MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xii

IME I PREZIME: MARIN MATEK **BROJ INDEKSA:** 17-1-0111-12

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti $y'' - y = -x + 1$ i odrediti posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. 20
2. Provjeriti da funkcija $f(x) = xe^x$ zadovoljava diferencijalnu jednadžbu $y'' - 2y' + y = 0$ i početne uvjete $y(0) = 0$ i $y'(0) = 1$. 15
3. Skicirati razinske krivulje funkcije $f(x, y) = \frac{1}{1+x^2+y^2}$. 15
4. Numeričkom integracijom odrediti vrijednost $\int_1^2 \frac{e^x}{x} dx$. (bodovanje: 20 za rel. grešku$\leq 3\%$, 15 za rel. grešku $\lessapprox 6\%$, 8 za rel. grešku $\lessapprox 10\%$) 20
5. $\int_0^{\pi} \frac{dx}{2\sin x - \cos x + 5} = ?$ 15
6. Integriranjem izračunati površinu trokuta zadanog točkama $A(0,0)$, $B(-2,2)$, $C(-1,-1)$. 15

Ukupno: 20

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

3. $f(x,y) = \frac{1}{1+x^2+y^2}$

$1+x^2+y^2 \neq 0$

$x^2 \neq -y^2 - 1$

$C = 2$

$\frac{1}{1+x^2+y^2} = 2 \mid \cdot$

$C = 1$

$\frac{1}{1+x^2+y^2} = 1 \mid \cdot$

$C = 4$

$\frac{1}{1+x^2+y^2} = 4 \mid \cdot$

$\frac{1}{1+x+y} = \sqrt{2} \mid \cdot 1+x+y$

$\frac{1}{1+x+y} = 1 \mid \cdot 1+x+y$

$\frac{1}{1+x+y} = 2 \mid \cdot 1+x+y$

$1 = \sqrt{2} + \sqrt{2}x + \sqrt{2}y$

$1 = 1+x+y$

$1 = 2 + 2x + 2y$

$\sqrt{2}x = 1 - \sqrt{2} - \sqrt{2}y$

$x = -y$

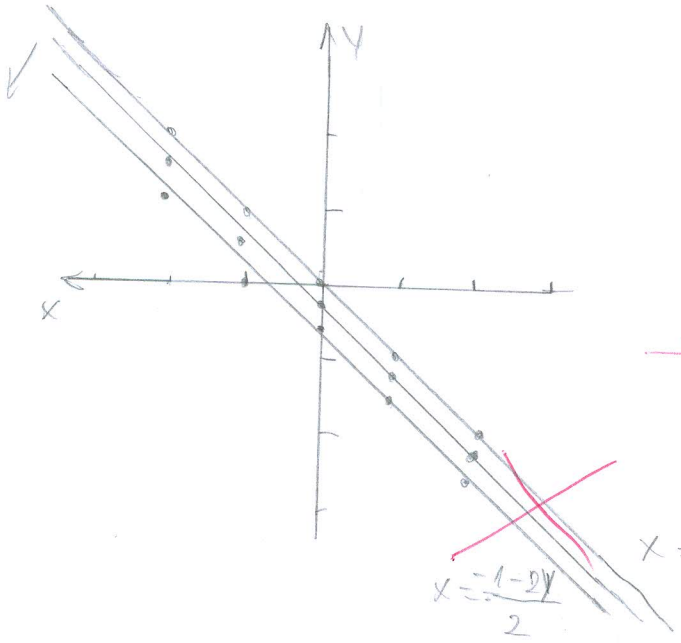
$x = \frac{1-\sqrt{2}-\sqrt{2}y}{\sqrt{2}}$

$x = \frac{-1-2y}{2}$

-2	-1	0	1	2
2	1	0	-1	-2

-2	-1	0	1	2
1,70	0,70	-0,3	-1,3	-2,29

-2	-1	0	1	2
1,5	0	-0,6	-1,5	-2,5



NIJE DOBRO

$x = -y$

$x = \frac{-1-2y}{2}$

$x = \frac{1-\sqrt{2}-\sqrt{2}y}{\sqrt{2}}$

4. $\int_1^2 \frac{e^x}{x}\,dx$

i	0	1	2
x_i	1	1,5	2
f_i	2,7182	2,98779	3,6945

$\frac{1}{6}\left(2{,}718281828 + 4 \cdot 2{,}98779274 + 3{,}694528049\right)$

$= 3{,}06066$ 20

5. $\int_0^{\pi} \frac{dx}{2\sin x - \cos x + 5}$

i	0	1	2
x_i	0	$\frac{\pi}{2}$	π
f_i	$\frac{1}{4}$	$\frac{1}{7}$	$\frac{1}{6}$

$\frac{\pi}{6}\left(\frac{1}{4} + 4 \cdot \frac{1}{7} + \frac{1}{6}\right)$

$= 0{,}51736$ 0

NE TRAŽI SE ~~A~~NUMERIČKA APROKSIMACIJA!

2. $y'' - 2y' + y = 0$

$$r_{1,2} = \frac{4 \pm \sqrt{4-4}}{2}$$

$r_1 = r_2 = 2$

$$y(x) = C_1 e^{2x} + C_2 x e^{2x}$$

$$y'(x) = 2C_1 e^{2x} + C_2 e^{2x} + 2C_2 x e^{2x}$$

$y'(0) = 1$ $\quad$ $y(0) = 0$

$0 = C_1 e^{2\cdot 0} + C_2 \cdot 0 e^{2\cdot 0}$

$1 = 2C_1 e^{2\cdot 0} + C_2 e^{2\cdot 0} + 2C_2 \cdot 0 e^{2\cdot 0}$

$0 = C_1$

$1 = 2C_1 + C_2$

OVO NIJE PROJETRA.

$-C_2 = 2C_1 - 1 \quad / \cdot (-1)$

$C_2 = -2C_1 + 1$

$C_2 = 1$

1. $y'' - y = -x + 1$

$y'' - y = 0$

$r^2 - 1 = 0$

$r = \pm 1$

$y_H(x) = C_1 e^{x} + C_2 e^{-x}$

$-x = e^{\alpha x}(P_m(x)\cos(\beta x) + Q_m(x)\sin(\beta x))$

$\alpha = 0$ $\quad$ $k = \max\{1, n/p\}$ $\quad$ $y_p = 0 + 0x = 0$

$\beta = 0$ $\quad$ $= 1$

$P_m = -x \quad m = 1$ $\quad$ $y_p = 0$

$1 = e^{\alpha x}(P_m(x)\cos(\beta x) + Q_m(x)\sin(\beta x))$

$\alpha = 0$ $\quad$ $k = \max\{0, n/p\}$ $\quad$ $y_p = 0$

$\beta = 0$ $\quad$ $= 0$

$P_m = 1 \quad m = 0$