01.07.

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox

IME I PREZIME: Radovčić Rikardo **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = \mathrm{e}^{-x} + 1$. 15
2. Odrediti lokalne ekstreme funkcije $f(x, y) = \mathrm{e}^{y} - y + x^2$. 15
3. Izračunati tangencijalnu ravninu plohe $z = \sin(x^2 y)$ u točki $(2, 1, \sin(4))$. 15
4. Numeričkom integracijom procijeniti vrijednost $\int_1^2 (x+2)\ln x \,\mathrm{d}x$ i dati procjenu greške ili neku drugu kvalitetnu informaciju o greški. 10+5
5. $\int_0^{\pi} \frac{\mathrm{d}x}{\sin x + 2} = ?$ 20
6. $\int_0^{2} \frac{x-1}{x^2+x-2}\mathrm{d}x = ?$ 20

Ukupno: 35

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x \mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

(1) $y'' - 2y' - 3y = e^{-x} + 1$

$y'' - 2y' - 3y = e^{-x}(e^x + 1)$

$y'' - 2y' - 3y = 0$

$\lambda^2 - 2\lambda - 3 = 0$

$\lambda_{1,2} = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} \to 3, -1$

$y_H = C_1 e^{3x} + C_2 e^{-x},\ C_1, C_2 \in \mathbb{R}$

~~$P(x) = e^{-x}(e^x + 1)$~~

$f_1(x) = e^{-x}$

$f_2(x) = 1$

$y'' - 2y' - 3y = 1$

$y_{A_3} = B$

$y'_{A_1} = 0 \quad y''_{A_2} = 0$

$0 - 2 \cdot 0 - 3B = 1$

$B = -\frac{1}{3}$

$y'' - 2y' - 3y = e^{-x}$

$y_A = A_{AC}^{-1}$

$y_A = A e^{-x} - A x e^{-x}$

$= e^{-x}(A - Ax)$

$y'_A = -e^{-x}(A - Ax) - Ae^{-x} =$

$= -e^{-x}(2A - Ax)$

$-e^{-x}(2A - Ax) - 2(A - Ax)e^{-x} - 3Axe^{-x} - e^{-1}Ae - 2A - 2A + 2Ax - 3Ae = 1$

$-4A = 1 \quad A = -\frac{1}{4}$

KONAČNO RJEŠENJE?

② $f(x,y) = e^y - y + x^2$

$f_x(x,y) = 2x$

$f_2(x,y) = e^y - 1$

$2x = 0 \Longrightarrow x = 0$

$e^y - 1 = 0 \Rightarrow e^y = 1$

$e^y = e^0 \Rightarrow y = 0$

*Stacionarna točka je $(0,0)$

Promjena:

$f_{xx}(x,y) = 2$ $\qquad$ $f_{yy}(x,y) = e^y$

$f_{xy}(x,y) = 0$ $\qquad$ $f_{yx}(x,y) = 0$

Determinanta:

$$\Delta_2(x,y) = \begin{vmatrix} 2 & 0 \\ 0 & e^y \end{vmatrix}$$

$$\Delta_2(0,0) = \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} = 2 \qquad \Delta_1 = 2$$

$\underbrace{\Delta_2 > 0, \Delta_1 > 0}_{\Delta_2 \text{ pozitivna}}$ $\Rightarrow$ lokalni minimum ✓

$f(0,0) = e^0 - 0 + 0 = 1$

③ Izračunati tangencijalnu ravninu plohe

$z = \sin(x^2 y)$ u točki $(2, 1, \sin(4))$.

$$z = f(x,y) = \sin(x^2 y)$$

$$P(x_0, y_0) = P(2,1)$$

$$\frac{\partial f(x+y_0)}{\partial x}(x - x_0) - \frac{\partial f(x_0, y_0)}{\partial y}(y - y_0) - (y + y_0) = 0$$

$$\frac{\partial f}{\partial x} = \sin(x^2 - y_0)\Big|_{(2,1)} = \sin(4) \qquad \frac{\partial f}{\partial x} = \cos(x^2 y) \cdot y \cdot 2x$$

$$\frac{\partial f(2,1)}{\partial x} = 2x \cos(x^2 y)\Big|_{(2,1)} = 4\cos(4)$$

$$\frac{\partial f}{\partial y} = \cos(x^2 y) \cdot x^2$$

$$\frac{\partial f(2,1)}{2y} = \cos(x^2 y)\Big|_{(2,1)} = \cos(4)$$

$\pi \ldots 4\cos(4)(x-2) + \cos(4)(y-1) - \{z - \sin(4)\} = 0$

$\pi \ldots \cos(4)(4x - 8 + y - 1) - z + \sin(4) = 0$

$\pi \ldots \cos(4)(4x + y - 9) - z + \sin(4) = 0$

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(4) Numeričkom integracijom procijeniti vrijednost

$$\int_1^2 (x+2)\ln x\,dx$$

i dati procjenu greške.

(6) $\int_0^2 \frac{x-1}{x^2+x-2}\,dx =$

$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1+3}{2}$ $\to -2$, $\to 1$

$= \int_0^2 \frac{x-1}{(x-1)(x+2)}\,dx = \int_0^2 \frac{dx}{x+2} = \ln(x+2)\Big|_0^2$

NIJE NEPRAVI ✓

$= \ln 4 - \ln 2$

$= \ln \frac{4}{2}$

$= \ln 2$ ✓

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01.07.2014.

⑤ $\int_0^{\pi} \frac{dx}{\sin x + 2} =$

$\int f(g(x+1))\, g'(x)\,dx = \int f(u)\,du$

$u = \tan\left(\frac{x}{2}\right)$

$dx = \frac{2}{1+u^2}\,du \qquad \sin x = \frac{2u}{1+u^2}$

$= \int_0^{\pi} \frac{1}{\frac{2u}{1+u^2} + 2} \cdot \frac{2}{1+u^2}\,du = \int_0^{\pi} \frac{1}{u^2+u+1}\,du$

$u^2+u+1 = \left(u+\frac{1}{2}\right)^2 + \frac{3}{4} \qquad \left(x^2+ax = \left(x+\frac{x}{2}\right) - \left(\frac{u}{2}\right)^2\right.$

$= \int_0^{\pi} \frac{1}{\left(u+\frac{1}{2}\right)^2 + \frac{3}{4}}\,du \qquad v = u+\frac{1}{2},\ dv = 1\,du \Rightarrow du = dv$

$= \int_0^{\pi} \frac{1}{v^2+\frac{3}{4}}\,dv \qquad v^2 + \frac{3}{4} = \frac{3}{4}\left(\frac{v^2}{\frac{3}{4}} + 1\right)$

$= \int_0^{\pi} \frac{1}{\frac{3}{4}\left(\frac{v^2}{\frac{3}{4}}+1\right)}\,dv \qquad \frac{v^2}{\frac{3}{4}} = \frac{4v^2}{3} = \left(\frac{2v}{\sqrt{3}}\right)^2$

$w = \frac{2v}{\sqrt{3}},\ dw = \frac{2}{\sqrt{3}}\,dv,$

$= \int_0^{\pi} \frac{1}{\frac{3}{4}\left(\left(\frac{2v}{\sqrt{3}}\right)^2+1\right)}\,dv \qquad dv = \frac{\sqrt{3}}{2}\,dw$

NASTAVAK

Nastavak ⑤ zadatka

$$= \int_0^{\pi} \frac{1}{\frac{3}{4}(w^2+1)} \cdot \frac{\sqrt{3}}{2}\, dw$$

$$= \int_0^{\pi} \frac{4^{2}}{\cancel{3}_{\sqrt{3}}(w^2+1} \cdot \frac{\sqrt{3}^{1}}{\cancel{2}}\, dw$$

$$= \int_0^{\pi} \frac{2}{\sqrt{3}\,(w^2+1)}\, dw$$

$$\frac{2}{\sqrt{3}} \int_0^{\pi} \frac{1}{w^2+1}\, dw$$

OVE GRANICE U TERMINIMA VARIJABLE x, NE VARIJABLE w

$$= \frac{2}{\sqrt{3}} \cdot \left(\frac{1}{1} \arctan \frac{w}{1}\right)\Big|_0^{\pi} = \frac{2\sqrt{3}}{3}\left(\arctan w\right)\Big|_0^{\infty}$$

$$\tan \frac{\pi}{2} = -$$

$$\tan \frac{\pi}{2} = \infty$$

divergira

Radovčić Rikardo

01.07.

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox

IME I PREZIME: MATE ĆOSIĆ **BROJ INDEKSA:** 55924

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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5. $\int_0^{\pi} \frac{\mathrm{d}x}{\sin x + 2} = ?$ 20
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Ukupno: 15

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$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
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$\int x^\alpha \,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
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6) $\int_0^2 \frac{x-1}{x^2+x-2}\,dx =$

$x^2+x-2=0$

$x_{1,2} = \frac{-1 \pm \sqrt{1^2-4\cdot 1\cdot(-2)}}{2} = \frac{-1 \pm \sqrt{9}}{2} = \frac{-1 \pm 3}{2}$

$x_1 = 1$

$x_2 = -2$

$\frac{x-1}{x^2+x-2} = \frac{A}{x-1} \cdot \frac{B}{x+2}$

$x-1 = A(x+2) + B(x-1)$

$x-1 = Ax + 2A + Bx - B$

$1 = A + B \Rightarrow B = -A$

$-1 = 2A - B$

$-1 = -2 - B$

$1 = -B$

$B = -1 \quad A = 0$

$\int_0^2 \frac{dx}{x-1} - 1\int_0^2 \frac{dx}{x+2}$

$\frac{1}{2}\int_0^2 \frac{dx}{x} = \frac{1}{2}\ln(x)\Big|_0^2$

$\frac{1}{2}\ln 2 - \frac{1}{2}\ln(0) = 0{,}34$

NIJE NEPRAVI INTEGRAL ✓

2) $f(x,y) = e^y - y + x^2$

$f_x = 2x$ $\quad 2x = 0$ $\quad \boxed{x = 0}$

$f_y = e^y - 1$ $\quad e^y = 1$ $\quad \boxed{y = 0}$

A $f_{xx} = 2$ ✓

B $f_{xy} = 0$

C $f_{yy} = e^y$ ✓

$\Delta = A \cdot C - B^2$

$\Delta = 2$ EXS. POSITIVO ✓

$A > 0$ MINIMUM ✓

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox

IME I PREZIME: ANĐELA UHODA **BROJ INDEKSA:** 17-2-0106-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = \mathrm{e}^{-x} + 1$. 15
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5. $\int_0^{\pi} \frac{\mathrm{d}x}{\sin x + 2} = ?$ 20
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Ukupno: 0

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$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
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$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
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1.

2. Odredi lokalne ekstreme $f(x,y) = e^y - y + x^2$

$f(x,y) = e^y - y + x^2 =$

3. Izračunaj tangencijalnu ravninu plohe $z = \sin(x^2 y)$ u točki $(2, 1, \sin(4))$

Rudela Uhoda

BROJ INDEKSA: 17-2-0106-2011

5. $\int_0^{\pi} \frac{dx}{\sin x + 2} = ?$

$\int = \frac{1}{\cos x}$

6. $\int_0^{2} \frac{x-1}{x^2+x-2} dx = ?$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox

IME I PREZIME: MARGARITA UHODA **BROJ INDEKSA:** 17-2-0175-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 0

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(3.) T (2, 1 sin(4))

2. $f(x,y) = e^y - y + x^2$

4. $\int_1^2 (x+2)\ln x \, dx$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

xox

IME I PREZIME: MARIN GALOŠIĆ **BROJ INDEKSA:** 17-2-0001

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = \mathrm{e}^{-x} + 1$. 15
2. Odrediti lokalne ekstreme funkcije $f(x, y) = \mathrm{e}^y - y + x^2$. 15
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4. Numeričkom integracijom procijeniti vrijednost $\int_1^2 (x+2)\ln x \mathrm{d}x$ i dati procjenu greške ili neku drugu kvalitetnu informaciju o greški. 10+5
5. $\int_0^{\pi} \frac{\mathrm{d}x}{\sin x + 2} = ?$ 20
6. $\int_0^2 \frac{x-1}{x^2+x-2}\mathrm{d}x = ?$ 20

Ukupno: 0

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x \mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox

IME I PREZIME: Luka Radaš **BROJ INDEKSA:** 57662

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = \mathrm{e}^{-x} + 1$. 15
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5. $\int_0^{\pi} \frac{\mathrm{d}x}{\sin x + 2} = ?$ 20
6. $\int_0^{2} \frac{x-1}{x^2+x-2}\mathrm{d}x = ?$ 20

Ukupno: 0

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
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Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x \,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

2) $f(x,y) = e^y - y + x^2$

$f_x = 2x$

$f_y = e^y - 1$

$f_{xx} = 2$

$f_{yy} = e^y$

$f(x,y) =$

6) $\int_0^2 \frac{x-1}{x^2+x-2}\,dx = \int_0^2 \frac{x+1-2}{x(x+1)-2}\,dx = \int_0^2 \frac{-2\,dx}{x-2}$

$= -2\int_0^2 \frac{dx}{x} - \int_0^2 dx = \left(-2\ln|x| - x\right)\Big|_0^2$ NEPRAVI INTEGRAL

$-2\ln|2| - 2 - \left(-2\ln|0| - 0\right) =$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox

IME I PREZIME: VESNA HLOŠ **BROJ INDEKSA:** 0269063872

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = \mathrm{e}^{-x} + 1$. 15
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Ukupno: 0

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$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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Tablica nekih integralа

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
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(5.) $\int_0^{\pi} \frac{\mathrm{d}x}{\sin x + 2} = \left\{ \begin{array}{l} t = \sin x + 2 \\ \mathrm{d}t = \cos x\,\mathrm{d}x \\ \mathrm{d}x = \frac{\mathrm{d}t}{\cos x} \end{array} \right\} = \int \frac{\frac{\mathrm{d}t}{\cos x}}{t} = \int \frac{\mathrm{d}t}{\cos x \cdot t} = \ln\lvert\cos x \sin x + 2\rvert + c$

✗ NE MOŽE BITI t i OSTALI x U INTEGRALU

$\left[\ln\lvert\cos x \sin x + 2\rvert\right]_0^{\pi} = \frac{1}{2}\left(\cos\pi \sin\pi + 2\right) - \frac{1}{2}\left(\cos 0 \sin 0 + 2\right)$

$= \frac{1}{2}\left(\cos\pi \sin\pi + 2\right) - \frac{1}{2}\left(\cos 0 + 2\right)$

(6.) $\int_0^2 \frac{x-1}{x^2+x-2}\mathrm{d}x = \left\{ \begin{array}{ll} u = x^2+x-2 & \mathrm{d}u = 2x+1\,\mathrm{d}x \\ \mathrm{d}v = x-1\,\mathrm{d}x & v = \int x-1\,\mathrm{d}x = x \end{array} \right\} = (x^2+x-2)\cdot x - \int (2x+1)\cdot x\,\mathrm{d}x$ ✗

$= x^2 x + x^2 - 2x - \left(\frac{2x^3}{3} + \frac{x^2}{2}\right) + c$

$\int 2x^2 + x\,\mathrm{d}x = 2\int x^2\mathrm{d}x + \int x\mathrm{d}x$

$= 2\frac{x^3}{3} + \frac{x^2}{2}$

$= \frac{2x^3}{3} + \frac{x^2}{2}$

$\left[x^2 x + x^2 - 2x - \frac{2x^3}{3} + \frac{x^2}{2}\right]_0^2 = \left(2^2\cdot 2 + 2^2 - 2\cdot 2 - \frac{2\cdot 2^3}{3} - \frac{2^2}{2}\right) - 0$

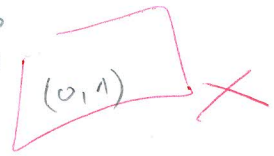
NE KRAJNI INTEGRAL ✓

$= 8 + 4 - 4 - \frac{16}{3} + \frac{4}{2} = 8 - \frac{16}{3} + 2$

$= \frac{24 - 16 + 6}{3} = \frac{14}{3}$

(2.) $f(x,y) = e^y - y + x^2$

∂xy	$2x$	$x=0$
∂yx	$e^y - 1$	$e^y = 1$



$$\begin{vmatrix} \partial xxy & \partial xyx \\ \partial xyx & \partial yyx \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & e^y \end{vmatrix} = 2e^y - 0 = 2e^y$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox

IME I PREZIME: DENI MILETIĆ **BROJ INDEKSA:** 57193

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = \mathrm{e}^{-x} + 1$. 15
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Ukupno:

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
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Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
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$\int \mathrm{e}^x \mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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1. $y'' - 2y' - 3y = e^{-x} + 1$

$r^2 - 2r - 3 = 0$

$r_{1/2} = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2}$

$r_1 = -1 \quad r_2 = 3$

$y_0 = C_1 \cdot e^{-x} + C_2 \cdot e^{3x}$

$g(x) = (P_m(x)\cos(\beta x) + Q_n(x)\sin(\beta x))$

1. $e^{-x} = e^{\alpha x}(P_m(x)\cos(\beta x) + Q_n(x)\sin(\beta x))$
2. $1 = e^{\alpha x}(P_m(x)\cos(\beta x) + Q_n(x)\sin(\beta x))$

1. $\alpha = -1 \quad \beta = 0 \quad n = 0 \quad N = \max\{m, n\} = 1$
   $k = \alpha + i\beta = -1$
2. $\alpha = 0 \quad \beta = 0 \quad n = 0 \quad N = \max\{m, n\} = 1$
   $k = \alpha + i\beta = 0$

$S_N = Ax + B \quad T_N = Cx + D$

1. $y_h = x^1 \cdot e^{-1 \cdot 0} \cdot (S_N(x)\cos(\beta x) + T_N(x)\sin(\beta x))$
   $= (Ax + B)\cos(0 \cdot x) + (Cx + D)\sin(0 \cdot x)$

$f_h = -Ax + B$

$f_h' = -A$

$f_h'' = 0$

REZULTAT?

2. $f(x,y) = e^y - y + x^2$

$\partial_x f = 2x \Rightarrow x = 0$ $\quad T(0,0)$

$\partial_y f = e^y - 1 \Rightarrow e^y = 1 \Rightarrow y = 0$

$\partial_{xx} f = 2 > 0$

$\partial_{xy} f = 0$

$\partial_{yy} f = e^y$

$\Delta = A \cdot C - B^2 = 2 \cdot e^0 - 0^2 = 2 > 0$ ZAKLJUČAK ?

5. $\int_0^{\pi} \frac{dx}{\sin x + 2} = \left\{ t = \tan\frac{x}{2};\ x = \arctan t \right\} = \int_0^{\pi} \frac{1}{\frac{2t}{1+t^2} + 2} \cdot \frac{2dt}{1+t^2} = \int_0^{\pi} \frac{1}{\frac{2t^2+2t+2}{1+t^2}} \cdot \frac{2dt}{1+t^2}$

$dx = \frac{2dt}{1+t^2}$

$= \int_0^{\pi} \frac{1+t^2}{2t^2+2t+2} \cdot \frac{2\,dt}{1+t^2} = \int_0^{\pi} \frac{2dt}{2t^2+2t+2} = \int_0^{\pi} \frac{2\,dt}{2(t^2+t+1)} = \int_0^{\pi} \frac{dt}{(t+1)^2} = \left\{ u = t+1;\ du = dt \right\}$

$= \int_0^{\pi} \frac{du}{u^2} = -\frac{1}{u}\Big|_0^{\pi} = -\frac{1}{t+1}\Big|_0^{\pi} = -\frac{1}{\tan\frac{x}{2}+1}\Big|_0^{\pi} = -\left(\frac{1}{\tan\frac{\pi}{2}+1}\right) + \left(\frac{1}{\tan\frac{0}{2}+1}\right)$

6. $\int_0^2 \frac{x-1}{x^2+x-2}\,dx =$ $\qquad x^2+x-2;\ x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2}$

$x_1 = -2 \quad x_2 = 1$ $\qquad \lim_{x \to 1} \frac{x-1}{x^2+x-2} = \frac{1-1}{1+1-2} = 0$

$= \int_0^a \lim_{a \to 1^-} \frac{x-1}{x^2+x-2}\,dx + \int_b^2 \lim_{b \to 1^+} \frac{x-1}{x^2+x-2}\,dx$

$= \int_0^a \lim_{a \to 1^-} \frac{x-1}{(x+1)(x-2)}\,dx = \frac{A}{x+1} + \frac{B}{x-2} \Big| \cdot (x+1)(x-2)$

$x - 1 = A(x-2) + B(x+1)$

$x - 1 = x(A+B) - 2A + B$

za x: $A + B = 1 \Rightarrow A - 2A + 1 = 1$

$A = 0$

za 1: $-2A + B = 1$

$B = -2A + 1$

$B = 1$

$\Rightarrow \int \frac{0}{x+1}\,dx + \int \frac{dx}{x-2} = \ln|x-2|$

$\Rightarrow \lim_{a \to 1_-} \left[\ln|x-2| \right]_0^a + \lim_{b \to 1_+} \left[\ln|x-2| \right]_b^2 = ?$

KOJI JE REZULTAT?

$\Rightarrow$ nastavak 1.

? $\cdot f_h = x^0 \cdot e^0 \cdot (S_N(x)\cos(\beta x) + T_N(x)\sin(\beta x))$

$= (Ax+B)\cos(0 \cdot x) + (Cx+D)\cdot \sin(0 \cdot x)$

$f_h = Ax+B$

$f_h' = A$

$f_h'' = 0$

REZULTAT?

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox

IME I PREZIME: FLAVIO ABATINOVIĆ **BROJ INDEKSA:** 17-2-0162-202

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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5. $\int_0^{\pi} \frac{\mathrm{d}x}{\sin x + 2} = ?$ 20
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Ukupno: 0

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
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$\int \cos x \mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

5. $\int_0^{\pi} \frac{\mathrm{d}x}{\sin x + 2} = \left\{ t = \sin x + 2 \right.$

6. $\int_0^2 \frac{x-1}{x^2+x-2}\mathrm{d}x =$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xox

IME I PREZIME: MARIN MATEK **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 20

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2. $f(x,y) = e^y - y + x^2$

$\Delta = \begin{vmatrix} 2 & 0 \\ 0 & e^y \end{vmatrix} = 2e^y$

$\frac{\partial f}{\partial x} = 2x$ ✓ $\quad \frac{\partial f}{\partial y} = e^y - 1$ ✓

EKSTREM JE MINIMUM

KOJI EKSTREM?

$\frac{\partial^2 f}{\partial x^2} = 2$ ✓ $\quad \frac{\partial^2 f}{\partial y^2} = e^y$ ✓

$\frac{\partial f}{\partial x \partial y} = 0$ ✓

$e^y - y + x^2 = 0$?

$x^2 = y - e^y$

4. $\int_1^2 (x+2)\ln x\,dx$

k	0	1	2
x_k	1	1,5	2
l_k	0	1,491278	2,772588

$\frac{1}{6}(l_0 + 4l_1 + l_2)$

$\frac{1}{6}(0 + 4\cdot 1{,}491 + 2{,}7725)$

$= 1{,}40818$ ✓

PROCJENA GREŠKE?

6. $\int_0^2 \frac{x-1}{x^2+x-2}\,dx = \lim_{\varepsilon \to 0} \frac{x-1}{x^2+x-2} = \frac{1}{2}$?

5. $\int_0^{\pi} \frac{dx}{\sin x + 2} =$

TOČAN REZULTAT INTEGRACIJOM ?

k	0	1	2
x_k	0	$\frac{\pi}{2}$	π
l_k	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$

$\frac{\pi}{6}(l_0 + 4\cdot l_1 + l_2)$

$\frac{\pi}{6}\left(\frac{1}{2} + 4\cdot\frac{1}{3} + \frac{1}{2}\right)$

$= \frac{\pi\pi}{18} = 1{,}221730$

ZA NUMERIČKI REZULTAT BEZ OCJENE GREŠKE POLA BODOVA

10

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IME I PREZIME: MARIN BEKAM **BROJ INDEKSA:** 0269079117

POPUNJAVA NASTAVNIK Broj ↓ bodova

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Ukupno: 0

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$$\int_0^2 \frac{0}{x-1} + \frac{1}{x+2}\mathrm{d}x =$$

$$\int_0^2 \frac{0}{x-1}\mathrm{d}x + \int_0^2 \frac{1}{x+2}\mathrm{d}x =$$

$$= \int \frac{0}{2-1}\mathrm{d}x + \int \frac{1}{2+2}\mathrm{d}x - \int \frac{0}{0-1}\mathrm{d}x - \int \frac{1}{0+2}\mathrm{d}x$$

$$= \int \frac{0}{1}\mathrm{d}x + \int \frac{1}{4}\mathrm{d}x - \int \frac{0}{-1}\mathrm{d}x - \int \frac{1}{2}\mathrm{d}x$$

$$= \int \frac{1}{4}\mathrm{d}x - \int \frac{1}{2}\mathrm{d}x$$

$$= \frac{1}{4} - \frac{1}{2} = \frac{1-2}{4} = -\frac{1}{4}$$ ✓ ✗

6. $\int_0^2 \frac{x-1}{x^2+x-2}\mathrm{d}x =$ $(x-1)(x+2)$ $x^2+2x-x-2$

$$\int_0^2 \frac{x-1}{(x-1)(x+2)}\mathrm{d}x =$$

NIJE NEPRAVI INTEGRAL ✓

0

$x-1=0 \quad x+2=0$

$x=1 \quad x=-2$

$$\frac{x-1}{x^2+x-2} = \frac{A}{x-1} + \frac{B}{x+2} \quad / \cdot (x^2+x-2)$$

$$x-1 = A(x+2) + B(x-1)$$

$$x-1 = Ax + 2A + Bx - B$$

$$= A(A+B) + 2A - B$$

$$1 = A+B \Rightarrow A = B-1$$

$$-1 = 2A - B$$

$$-1 = 2(B-1) - B$$

$$-1 = 2B - 2 - B$$

$$1 = B$$

$$1 = A+1$$

$$A = 0$$

5. $\int_0^{\pi} \frac{dx}{\sin x + 2} = \begin{cases} \sin x + 2 = t \\ dt = \cos x \, dx \end{cases}$

$= \int (\sin x + 2)^{-1} dt$

$= 0$