MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** 000

Ime i prezime: ANTONIO BABARA **Broj indeksa:** 17-2-0282-2013
0269078055

Popunjava nastavnik
Broj ↓ bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 2y' = 1$, uz $y(0) = 1$ i $y'(0) = 0$. Na kraju provjeri rješenje. ~~15~~
2. Pronađi funkciju koja zadovoljava $xy' = x + y$ i $y(1) = 1$. ~~15~~
3. Zadana je funkcija a $f(x, y) = \sqrt{(x-1)^2 + (y+1)^2} - 1$. Ispitati domenu, kodomenu i razinske krivulje. 15
4. $\int_0^{\pi} \cos(x)\sin(x)\,dx = ?$ 15
5. $\int_0^{2} \frac{2x^2+1}{x^2+1}dx = ?$ ~~20~~
6. Izračunati površinu lika omeđenog pravcem $y = x - 1$ i parabolom $y = x^2 - x - 4$. 20

Ukupno: 35

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

6. $y = x - 1$ i $y = x^2 - x - 4$

x	0	1
y	-1	0

x	0	1	-1	2	-2
y	-4	-4	-2	-2	2

$x^2 - x - 4 = x - 1$

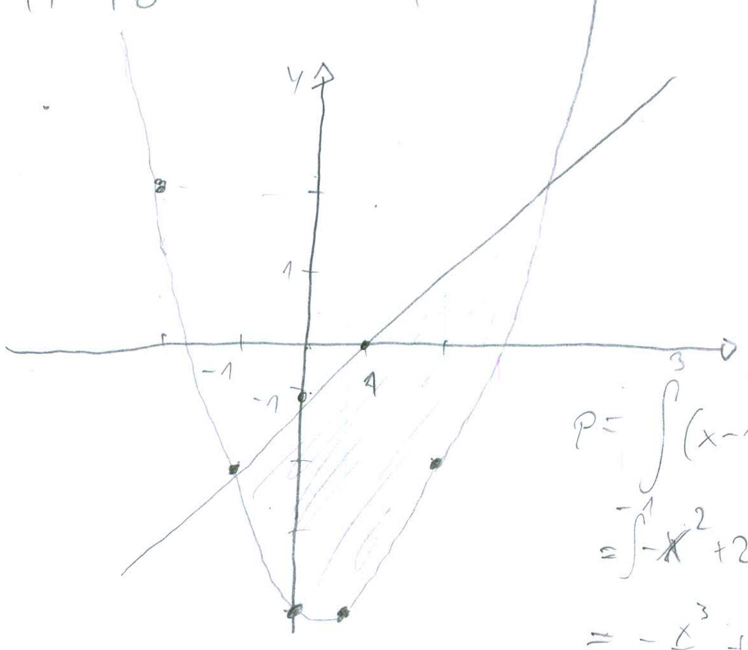
$x^2 - 2x + 3 = 1$

$x_{1,2} = \frac{2 \pm \sqrt{4+12}}{2}$

$x_{1,2} = \frac{2 \pm 4}{2}$

$x_1 = 3$

$x_2 = -1$



$P = \int_{-1}^{3} (x-1) - (x^2 - x - 4)\,dx = \int x - 1 - x^2 + x + 4\,dx$

$= \int -x^2 + 2x + 3\,dx = -\int x^2\,dx + 2\int x\,dx + 3\int dx$

$= -\frac{x^3}{3} + 2\frac{x^2}{2} + 3x \Big|_{-1}^{3} = -\frac{3^3}{3} + 2\cdot\frac{3^2}{2} + 3\cdot 3 - \left(-\frac{(-1)^3}{3} + 2\cdot\frac{(-1)^2}{2} - 3\right)$

$= \cancel{-9} + \cancel{9} + 9 - \left(\frac{1}{3} + 1 - 3\right)$

$= 9 - \frac{1}{3} - 1 + 3 = 11 - \frac{1}{3} = \frac{32}{3} \approx 10.6666$ ✓

4. $\int_0^{\pi} \cos(x) \sin(x) dx = \left| \begin{array}{c} \sin x = t \\ \cos x \, dx = dt \\ dx = \frac{dt}{\cos x} \end{array} \right| = \int \cos(x) \cdot t \cdot \frac{dt}{\cancel{\cos x}} = \int t \, dt$

$= \frac{t^2}{2} + c = \frac{(\sin x)^2}{2} + c$

$\left. \frac{(\sin x)^2}{2} \right|_0^{\pi} = \frac{(\sin \pi)^2}{2} - \left(\frac{(\sin 0)^2}{2}\right) = \frac{(\sin \pi)^2}{2} \simeq 0,00150172$

$\downarrow$ 0

$\boxed{\sin \pi = 0}$

$4x$

5. $\int_0^2 \frac{2x^2+1}{x^2+1} dx = 2\int \frac{2x^2+1}{2x^2+2} dx = 2\int \frac{2x^2+1}{2x^2+2-1+1} dx = 2\int \frac{2x^2+1}{2x^2+1} dx + 2\int \frac{2x^2+1}{1} dx$

$\emptyset$

$2x$

$= 2\int dx + 4\int 2x^2 dx + 2\int dx = 2 \cdot x + 4 \cdot \frac{x^3}{3} + 2x = 4x + \left. \frac{4x^3}{3} \right|_0^2$

$= 8 + 4 \cdot \frac{8}{3} - 0 = 8 + \frac{32}{3} = \frac{24+32}{3} = \frac{56}{3} = 18\frac{2}{3}$

ANTONIO BARARA

1. $y'' - 2y' = 1$ $\quad y(0)=1 \quad y'(0)=0$

$k^2 - 2k = 0$

$k(k-2) = 0$

$\Downarrow \quad k-2=0$

$k_1 = 0 \quad \underline{k_2 = 2}$

$\alpha = 0, \quad r = 1$

$y = C_1 e^{0 \cdot x} + C_2 e^{2x}$

$y = C_2 e^{2x}$

$y' = C_2 \cdot e^{2x} \cdot 2x$

$y'' = C_2 \cdot e^{2x} \cdot 2x +$

0

2. $xy' = x + y \,/\, x$ $\quad y(1) = 1$

$y' = \frac{x+y}{x}$

0

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** 000
IME I PREZIME: RENATA RUBČIĆ **BROJ INDEKSA:** 17-2-0160-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 2y' = 1$, uz $y(0) = 1$ i $y'(0) = 0$. Na kraju provjeri rješenje. 15
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(1) $y'' - 2y' = 1$ $\quad y(0) = 1 \quad y'(0) = 0$

$y'' - 2y' = 0$

$k^2 - 2k = 0$

$k(k-2) = 0$

$k = 0$

$k = 2$

$y_H = C_1 + C_2 e^{2x}$

$y_p = a_0 x$

$y_p' = a_0$

$y_p'' = 0$

$-2a_0 = 1$

$a_0 = -\frac{1}{2}$

$y_p = -\frac{1}{2}x$

$\boxed{y = y_H + y_p = C_1 + C_2 e^{2x} - \frac{1}{2}x}$

$y_0 = C_1 + C_2 = 1$

$y' = 2C_2 e^{2x} - \frac{1}{2}$

$y'(0) = 2C_2 - \frac{1}{2} = 0$

(1)

$2c_2 = \frac{1}{2} \Rightarrow c_2 = 1$

$c_1 + 1 = 1$

$c_1 = 0$

PARTIKULARNO RJEŠENJE : $\boxed{y = e^{2x} - \frac{1}{2}x}$

PROVJERA

$y(0) = 1$

$y' = 2e^{2x} - \frac{1}{2}$

$y'(0) = 2 - \frac{1}{2} = \frac{3}{2} \neq 0$ ↯

$\emptyset$

④ $$\int_0^{\pi} \cos(x)\sin(x)\,dx = \left\{ \begin{array}{l} \sin x = t \\ \cos dx = dt \\ dx = \frac{dt}{\cos x} \end{array} \quad \begin{array}{c|c|c} x & 0 & \pi \\ \hline t & 0 & 1 \times \end{array} \right\} \times$$

$$= \int_0^1 \cancel{\cos x} \cdot t \cdot \frac{dt}{\cancel{\cos x}} = \int_0^1 t\,dt = \frac{t^2}{2}\Big|_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$$

⑤ $$\int_0^2 \frac{2x^2+1}{x^2+1}\,dx =$$

$(\cancel{2x^2} + 1) : (x^2+1) = 2$

$-2x \pm 2$

(-1)

$$= \int_0^2 2\,dx - \int_0^2 \frac{dx}{x^2+1} \quad ✓$$

$$= 2x\Big|_0^2 - \frac{1}{1}\,\mathrm{arctg}\left(\frac{x}{1}\right)\Big|_0^2$$

$= 2 \cdot 2 - \mathrm{arctg}\,2$

$= 4 - \mathrm{arctg}\,2$ ✓

(3.) $f(x,y) = \sqrt{(x-1)^2 + (y+1)^2} - 1$

$(x-1)^2 + (y+1)^2 \geq 0$

$D_f = [(x,y) \in \mathbb{R}^2 : (x-1)^2 + (y+1)^2 \geq 0]$? $D_f = \mathbb{R} \times \mathbb{R} = \mathbb{R}^2$

KODOMENA : skup $\mathbb{R}$

(2) $xy' = x + y \,/ : x \qquad y(1) = 1$

$y' = 1 + \frac{y}{x}$

$u = \frac{y}{x} \qquad y = ux \qquad y' = u'x + u$

$u'x + \cancel{u} = 1 + \cancel{u}$

$u'x = 1$

$\frac{du}{dx} = \frac{1}{x} \,\Big/ \, dx$

$du = \frac{dx}{x} \,\Big/ \int$

$\int du = \int \frac{dx}{x}$

$u = \ln|x| + c$

$\frac{y}{x} = \ln x + c$ → LAKŠE : $y = x(\ln x + c)$

$xc = e^{\frac{y}{x}}$ OPĆE RJEŠENJE

$y(1) = 1$

$c = e^{1}$

$xe = e^{\frac{y}{x}}$ PARTIKULARNO RJEŠENJE ✓

RENATA RUBČIĆ
17-2-0160-2012

(6.) $y = x - 1$

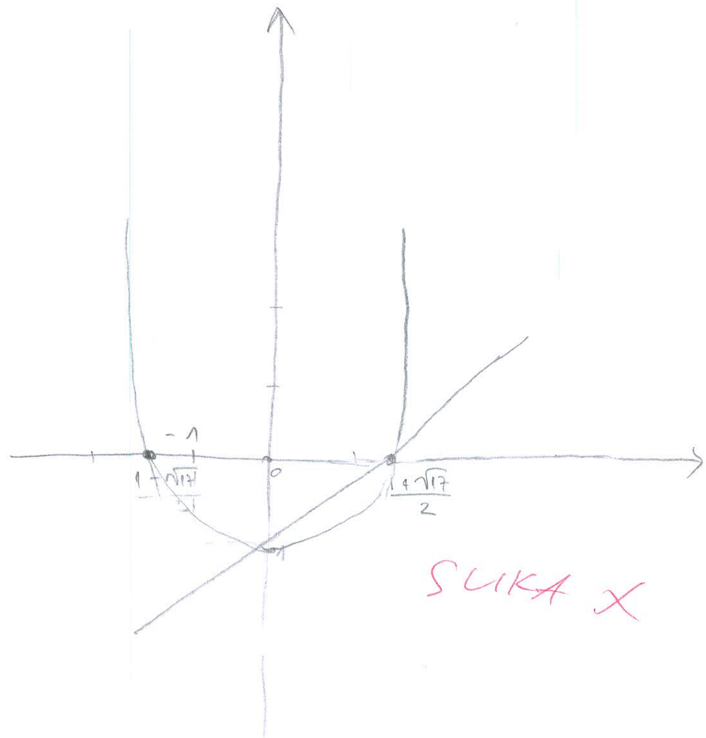
$y = x^2 - x - 4$

$x^2 - x - 4 = 0$

$$x = \frac{-1 \pm \sqrt{1+16}}{2} = \frac{1 \pm \sqrt{17}}{2}$$

$y = x - 1$

y	-1	0
x	0	1



SLIKA X

$x - 1 = x^2 - x - 4$

$x^2 - 2x - 3 = 0$

$$x = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2}$$

$x_1 = 3 \qquad x_2 = -1$

$$P = \int_{-1}^{3} (x - 1 - x^2 + \cancel{x} + 4)\,dx = \int_{-1}^{3} (3 - x^2)\,dx$$

$$= 3\int_{-1}^{3} dx - \int_{-1}^{3} x^2 dx = 3x \Big|_{-1}^{3} - \frac{x^3}{3}\Big|_{-1}^{3}$$

$$= 3 \cdot (3+1) - \left(\frac{3^3}{3} - \frac{(-1)^3}{3}\right)$$

$$= 12 - \frac{28}{3} = \frac{8}{3}$$ X

0

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** 000 **IME I PREZIME:** IVAN VUKAŠINA **BROJ INDEKSA:** 1720182-12	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 2y' = 1$, uz $y(0) = 1$ i $y'(0) = 0$. Na kraju provjeri rješenje. 15

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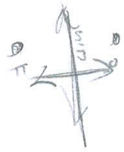
4)

$$\int_0^\pi \cos(x)\sin(x)\,dx \quad \left[\begin{array}{l}\sin x = t\\ \cos x\,dx = dt\end{array}\right] = \int_0^\pi t\,dt = \left[\frac{t^2}{2}\right]_0^\pi = \left[\frac{\sin^2(x)}{2}\right]_0^\pi = \left[\frac{\frac{1-\cos(2x)}{2}}{\frac{2}{1}}\right]_0^\pi$$



$$\left|\frac{1-\cos(2x)}{4}\right|_0^\pi = \frac{1-\overset{(\cos 2\pi)}{1}}{4} - \frac{1-\overset{(\cos(2\cdot 0))}{1}}{4} = 0 - 0 = 0$$

5)

$$\int_0^2 \frac{2x+1}{x^2+1}dx = \int_0^2 \frac{2x}{x^2+1} + \int_0^2 \frac{dx}{x^2+1} \Rightarrow \overset{*}{\int_0^2 \frac{2x}{x^2+1}} \left[\begin{array}{l}x^2+1 = t\\ 2x = dt\end{array}\right] = \int_0^2 \frac{dt}{t} = \ln|x^2+1|^2 - \ln|x^2+1|^0$$

$$= \ln|4+1| - \ln|0+1| = \ln|5| = 1.609$$

DER Nije napisao

$$\int_0^2 \frac{dx}{x^2+1} = \left|\arctan\frac{x}{1}\right|_0^2 = \arctan 2 - \arctan 0 \approx 1.107$$

$$\Rightarrow 0 + * \approx 1.609 + 1.107 = 2.716$$

IVAN VUKASINA

⑥ $y = x - 1$

$y = x^2 - x - 4$

$x^2 - x - 4 = x - 1$

$x^2 - x - 4 - x + 1 = 0$

$x^2 - 2x - 3 = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_1 = 3$

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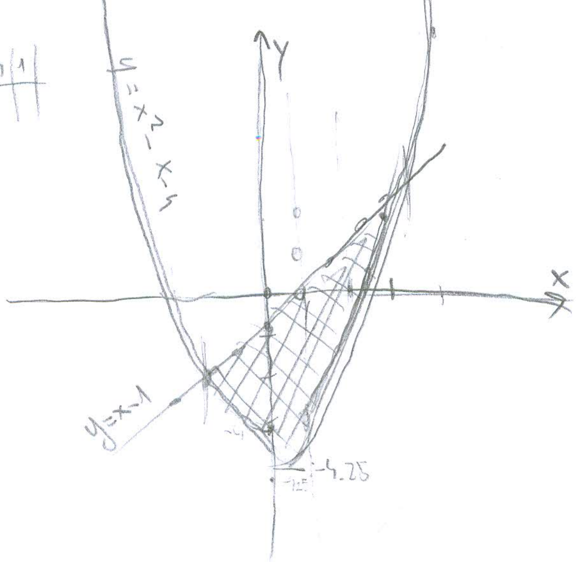

$$\int_{-1}^{3} x^2 - x - 4 - (x - 1)\, dx = \int_{-1}^{3} x^2 - 2x - 3\, dx = \int_{-1}^{3} x^2 dx + \int_{-1}^{3} -2x\, dx + \int_{-1}^{3} -3\, dx$$

$$= \int_{-1}^{3} x^2 - 2\int_{-1}^{3} x\, dx - 3\int_{-1}^{3} dx = \left| \frac{x^3}{3} - \cancel{2}\frac{x^2}{\cancel{2}} - 3x \right|_{-1}^{3}$$

$$= \left| \frac{x^3}{3} - x^2 - 3x \right|_{-1}^{3} = \left| \frac{(3)^3}{3} - (3)^2 - 3 \cdot 3 \right| - \left| \frac{(-1)^3}{3} - (-1)^2 - 3 \cdot (-1) \right|$$

$$\left| \frac{27}{3} - 9 - 9 \right| - \left| \frac{-1}{3} - 1 + 3 \right| = |-9| - \left| \frac{5}{3} \right| = -\frac{32}{3} = \frac{32}{3}$$

$$\int_{-1}^{3} x-1-(x^2-x-4)\,dx = \int_{-1}^{3} -x^2+x+4+x-1 = \int_{-1}^{3} -x^2+2x+3$$

$$-\int_{-1}^{3} x^2\,dx + 2\int_{-1}^{3} x\,dx + 3\int_{-1}^{3} dx = \left| -\frac{x^3}{3} + \cancel{2}\cdot\frac{x^2}{\cancel{2}} + 3x \right|_{-1}^{3}$$

$$= \left| -\frac{\overset{9}{27}}{3} + 9 + 9 \right| - \overset{\left(-\frac{5}{3}\right)}{\left| \frac{1}{3} + 1 + (-3) \right|} = 9 + \frac{5}{3} = \frac{32}{3}$$

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IME I PREZIME: ANĐELO ŽMIRE **BROJ INDEKSA:**

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4. $\int\limits_0^\pi \cos(x)\sin(x)\,\mathrm{d}x = ?$ ~~15~~
5. $\int\limits_0^2 \frac{2x^2+1}{x^2+1}\mathrm{d}x = ?$ ~~20~~
6. Izračunati površinu lika omeđenog pravcem $y = x - 1$ i parabolom $y = x^2 - x - 4$. 20

Ukupno: 37

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

2. $xy' = x + y$ $\qquad y(1) = 1$

$xy' - y = x \ /:x$

$y' - \frac{1}{x}\cdot y = 1$

$f(x) = -\frac{1}{x} \qquad g(x) = 1$

$\int f(x)\,dx = \int -\frac{1}{x}\,dx = -\int \frac{1}{x}\,dx = -\ln|x| + c$

$y = e^{-\int f(x)}\left[\int e^{\int f(x)}\cdot g(x)\,dx + c\right]$

$y = e^{\ln|x|}\cdot\left[\int e^{-\ln|x|}\cdot 1\,dx + c\right]$

$y = x\cdot\left[\int \frac{1}{x}\,dx + c\right]$

$y = x\cdot\ln|x| + c$ ✓

$1 = 1\cdot\ln|1| + C$

$\boxed{1 = C}$ ✓

6. $y = x - 1$

$y = x^2 - x - 4$

x	$f(x)$
0	-1
1	0
-1	-2
2	1
-2	-3

x	$g(x)$
0	-4
1	-4
-1	-2
2	-2
-2	2
3	2

$x - 1 = x^2 - x - 4$

$x - 1 - x^2 + x + 4 = 0 \quad / \cdot (-1)$

$x^2 - 2x - 3 = 0$

$x_{1,2} = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2}$

$x_1 = 3 \quad x_2 = -1$

$$P = \int_{-1}^{3} (x-1) - (x^2 - x - 4)\, dx$$

$$= \int_{-1}^{3} (x - 1 - x^2 + x + 4)\, dx$$

$$= \int_{-1}^{3} (-x^2 + 2x + 3)\, dx$$

$$= -\int_{-1}^{3} x^2 dx + 2\int_{-1}^{3} x\, dx + 3\int_{-1}^{3} dx$$

$$= \left[-\frac{x^3}{3} + x^2 + 3x\right]_{-1}^{3}$$

$$= \left(-\frac{27}{3} + 9 + 9\right) - \left(-\frac{-1}{3} + 1 - 3\right)$$

$$= 9 - \left(\frac{1}{3} + 1 - 3\right)$$

$$= 9 - \frac{1}{3} - 1 + 3$$

$$= 10\frac{2}{3}$$

③ $f(x,y) = \sqrt{(x-1)^2 + (y+1)^2} - 1$

$$\left((x-1)^2 + (y+1)^2\right) \geq 0$$

$D_f : \mathbb{R}^2$ ✓

2

1. $y'' - 2y' = 1$

$r^2 - 2r = 0$

$r(r-2) = 0$

$\boxed{r = 0}$

$r - 2 = 0$

$\boxed{r = 2}$

$y = C_1 e^0 + C_2 e^{2x}$

$1 = C_1 + C_2 e^2$

$1 = C_1 + C_2 \cdot 7{,}39$

∅

ANĐELO ŽMIRE

4. $\int_0^{\pi} \cos(x)\sin(x)\,dx = \begin{bmatrix} \cos x = t \,/' \\ -\sin(x)dx = dt \end{bmatrix}$

$= -\int t\,dt = -\frac{t^2}{2} + C$

$= \frac{(\cos(x))^2}{2}\Big|_0^{\pi}$

$= \frac{(\cos(x))^2}{2}\Big|_0^{\pi} = \frac{(\cos(\pi))^2}{2} - \frac{(\cos(0))^2}{2}$

$= \frac{1}{2} - \frac{1}{2}$

$= 0$

5. $\int_0^2 \frac{2x^2+1}{x^2+1}\,dx =$

$2x^2+1 : (x^2+1) = 2 - \frac{1}{x^2+1}$

$\underline{-2x^2 \pm 2}$

$0 \quad -1$

$\int \frac{1}{x^2-1}\,dx = \int \frac{dx}{x^2-1^2} = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|$

$= \int_0^2 2\,dx - \int_0^2 \frac{1}{x^2-1}\,dx\ ^{*}$

$= 2x\Big|_0^2 - \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|\Big|_0^2$

$= \left(4 - \frac{1}{2}\ln\left|\frac{2-1}{2+1}\right|\right) - \left(0 - \frac{1}{2}\ln\left|\frac{-1}{1}\right|\right)$

$= \left(4 - \frac{1}{2}\ln\left|\frac{1}{3}\right|\right) + \frac{1}{2}\ln|1|$

$\approx 4,55 + 0$

$= 4,55$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** 000	POPUNJAVA NASTAVNIK
IME I PREZIME: Franko Bakarić **BROJ INDEKSA:** 0269070613	Broj ↓ bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 2y' = 1$, uz $y(0) = 1$ i $y'(0) = 0$. Na kraju provjeri rješenje. 15

2. Pronađi funkciju koja zadovoljava $xy' = x + y$ i $y(1) = 1$. 15

3. Zadana je funkcija a $f(x, y) = \sqrt{(x-1)^2 + (y+1)^2} - 1$. Ispitati domenu, kodomenu i razinske krivulje. 15

4. $\displaystyle\int_0^\pi \cos(x)\sin(x)\,\mathrm{d}x = ?$ 15

5. $\displaystyle\int_0^2 \frac{2x^2+1}{x^2+1}\,\mathrm{d}x = ?$ 20

6. Izračunati površinu lika omeđenog pravcem $y = x - 1$ i parabolom $y = x^2 - x - 4$. 20

Ukupno: Ø

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** 000

IME I PREZIME: TONI STOŠIĆ

BROJ INDEKSA: 51817-2009

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 2y' = 1$, uz $y(0) = 1$ i $y'(0) = 0$. Na kraju provjeri rješenje. 15

2. Pronađi funkciju koja zadovoljava $xy' = x + y$ i $y(1) = 1$. 15

3. Zadana je funkcija a $f(x,y) = \sqrt{(x-1)^2 + (y+1)^2} - 1$. Ispitati domenu, kodomenu i razinske krivulje. 15

4. $\displaystyle\int_0^{\pi} \cos(x)\sin(x)\,\mathrm{d}x = ?$ 15

5. $\displaystyle\int_0^{2} \frac{2x^2+1}{x^2+1}\mathrm{d}x = ?$ 20

6. Izračunati površinu lika omeđenog pravcem $y = x - 1$ i parabolom $y = x^2 - x - 4$. 20

Ukupno: 20

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

5.

$(2x^2+1):(x^2+1) = 2$
$\underline{-2x^2 \mp 2}$
-1

$\int\left(2 + \frac{-1}{x^2+1}\right)\mathrm{d}x = \int 2\,\mathrm{d}x - 1\int\frac{\mathrm{d}x}{x^2+1^2} = 2x - 1\cdot\frac{1}{1}\,\mathrm{arctan}\,\frac{x}{1} -$

$= 2x - 1\,\mathrm{arctan}\,x$

$2x - 1\,\mathrm{arctan}\,x\,\Big|_0^2 = 2\cdot 2 - 1\,\mathrm{arctan}\,2 - (2\cdot 0 - 1\,\mathrm{arctan}\,0)$ ✓

$= 2.893 - 0 = 2.893$

(4.) $\int_0^{\pi} \cos(x)\sin(x)\,dx =$

0

$\int \cos x \cdot \sin x \cdot dx =$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** 000

IME I PREZIME: [redacted] **BROJ INDEKSA:** [redacted]

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 2y' = 1$, uz $y(0) = 1$ i $y'(0) = 0$. Na kraju provjeri rješenje. 15
2. Pronađi funkciju koja zadovoljava $xy' = x + y$ i $y(1) = 1$. ~~15~~
3. Zadana je funkcija a $f(x,y) = \sqrt{(x-1)^2 + (y+1)^2} - 1$. Ispitati domenu, kodomenu i razinske krivulje. 15
4. $\int_0^\pi \cos(x)\sin(x)\,dx = ?$ 15
5. $\int_0^2 \frac{2x^2+1}{x^2+1}dx = ?$ ~~20~~
6. Izračunati površinu lika omeđenog pravcem $y = x - 1$ i parabolom $y = x^2 - x - 4$. ~~20~~

Ukupno: $\varnothing$

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
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$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

② $xy' = x + y$ za $y(1) = 1$

$x \cdot \frac{dy}{dx} = x + y \quad / \cdot dx$

$x\,dy = x + y\,dx$

$\frac{dy}{y} = dx \quad / \int$

$\ln\lvert y\rvert = x + C$

$C = -1$

$\varnothing$

⑤ $\int_0^2 \frac{2x^2+1}{x^2+1}dx$

$2\int_0^2 \frac{x^2}{x^2+1}dx + 2\int_0^2 \frac{1}{x^2+1}dx = ?$

$\varnothing$

⑥ $y = x - 1$ $\quad y = x^2 - x - 4$

x	1	2	4
y	0	1	3

x	1	2	4
y	-4	-2	8

$x - 1 = x^2 - x - 4$

$x^2 - x - 4 - x + 1 = 0$

$x^2 - 2x - 3 = 0$

$x_{1,2} = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-3)}}{2}$

$x_{1,2} = \frac{2 \pm 4}{2}$

$x_1 = 3$

$x_2 = -1$

$P = \int_{-1}^{3} x^2 - 2x - 3\,dx$

$= \frac{x^3}{3} - 2\frac{x^2}{2} - 3x \Big|_{-1}^{3}$

$= \frac{x^3}{3} - x^2 - 3x \Big|_{-1}^{3}$

$= (9 - 9 - 9) - \left(-\frac{1}{3} + 1 + 4\right)$

$= -13{,}66$ ✗

VIDI STANIŠIĆ

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** 000

IME I PREZIME: MARIO IVANAC **BROJ INDEKSA:** 17-1-0096-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y''-2y'=1$, uz $y(0)=1$ i $y'(0)=0$. Na kraju provjeri rješenje. 15
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3. Zadana je funkcija a $f(x,y)=\sqrt{(x-1)^2+(y+1)^2}-1$. Ispitati domenu, kodomenu i razinske krivulje. 15
4. $\int\limits_0^{\pi} \cos(x)\sin(x)\,\mathrm{d}x = ?$ 15
5. $\int\limits_0^{2} \frac{2x^2+1}{x^2+1}\mathrm{d}x = ?$ 20
6. Izračunati površinu lika omeđenog pravcem $y=x-1$ i parabolom $y=x^2-x-4$. 20

Ukupno: 0

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^{\alpha}\ (\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_{\alpha} x\ (\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha>0)$	$\alpha^x\ln\alpha$
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$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x+C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a}+C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert+C$
$\int x^{\alpha}\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert+C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert+C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert+C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert+C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert+C$
$\int e^x\,\mathrm{d}x = e^x+C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x+C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a}+C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a}+C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x+C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right)+C$
$\int \sin x\,\mathrm{d}x = -\cos x+C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]+C$	
$\int \cos x\,\mathrm{d}x = \sin x+C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$	

6)

$y = x-1$

$y = x^2-x-4$

x	0	1	2
y	-1	0	1

$x-1 = x^2-x-4$

$x-1-x^2+x+4=0$

$-x^2+2x+3=0$

$x_{1/2} = \frac{-2\pm\sqrt{4-4\cdot(-1)\cdot 3}}{-2}$

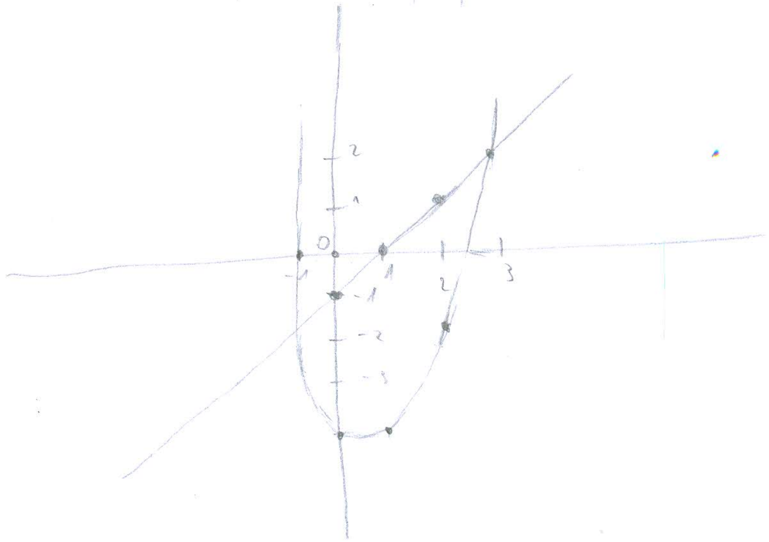
$x_{1,2} = \frac{-2\pm 4}{-2}$

$x_2 = -1$

$x_1 = 3$

x	-1	0	1	2	3
y	-4	-4	-4	-2	2

VIDI STANIŠIĆ



$$P = \int_{-1}^{3} \left((x-1) - (x^2 - x - 4) \right) dx$$

$$= \int_{-1}^{3} (x - 1 - x^2 + x + 4)\, dx$$

$$= \int_{-1}^{3} (-x^2 + 2x + 3)\, dx$$

$$= \left(-\frac{3x^2}{3} + x^2 + 3x \right) \Big|_{-1}^{3}$$

$$= \left(-\frac{3 \cdot 3^2}{3} + 3^2 + 3 \cdot 3 \right) - \left(-\frac{3 \cdot (-1)^2}{3} + (-1)^2 + 3 \cdot (-1) \right)$$

$$= (-9 + 9 + 9) - (-1 + 1 - 3)$$

$$= 9 - 3$$

$$\boxed{P = 6}$$

Ø

5) $$\int_0^2 \frac{2x^2+1}{x^2+1}\, dx = \begin{bmatrix} t = x^2+1 & dx = dt \\ t' = 2x & \end{bmatrix} = \int_0^2 = 2\frac{t}{t}\, dt = 2\int_0^2 \frac{t}{t}\, dt$$ Ø

$$= 2\int_0^2 t\, dt = 2t^2 \Big|_0^2 = \left(2 \cdot (x^2+1)\right) \Big|_0^2 = (2x^2+2) \Big|_0^2$$

$$= (2 \cdot 2^2 + 2) - (2 \cdot 0^2 + 2) = 10 - 2 = 8$$

4) $$\int_0^{\pi} \cos x \sin x\, dx = \begin{bmatrix} u = \sin x & v = \cos x \\ du = -\cos x & \end{bmatrix}$$

$$= \sin x \cdot \cos x - \int \cos x \cdot (-\cos x) = \sin x \cdot \cos x - \int -2\cos x$$ Ø

$$= \sin x \cdot \cos x - 2\int -\cos x = \sin x \cdot \cos x - 2 \cdot (-\sin x) = (\sin x \cdot \cos x + 2\sin x) \Big|_0^{\pi}$$

$$= (\sin \pi \cdot \cos \pi + 2 \sin \pi) - (\sin 0 \cdot \cos 0 + 2 \sin 0)$$

$$= (0{,}054721303 + 0{,}10960733) - (0 \cdot 1 + 0) = 0{,}164328633$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** 000	POPUNJAVA NASTAVNIK
IME I PREZIME: ANTE STANIŠIĆ **BROJ INDEKSA:** 17-1-0066-2011	Broj ↓ bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y''-2y'=1$, uz $y(0)=1$ i $y'(0)=0$. Na kraju provjeri rješenje.	15
2. Pronađi funkciju koja zadovoljava $xy'=x+y$ i $y(1)=1$.	15
3. Zadana je funkcija a $f(x,y)=\sqrt{(x-1)^2+(y+1)^2}-1$. Ispitati domenu, kodomenu i razinske krivulje.	~~15~~ 2
4. $\int_0^{\pi}\cos(x)\sin(x)\,dx=?$	~~15~~
5. $\int_0^{2}\frac{2x^2+1}{x^2+1}dx=?$	20
6. Izračunati površinu lika omeđenog pravcem $y=x-1$ i parabolom $y=x^2-x-4$.	20
	Ukupno: 20

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha>0)$	$\alpha^x\ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx=x+C$	$\int\frac{dx}{a^2+x^2}=\frac{1}{a}\arctan\frac{x}{a}+C$	$\int\frac{dx}{a^2-x^2}=\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert+C$
$\int x^\alpha dx=\frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq-1$	$\int\tan x\,dx=-\ln\lvert\cos x\rvert+C$	$\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert+C$
$\int\frac{dx}{x}=\ln\lvert x\rvert+C$	$\int\cot x\,dx=\ln\lvert\sin x\rvert+C$	$\int\frac{dx}{\sqrt{x^2\pm a^2}}=\ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert+C$
$\int e^x dx=e^x+C$	$\int\frac{dx}{\cos^2 x}=\tan x+C$	$\int\frac{dx}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$
$\int a^x dx=\frac{a^x}{\ln a}+C$	$\int\frac{dx}{\sin^2 x}=-\cot x+C$	$\int\frac{dx}{\sqrt{2ax-x^2}}=\arccos\left(1-\frac{x}{a}\right)+C$
$\int\sin x\,dx=-\cos x+C$	$\int\sqrt{x^2\pm a^2}\,dx=\frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]+C$	
$\int\cos x\,dx=\sin x+C$	$\int\sqrt{a^2-x^2}\,dx=\frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$	

3) $f(x,y)=\sqrt{(x-1)^2+(y+1)^2}-1$

1° Domena $D=R^2$ ✓

2° RAZINSKE KRIVULJE

$f(x,y)=c$

$\sqrt{(x-1)^2+(y+1)^2}=1+c \quad /^2$

$(x-1)^2+(y+1)^2=(1+c)^2 \quad /\sqrt{}$

$x-1+y+1=1+C$ ✗

$C=1$

$x\cancel{-1}+y\cancel{+1}=1+1$

$x\cancel{-1}+y\cancel{+1}=2$

$x=2-y$

x	-1	0	1
y	3	2	1

$C=2$

$x\cancel{-1}+y\cancel{+1}=1+2$

$x=3-y$

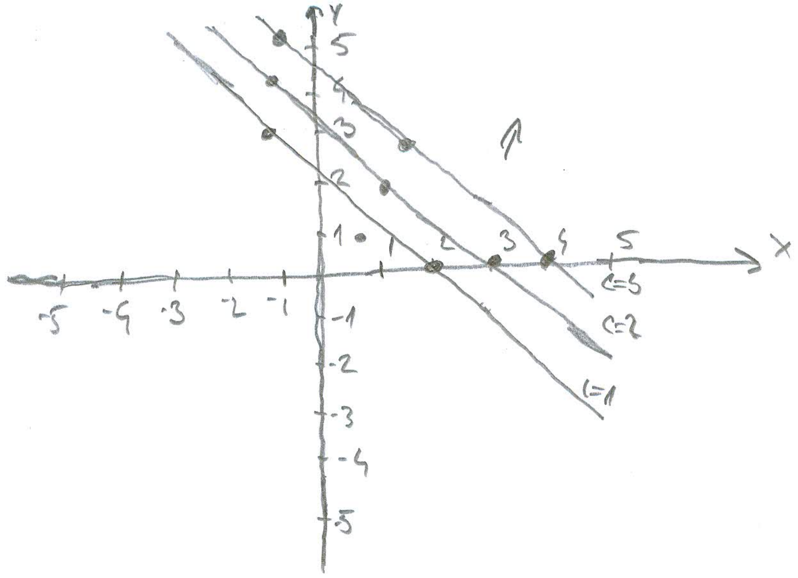
x	-1	0	1
y	4	3	2

$C=3$

$x+y=1+3$

$x=4-y$

x	-1	0	1
y	5	4	3


Y
X
c=2
c=1

ANTE STANISIC

4)

$$\int_0^{\pi} \cos(x)\sin(x)\,dx \qquad \left|\begin{array}{l} \cos x = t \;/' \\ -\sin x\,dx = dt \\ \sin x\,dx = -dt \end{array}\right.$$

$$-\int_0^{\pi} t\,dt = -\frac{t^2}{2}\Big|_0^{\pi} = \left(-\frac{\pi^2}{2}\right) - \left(\cancel{-\frac{0^2}{2}}\right)$$

$$= -4{,}9348$$

$\varnothing$

---

6)

$y = x - 1$

$y = x^2 - x - 4 \Rightarrow a > 0 \Rightarrow \cup$

$x^2 - x - 4 = x - 1$

$x^2 - x - 4 - x + 1 = 0$

$x^2 - 2x - 3 = 0$

$$x_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{12} = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-3)}}{2}$$

$$x_{12} = \frac{2 \pm \sqrt{16}}{2} = \frac{2+4}{2} = \frac{6}{2} = x_2 = 3$$

$$x_1 = \frac{2-4}{2} = x_1 = -1$$

$\cancel{\cup}$

$$P = \int_{-1}^{3} \left|(x-1) - (x^2 - x - 4)\right| dx$$

$$P = \int_{-1}^{3} (x - 1 - x^2 + x + 4)\,dx = \int_{-1}^{3} (-x^2 + 2x + 3)\,dx$$

$$P = -\frac{x^3}{3} + \cancel{2}\frac{x^2}{\cancel{2}} + 3x \Big|_{-1}^{3}$$

$$P = \left(-\frac{3^3}{3} + 3^2 + 3 \cdot 3\right) - \left(-\frac{(-1)^3}{3} + (-1)^2 + 3 \cdot (-1)\right)$$

$$P = 9 + \frac{5}{3} = \frac{32}{3} = \boxed{10{,}6667}$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** 000

IME I PREZIME: Goran Marjanović **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 2y' = 1$, uz $y(0) = 1$ i $y'(0) = 0$. Na kraju provjeri rješenje. 15

2. Pronađi funkciju koja zadovoljava $xy' = x + y$ i $y(1) = 1$. 15

3. Zadana je funkcija a $f(x, y) = \sqrt{(x-1)^2 + (y+1)^2} - 1$. Ispitati domenu, kodomenu i razinske krivulje. 15

4. $\displaystyle\int_0^{\pi} \cos(x)\sin(x)\,\mathrm{d}x = ?$ 15

5. $\displaystyle\int_0^{2} \frac{2x^2+1}{x^2+1}\mathrm{d}x = ?$ 20

6. Izračunati površinu lika omeđenog pravcem $y = x - 1$ i parabolom $y = x^2 - x - 4$. 20

Ukupno: 0

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	