MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxx

IME I PREZIME: ANIZA ŠTUC

BROJ INDEKSA: 17-1-0173-2013

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y = x^2 - e^x$ uz početne uvjete $y(0) = 1$ i $y'(0) = 0$. — 15
2. Odrediti lokalne ekstreme funkcije: $f(x, y) = x\sqrt{y} - x^2 - y + 3x$. — 15
3. Odredi tangencijalnu ravninu i normalu na plohu $z = \arctan(xy)$ u točki $M(1, 1, z_0)$. — 15
4. $\int_0^{2\pi} \sin^4 x \, dx = ?$ — 20
5. $\int_0^1 x^2 \tan(3x^3 + 1) \, dx = ?$ — 15
6. Neka je $f(x) = \frac{1}{x^2}$. Odrediti $\int_1^{+\infty} f(x)dx$. Skicirati graf funkcije f i površinu određenu integralom. — 20

Ukupno: 70

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

3. $z_0 = \operatorname{arctg}(1) = \frac{\pi}{4}$ ✓

$\frac{\partial f}{\partial x} = \frac{1}{1+(xy)^2} \cdot y$ ✓

$\frac{\partial f}{\partial x}(1,1) = \frac{1}{2}$ ✓

$\frac{\partial f}{\partial y} = \frac{1}{1+(xy)^2} \cdot x$ ✓

$\frac{\partial f}{\partial y}(1,1) = \frac{1}{2}$ ✓

$R: R_t \dots \quad z - \frac{\pi}{4} = \frac{1}{2}(x-1) + \frac{1}{2}(y-1)$ ✓

→ IMA IZA

4.

$$\int_0^{2\pi} \sin^4 x\,dx = \int_0^{2\pi} (\sin^2 x)^2\,dx =$$

$$= \int_0^{2\pi} \left(\frac{1-\cos(2x)}{2}\right)^2 dx =$$

$$= \int_0^{2\pi} \frac{1-2\cos(2x)+\cos^2(2x)}{4}\,dx \quad \checkmark$$

$$= \int_0^{2\pi} \left(\frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{1}{4}\cdot\frac{1+\cos(4x)}{2}\right) dx \quad \checkmark$$

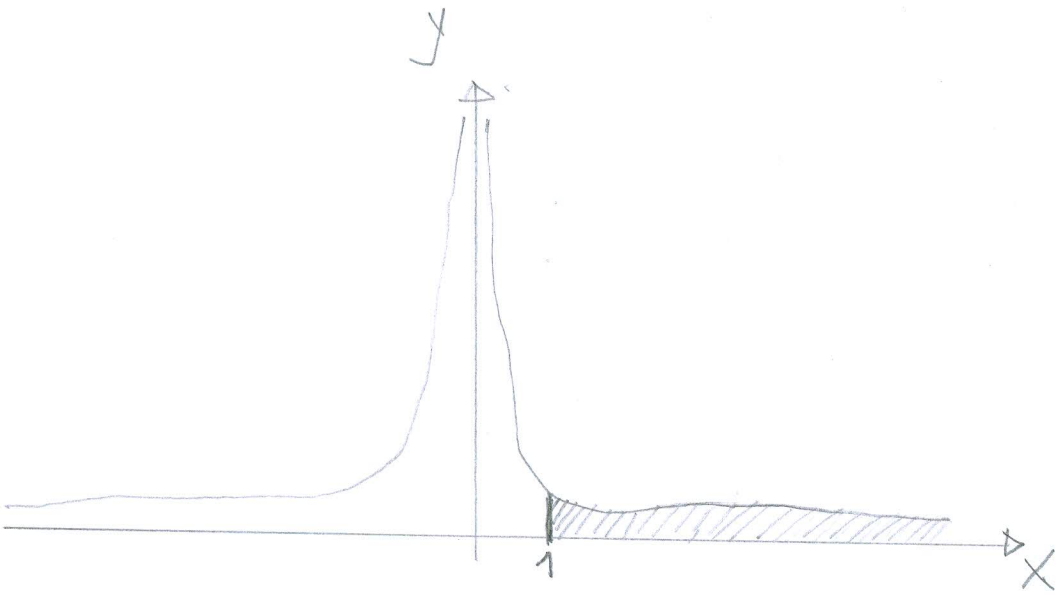
$$= \int_0^{2\pi} \left(\frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{1}{8} + \frac{1}{8}\cos(4x)\right) dx \quad \checkmark$$

$$= \left(\frac{1}{4}x - \frac{1}{4}\sin(2x) + \frac{1}{8}x + \frac{1}{32}\sin(4x)\right)\Big|_0^{2\pi} \quad \checkmark$$

$$= \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4} \quad \checkmark$$

ANDREA STOJ

(6.)



$$\int_1^{+\infty} \frac{1}{x^2}\,dx = \lim_{b\to+\infty} \int_1^b \frac{1}{x^2}\,dx = \lim_{b\to+\infty} \left(-\frac{1}{x}\right)\Big|_1^b$$

$$= \lim_{b\to\infty} \left(-\frac{1}{b}+1\right) = 1 \quad \checkmark$$

---

(2) $\frac{\partial f}{\partial x} = \sqrt{y} - 2x + 3 = 0$

$\frac{\partial f}{\partial y} = \frac{x}{2\sqrt{y}} - 1 = 0 \rightarrow x = 2\sqrt{y}$

$\sqrt{y} - 4\sqrt{y} + 3 = 0$

$-3\sqrt{y} = -3$

$\sqrt{y} = 1 \Rightarrow y = 1$

$x = 2$ ✓

$\frac{\partial^2 f}{\partial x^2} = -2 \quad \frac{\partial^2 f}{\partial x \partial y} = \frac{1}{2\sqrt{y}} \quad \frac{\partial^2 f}{\partial y^2} = -\frac{1}{4} y^{-\frac{3}{2}}$

$\Delta = -2 \cdot \left(-\frac{1}{4}\right) - \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

IMA IZA →