MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

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POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = \cos(2x)$. 15

2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2} - 2$. 15

3. Izračunati tangencijalnu ravninu na graf funkcije $f(x,y) = \frac{y^2}{x+1}$ u točki $T(1,3,z_0)$. 15

4. $\int_0^1 xe^x\,dx = ?$ 20

5. $\int_0^2 \frac{3x}{x^2-2x+1} = ?$ 15

6. $\int_2^{+\infty} \frac{dx}{1-x^2} = ?$ 20

Ukupno: 20

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

4) $\int_0^1 xe^x dx = xe^x\Big|_0^1 - \int_0^1 e^x dx = (e-0) - e^x\Big|_0^1 =$

$= e - (e-1) = 1$ ✓

5) $\int_0^2 \frac{3x}{x^2-2x+1} = \int_0^2 \frac{3x}{x^2-2x+1}\,dx = \int_0^2 \frac{3x}{(x-1)^2}\,dx =$ $\left|\begin{array}{l} t = x-1 \\ dt = dx \\ x = t+1 \end{array}\right|$ ∅

$= \int_0^2 \frac{3t+3}{t^2}\,dt = \int_0^2 \frac{3t}{t^2}\,dt + \int_0^2 \frac{3}{t^2}\,dt =$

NEPRAVI INTEGRAL

$= 3\int_0^2 \frac{1}{t}dt + 3\int_0^2 \frac{1}{t^2}dt = 3\ln|t|\Big|_0^2 + 3\left(-\frac{1}{t}\right)\Big|_0^2$

$= 3\ln|x-1|\Big|_0^2 + 3\left(\frac{1}{1-x}\right)\Big|_0^2$ →

$= 3(\ln|1| - \ln|-1|) + 3\left(\frac{1}{-1} - \frac{1}{1}\right) = -6$

⑥ $\int_2^{+\infty} \frac{dx}{1-x^2} = \int_2^{+\infty} \frac{1}{1-x^2}\,dx = \frac{1}{2}\int_2^{+\infty} \frac{1}{1-x} + \frac{1}{2}\int_2^{+\infty} \frac{1}{1+x} = -\frac{1}{2}\ln|1-x|\Big|_2^{+\infty} + \frac{1}{2}\ln|x+1|\Big|_2^{+\infty}$

↳ $(1-x)(1+x)$

$= +\infty$

$\frac{A}{1-x} + \frac{B}{1+x} = \frac{1}{1-x^2} \;/\cdot (1-x^2)$

$A(1+x) + B(1-x) = 1$

$A + Ax + B - Bx = 1$

$A + B = 1$

$A - B = 0 \Rightarrow A = \frac{1}{2} \quad B = \frac{1}{2}$

① $y'' + 2y' + 5y = \cos(2x)$