

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: MARIN MATEK BROJ INDEKSA: 17-1-0111-12

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

xii

- Riješiti $y'' - y = -x + 1$ i odrediti posebno rješenje koje udovoljava početnom uvjetu $x = 0, y = 0, y' = 0$. 20
- Provjeriti da funkcija $f(x) = xe^x$ zadovoljava diferencijalnu jednadžbu $y'' - 2y' + y = 0$ i početne uvjete $y(0) = 0$ i $y'(0) = 1$. 15
- Skicirati razinske krivulje funkcije $f(x, y) = \frac{1}{1+x^2+y^2}$. 15
- Numeričkom integracijom odrediti vrijednost $\int_1^2 \frac{e^x}{x} dx$. (bodovanje: 20 za rel. grešku $\leq 3\%$, 15 za rel. grešku $\leq 6\%$, 8 za rel. grešku $\leq 10\%$) 20
- $\int_0^\pi \frac{dx}{2 \sin x - \cos x + 5} = ?$ 15
- Integriranjem izračunati površinu trokuta zadanog točkama $A(0, 0), B(-2, 2), C(-1, -1)$. 15

Ukupno: 20

f	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$a^x (\alpha > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

3. $f(x, y) = \frac{1}{1+x^2+y^2}$

$1+x^2+y^2 \neq 0$

$C=2$

$x^2 \neq -y^2 - 1$

$C=1$

$\frac{1}{1+x^2+y^2} = 2 / \sqrt{\quad}$

$C=4$

$\frac{1}{1+x^2+y^2} = 1$

$\frac{1}{1+x+y} = \sqrt{2} / \cdot (1+x+y)$

$\frac{1}{1+x^2+y^2} = 4 / \sqrt{\quad}$

$\frac{1}{1+x+y} = 1 / \cdot (1+x+y)$

$1 = \sqrt{2} + \sqrt{2}x + \sqrt{2}y$

$\frac{1}{1+x+y} = 2 / \cdot (1+x+y)$

$1 = 1 + x + y$

$\sqrt{2}x = 1 - \sqrt{2} - \sqrt{2}y$

$1 = 2 + 2x + 2y$

$x = -y$

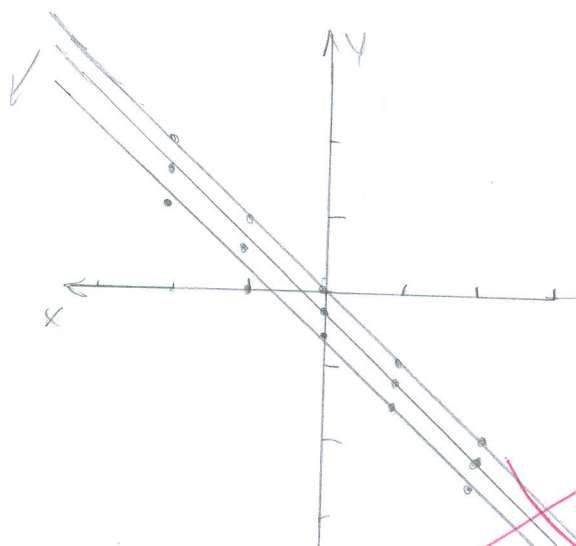
$y = \frac{1 - \sqrt{2} - \sqrt{2}y}{\sqrt{2}}$

$x = \frac{-1 - 2y}{2}$

$$\begin{array}{c|ccc|c|c} -2 & -1 & 0 & 1 & 2 \\ \hline 2 & 1 & 0 & -1 & -2 \end{array}$$

$$\begin{array}{c|ccc|c|c} -2 & -1 & 0 & 1 & 2 \\ \hline 1,70 & 0,70 & -0,3 & -1,9 & -2,29 \end{array}$$

$$\begin{array}{c|ccc|c|c} -2 & -1 & 0 & 1 & 2 \\ \hline 1,5 & 0 & -0,6 & -1,6 & -2,5 \end{array}$$



NIJE DOPRO

4. $\int_1^2 \frac{e^x}{x} dx$

l	0	1	2
x_0	1	1,5	2
f_0	2,718281828	2,9877492714	3,694528049

$$\frac{1}{6} \left(2,718281828 + 4 \cdot 2,9877492714 + 3,694528049 \right)$$

= 3,06066

20

5. $\int_0^\pi \frac{dx}{2 \sin x - \cos x + 5}$

l	0	1	2
x_0	0	$\frac{\pi}{2}$	π
f_0	$\frac{1}{4}$	$\frac{1}{7}$	$\frac{1}{6}$

$$\frac{\pi}{6} \left(\frac{1}{4} + 4 \cdot \frac{1}{7} + \frac{1}{6} \right)$$

= 0,51736

NE TRAZI SE ~~ANUMERIČKA~~
APROKSIMACIJA!

$$2y'' - 2y' + y = 0$$

$$r_{1,2} = \frac{4 \pm \sqrt{4-4}}{2}$$

$$r_1 = r_2 = 2$$

$$y(x) = c_1 e^{2x} + c_2 x e^{2x}$$

$$y'(x) = 2c_1 e^{2x} + c_2 e^{2x} + 2c_2 x e^{2x}$$

$$y'(0) = 1$$

$$y(0) = 0$$

$$1 = 2c_1 e^{2 \cdot 0} + c_2 e^{2 \cdot 0} + 2c_2 \cdot 0 e^{2 \cdot 0}$$

$$0 = c_1 e^{2 \cdot 0} + c_2 \cdot 0 e^{2 \cdot 0}$$

$$0 = c_1$$

$$1 = 2c_1 + c_2$$

$$-c_2 = 2c_1 - 1 \quad | \cdot (-1)$$

$$c_2 = -2c_1 + 1$$

$$c_2 = 1$$

OVO NIJE PROJEKTA.

$$1. y'' - y = -x + 1$$

$$y'' - y = 0$$

$$r^2 - 1 = 0$$

$$r = \pm 1$$

$$y_H(x) = c_1 e^x + c_2 e^{-x}$$

$$-x = e^{\alpha x} (P_m(x) \cos(\beta x) + Q_m(x) \sin(\beta x))$$

$$\alpha = 0$$

$$\beta = 0$$

$$P_m = -x \quad m = 1$$

$$N = \max\{1, \omega/p\}$$

$$= 1$$

$$b_2 = 0 + 0i = 0$$

$$b_1 = 0$$

$$1 = e^{\alpha x} (P_m(x) \cos(\beta x) + Q_m(x) \sin(\beta x))$$

$$\alpha = 0$$

$$\beta = 0$$

$$P_m = 1 \quad m = 0$$

$$N = \max\{0, \omega/p\}$$

$$= 0$$

$$b_2 = 0$$

MARIU MATEK

$$A(0, 0)$$

$$B(-2, 2)$$

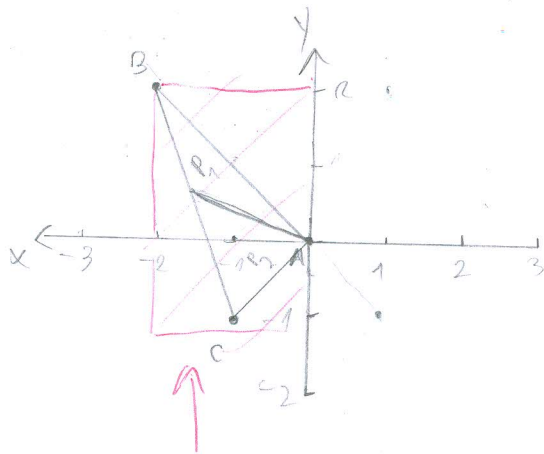
$$C(-1, -1)$$

$$\overline{AB} = y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1)$$

$$= y - 0 = \frac{2 - 0}{-2 - 0} \cdot (x - 0)$$

$$= y = -x$$

$$= x = -y$$



OVAJ PRAVOKUTNIK JE POUVRŠINE $P=6$,
PA TROKUT MORA BITI MANJE POUVRŠINE

$$\int_{-2}^0 P_1 + P_2$$

$$= \int_{-2}^0 (x + y + \frac{3}{2}x - \frac{y}{2} - 2) + (x - y - 1 + \frac{3}{2}x - \frac{y}{2} - 2) = \int_{-2}^0 (\frac{5}{2}x + \frac{y}{2} - 2) + (\frac{5}{2}x - \frac{3}{2}y - 3)$$

$$= \int_{-2}^0 5x - y - 5 = 5 \int x dx - \int y dy - \int 5 dx = \frac{5x^2}{2} - \frac{y^2}{2} - 5x \Big|_{-2}^0 = 0 - 13 = 13$$

$$\overline{AC} = y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y + 1 = \frac{-1 - 0}{-1 - 0} (x - 0)$$

$$y + 1 = x$$

$$x = y + 1$$

$$\overline{BC} = y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$= y - 2 = \frac{-1 - 2}{-1 + 2} (x + 2)$$

$$= y - 2 = -3x - 6$$

$$3x = -4 - y$$

$$x = \frac{-4 - y}{3}$$

~~0~~

MARIN MATEK

$$y_H(x) = x^0 e^{\lambda x} (S_n(x) \cos(\beta x) + T_n(x) \sin(\beta x))$$

$$= Ax + B$$

$$y' = A$$

$$y'' = 0$$

$$y_P(x) = x^2 e^{\lambda x} (S_n(x) \cos(\beta x) + T_n(x) \sin(\beta x))$$

$$= A$$

$$y' = 0$$

$$y'' = 0$$

KOJE JE RJEŠENJE?

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: **DINO CUITAN**

BROJ INDEKSA: **17-2-0068**

xii

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

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$A(0,0), B(-2,2), C(-1,-1)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$A(0,0)$
 $B(-2,2)$
 x_1, y_1
 x_2, y_2

$$y - 0 = \frac{2 - 0}{-2 - 0} (x - 0)$$

$$y - 0 = -1(x - 0)$$

$$y - 0 = -1x$$

$$y = -1x$$

$B(-2,2)$
 $C(-1,-1)$
 x_1, y_1
 x_2, y_2

$$y - 2 = \frac{-1 - 2}{-1 + 2} (x + 2)$$

$$y - 2 = \frac{-3}{1} (x + 2)$$

$$y - 2 = -3x - 6$$

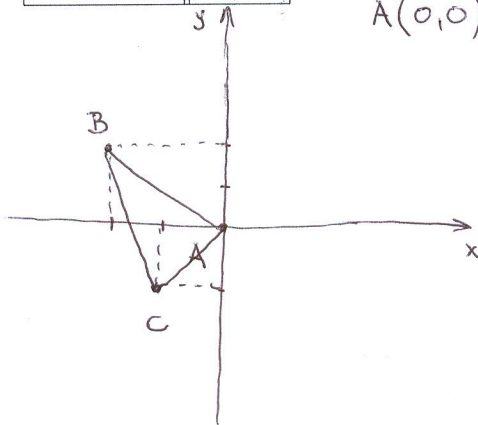
$$y = -3x - 4$$

$A(x_1, y_1)$
 $A(0,0)$
 $C(x_2, y_2)$
 $C(-1,-1)$

$$y - 0 = \frac{-1 - 0}{-1 - 0} (x - 0)$$

$$y - 0 = 1x$$

$$y = 1x$$



6.

$$P_1 = \int_{-1}^0 (1-x) dx + \int_0^1 (-1-x) dx$$

$$= \left[\frac{x}{2} - \frac{x^2}{2} \right]_{-1}^0 + \left[-x - \frac{x^2}{2} \right]_0^1$$

$$= \frac{1}{2} + \left(-1 - \frac{1}{2} \right)$$

$$P_2 = \int_0^2 = ?$$

$$P_1 = \int_{-1}^0 (1-x) dx + \int_0^1 (-3x-4) dx$$

$$= \left[\frac{x}{2} - \frac{x^2}{2} \right]_{-1}^0 + \left[-\frac{3x^2}{2} - 4x \right]_0^1$$

$$= \frac{1}{2} + \left(-\frac{3}{2} - 4 \right)$$

$$= \frac{1}{2} + \frac{11}{2}$$

$$= \frac{6}{2} = 3 //$$

$$\textcircled{3} \int_C (-y) = \frac{1}{1+x^2+y^2} \quad N \neq 0$$

$$1+x^2+y^2 \neq 0$$

$$x^2+y^2 \neq -1$$

$$r^2 = -1/v$$

$$r = i$$

SKICA?

$$5. \int_0^\pi \frac{dx}{2\sin x - \cos x + 5} = \int_0^\pi \frac{1}{2\sin x} dx - \int_0^\pi \frac{1}{\cos x} dx + \int_0^\pi 5 dx$$

$$= -\frac{1}{2} \cos x$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: DOMAGOJ GROZAN BROJ INDEKSA: 19-1-0056-2011

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

xii

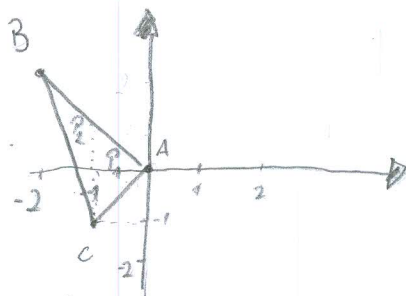
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ⓐ $A(0,0) \quad B(-2,2) \quad C(-1,-1)$
 $(y - y_1)(x_2 - x_1) = (y_2 - y_1)(x - x_1)$



AB $(y-0)(-2-0) = (2-0)(x-0)$

$-2y = 2x \quad | : -2$

$y = -x$

AC $(y-0)(-1-0) = (-1-0)(x-0)$

$-y = -x$

$y = x$

BC $(y-2)(-1+2) = (-1-2)(x-2)$

$-y+2y+2-4 = -x+2-2x+4$

$y-2 = -3x+6 \quad y = -3x+8$

$P_1 = \int_0^{-1} AC - AB$

$= ?$

$P_2 = \int_{-1}^{-2} AC - BC$

$\int_{-1}^{-2} -2x + 8 dx$

$= -2 \frac{x^2}{2} \Big|_{-1}^{-2}$

$= -4 - 1$

$= -5$

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IME I PREZIME: ANTE PAULOUIC

BROJ INDEKSA: 54959/2007

POPUNJAVA
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xii

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$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

3. $f(x, y) = \frac{1}{1+x^2+y^2}$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: DENIS ILIC

BROJ INDEKSA: 56194-2008

xii

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

- Riješiti $y'' - y = -x + 1$ i odrediti posebno rješenje koje udovoljava početnom uvjetu $x = 0, y = 0, y' = 0$. 20
- Provjeriti da funkcija $f(x) = xe^x$ zadovoljava diferencijalnu jednadžbu $y'' - 2y' + y = 0$ i početne uvjete $y(0) = 0$ i $y'(0) = 1$. 15
- Skicirati razinske krivulje funkcije $f(x, y) = \frac{1}{1+x^2+y^2}$. 15
- Numeričkom integracijom odrediti vrijednost $\int_1^2 \frac{e^x}{x} dx$. (bodovanje: 20 za rel. grešku $\leq 3\%$, 15 za rel. grešku $\leq 6\%$, 8 za rel. grešku $\leq 10\%$) 20
- $\int_0^\pi \frac{dx}{2 \sin x - \cos x + 5} = ?$ 15
- Integriranjem izračunati površinu trokuta zadanog točkama $A(0, 0), B(-2, 2), C(-1, -1)$. 15

Ukupno:

f	$\frac{df}{dx}$	Tablica nekih integrala		
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\ln x$	$\frac{1}{x}$	$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$	$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
e^x	e^x	$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\alpha^x (\alpha > 0)$	$\alpha^x \ln \alpha$	$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
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$\cos x$	$-\sin x$	$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	
$\tan x$	$\frac{1}{\cos^2 x}$			
$\cot x$	$\frac{-1}{\sin^2 x}$			
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$			
$\arctan x$	$\frac{1}{1+x^2}$			

③ $f(x, y) = \frac{1}{1+x^2+y^2}$

① $y'' - y = -x + 1$ poč. uvj. $x=0, y=0, y'=0$

⑤ $\int_0^\pi \frac{dx}{2 \sin x - \cos x + 5}$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: ROKO DUŠEVIĆ

BROJ INDEKSA: 57351-2009

xii

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

- Riješiti $y'' - y = -x + 1$ i odrediti posebno rješenje koje udovoljava početnom uvjetu $x = 0, y = 0, y' = 0$. 20
- Provjeriti da funkcija $f(x) = xe^x$ zadovoljava diferencijalnu jednadžbu $y'' - 2y' + y = 0$ i početne uvjete $y(0) = 0$ i $y'(0) = 1$. 15
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Ukupno:

f	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
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5) $\int_0^\pi \frac{dx}{2 \sin x - \cos x + 5} =$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

xii

IME I PREZIME: **ANDRO KLARIN**

BROJ INDEKSA:

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

- Riješiti $y'' - y = -x + 1$ i odrediti posebno rješenje koje udovoljava početnom uvjetu $x = 0, y = 0, y' = 0$. 20
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$\alpha^x (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
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