

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

odgovornosti studenata. **PIŠITE DVOSTRANO!**

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POPUNJAVA

NASTAVNIK

Broj ↓

bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 2y' = 1$, uz $y(0) = 1$ i $y'(0) = 0$.
Na kraju provjeri rješenje.

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2. Pronađi funkciju koja zadovoljava $xy' = x + y$ i $y(1) = 1$.

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3. Zadana je funkcija a $f(x, y) = \sqrt{(x-1)^2 + (y+1)^2} - 1$. Ispitati domenu, kodomenu i razinske krivulje.

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4. $\int_0^{\pi} \cos(x) \sin(x) dx = ?$

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5. $\int_0^2 \frac{2x^2 + 1}{x^2 + 1} dx = ?$

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6. Izračunati površinu lika omeđenog pravcem $y = x - 1$ i parabolom $y = x^2 - x - 4$.

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Ukupno:

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f	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$a^x (\alpha > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

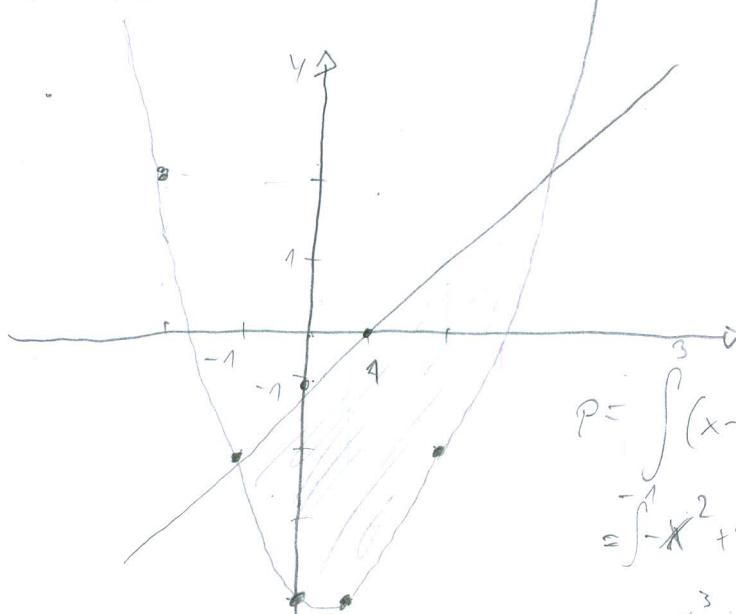
Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2} + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

6. $y = x - 1$

$y = x^2 - x - 4$

x	0	1
y	-1	0

x	0	1	-1	2	-2
y	-4	4	-2	-2	2



$x^2 - x - 4 = x - 1$

$x^2 - 2x + 3 = 0$

$x_{1,2} = \frac{2 \pm \sqrt{4-12}}{2}$

$x_{1,2} = \frac{2 \pm 2i}{2}$

$x_1 = 1 + i$

$x_2 = 1 - i$

$x_1 = 3$

$x_2 = -1$

$P = \int_{-1}^3 (x-1) - (x^2 - x - 4) dx = \int_{-1}^3 x - 1 - x^2 + x + 4 dx$

$= \int_{-1}^3 -x^2 + 2x + 3 dx = -\int_{-1}^3 x^2 dx + 2 \int_{-1}^3 x dx + 3 \int_{-1}^3 dx$

$= -\frac{x^3}{3} + 2 \frac{x^2}{2} + 3x \Big|_{-1}^3 = -\frac{3^3}{3} + 2 \cdot \frac{3^2}{2} + 3 \cdot 3 - \left(-\frac{(-1)^3}{3} + 2 \cdot \frac{(-1)^2}{2} + 3 \cdot (-1) \right)$

$$= -5 + 8 + 9 - \left(\frac{4}{3} + 1 - 3\right)$$

$$= 9 - \frac{1}{3} - 1 + 3 = 11 - \frac{1}{3} = \frac{32}{3} \approx 10.6666 \checkmark$$

$$4. \int_0^{\pi} \cos(x) \sin(x) dx = \left| \begin{array}{l} \sin x = t \\ \cos x dx = dt \\ dx = \frac{dt}{\cos x} \end{array} \right| = \int \cancel{\cos(x)} \cdot t \cdot \frac{dt}{\cancel{\cos x}} = \int t dt$$

$$= \frac{t^2}{2} + c = \frac{(\sin x)^2}{2} + c$$

$$= \frac{(\sin x)^2}{2} \Big|_0^{\pi} = \frac{(\sin \pi)^2}{2} - \underbrace{\left(\frac{(\sin 0)^2}{2}\right)}_{\downarrow 0} = \frac{(\sin \pi)^2}{2} \approx 0,00150172 \quad \text{😊}$$

$\sin \pi = 0$

$$5. \int_0^2 \frac{2x^2+1}{x^2+1} dx = 2 \int \frac{2x^2+1}{2x^2+2} dx = 2 \int \frac{2x^2+1}{2x^2+2-1+1} dx = 2 \int \frac{2x^2+1}{2x^2+1} dx + 2 \int \frac{1}{1} dx$$

$$= 2 \int dx + 4 \int 2x^2 dx + 2 \int dx = 2 \cdot x + 4 \cdot \frac{x^3}{3} + 2x = 4x + \frac{4x^3}{3} \Big|_0^2$$

$$= 8 + 4 \cdot \frac{8}{3} - 0 = 8 + \frac{32}{3} = \frac{24+32}{3} = \frac{56}{3} = 18 \frac{2}{3}$$

$$2. x^2 = 2 \cdot 4 \quad 4 \cdot 2 = 8 \quad 2 \cdot 4 = 8$$

$$8 \cdot 4 = 32 \neq 8$$

$$4 \cdot 4 = 16$$

$$4 =$$

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$$1. \quad y'' - 2y' = 1$$

$$y(0) = 1 \quad y'(0) = 0$$

$$k^2 - 2k = 0$$

$$k(k-2) = 0$$

$$\downarrow \quad \downarrow$$
$$k = 0 \quad k = 2$$

$$k_1 = 0 \quad k_2 = 2$$

$$d = 0, \quad r = 1$$

$$y = C_1 e^{0 \cdot x} + C_2 e^{2x}$$

$$y = C_2 e^{2x}$$

$$y' = C_2 \cdot e^{2x} \cdot 2$$

$$y'' = C_2 \cdot e^{2x} \cdot 2 \cdot 2$$

$$2. \quad xy' = x + y \quad | \quad x \neq 0$$

$$y(1) = 1$$

$$\frac{y'}{y} = \frac{x+y}{x}$$

