = \frac{y^2}{x+1} = \frac{3^2}{1+1} = \frac{9}{2}$ $\quad T(1,3,\frac{9}{2})$

$\cancel{f_x = x^{-2}}$

$f_y = 2y$

$f_x(T) = 1$

$f_y(T) = 6$

$z - z_0 = f_x(T)(x-x_0) + f_y(T)(y-y_0)$

$z - \frac{9}{2} = 1(x-x_0) + 6(y-y_0)$

$\frac{9}{2} z = x - 1 + 6y - 18$

$-x - 6y + \frac{9}{2} z + 19 = 0$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxo

IME I PREZIME: Vedran Ivanković **BROJ INDEKSA:** 17-2-0235-2012

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \cos(2x)$. 15
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4. $\int_0^1 xe^x \, dx = ?$ 20 (20)
5. $\int_0^2 \frac{3x}{x^2 - 2x + 1} = ?$ 15
6. $\int_2^{+\infty} \frac{dx}{1 - x^2} = ?$ 20

Ukupno: 20

f	$\frac{df}{dx}$
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Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
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$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

~~1. $y'' + 2y' + 5y = \cos(2x)$~~

4. $\int_0^1 xe^x dx = ?$ $\begin{Bmatrix} u = x & du = 1 \\ dv = e^x & v = e^x \end{Bmatrix} = (x \cdot e^x)\Big|_0^1 - \int_0^1 1e^x dx$

$= (xe^x)\Big|_0^1 - \int_0^1 e^x dx = (xe^x)\Big|_0^1 - (e^x)\Big|_0^1$

$= (xe^x - e^x)\Big|_0^1 = (e^x(x-1))\Big|_0^1$

$= e^1(1-1) - e^0(0-1)$

$= 0 - 1 \cdot (-1)$

$= 1$ //

5.) $\int_0^2 \frac{3x}{x^2-2x+1}$

6.) $\int_2^{+\infty} \frac{dx}{1-x^2} = ?$

$$\int_2^{+\infty} \frac{dx}{1-x^2}\,dx = \left(\cancel{1}\,\frac{1}{1}\ln\left|\frac{1+x}{1+x}\right|\right)\Big|_2^{+\infty} = \left(\ln\left|\frac{1+x}{1-x}\right|\right)\Big|_2^{-\infty}$$

$$= \ln\left|\frac{\infty}{-\infty}\right| - \ln\left|\frac{\cancel{1}+2}{1-2}\right| = -\ln 3$$

?

5.) $\int_0^2 \frac{3x}{x^2-2x+1}\,dx = 3\int_0^2 \frac{x\,dx}{x^2-2x+1} = \begin{cases} u = x, & du = 1 \\ dv = \frac{1}{x^2-2x+1}, & v = \frac{1}{1-x} \end{cases}$

$$= 3\left(\left(x\cdot\frac{1}{1-x}\right)\Big|_0^2 - \int_0^2 1\cdot\frac{1}{1-x}\,dx\right.$$

$$= 3\left(\frac{x}{1-x}\Big|_0^2 - \int_0^2 \frac{1}{1-x}\,dv\right) = \begin{cases} u = \varphi(x) = 1-x \\ du = -1dx \Rightarrow dx = -du \end{cases}$$

$$= 3\left(\frac{x}{\cancel{1}-x}\right)\Big|_0^2 + \int_1^{-1} \frac{1}{u}\,du\right)$$

NEPRAVI INTEGRAL

$$= 3\left(\left(\frac{x}{1-x}\right)\Big|_0^2 - \ln x\Big|_{-1}^{1}\right)$$

$$= 3\cdot(-2)$$

$$= -6$$

Vedran Ivanković

1.) $y'' + 2y' + 5y = \cos(2x)$

$$\frac{d^2y(x)}{dx^2} + 2\,\frac{dy(x)}{dx} + 5y(x) = \cos 2x$$

$$\frac{d^2y(x)}{dx^2} + 2\,\frac{dy(x)}{dx^2} + 5y(x) = 0 \quad , \; y(x) = e^{\lambda x}$$

$$\frac{d^2}{dx^2}(e^{\lambda x}) + 2\,\frac{d}{dx}(e^{\lambda x}) + 5e^{\lambda x} = 0$$

$$\frac{d^2}{dx^2}(e^{\lambda x}) = \lambda^2 e^{\lambda x}$$

$$\frac{d}{dx}(e^{\lambda x}) = \lambda e^{\lambda x}$$

$$\lambda^2 e^{\lambda x} + 2\lambda e^{\lambda x} + 5e^{\lambda x} = 0$$

$$(\lambda^2 + 2\lambda + 5)e^{\lambda x} = 0$$

$$\lambda^2 + 2\lambda + 5 = 0$$

$$y(x) = \frac{c_1}{e^{(1-2i)}} + \frac{c_2}{e^{(1+2i)x}}$$

?

$$y(x) = c_1\left(\frac{\cos 2x}{e^x} + \frac{i\sin 2x}{e^x}\right) + c_2\left(\frac{\cos 2x}{e^x} - \frac{i\sin 2x}{e^x}\right)$$

$$y(x) = \frac{(c_1 + c_2)\cos 2x}{e^x} + \frac{i(c_1 - c_2)\sin 2x}{e^x}$$

$$y(x) = \frac{c_1\cos 2x}{e^x} + \frac{c_2 \sin 2x}{e^x}$$

$$\frac{d^2y(x)}{dx^2} + 2\,\frac{dy(x)}{dx} + 5y\cancel{x} - \cos 2x \Rightarrow y_p = a_1 \cos 2x + a_2 \sin 2x$$

$$\frac{d^2 y_p(x)}{dx^2} = \frac{d^2}{dx^2}(a_1\cos 2x + a_2 \sin 2x) = -4a_1\cos 2x - 4a_2 \sin 2x$$

$$\frac{dy_p(x)}{dx} = \frac{d}{dx}(a_1\cos 2x + a_2\sin 2x) = -2a_1\sin 2x + 2a_2\cos 2x$$

$$(a_1 + 4a_2)\cos 2x + (-4a_1 + a_2)\sin 2x = \cos 2x$$

$$a_1 + 4a_2 = 1 \Rightarrow a_1 = 1 - 4a_2$$

$$-4a_1 + a_2 = 0 \qquad -4(1 - 4a_2) + a_2 = 0$$

$$-4 + 16a_2 + a_2 = 0$$

$$17a_2 = 4 \;/:17$$

$$a_2 = \frac{4}{17} \qquad a_1 = \frac{1}{17}$$

$$a_1 = 1 - 4\,\frac{4}{17}$$

$$y_p = \frac{1}{17}\cos 2x + \frac{4}{17}\sin 2x$$

$$a_1 = \frac{17}{17} - \frac{16}{17}$$

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IME I PREZIME: ANTON RAŠIĆ   **BROJ INDEKSA:** 17-2-0084-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 0

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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

(3) $f(x,y) = \frac{y^2}{x+1}$

$T(1,3,z_0)$

$z_0 = \frac{y^2}{x+1} = \frac{3^2}{1+1} = \frac{9}{2}$ ✓

$\frac{\partial f}{\partial y} = \frac{2y}{x+1}$

$\frac{\partial f}{\partial y}(1,3) = \frac{6}{2} = 3$

$-\frac{3}{2} + \frac{6}{1} + \frac{9}{2}$

$\frac{-3+12+9}{2} = \frac{18}{2} = 9$

$f_x = \frac{\partial f}{\partial x} = \frac{(y^2)'(x+1) - y^2(x+1)'}{(x+1)^2} = \frac{2y(x+1) - y^2 \cdot 1}{(x+1)^2} = \frac{2xy + 2y - y^2}{x^2+2x+1}$

0

$= \frac{2y}{2x+2} = \frac{\cancel{6}^3}{\cancel{4}_2} = \boxed{\frac{3}{2}} = f_x(T)$

$f_x(T) = \frac{3}{2}$

$f_y(T) = -2$

$f_y = \frac{2x+2-2y}{1} = \frac{2+2-6}{1} = \frac{4-6}{1} = \boxed{-2}\ f_y(T)$ ✗

$z - \frac{9}{2} = \frac{3}{2}x - \frac{3}{2} - 2y + 6$

$\boxed{z = \frac{3}{2}x - \frac{2y}{1} + \frac{9}{1}}$

$z - \frac{9}{2} = \frac{3}{2}(x-1) + (-2)(y-3)$ ✓ ✗

$z = \frac{3}{2}x - \frac{3}{2} + \frac{6}{1} + \frac{9}{2} - \frac{2y}{1}$

① $\int_0^1 xe^x dx = \left[\begin{matrix} e^x = t \\ e^x dx = dt \\ dx = \frac{1}{e^x} dt\end{matrix}\right] = \int_0^1 xt \cdot \frac{1}{e^x} dt = \int_0^1 \frac{xe^x}{e^x} dt$

$= x\,dt \Big|_0^1 = \frac{x^2}{2}\Big|_0^1 = \frac{1^2}{2} - \frac{0^2}{2} = \frac{1}{2}$ ×

④ $\int_0^1 xe^x dx = \left[\begin{matrix} u = e^x & dv = x \\ du = e^x & v' = \frac{x^2}{2}\end{matrix}\right] = e^x \cdot \frac{x^2}{2} - \int_0^1 \frac{x^2}{2} \cdot e^x$

$= \frac{e^x x^2}{2} - \frac{1}{2}\int_0^1 x^2 e^x =$

② $f(x,y)\ \frac{1}{x^2+y^2+1} - \frac{2}{1} = \frac{1-2(x^2+y^2+1)}{x^2+y^2+1} = \frac{1-2x^2-2y^2-2}{x^2+y^2+1} = \frac{-2x^2-2y^2-1}{x^2+y^2+1}$

$f'(x,y) = \frac{(-2x^2-2y^2-1)'(x^2+y^2+1) - (-2x^2-2y^2-1)(x^2+y^2+1)'}{(x^2+y^2+1)^2}$

$= \frac{(-4x-4y)\cancel{(x^2+y^2+1)} + (2x^2+2y^2+1)(2x+2y)}{(x^2+y^2+1)^2}$

$= \frac{\cancel{-4x^3}\cancel{-4y^2x}-4x\cancel{-4x^2y}\cancel{-4y^3}-4y+\cancel{4x^3}+\cancel{4x^2y}+\cancel{4y^2x}+\cancel{4y^3}+2x+2y}{(x^2+y^2+1)^2}$

$= \frac{-4x-4y+2x+2y}{(x^2+y^2+1)^2} = \frac{-2x-2y}{(x^2+y^2+1)^2} \Big/ \sqrt{\ } = \frac{\sqrt{(-2x-2y)}}{x^2+y^2+1}$

$= \frac{\frac{x^2+y^2+1}{2\cancel{\sqrt{(-2x-2y)}}} - \cancel{\sqrt{(-2x-2y)}}(2x+2y)}{(x^2+y^2+1)^2} = \frac{\frac{x^2+y^2+1}{2} - \frac{2x+2y}{1}}{\frac{(x^2+y^2+1)^2}{1}}$

$= \frac{\frac{(x^2+y^2+1)-4x+4y}{2}}{\frac{(x^2+y^2+1)^2}{1}} = \frac{\cancel{(x^2+y^2+1)}-4x+4y}{2(x^2+y^2+1)^2} = \frac{-4x+4y}{2x^2+2y^2+2}$

ANTON RAŠIĆ

② NASTAVAK

$$\frac{\delta f}{\delta x} = \frac{-\cancel{4}}{\cancel{4}x} = \frac{-1}{x} \qquad -\frac{1}{x} = 0 \qquad x = 0$$

$$\frac{\delta f}{\delta y} = \frac{4}{4y} = \frac{1}{y} \qquad \frac{1}{y} = 0 \qquad y = 0$$

$T(0,0)$

$$\frac{\delta^2 f}{\delta^2 x} = \frac{0}{x^2} = 0 \qquad \frac{\delta f}{\delta x \delta y} = 0$$

$$\frac{\delta^2 f}{\delta^2 y} = \frac{0 \cdot y - 1 \cdot 0}{y^2} = \frac{0}{y^2} = 0 \qquad \frac{\delta f}{\delta y \delta x} = 0$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxo

IME I PREZIME: Matija Miočić **BROJ INDEKSA:** 17-1-0110-2012

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \cos(2x)$. 15
2. Odrediti lokalne ekstreme funkcije $f(x, y) = \frac{1}{1+x^2+y^2} - 2$. 15
3. Izračunati tangencijalnu ravninu na graf funkcije $f(x, y) = \frac{y^2}{x+1}$ u točki $T(1, 3, z_0)$. 15
4. $\int_0^1 x\mathrm{e}^x \,\mathrm{d}x = ?$ 20
5. $\int_0^2 \frac{3x}{x^2 - 2x + 1} = ?$ 15
6. $\int_2^{+\infty} \frac{\mathrm{d}x}{1 - x^2} = ?$ 20

Ukupno: 20

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_a x \ (a > 0)$	$\frac{1}{x \ln a}$
e^x	e^x
$a^x \ (a > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\| \frac{a+x}{a-x} \right\| + C$
$\int x^\alpha \,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\|\cos x\| + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\| \frac{x-a}{x+a} \right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x \,\mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \mathrm{e}^x \,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2} \,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4. $\int_0^1 xe^x dx = xe^x\Big|_0^1 - e^x\Big|_0^1 = e - e + 1 = \underline{1}$

5. $\int_0^2 \frac{3x}{x^2-2x+1} = \int_0^2 \frac{3x}{(x-1)^2} = \frac{3x}{(x-1)^2}\Big|_0^2 - 3\underbrace{\int \frac{x}{(x-1)^2}}$

$= \frac{3x}{(x-1)^2}\Big|_0^2 - \frac{x^2}{2(x-1)}\Big|_0^2 = 6 - 0 = \underline{6}$

NEPRAVI INTEGRAL

⑥ $\int_2^{+\infty} \frac{dx}{1-x^2} = \int_2^{+\infty} \frac{dx}{(1-x)(1+x)}$

$= \int \frac{1}{1-x} \cdot \int \frac{dx}{1+x} =$

$= \frac{1}{2}((1-x)) \cdot \ln|x+1|$

$= \frac{1}{1-x^2} \Big|_2^{+\infty} - \frac{1}{2}(1-x) \cdot \ln|x+1| \Big|_2^{+\infty}$?

② $f(x,y) = \frac{1}{1+x^2+y^2} - 2$?

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO

IME I PREZIME: MARTIN ŠOŠA **BROJ INDEKSA:** 17-1-0037-2011

POPUNJAVA NASTAVNIK

	Broj ↓ bodova
1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \cos(2x)$.	15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2} - 2$.	15
3. Izračunati tangencijalnu ravninu na graf funkcije $f(x,y) = \frac{y^2}{x+1}$ u točki $T(1,3,z_0)$.	15
4. $\int_0^1 xe^x \, dx = ?$	20
5. $\int_0^2 \frac{3x}{x^2-2x+1} = ?$	15
6. $\int_2^{+\infty} \frac{dx}{1-x^2} = ?$	20
Ukupno:	0

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

6.

$$\int_2^{+\infty} \frac{dx}{1-x^2} = \frac{1}{2}\ln\left\lvert\frac{1+x}{1-x}\right\rvert + C$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO

IME I PREZIME: Antonio Pribil **BROJ INDEKSA:** 57666

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \cos(2x)$. 15
2. Odrediti lokalne ekstreme funkcije $f(x, y) = \frac{1}{1+x^2+y^2} - 2$. 15
3. Izračunati tangencijalnu ravninu na graf funkcije $f(x, y) = \frac{y^2}{x+1}$ u točki $T(1, 3, z_0)$. 15
4. $\int_0^1 x e^x \, dx = ?$ $\frac{x^2}{2} \cdot e^x + C = \frac{x^2 e^x}{2} + C$ 20
5. $\int_0^2 \frac{3x}{x^2 - 2x + 1} = ?$ 15
6. $\int_2^{+\infty} \frac{dx}{1 - x^2} = ?$ $\frac{1}{2 \cdot 1} \ln\left|\frac{1+x}{1-x}\right| + C$ 20

Ukupno: 0

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\| \frac{a+x}{a-x} \right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\| \frac{x-a}{x+a} \right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\| x + \sqrt{x^2 \pm a^2} \right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: MARKO ČULINA **BROJ INDEKSA:** 17-1-0008-2010	

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \cos(2x)$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2} - 2$. 15
3. Izračunati tangencijalnu ravninu na graf funkcije $f(x,y) = \frac{y^2}{x+1}$ u točki $T(1,3,z_0)$. 15
4. $\displaystyle\int_0^1 xe^x\,dx = ?$ 20
5. $\displaystyle\int_0^2 \frac{3x}{x^2-2x+1} = ?$ 15
6. $\displaystyle\int_2^{+\infty} \frac{dx}{1-x^2} = ?$ 20

Ukupno: 0

f	$\frac{df}{dx}$
x^α ($\alpha \neq 0$)	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ ($\alpha > 0$)	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x ($\alpha > 0$)	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\,dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\,dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\|x + \sqrt{x^2\pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

4) $\displaystyle\int_0^1 xe^x dx = \left| \begin{matrix} u = x & dv = \int e^x \\ du = dx & v = e^x \end{matrix} \right| = \int_0^1 x\cdot e^x - \int e^x dx = \int_0^1 x\cdot e^x - e^x =$

$= (1\cdot e^1 - e^1) - 0\cdot e^0 - e^0 = -2$ ✗

6) $\displaystyle\int_2^{+\infty} \frac{dx}{1-x^2} = \frac{1}{2}\ln\left|\frac{1+x^2}{1-x^2}\right| \Bigg|_2^{+\infty} = \frac{1}{2}\ln\left|\frac{1+\infty}{1-\infty}\right| - \frac{1}{2}\ln\left|\frac{1+2^2}{1-2^2}\right|$

$= \infty - \infty = \infty$ 0

3) $f(x,y) = \frac{y^2}{x+1}$ $T(1, 3, z_0)$

$z_0 = \frac{y^2}{x+1}$

$z_0 = \frac{9}{1+1} = \frac{9}{2}$

2) $f(x,y) = \frac{1}{1+x^2+y^2} - 2$

$f(x,y) = 1 + x^{-2} + y^{-2} - 2$

$f(x,y) = -1 + x^{-2} + y^{-2}$

$\frac{df}{dx} = -2x^{-1} + y^{-2} = -2x^{-1} + \frac{1}{y^2} = -2\frac{1}{x} + \frac{1}{y^2}$ ×

$\frac{df}{dy} = x^{-2} - 2y^{-1} = \frac{1}{x^2} - 2y^{-1}$

$a(x-x_1)(x-x_2)$

5) $\int_0 \frac{3x}{x^2-2x+1} =$

$x_{1,2} = \frac{2 \pm \sqrt{2^2-4 \cdot}}{2}$

$x_{1,2} = \frac{2}{2} = 1$

$I_1 \quad I_2$

$\int_0^2 \frac{3x}{(x-1)(x-1)} = \int_0^2 \frac{3x}{(x-1)} + \frac{3x}{(x-1)}$ X

$\emptyset$

$I_1 \int_0^2 \frac{3x\,dx}{(x-1)} \left| \begin{matrix} 3x = t \\ dx = dt \end{matrix} \quad \begin{matrix} x=2 \ldots t=6 \\ x=0 \quad t=0 \end{matrix} \right| = \int_0^6 \frac{dt}{(x-1)} = \frac{1}{2} \ln\left|\frac{x-1}{x+1}\right|$

$= -\infty - \infty$

$I = I_1 + I_2 = -\infty - \infty$

MARKO ČULINA

1)

$$y'' + 2y' + 5y = \cos(2x)$$

$r^2 + 2 + 5 = 0$

$a = 1 \quad b = 2 \quad c = 5$

$$r_{1,2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 5}}{2}$$

$$r_{1,2} = \frac{-2 \pm \sqrt{-16}}{2}$$

$$r_{1,2} = \frac{-2 + (-4i)}{2} = \underset{\alpha}{-1} \pm \underset{\beta}{(-4i)}$$

$y = e^{\alpha x}(C_1 \cos \beta + C_2 \sin \beta)$

$g(x) = \cos(2x) \quad = e^{0}(1 \cdot \cos(2x) + \sin(2x))$ $\qquad \alpha + \beta i = 0 + 2 = 2i$

$\alpha = 0 \quad m = 0 \quad \beta = 2 \quad n = 0 \qquad k = 0$

$y_p = x^0 e^{x}(A \cos(2x) + B \sin(2x)) = e^{x}(A\cos(2x) + B\sin(2x))$

$y_p' = e^{x} \cdot (A\cos(2x) + B\sin(2x)) + e^{x}(-2A\sin(2x) + 2B\cos(2x))$

$y_p'' = e^{x}(A\cos(2x) + B\sin(2x)) + e^{x}(-2A\sin 2x + 2B\cos(2x)) + e^{x}(-2A\sin(2x) + 2B\cos(2x)) +$

$e^{x}(-2A\cos(2x) - 2B\sin(2x))$

$e^{x}(A\cos(2x) + B\sin(2x) - 2A\sin(2x) + 2B\cos(2x) - 2A\sin(2x) + 2B\cos(2x) - 2A\cos(2x) - 2B\sin(2x)) +$

$+2(e^{x}(-2A\sin(2x) + B\sin(2x))) + 5 \cdot (e^{x}(A\cos(2x) + B\sin(2x))) = \cos(2x)$

DALJE ?

$e^{x}($

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxo
IME I PREZIME: IVAN RADOVIĆ   **BROJ INDEKSA:** 57230

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \cos(2x)$. 15
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4. $\int\limits_0^1 x\mathrm{e}^x\,\mathrm{d}x = ?$ 20
5. $\int\limits_0^2 \frac{3x}{x^2-2x+1} = ?$ 15
6. $\int\limits_2^{+\infty} \frac{\mathrm{d}x}{1-x^2} = ?$ 20

Ukupno: 0

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
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2. $f(x,y) = \frac{1}{1+x^2+y^2} - 2$

$\frac{df}{dx} = \frac{1}{2x}$ X $\quad \frac{\partial f}{\partial x} = \frac{-2x}{(1+x^2+y^2)^2}$

$\frac{\delta f}{\delta y} = \frac{1}{2y}$

$\frac{\delta^2 f}{\delta x^2} = \frac{1}{2}$

$\frac{\delta^2 f}{\delta y^2} = \frac{1}{2}$

$\frac{\delta^2 f}{\delta x^2 \delta y^2} = \frac{\delta^2 f}{\delta y^2 \delta x^2} = 0$

$D = \begin{vmatrix} \frac{1}{2x} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2y} \end{vmatrix} = \frac{1}{4xy} - \frac{1}{4}$

$\frac{1}{2}\cdot\frac{1}{x} = 0$

$(2x)^{-1} = 0 /:2$

$x^{-1} = 0$

$\underline{x = 0}$

$\frac{1}{2y} = 0$

$\underline{y = 0}$

$A(0,0)$

$D = \frac{1}{4\cdot 0\cdot 0} - \frac{1}{4} = -\frac{1}{4} < 0$ NEMA EKSTREMA

$$\int_0^1 x e^x dx = \begin{bmatrix} u = x & dv = e^x / \int \\ du = dx & v = e^x \end{bmatrix} = x \cdot e^x \Big|_0^1 - \int_0^1 e^x dx$$

$u \cdot dv = u \cdot v - \int v \cdot du$

$$= 1 \cdot 1.718281828 - e^x x \Big|_0^1$$

$$= 0 \times$$

$$\int_2^{+\infty} \frac{dx}{1-x^2} = \int_2^{+\infty} \frac{dx}{1^2 - x^2} = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| \Bigg|_2^{+\infty} = \frac{1}{2} \ln \left| \frac{1+10}{1-10} \right| - \frac{1}{2} \ln \left| \frac{1+2}{1-2} \right|$$

?

$x^2 - 2x + 1 = t$

$2x - 2 = dt$

$$= 0.100335347 - 1.5939612289 = -1.49327$$

$$\int_0^2 \frac{3x}{x^2-2x+1} = 3 \int_0^2 \frac{x\,dx}{x^2-2x+1} = 3 \int_0^2 \frac{x}{x^2-2x+1} \cdot \int_0^2 \frac{dx}{(x-1)^2} = \begin{bmatrix} (x-1) = t \\ dx = dt \end{bmatrix}$$

$$= 3 \int_0^2 \frac{x}{x^2-2x+1} \cdot \int_0^2 \frac{dt}{t^2}$$

$$= 3 \int_0^2 \cdot$$

NEPRAVI INTEGRAL

$(x-1)^2$

$x - 1 = t$

$dx = dt$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO

IME I PREZIME: FILIP MEŠTROVIĆ **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK

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6. $\int\limits_2^{+\infty} \frac{dx}{1-x^2} = ?$ 20

Ukupno: 20

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
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Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
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$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

1) $r^2 + 2r + 5 = 0$ $\quad \boxed{r_1 = -1 + 2i \quad r_2 = -1 - 2i}$

$r_{12} = \frac{-2 \pm \sqrt{4-20}}{2}$

$y_H = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$

$y_H = e^{-x}(C_1 \cos 2x + C_2 \sin 2x)$

$y_p = e^{\alpha x}(A \cos(\beta x) + B \sin(\beta x))$

$y_p = x e^0 (A \cos(2x) + B \sin(2x))$

(4) $\int_0^1 xe^x dx = \left| \begin{matrix} u = x & dv = e^x dx \\ du = dx & v = e^x \end{matrix} \right| = xe^x - \int e^x dx$

$= \underline{xe^x - e^x} \Big|_0^1$

$= (1 \cdot e^1 - e^1) - (-1) = +1$

5.)

$\overset{I_1}{} \quad \overset{I_2}{}$

$\int_0^2 \frac{3x}{x^2-2x+1} dx = \int_0^2 \frac{3}{x-1} dx + \int_0^2 \frac{3}{(x-1)^2}$

$x^2 - 2x + 1 = 0$

$x_{12} = \frac{2 \pm \sqrt{4-4}}{2}$

$x_{12} = 1$

$(x-1)(x-1)$

$x^2 - x - x + 1$

$\frac{3x}{x^2-2x+1} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} \Big| \cdot N$

$3x = A(x-1) + B$

$3x = Ax - A + B$

$\boxed{3 = A} \quad -A + B = 0$

$\boxed{B = 3}$

prekid je u točki $x=1$

RJEŠENJE ?

$I_1 = 3\ln(x-1) + 3\ln$

~~5.~~

6. $\displaystyle\int_2^{+\infty} \frac{dx}{1-x^2} = \cancel{\begin{vmatrix} 1-x^2 = t \\ -2x\,dx = dt \\ x\,dx \end{vmatrix}} = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + C$

$$\int_2^{+\infty} \frac{dx}{1-x^2} = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|\Bigg|_2^{+\infty} =$$

$$= \frac{1}{2}\ln\frac{+\infty}{-\infty} - \frac{1}{2}\ln 3$$

?

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

XXO

IME I PREZIME: Tibor Rak **BROJ INDEKSA:** 17-1-0060-2011

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	Broj ↓ bodova
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5. $\displaystyle\int_0^2 \frac{3x}{x^2-2x+1} = ?$	15
6. $\displaystyle\int_2^{+\infty} \frac{dx}{1-x^2} = ?$	20
Ukupno:	0

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

2. $f(x,y) = \frac{1}{1+x^2+y^2} - 2 \Big/ \cdot (1+x^2+y^2)$

$= 1 - 2(1+x^2+y^2)$

$= 1 - 2 - 2x^2 - 2y^2$

$= -1 - 2x^2 - 2y^2$

$\frac{\partial f}{\partial x} = -4x \qquad \frac{\partial f}{\partial y} = -4y$

$\frac{\partial^2 f}{\partial x^2} = -4 \qquad \frac{\partial^2 f}{\partial y^2} = -4$

$\frac{\partial f}{\partial x \partial y} = 0 \qquad \frac{\partial f}{\partial y \partial x} = 0$

$$\Delta = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial f}{\partial y \partial x} \\ \frac{\partial f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{vmatrix} = \begin{vmatrix} -4 & 0 \\ 0 & -4 \end{vmatrix} = 16 > 0$$

LOKALNI MINIMUM

GDJE?

4. $\int_0^1 x e^x \, dx =$