MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OXX
IME I PREZIME: TONI PASTUOVIĆ **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti $y'' - y = -x + 1$ i odrediti posebno rješenje koje udovoljava početnom uvjetu $x = 0,\ y = 1,\ y' = 0$. 20
2. Provjeriti da funkcija $f(x) = x\sin(2x) + 1$ zadovoljava diferencijalnu jednadžbu $y'' + 4y = 4\cos(2x) + 4$ i početne uvjete $y(0) = 1$ i $y'(0) = 0$. 10
3. Skicirati razinske krivulje, na skici označiti smjer rasta i diskutirati diferencijabilnost funkcije $f(x,y) = \frac{1}{x^2+y^2}$. ~~10+5+5~~ 0
4. Numeričkom integracijom odrediti vrijednost $\int_0^2 \sin(x^2)\,\mathrm{d}x$. (bodovanje: 20 za rel. grešku $\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$) 20
5. $\displaystyle\int_0^{\pi/2} \frac{\sin x\,\mathrm{d}x}{\cos x + 1} = ?$ ~~15~~
6. Integriranjem izračunati površinu trokuta zadanog točkama $A(0,-1)$, $B(1,0)$, $C(-1,1)$. ~~15~~

Ukupno: 45

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

3 $f(x,y) = \frac{1}{x^2+y^2}$

$D_{f(x,y)} = \mathbb{R}^2 / \{0\}$

$C = \frac{1}{x^2+y^2}$

$C=1$

$1 = \frac{1}{x^2+y^2}$

$x^2+y^2=1$

$y^2 = 1-x^2$

$y = \sqrt{1-x^2}$

$C=2$

$2 = \frac{1}{x^2+y^2} \ / \cdot x^2+y^2$

$2x^2+2y^2=1$

$y^2 = \frac{1}{2} - x^2$

$y = \sqrt{\frac{1}{2} - x^2}$

$C=0$

$0 = \frac{1}{x^2+y^2} \ / \cdot x^2+y^2$

$0 \neq 1$ nije u domeni

$C=3$

$3x^2+3y^2=1$

$3y^2 = 1-3x^2$

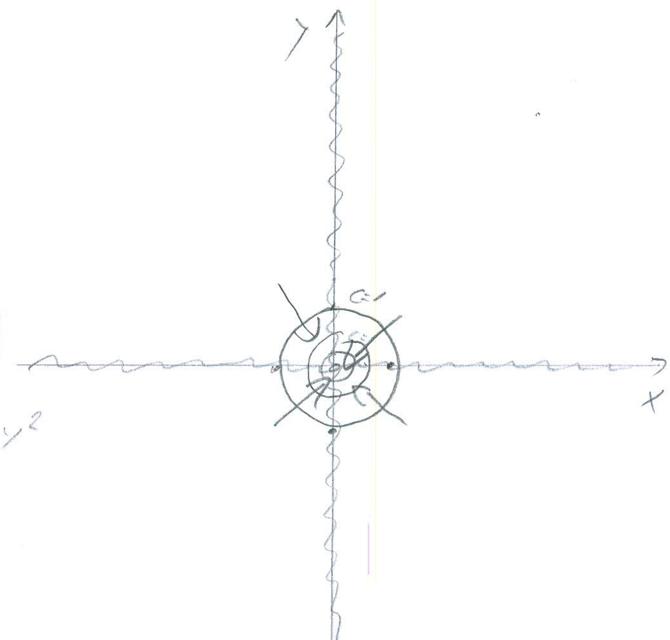
$y = \sqrt{\frac{1}{3} - x^2}$

$C=-1$

$-1 = \frac{1}{x^2+y^2} \ / \cdot x^2+y^2$

$-x^2-y^2=1$

$\sqrt{-x^2-1} = y$

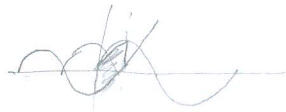


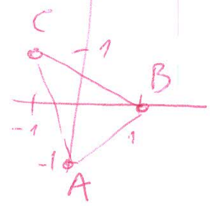
nije u domeni

(5) $\int_0^{\frac{\pi}{2}} \frac{\sin x\,dx}{\cos x+1} = \Big/ \begin{matrix} t=\cos x+1 \\ dt=-\sin x\,dx \\ dx=\frac{dt}{\sin x} \end{matrix} \Big/ = -\int_0^{\frac{\pi}{2}} \frac{\sin x}{t}\frac{dt}{\sin x} = -\int_0^{\frac{\pi}{2}} \frac{dt}{t} = -\ln|t|\Big|_0^{\frac{\pi}{2}} = -\ln|\cos x+1|\Big|_0^{\frac{\pi}{2}}$

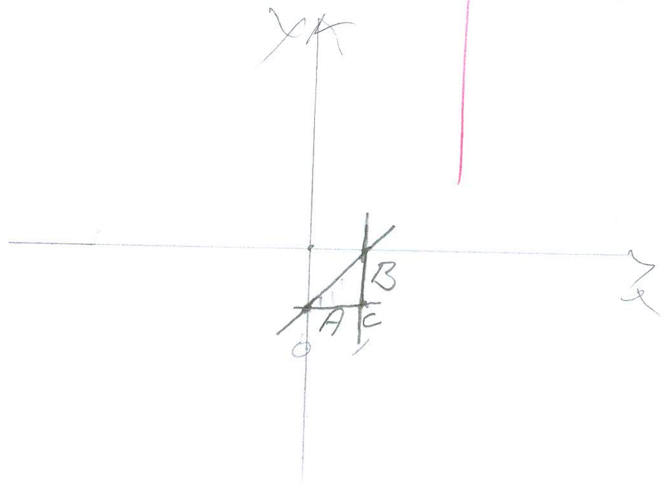
$= \ln|\cos\frac{\pi}{2}+1| - \ln|\cos 0+1| = \ln|0+1| - \ln|1+1| = 0 - 0{,}6931471 8$

$= -0{,}693$ ✗





(6) $A(0,-1)$
$B(1,0)$
$C(-1,1)$



$(x-x_1)(y_2-y_1) = (y-y_1)(x_2-x_1)$

AB

$(x-0)(0+1) = (y+1)(1-0)$

$x = y+1$

$y = x-1$

BC

$(x-1)(1-0) = (y-0)(-1-1)$

$x-1 = -2y$

$-2y = x-1$

$y = \frac{x-1}{-2}$

AC

$(x-0)(1+1) = (y+1)(-1-0)$

$2x = -y-1$

$y = -1-2x$

$P = \int_0^1 (AB-AC) = \int_0^1 ((x-1)-(-1-2x))\,dx$

$= \int_0^1 ((x-1)+1+2x)\,dx = \int_0^1 x\,dx - \cancel{1\int dx} + \cancel{\int dx} + 2\int x\,dx$

$= \frac{x^2}{2} - \cancel{x} + \cancel{x} + 2\frac{x^2}{2} = 0{,}5$ ✗

TONI PASTUOVIĆ

2)

$f(x) = x\sin(2x) + 1$

$y'' + 4y = 4\cos(2x) + 4$ $\quad y(0) = 1$

$y'(0) = 0$

$(x\sin(2x) + 1)'$

$x'\sin(2x) + x(\sin(2x))' \cdot 2$

$(\sin(2x) + 2x\cos(2x))'$

$\cos(2x)\cdot(2x)' + 2x(-\sin(2x)\cdot 2$

$4\cos(2x) - 4x\sin(2x) = y''$

$y(0) = x\sin(2x) + 1$

$y(0) = 0 \cdot \sin(0) + 1$

$y(0) = 1$ zadovoljava prvi uvjet

$y'(0) = \sin(2x) + 2x\cos(2x)$

za $y'(0) = 0 + 0 \cdot 1$

$y'(0) = 0$ zadovoljava drugi uvjet

$4\cos(2x) - \cancel{4x\sin(2x)} + \cancel{4x\sin(2x)} + 4 = 4\cos(2x) + 4$

$4\cos(2x) + 4 = 4\cos(2x) + 4$

funkcija zadovoljava diferencijalnu jednadžbu

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IME I PREZIME: MATE RADAŠ **BROJ INDEKSA:** 17-2-1083-2012	

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Ukupno: 0

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1. $y'' - y + x - 1 = 0$ $x = 0$

$y = 1$

$y' = 0$