

01.07.

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: Radošević Rikardo BROJ INDEKSA: _____

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

1. Riješiti diferencijalnu jednačinu: $y'' - 2y' - 3y = e^{-x} + 1$. 15
2. Odrediti lokalne ekstreme funkcije $f(x, y) = e^y - y + x^2$. 15
3. Izračunati tangencijalnu ravninu plohe $z = \sin(x^2y)$ u točki $(2, 1, \sin(4))$. 15
4. Numeričkom integracijom procijeniti vrijednost $\int_1^2 (x+2) \ln x dx$ i dati procjenu greške ili neku drugu kvalitetnu informaciju o greški. 10+5
5. $\int_0^\pi \frac{dx}{\sin x + 2} = ?$ 20
6. $\int_0^2 \frac{x-1}{x^2+x-2} dx = ?$ 20

Ukupno:
35

f	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$a^x (a > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2} + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2-x^2} dx = \frac{1}{2} [x\sqrt{a^2-x^2} + a^2 \arcsin \left(\frac{x}{a} \right)] + C$	

① $y'' - 2y' - 3y = e^{-x} + 1$
 $y'' - 2y' - 3y = e^{-x}(e^x + 1)$
 $y'' - 2y' - 3y = 0$
 $\lambda^2 - 2\lambda - 3 = 0$
 $\lambda_{1,2} = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} \rightarrow -1$
 $y_H = C_1 e^{3x} + C_2 e^{-x}, C_1, C_2 \in \mathbb{R}$
 ~~$y_{part} = e^{-x}(e^x + 1)$~~

$f_1(x) = e^{-x}$
 $f_2(x) = 1$
 $y'' - 2y' - 3y = 1$
 $y_{A3} = B$
 $y'_{A1} = 0, y''_{A2} = 0$
 $0 - 2 \cdot 0 - 3B = 1$
 $B = -\frac{1}{3}$

$y'' - 2y' - 3y = e^{-x}$
 $y_A = A e^{-x}$
 $y_A = A e^{-x} - A x e^{-x}$
 $= e^{-x} (A - A x)$
 $y''_A = -e^{-x} (A - A x)$
 $= -A e^{-x} =$
 $= -e^{-x} (2A - A x) - 2(A - A x) e^{-x}$
 $= -3A x e^{-x} - e^{-x} (2A - 2A - 2A + 2A x - 3A e) = 1$
 $-4A = 1 \quad A = -\frac{1}{4}$

KONAČNO RJEŠENJE

$$\textcircled{2} f(x,y) = e^y - y + x^2$$

$$f_x(x,y) = 2x$$

$$f_y(x,y) = e^y - 1$$

$$2x = 0 \implies x = 0$$

$$e^y - 1 = 0 \implies e^y = 1$$

$$e^y = e^0 \implies y = 0$$

* Stacionarna točka je $(0,0)$

Promjena:

$$f_{xx}(x,y) = 2$$

$$f_{yy}(x,y) = e^y$$

$$f_{xy}(x,y) = 0$$

$$f_{yx}(x,y) = 0$$

Determinanta:

$$\Delta_2(x,y) = \begin{vmatrix} 2 & 0 \\ 0 & e^y \end{vmatrix}$$

$$\Delta_2(0,0) = \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} = 2$$

$$\Delta_1 = 2$$

$\Delta_2 > 0, \Delta_1 > 0 \implies$ lokalni minimum ✓
 Δ_2 pozitivna

$$f(0,0) = e^0 - 0 + 0 = 1$$

