

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

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1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' + 2y' = 1$, uz $y(0) = 0$ i $y'(0) = 0$.
Na kraju provjeri rješenje. 15

2. Nađi koliko iznosi $f(5)$ ako f zadovoljava $dy = \left(1 + x + \frac{y}{1-x^2}\right) dx$ i $y(0) = 2$. 15

3. Za funkciju $f(x, y) = \ln\left(\frac{y}{x}\right)$ odrediti domenu, kodomenu, razinske krivulje i limes u ishodištu (ako postoji). 20 8

4. $\int_0^2 xe^x dx = ?$ 15

5. Zadana je funkcija $f(x) = \sqrt{|x|}$. Traži se površina ispod grafa funkcije (do osi apcise) na segmentu $[-1, 2]$. Podijeliti segment na nekoliko dijelova i preko trapezne formule procijeniti traženu površinu. Skicirati graf funkcije, površinu koja je dobivena procjenom i vizualno ocijeniti grešku numeričkog postupka. 20

6. Integriranjem izračunati površinu trokuta zadanog točkama $A(-2, 1)$, $B(0, 2)$, $C(1, -1)$. 15

Ukupno:

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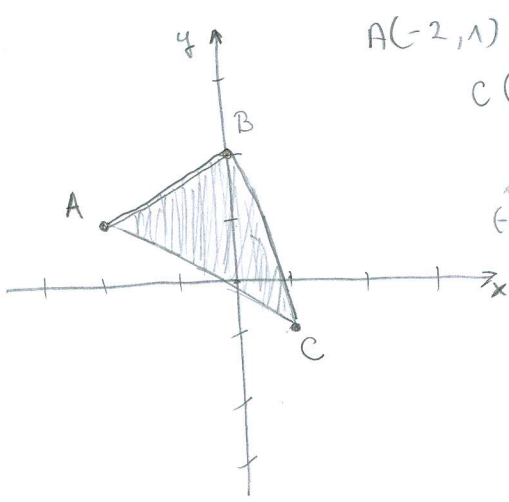
| | |
|------------------------------|--------------------------|
| f | $\frac{df}{dx}$ |
| $x^\alpha (\alpha \neq 0)$ | $\alpha x^{\alpha-1}$ |
| $\ln x$ | $\frac{1}{x}$ |
| $\log_\alpha x (\alpha > 0)$ | $\frac{1}{x \ln \alpha}$ |
| e^x | e^x |
| $a^x (\alpha > 0)$ | $a^x \ln a$ |
| $\sin x$ | $\cos x$ |
| $\cos x$ | $-\sin x$ |
| $\tan x$ | $\frac{1}{\cos^2 x}$ |
| $\cot x$ | $\frac{-1}{\sin^2 x}$ |
| $\arcsin x$ | $\frac{1}{\sqrt{1-x^2}}$ |
| $\arctan x$ | $\frac{1}{1+x^2}$ |

| Tablica nekih integrala | | |
|--|---|--|
| $\int dx = x + C$ | $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$ | $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$ |
| $\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$ | $\int \tan x dx = -\ln \cos x + C$ | $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$ |
| $\int \frac{dx}{x} = \ln x + C$ | $\int \cot x dx = \ln \sin x + C$ | $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$ |
| $\int e^x dx = e^x + C$ | $\int \frac{dx}{\cos^2 x} = \tan x + C$ | $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$ |
| $\int a^x dx = \frac{a^x}{\ln a} + C$ | $\int \frac{dx}{\sin^2 x} = -\cot x + C$ | $\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$ |
| $\int \sin x dx = -\cos x + C$ | $\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$ | |
| $\int \cos x dx = \sin x + C$ | $\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$ | |

4. $\int_0^2 xe^x dx = \left[\begin{array}{l} u=x \\ du=dx \\ dv=e^x dx \\ v=e^x \end{array} \right] = x \cdot e^x - \int e^x dx = x \cdot e^x - e^x \Big|_0^2 = (2 \cdot e^2 - e^2) - (0 \cdot e^0 - e^0)$

= 8,389 ✓

6.



$A(-2, 1), B(0, 2)$
 $C(1, -1)$

$$(y - y_1)(x_2 - x_1) = (y_2 - y_1)(x - x_1)$$

\overline{AB}
 $(-2, 1) (0, 2)$

$$(y - 1) \cdot (0 + 2) = (2 - 1) \cdot (x + 2)$$

$$(y - 1) \cdot 2 = 1 \cdot (x + 2)$$

$$2y - 2 = x + 2$$

$$2y = x + 4$$

$$y = \frac{1}{2}x + 2$$

$$\left[y = \frac{1}{2}x + 2 \right]$$

\overline{AC}
 $(-2, 1) (1, -1)$

$$(y - y_1)(x_2 - x_1) = (y_2 - y_1)(x - x_1)$$

$$(y - 1)(1 + 2) = (-1 - 1)(x + 2)$$

$$(y - 1) \cdot 3 = -2(x + 2)$$

$$3y - 3 = -2x - 4$$

$$3y = -2x - 1$$

$$y = -\frac{2}{3}x - \frac{1}{3}$$

$$\left[y = -\frac{2}{3}x - \frac{1}{3} \right]$$

\overline{BC}
 $(0, 2) (1, -1)$

$$(y - y_1)(x_2 - x_1) = (y_2 - y_1)(x - x_1)$$

$$(y - 2)(1 - 0) = (-1 - 2)(x - 0)$$

$$y - 2 = -3x$$

$$\left[y = -3x + 2 \right]$$

$$\int_{-2}^1 \left(-\frac{1}{2}x + \frac{2}{3}x + \frac{1}{3} \right) dx - \int_{-2}^0 \left(-\frac{1}{2}x + \frac{2}{3}x + \frac{1}{3} \right) dx + \int_0^1 \left(-3x + 2 + \frac{2}{3}x + \frac{1}{3} \right) dx =$$

$$= -\frac{1}{2} \left(\frac{x^2}{2} \right) \Big|_{-2}^1 + \frac{2}{3} \left(\frac{x^2}{2} \right) \Big|_{-2}^1 + \frac{1}{3} (x) \Big|_{-2}^1 - \left[-\frac{1}{2} \left(\frac{x^2}{2} \right) \Big|_{-2}^0 + \frac{2}{3} \left(\frac{x^2}{2} \right) \Big|_{-2}^0 + \frac{1}{3} (x) \Big|_{-2}^0 \right] + 3 \left(\frac{x^2}{2} \right) \Big|_0^1 + 2 \cdot (x) \Big|_0^1 + \frac{2}{3} \left(\frac{x^2}{2} \right) \Big|_0^1 + \frac{1}{3} \cdot 1$$

$$= -\frac{1}{2} \cdot \left(\frac{5}{2} \right) + \frac{2}{3} \cdot \left(\frac{5}{2} \right) + \frac{1}{3} \cdot (3) + \frac{1}{2} \cdot (1) + \frac{2}{3} \cdot (1) + \frac{1}{3} \cdot 2 + 3 \cdot \frac{1}{2} + 2 \cdot 1 + \frac{2}{3} \cdot \left(\frac{1}{2} \right) + \frac{1}{3} \cdot 1$$

$$= -\frac{5}{4} + \frac{5}{3} + 1 + \frac{1}{2} + \frac{2}{3} + \frac{2}{3} + \frac{3}{2} + 2 + \frac{1}{3} + \frac{1}{3}$$

$$= \frac{89}{12}$$

VIDI NOVOSZ

