MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxx

IME I PREZIME: ANĐELA STUC **BROJ INDEKSA:** 17-1-0173-2013

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y = x^2 - e^x$ uz početne uvjete $y(0) = 1$ i $y'(0) = 0$. 15
2. Odrediti lokalne ekstreme funkcije: $f(x, y) = x\sqrt{y} - x^2 - y + 3x$. 15
3. Odredi tangencijalnu ravninu i normalu na plohu $z = \arctan(xy)$ u točki $M(1, 1, z_0)$. 15
4. $\displaystyle\int_0^{2\pi} \sin^4 x \, dx = ?$ 20
5. $\displaystyle\int_0^{1} x^2 \tan\left(3x^3 + 1\right) dx = ?$ 15
6. Neka je $f(x) = \dfrac{1}{x^2}$. Odrediti $\int_1^{+\infty} f(x)dx$. Skicirati graf funkcije f i površinu određenu integralom. 20

Ukupno: 70

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

3. $z_0 = \operatorname{arctg}(1) = \dfrac{\pi}{4}$

$$\frac{\partial f}{\partial x} = \frac{1}{1+(xy)^2} \cdot y$$

$$\frac{\partial f}{\partial x}(1,1) = \frac{1}{2}$$

$$\frac{\partial f}{\partial y} = \frac{1}{1+(xy)^2} \cdot x$$

$$\frac{\partial f}{\partial y}(1,1) = \frac{1}{2}$$

$$Rt \ldots z - \frac{\pi}{4} = \frac{1}{2}(x-1) + \frac{1}{2}(y-1)$$

→ IMA IZA

(4). $\int_0^{2\pi} \sin^4 x \, dx = \int_0^{2\pi} (\sin^2 x)^2 dx =$

$= \int_0^{2\pi} \left(\frac{1-\cos(2x)}{2} \right)^2 dx =$

$= \int_0^{2\pi} \frac{1-2\cos(2x)+\cos^2(2x)}{4} dx$

$= \int_0^{2\pi} \left(\frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{1}{4} \cdot \frac{1+\cos(4x)}{2} \right) dx$

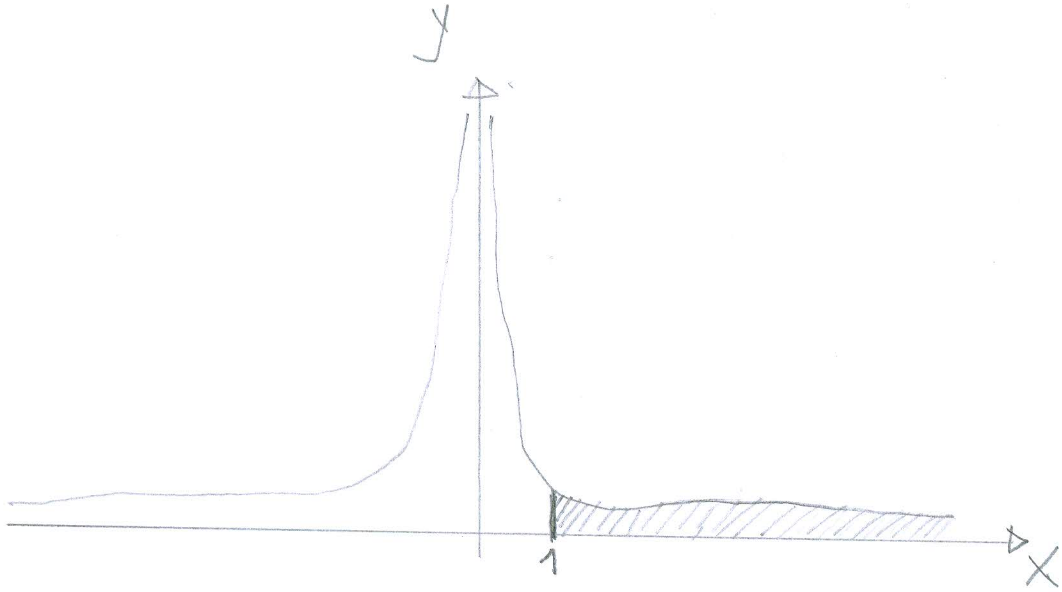
$= \int_0^{2\pi} \left(\frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{1}{8} + \frac{1}{8}\cos(4x) \right) dx$

$= \left(\frac{1}{4}x - \frac{1}{4}\sin(2x) + \frac{1}{8}x + \frac{1}{32}\sin(4x) \right) \Big|_0^{2\pi}$

$= \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

ANDEZA STUC

6.



$$\int_1^{+\infty} \frac{1}{x^2}\,dx = \lim_{b\to+\infty} \int_1^b \frac{1}{x^2}\,dx = \lim_{b\to+\infty} \left(-\frac{1}{x}\right)\Big|_1^b$$

$$= \lim_{b\to\infty} \left(-\frac{1}{b}+1\right) = 1$$

2. $$\frac{\partial f}{\partial x} = \sqrt{y} - 2x + 3 = 0$$

$$\frac{\partial f}{\partial y} = \frac{x}{2\sqrt{y}} - 1 = 0 \rightarrow x = 2\sqrt{y}$$

$$\sqrt{y} - 4\sqrt{y} + 3 = 0$$

$$-3\sqrt{y} = -3$$

$$\sqrt{y} = 1 \Rightarrow y = 1$$
$$x = 2$$

$$\frac{\partial^2 f}{\partial x^2} = -2 \qquad \frac{\partial^2 f}{\partial x \partial y} = \frac{1}{2\sqrt{y}} \qquad \frac{\partial^2 f}{\partial y^2} = -\frac{1}{4} y^{-\frac{3}{2}}$$

$$\Delta = -2 \cdot \left(-\frac{1}{4}\right) - \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

IMA 12A

→

$f(2,1) = 2-4-1+6 = 3$

TOČKA $T(2,1,3)$ JE MAKSIMUM

---

1. $r^2 - 2 = 0$

$r^2 = 2$

$r = \pm\sqrt{2}$

$y_H = C_1 e^{-\sqrt{2}x} + C_2 e^{\sqrt{2}x}$

$g_1(x) = x^2 \rightarrow \alpha = 0 \quad \beta = 0 \quad k = 0$

$N = 2$

$y_{p1} = Ax^2 + Bx + C$

$y'_{p1} = 2Ax + B \qquad y''_{p1} = 2A$

$2A - 2Ax^2 - 2Bx - 2C = x^2$

$-2A = 1 \rightarrow A = -\frac{1}{2}$

$-2B = 0 \rightarrow B = 0$

$2A - 2C = 0 \rightarrow C = A = -\frac{1}{2}$

ANĐELA STOC

(1.) $g_2(x) = -e^x$ $\quad \alpha = 1 \quad \beta = 0 \quad k = 0$
$N = 0$

$y_{p_2} = Ae^x$

$y'_{p_2} = Ae^x \quad y''_{p_2} = Ae^x$

$Ae^x - 2Ae^x = -e^x$

$-A = -1 \rightarrow A = 1$

$y = C_1 e^{-\sqrt{2}x} + C_2 e^{\sqrt{2}x} - \frac{1}{2}x^2 - \frac{1}{2} + e^x$

$y' = -\sqrt{2}C_1 e^{-\sqrt{2}x} + \sqrt{2}C_2 e^{\sqrt{2}x} - x + e^x$

$y(0) = 1 \rightarrow C_1 + C_2 - \frac{1}{2} + 1 = 1$

$y'(0) = 0 \rightarrow -\sqrt{2}C_1 + \sqrt{2}C_2 + 1 = 0 \,/ : \sqrt{2}$

$C_1 + C_2 = \frac{1}{2}$

$-C_1 + C_2 = -\frac{1}{\sqrt{2}}$

$2C_2 = \frac{1}{2} - \frac{\sqrt{2}}{2}$

$C_2 = \frac{1}{4} - \frac{\sqrt{2}}{4}$

$C_1 = \frac{1}{2} - \frac{C}{2} = \frac{1}{2} - \frac{1}{4} + \frac{\sqrt{2}}{4} = \frac{1}{4} + \frac{\sqrt{2}}{4}$

$y = \left(\frac{1}{4} + \frac{\sqrt{2}}{4}\right)e^{-\sqrt{2}x} + \left(\frac{1}{4} - \frac{\sqrt{2}}{4}\right) - \frac{1}{2}x^2 - \frac{1}{2} + e + e^{\sqrt{2}x}$

PROVJERA

$y(0) = \frac{1}{4} + \cancel{\frac{\sqrt{2}}{4}} + \frac{1}{4} - \cancel{\frac{\sqrt{2}}{4}} - \frac{1}{2} + e + 1 \neq 1$

$\varnothing$

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IME I PREZIME: JURAJ POLJAK **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK

Broj ↓ bodova

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① $y'' - 2y = x^2 - e^x$

POČ. UVJ.

$\boxed{y(0) = 1 \quad y'(0) = 0}$

1) HOMOGENO RJ.

$y'' - 2y = 0$

$\lambda^2 - 2 = 0$

$\lambda^2 = 2$

$\lambda = \pm\sqrt{2}$

$\hookrightarrow \boxed{y_H(x) = C_1 e^{\sqrt{2}x} + C_2 e^{-\sqrt{2}x}}$ ✓

2) PARTIKULARNO RJ.

$f_1(x) = x^2,\ f_2(x) = -e^x$

općenito: $f(x) = e^{\alpha(x)}$

za $f_1(x) = x^2$ $\alpha = 0$ $\beta = 1$

$Q_n(x) = 0$ $n = 0$

$P_m(x) = x^2$ $m = 0$

za $f_1(x) = x^2$ $\alpha = 0$ $\beta = 1$

$Q_n(x) = 0$ $n = 0$

$P_n(x) = x^2$ $n = 0$

$\lambda = \max\{m, n\} = 0$

$\lambda = 0 + 1i \Rightarrow r = 0$

KONAČNO RJEŠENJE?

$\Rightarrow y_{p1}(x) = Ax^2 + Bx + C$

$y'_{p1}(x) = 2Ax + B$

$y''_{p1}(x) = 2A$

$y'' - 2y = x^2$

$2A - 2Ax^2 - 2Bx - 2C = x^2$

$-2A = 1$

$-2B = 0$

$2A - 2C = 0$

$\boxed{A = -\frac{1}{2}}$

$\boxed{B = 0}$

$\boxed{2A - 2C = 0}$

$\boxed{C = -\frac{1}{2}}$

$f_2(x) = -e^x = e^x(-1)$

$\boxed{y_{p1}(x) = -\frac{1}{2}x^2 - \frac{1}{2}}$

5. $\int_0^1 x^2 \tan(3x^3+1)\,dx = \left| \begin{array}{ll} u = 3x^3+1 & 0 \to 1 \\ du = 9x^2\,dx & 1 \to 4 \\ \frac{1}{9}\,du = x^2 dx & \end{array} \right| =$

$= \frac{1}{9}\int_1^4 \tan u\,du = \frac{1}{9}\left(-\ln|\cos u|\right)\Big|_1^4 =$

$= \frac{1}{9}\left(-\ln|\cos 4| + \ln|\cos 1|\right) =$

$= \frac{1}{9}\left(-\ln(0.6535) + \ln(0.5403)\right)$

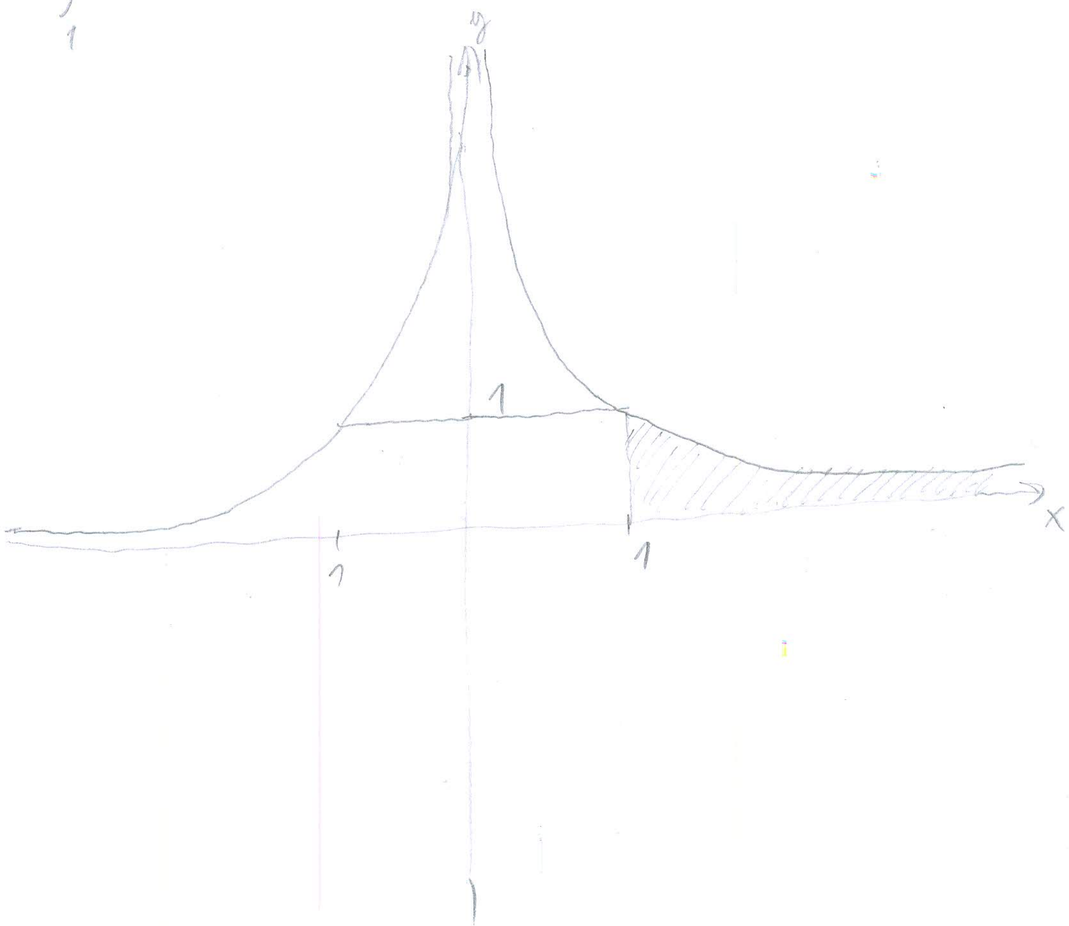
$= \underline{-0.02111}$

$\emptyset$

$\boxed{\text{NEPRAVI INTEGRAL}}$

16) $f(x) = \frac{1}{x^2}$

$$\int_1^{\infty} \frac{1}{x^2}\,dx = \int_1^{\infty} x^{-2}\,dx = \frac{x^{-2+1}}{-2+1}\Big|_1^{\infty} = -\frac{1}{x}\Big|_1^{\infty} = \left(-\frac{1}{\infty} - 1\right) = 1$$ ✓



$V=1$, $B=1$, $Q_n(x)=0$, $m=0$

nastavak 1og zadatka

$P_n(x)=-1$, $n=0$

JURAJ POLJAK

$\lambda=0$

$\lambda=i$ $r=0$

$y_{p_0}(x)=e^x\cdot D$

$y'_{p_0}(x)=De^x$

$y''_{p_0}(x)=De^x$

$De^x-2De^x=-e^x$

$-De^x=-e^x$

$-D=-1$

$\boxed{D=1}$

$\boxed{y_{p_2}(x)=e^x}$

$y_{p_0}(x)=e^x$

$y(x)=y_H(x)+y_{p_1}(x)+y_{p_2}(x)$

$y(x)=C_1e^{\sqrt{2}x}+C_2e^{-\sqrt{2}x}-\frac{1}{2}x^2+\frac{1}{2}+e^x$

$1=y(0)=C_1+C_2+\frac{1}{2}+1 \Rightarrow C_1+C_2=-\frac{1}{2}$

$y'(x)=\sqrt{2}C_1e^{-\sqrt{2}x}-\sqrt{2}C_2e^{-\sqrt{2}x}-x+e^x$

$0=y'(0)=\sqrt{2}C_1-\sqrt{2}C_2+1$

$\sqrt{2}C_1-\sqrt{2}C_2=-1 \mid \frac{1}{\sqrt{2}}$

$\left.\begin{array}{l} C_1-C_2=-\frac{1}{\sqrt{2}} \\ C_1+C_2=-\frac{1}{2} \end{array}\right\} +$

$2C_1=\frac{1}{-\sqrt{2}}-\frac{1}{2} \mid \frac{1}{2}$

$C_1=\frac{-1}{2\sqrt{2}}-\frac{1}{4} \Rightarrow C_2=-\frac{1}{2}+\frac{1}{2\sqrt{2}}+\frac{1}{4}$?

$y(x)=\left(-\frac{1}{2\sqrt{2}}-\frac{1}{4}\right)e^{\sqrt{2}x}+\left(-\frac{1}{2}+\frac{1}{2\sqrt{2}}+\frac{1}{4}\right)e^{-\sqrt{2}x}\frac{1}{2x^2}+\frac{1}{2}+e^x$

ŠTO JE OVO?

PROVJERA

$y(0)=\left(-\cancel{\frac{1}{2\sqrt{2}}}-\cancel{\frac{1}{4}}\right)+\left(-\cancel{\frac{1}{2}}+\cancel{\frac{1}{2\sqrt{2}}}+\cancel{\frac{1}{4}}\right)+?+\cancel{\frac{1}{2}}+1=1$?

(4) $\int_0^{2\pi} \sin^4 x\,dx = \int_0^{2\pi} (\sin^2 x)^2 dx = \int_0^{2\pi} \left(\frac{1-\cos 2x}{2}\right)^2 dx =$

$= \int_0^{2\pi} \frac{1-2\cos 2x+\cos^2 2x}{4}\,dx = \int_0^{2\pi} \frac{1}{4}dx - \frac{1}{2}\int_0^{2\pi} \cos 2x\,dx + \frac{1}{4}\int_0^{2\pi} \cos^2 2x\,dx =$

$= \frac{1}{4}x \Big|_0^{2\pi} - \frac{1}{2}\,\frac{\sin 2x}{2}\Big|_0^{2\pi} + \frac{1}{4}\int_0^{2\pi} \frac{1+\cos 4x}{2}\,dx =$

$= \int_0^{2\pi} \left(\frac{1-\cos 2x}{2}\right)^2 dx = \int_0^{2\pi} \frac{1-2\cos 2x + \cos^3 dx}{4}\,dx =$

$= \int_0^{2\pi} \frac{1}{4}\,dx - \frac{1}{2}\int_0^{2\pi} \cos 2x\,dx + \frac{1}{4}\int_0^{2\pi} \cos^2 2x\,dx =$

$= \frac{1}{4}x \Big|_0^{2\pi} - \frac{1}{2}\,\frac{\sin 2x}{2}\Big|_0^{2\pi} + \frac{1}{4}\int_0^{2\pi} \frac{1+\cos 4x}{2}\,dx =$

$= \frac{\pi}{2} + \frac{1}{4}\int_0^{2\pi} \frac{1}{2}\,dx + \frac{1}{4}\int_0^{2\pi} \frac{\cos 4x}{2}\,dx =$

$= \frac{\pi}{2} + \frac{1}{8}x \Big|_0^{2\pi} + \frac{1}{8}\,\frac{\sin 4x}{4}\Big|_0^{2\pi}$

$= \frac{\pi}{2} + \frac{\pi}{4} = \frac{4\pi + 2\pi}{8} = \frac{6\pi}{8} = \frac{3\pi}{4}$ ✓

(2) lok. ekstrem

JURAJ POLJAK

$f(x,y) = x\sqrt{y} - x^2 - y + 3x$

1° parcijalne derivacije

$\frac{\partial f}{\partial x}(x,y) = \sqrt{y} - 2x + 3$ ✓

$\frac{\partial f}{\partial x}(x,y) = \frac{x}{2\sqrt{y}} - 1$ ✓

mora riješiti:

$\sqrt{y} - 2x + 3 = 0$

$\frac{x}{2\sqrt{y}} - 1 = 0 \implies \frac{x}{2\sqrt{y} - 1}$

$x = 2\sqrt{y}$

$\sqrt{y} - 4\sqrt{y} + 3 = 0$

$-3\sqrt{y} = -3$

determinanta u točki $S(2,1)$ ✓

$$\Delta_2(2,1) = \begin{vmatrix} -2 & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -2 \cdot \left(-\frac{1}{2}\right) - \frac{1}{2} \cdot \frac{1}{2}$$

$$= -1 - \frac{1}{4} < 0 \quad \times$$

$\Delta_1 = -2 < 0$

$\Delta_2 = -\frac{5}{4} < 0$

funkcija u točki S nema lokalni ekstrem

∅

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IME I PREZIME: NIKOLA TOMAŠOV **BROJ INDEKSA:** 17-2-0161-2012	

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$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

5.
$$\int_0^1 x^2 \tan(3x^3+1)\,dx = \left\{\begin{array}{ll} t = 3x^3+1 & x=1, \ t=4 \\ dt = 9x^2\,dx & x=0, \ t=1 \\ dx = \frac{dt}{9x^2} & \end{array}\right\}$$

$$\int_1^4 \cancel{x^2}\cdot \tan(t)\cdot \frac{1\,dt}{9\cancel{x^2}} = \frac{1}{9}\int_1^4 \tan(t)\,dt = \frac{1}{9}\cdot\left(-\ln|\cos t|\right)\Big|_1^4$$

$$= \frac{1}{9}\left(-\ln|\cos 4| - \ln|\cos 1|\right) = \frac{1}{9}\left(-\ln(0{,}6536436209) - \ln(0{,}5403023059)\right)$$

$$= 0{,}115646 6076$$

NEPRAVI INTEGRAL

③ $Z = \operatorname{arctg}(xy)$, $M(1,1,Z_0)$

$Z_0 = \operatorname{arctg}(1)$

$Z_0 = \underline{\underline{45}}^\circ$

$M(1,1,45)$

$f_x = \frac{\partial f}{\partial x} = \frac{1}{1+1^2} = \frac{1}{1} = \underline{\underline{1}}$

$f_y = \frac{\partial f}{\partial y} = \frac{1}{1+1^2} = \underline{\underline{1}}$

$f_x(T) = \operatorname{arctg}(1) = 45^\circ$

$f_y(T) = \operatorname{arctg}(1) = 45$

$Z - Z_0 = f_x(T)(x-x_0) + f_y(T)(y-y_0)$

$Z - 45 = 45(x-1) + 45(y-1)$

$Z - 45 = 45x - 45 + 45y - 45$

$-45x - 45y + 45 + Z = 0 \;/\cdot(-1)$

$45x + 45y - 45 - Z = 0 \; \ldots \; Rt$

$\operatorname{arctan}(1) \neq 45$

$\rightarrow 45^\circ$

$\frac{\partial f}{\partial x} = ?$ $\varnothing$

$\frac{x-x_0}{f_x(T)} = \frac{y-y_0}{f_y(T)} = \frac{z-z_0}{-1}$

$\frac{x-1}{45} = \frac{y-1}{45} = \frac{z-45}{-1}$

② $f(x,y) = x\sqrt{y} - x^2 - y + 3x$

$\frac{\partial f}{\partial x} = \sqrt{y} - 2x + 3$

$\frac{\partial f}{\partial y} = \frac{x}{2\sqrt{y}} - 1 = 0$

$\frac{x}{2\sqrt{y}} - 1 = 0 \;/\cdot 2\sqrt{y}$

$x - 2\sqrt{y} = 0 \;/\sqrt{\ }$

$y(x-2) = 0$

$x - 2 = 0$

$x = 2$

$\sqrt{y} - 2\cdot 2 + 3 = 0$

$\sqrt{y} - 4 + 3 = 0$

$\sqrt{y} = 4 - 3$

$\sqrt{y} = 1$

$y = 1$

$T(2,1)$ ✓

$\frac{\partial f}{\partial x^2} = -2 = \underline{\underline{A}}$

$\frac{\partial f}{\partial y^2} = \frac{x}{2}\cdot\left(-\frac{1}{2}\right)\cdot y^{-\frac{1}{2}-1} = \frac{x}{2}\cdot\left(-\frac{1}{2}\right)\cdot y^{-\frac{3}{2}} = -\frac{x}{4\sqrt{y^3}} = \underline{\underline{C}}$

$\frac{\partial f}{\partial x \partial y} = \frac{1}{2\sqrt{y}} = \underline{\underline{B}}$

$\Delta \begin{vmatrix} A & B \\ B & C \end{vmatrix} = A\cdot C - B^2 = 1 - \frac{1}{2} = \frac{1}{2} > 0 \rightarrow$ POZITIVNO ZNAČI EKSTREM

$f_{MAX} = 2\sqrt{1} - 2^2 - 1 + 3\cdot 2 = 3 \rightarrow$ MAX. ✓

NIKOLA TOMASOV

(4.) $\int_0^{2\pi} \sin^4 x \, dx = \int_0^{2\pi} \left(\frac{1+\sin(2x)}{2} \right)^2$

$= \frac{1}{4} \int_0^{2\pi} \sin 4x \, dx = \frac{1}{4} \left(\sin 4 \cdot (2\pi) - \sin 4 \cdot 0 \right)$

$= -1,18878258$

0

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxx	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: ANTONIO SEKULA BROJ INDEKSA:	

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y = x^2 - e^x$ uz početne uvjete $y(0) = 1$ i $y'(0) = 0$. 15
2. Odrediti lokalne ekstreme funkcije: $f(x,y) = x\sqrt{y} - x^2 - y + 3x$. 15
3. Odredi tangencijalnu ravninu i normalu na plohu $z = \arctan(xy)$ u točki $M(1,1,z_0)$. 15
4. $\int_0^{2\pi} \sin^4 x \, dx = ?$ 20
5. $\int_0^1 x^2 \tan(3x^3+1) \, dx = ?$ 15
6. Neka je $f(x) = \frac{1}{x^2}$. Odrediti $\int_1^{+\infty} f(x)dx$. Skicirati graf funkcije f i površinu određenu integralom. 20

Ukupno: 15

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

5. $\int_0^1 x^2 \tan(3x^3+1)\,dx = \left| \begin{matrix} t = 3x^3+1 \\ dt = 9x^2 dx \\ dx = \frac{dt}{9x^2} \end{matrix} \right| = \int x^2 \tan(t) \frac{dt}{9x^2} =$

$\frac{1}{9}\int \cancel{x^2} \tan(t) \frac{dt}{\cancel{x^2}} = \frac{1}{9}\int \tan(t)\,dt = -\frac{1}{9}\ln|\cos t| =$

$-\frac{1}{9}\ln|\cos 3x^3+1|\Big|_0^1 = -\frac{1}{9}\ln|\cos 4| - \left(-\frac{1}{9}\ln|\cos 1|\right)$

$-\frac{1}{9}\ln|\cos 4| + \frac{1}{9}\ln|\cos 1| =$

$-\frac{1}{9}\ln|-0{,}65| + \frac{1}{9}\ln|0{,}54|$ 0

$3x^3+1 =$
$3x^3 = -1$
$x^3 = -\frac{1}{3}$
$x = \sqrt[3]{-\frac{1}{3}}$
$x = -0{,}69$

NEPRAVI INTEGRAL ✓

DA LI KONVERGIRA?

(2.) $f(x,y) = x\sqrt{y} - x^2 - y + 3x$

$\partial x = \sqrt{y} - 2x + 3$ ✓ $\quad \sqrt{y} = 2x-3$

$\partial y = \dfrac{x}{2\sqrt{y}} - 1$ ✓

$x\sqrt{y} = x \cdot y^{\frac{1}{2}}$

$x \cdot \frac{1}{2} y^{-\frac{1}{2}} = x \dfrac{1}{2\sqrt{y}}$

$\sqrt{y}' = y^{\frac{1}{2}} = \frac{1}{2} y^{-\frac{1}{2}} = \frac{1}{2\sqrt{y}}$

$\dfrac{x}{2\sqrt{y}} - 1 = 0$

$\dfrac{x}{2\sqrt{y}} = 1 \quad / \cdot 2\sqrt{y}$

$x = 2\sqrt{y}$

$x = 2 \cdot (2x-3)$

$x = 4x - 6$

$-3x = -6$

$x = 2$ ✓

$\sqrt{y} = 2 \cdot 2 - 3$

$\sqrt{y} = 4 - 3$

$\sqrt{y} = 1$

$y = 1$

$T(2,1)$ ✓

$\Delta \begin{vmatrix} \partial xx & \partial yx \\ \partial xy & \partial yy \end{vmatrix}$

$\begin{vmatrix} -2 & \frac{1}{2\sqrt{y}} \\ \frac{1}{2\sqrt{y}} & \frac{-x}{4\sqrt{y^3}} \end{vmatrix} = \begin{vmatrix} -2 & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix}$

$\dfrac{x}{2\sqrt{y}} = x \cdot \frac{1}{2} y^{-\frac{1}{2}} = \frac{1}{2} y^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{y}}$

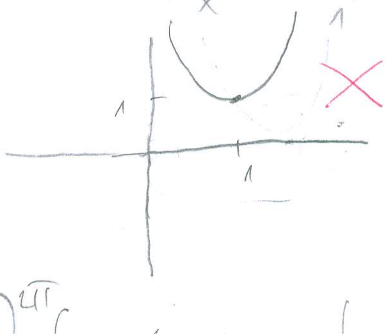
$\dfrac{x}{2\sqrt{y}} = x \cdot \frac{1}{2} y^{-\frac{1}{2}} = x \cdot -\frac{1}{4} y^{-\frac{3}{2}} = -x \dfrac{1}{4\sqrt{y^3}}$

$\Delta = \left(-2 \cdot \left(-\frac{1}{2}\right)\right) - \left(\frac{1}{2} \cdot \frac{1}{2}\right) = 1 - \frac{1}{4} = \frac{3}{4}$

$\partial xx < 0$ $\quad$ MAX. ✓

$\Delta > 0$

(6.) $f(x) = \dfrac{1}{x^2}$ $\quad \displaystyle\int_1^{+\infty} \frac{1}{x^2} = \int x^{-2}\, dx = \left.\frac{x^{-1}}{-1}\right|_1^{+\infty} = \frac{b^{-1}}{-1} - \left(\frac{1^{-1}}{-1}\right)$



$\lim_{b \to \infty}$

$= \dfrac{b^{-1}}{-1} + 1 = ?$

$\emptyset$

(4.) $\displaystyle\int_0^{4\pi} \sin^4 x\, dx = \int \sin^2 x \cdot \sin^2 x\, dx = \int \frac{1-\cos 2x}{2} \cdot \frac{1-\cos 2x}{2}\, dx$

$= \displaystyle\int \frac{1-\cos^4 x}{4} x = \frac{1}{4}\int \cos 4x = -\frac{1}{4}\sin 4x = -\frac{1}{4}\sin^4 2\pi + \left(-\frac{1}{4}\sin 0\right)$

$= \frac{1}{4} \neq 0 + 0 = 0$ ✗

$\emptyset$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxx	POPUNJAVA NASTAVNIK
IME I PREZIME: Ivan Klanac **BROJ INDEKSA:** 17-2-0098-2011	Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y = x^2 - e^x$ uz početne uvjete $y(0) = 1$ i $y'(0) = 0$.	15
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4. $\int_0^{2\pi} \sin^4 x \, dx = ?$	~~20~~
5. $\int_0^1 x^2 \tan(3x^3 + 1) \, dx = ?$	~~15~~
6. Neka je $f(x) = \frac{1}{x^2}$. Odrediti $\int_1^{+\infty} f(x)dx$. Skicirati graf funkcije f i površinu određenu integralom.	20
	Ukupno: ~~0~~

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2} dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

3. $z = \arctan(xy)$ u točki $M(1, 1, z_0)$

$f(x) = \arctan(xy)$

$z_0 = \arctan(1 \cdot 1) = \arctan 1 = \frac{\pi}{4}$

$M = (\overset{x_0}{1}, \overset{y_0}{1}, \overset{z_0}{\frac{\pi}{4}})$

~~0~~

$f_x = \arctan(xy) = \frac{1}{1+(xy)^2} \cdot (xy)' = \frac{1}{1+x^2+y^2} \cdot y = \frac{\cancel{y}}{1+x^2+\cancel{y^2}} = 1+x^2+y$ ×

$f_y = \arctan(xy) = \frac{1}{1+(xy)^2} \cdot (xy)' = \frac{1}{1+}$

(4) $\int_0^{2\pi} \sin^4 x\,dx =$

$\int \sin^4 x\,dx = \int \sin^3 x \sin x\,dx = \begin{bmatrix} u=\sin^3 x & dv=\sin x\,dx \\ du=3\sin^2 x\cos^3 x\,dx & v=-\cos x \end{bmatrix}$

$=-\sin^3 x\cos x+\int \cos x\, 3\sin^2 x\cos^3 x\,dx$

$=-\sin^3 x\cos x+3\int \sin^2 x+\cos^4 x\,dx$

$=-\sin^3 x\cos x+3\int \underbrace{\sin^2 \cancel{x}}_{i_1}+\int \cos^4 x\,dx$

$=-\sin^3 x\cos x+3\cdot\frac{1}{2}x+\frac{1}{4}\cos x+$

$U=\sin^2 x$ $\quad dv=\cos^4 x\,dx$

$du=2\sin x\cos x$ $\quad V=-\sin^4 x$

$i_1=\int \sin^2 x$

$=(\frac{1}{2}-\frac{1}{2}\cos 2x)dx$

$=\frac{1}{2}\int dx-\frac{1}{2}\int \sin 2x\,dx$

$=\frac{1}{2}x+\frac{1}{4}\cos x$

$\emptyset$

⑤ $\int_0^1 x^2 \tan(3x^3+1)\,dx =$

IVAN KLANAC

$-\frac{1}{9}\ln|\cos(3x^3+1)|\Big|_0^1$

OVO JE NEPRAVI INTEGRAL!

$= -\frac{1}{9}\ln|\cos(3\cdot 1^3+1)| - \left(-\frac{1}{9}\ln|\cos(3\cdot 0^3+1)|\right)$

$= -\frac{1}{9}\ln|\cos 4| + \frac{1}{9}\ln|\cos 1|$

$\emptyset$

$=$

$\int x^2 \tan(3x^3+1)\,dx = \begin{bmatrix} 3x^3+1=t & x^2\,dx = \frac{1}{9}dt \\ 9x^2\,dx = dt & \end{bmatrix}$

$\int \tan t \,\frac{1}{9}\,dt = \frac{1}{9}\int \tan t\,dt = -\frac{1}{9}\ln|\cos(3x^3+1)|$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxx

IME I PREZIME: TOMISLAV TUTA **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y = x^2 - e^x$ uz početne uvjete $y(0) = 1$ i $y'(0) = 0$. 15
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Ukupno: $\emptyset$

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
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$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

5.) $\int_0^1 x^2 \tan(3x^3+1)\, dx =$

$3x^3+1 - t$

$6x^2 dx - dt$

$dx = \frac{dt}{6x^2}$

$\int_0^1 \cancel{x^2} \tan(t) \frac{dt}{6\cancel{x^2}}$

$\int_0^1 \frac{1}{6} \tan(t)$ X X

X

$\int_0^1 \frac{1}{6} \cdot -\ln|\cos x|$

$0.1 - 0 = 0.1$ X $\emptyset$

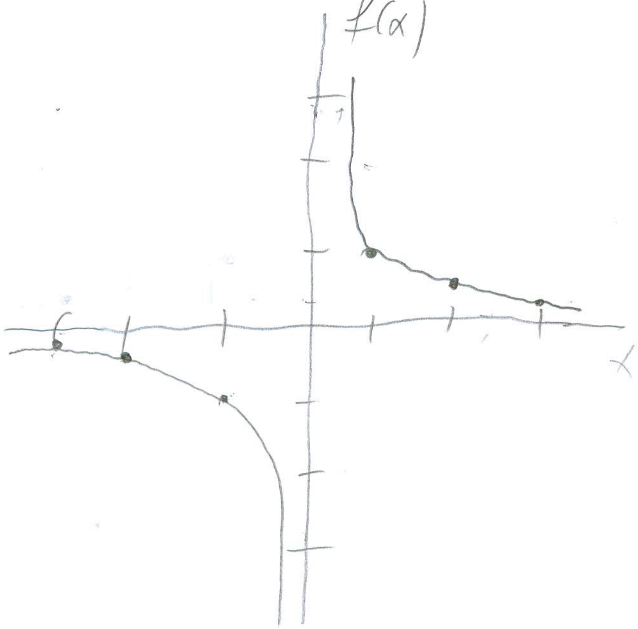
$= \left[-\frac{1}{6}\ln|\cos x|\right]_0^1 = -\frac{1}{6}\left[\ln|\cos 1| - \underbrace{\ln|\cos 0|}_{=0}\right]$

$\cos 1 = 0{,}54$

$\ln|\cos 1| = \ln 0{,}54 = -0.62$

6.) $f(x) = \frac{1}{x^2}$ $\int_{1}^{+\infty}$ $\int x^{-2} = \frac{x^{-1}}{-1}$ $\frac{1}{2\sqrt{x}}$

$\lim\limits_{x \to \infty} = \frac{x^{-1}}{-1} - \frac{x^{-1}}{-1} = \quad + 1$



f(x)	(x)
−3	$-\frac{1}{9}$
−2	$-\frac{1}{4}$
−1	−1
0	/
1	1
2	$\frac{1}{4}$
3	$\frac{1}{9}$

$\cos^2 x \cdot \sin^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

4) $\int_0^{2\pi} \sin^4 x \, dx =$

$\int_0^{2\pi} \sin^2 x \cdot \sin^2 x \, dx$

$\sin^2 x \cdot 1 - \int \cos^2 x \, dx$

$\sin x - t$

$\cos x \, dx - dt$

$\int_0^{2\pi} t^2 - dt$

$\left| \frac{t^3}{3} = \frac{\sin x^3}{3} = 0\right.$

TUTA

$$f(x,y) = x\sqrt{y} - x^2 - y + 3x$$

$$\frac{df}{dx} = \sqrt{y} - 2x + 3 \checkmark \qquad \frac{df}{dy} = \frac{x}{2\sqrt{y}} - 1 \checkmark$$

$$\frac{d^2f}{dx^2} = -2 \checkmark \qquad \frac{d^2f}{dy^2} = \frac{x}{2} \cdot \frac{1}{2\sqrt{y}} = \frac{x}{4\sqrt{y}} \times$$

$$\frac{df}{dx\,dy} = \frac{1}{2\sqrt{y}} \checkmark \qquad \Delta = \begin{vmatrix} -2 & \frac{1}{2\sqrt{y}} \\ \frac{1}{2\sqrt{y}} & \frac{x}{4\sqrt{y}} \end{vmatrix}$$

$$\sqrt{y} - 2x + 3 = 0 \qquad \frac{x}{2\sqrt{y}} - 1 = 0$$

$$\sqrt{y} = 2x - 3 \qquad \frac{x}{2\sqrt{y}} = 1$$

$$2x - 3 = \sqrt{y} \qquad \frac{x}{2(2x-3)} = 1$$

$$2 \cdot \frac{3}{2} - 3 = \sqrt{y} \qquad 2 = 0$$

$$0 = \sqrt{y} \qquad 2x - 3 = 0$$

$$y = 0 \qquad 2x = 3$$

$$x = \frac{3}{2}$$

$$\Delta = -2 \cdot \frac{x}{4\sqrt{y}} - \frac{1}{2\sqrt{y}} \cdot \frac{1}{2\sqrt{y}}$$

$\Delta = 0$ NEMA EKS

LOK MAX $\left(0, \frac{3}{2}\right)$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxx

IME I PREZIME: ROKO DUŠEVIĆ **BROJ INDEKSA:** 57351-2009

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y = x^2 - e^x$ uz početne uvjete $y(0) = 1$ i $y'(0) = 0$. 15
2. Odrediti lokalne ekstreme funkcije: $f(x, y) = x\sqrt{y} - x^2 - y + 3x$. 15
3. Odredi tangencijalnu ravninu i normalu na plohu $z = \arctan(xy)$ u točki $M(1, 1, z_0)$. 15
4. $\displaystyle\int_0^{2\pi} \sin^4 x \, dx = ?$ 20
5. $\displaystyle\int_0^{1} x^2 \tan\left(3x^3 + 1\right) dx = ?$ 15
6. Neka je $f(x) = \dfrac{1}{x^2}$. Odrediti $\int_1^{+\infty} f(x) dx$. Skicirati graf funkcije f i površinu određenu integralom. 20

Ukupno: Ø

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
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$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

2.) $f(x,y) = x\sqrt{y} - x^2 - y + 3x$

$$\frac{\partial f}{\partial x} = \sqrt{y} - 2x + 3$$

$$\frac{\partial f}{\partial y} = \frac{1}{2} y^{-\frac{1}{2}} x - 1$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxx	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: ŠIME-BORNA MAGAŠ **BROJ INDEKSA:** 17-2-0108-2011	

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y = x^2 - e^x$ uz početne uvjete $y(0) = 1$ i $y'(0) = 0$. 15

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Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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4) $\displaystyle\int_0^{2\pi} \sin^4 x\, dx$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	XXX	POPUNJAVA NASTAVNIK
IME I PREZIME: MARIO ŽMIRE	**BROJ INDEKSA:** 0269071808	Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y = x^2 - e^x$ uz početne uvjete $y(0) = 1$ i $y'(0) = 0$. 15
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Ukupno: 0

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$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
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