MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!**

xix

Ime i prezime: MIHOVIL PEDIŠIĆ

Broj indeksa: 17-2-0253-2012

Popunjava nastavnik

Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x\sin x$ i na kraju provjeriti rješenje. — 20
2. Riješi diferencijalnu jednadžbu $xyy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$. — 15
3. Obrazloži diferencijabilnost funkcije $f(x,y) = e^{xy} + 2y + x^2$. — 15
4. $\int_0^\pi \sin^5 x \cos^3 x \, dx = ?$ — 20
5. $\int_0^\pi \frac{\sin x \, dx}{1 - \sin x} = ?$ — 15
6. Integriranjem pronaći površinu trokuta koji je zadan točkama $A(3,3)$, $B(0,1)$, $C(3,-1)$. — 15

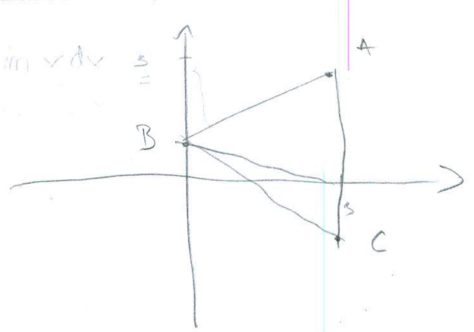
Ukupno: 30

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

6.



$(y-y_1)(x_2-x_1) = (y_2-y_1)(x-x_1)$

$BA \to (y-1)(3-0) = (3-1)(x-0)$

$3y - 3 = 2x$

$3y = 2x + 3 \ /:3$

$y = \frac{2}{3}x + 1$

$BC \to (y-1)(3-0) = (-2)(x-0)$

$3y - 3 = -2x$

$3y = -2x + 3$

$y = -\frac{2}{3}x + 1$

$\int_0^3 \left(\frac{2}{3}x+1\right) - \left(-\frac{2}{3}x+1\right) dx = \int_0^3 \frac{2}{3}x + 1 + \frac{2}{3}x - 1 dx = \int_0^3 \frac{4}{3}x \, dx = \frac{4}{3}\int_0^3 x \, dx =$

$\left.\frac{4}{3}\cdot\frac{x^2}{2}\right|_0^3 = \left(\frac{4}{3}\cdot\frac{9}{2}\right) - \left(\frac{4}{3}\cdot\frac{0}{2}\right) = 6$

2] $xyy' = 1 - x^2$ $\quad y(1) = 1$

$$yy' = \frac{1-x^2}{x}$$

$$y\frac{dy}{dx} = \frac{1-x^2}{x} \quad / \cdot dx$$

$$ydy = \frac{1-x^2}{x} \cdot dx \quad / \int$$

$$\int ydy = \int \underbrace{\frac{1-x^2}{x} dx}_{*}$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + \ln|x| + C \quad / \cdot 2$$

$$\boxed{y^2 = -x^2 + 2\ln|x| + C}$$

$$* \int \frac{1-x^2}{x} \; ; \quad \begin{array}{l} 1-x^2 : x = -x + \frac{1}{x} \\ \underline{-\;-x^2} \\ \quad 1 \end{array}$$

$$\int -x\,dx + \int \frac{1}{x}\,dx =$$

$$-\frac{x^2}{2} + \ln|x| + C$$

$$1 = -1 + 2\ln|1| + C$$

$$1 = -1 + C$$

$$C = 2$$

$y(1) = 1 \ldots \quad y^2 = -x^2 + 2\ln|x| + 2$ ✓

5] $\cancel{\int_0^{\pi} \frac{\sin x\, dx}{1 - \sin x}} = \int_0^{\pi} \sin x \cdot \frac{dx}{1 - \sin x} = \begin{cases} u = \sin x & dv = \frac{dx}{1 - \sin x} \;/ \int \\ du = \cos x & \end{cases} = \begin{cases} \sin x = t \\ dt = \end{cases}$

5) $$\int_0^{\pi} \frac{\sin x \, dx}{1-\sin x} = \left\{ \begin{array}{l} \sin x = \frac{2t}{1+t^2} \\ dx = \frac{2dt}{1+t^2} \end{array} \right\} = \int_0^{\pi} \frac{\frac{2t}{1+t^2} \cdot \frac{2dt}{1+t^2}}{1-\frac{2t}{1+t^2}} =$$

$t = \tan\frac{x}{2}$

OVO BI BIO DOBAR POSTUPAK.

$$= \int_0^{\pi} \frac{\frac{4t\,dt}{(1+t^2)^2}}{\frac{1+t^2-2t}{1+t^2}} = \int_0^{\pi} \frac{4t\,dt}{(1+t^2)(t-1)^2} = \int_0^{\pi} \underbrace{\frac{4t\,dt}{(1+t^2)}}_{x} \cdot \int_0^{\pi} \underbrace{\frac{4t\,dt}{(t-1)^2}}_{\#} =$$

$$x = \int \frac{dt}{1+t^2} = \operatorname{arctg} t$$

$$\# = \int \frac{4t\,dt}{(t-1)^2} = 4\int \frac{t+1-1}{(t-1)^2}dt = 4\int \frac{t-1}{(t-1)^2}dt + 4\int \frac{dt}{(t-1)^2}$$

$$= 4\ln|t-1| - \frac{1}{t-1}$$

$$= \left(\operatorname{arctg}\left(\operatorname{tg}\frac{x}{2}\right) \cdot \left(4\ln\left|\operatorname{tg}\frac{x}{2}-1\right| - \frac{1}{\operatorname{tg}\frac{x}{2}-1}\right) \right)\Bigg|_0^{\pi}$$

0

=

MIHOVIL PEDISIC

5] $\int_0^{\pi} \frac{1-1+\sin x}{1-\sin x}dx = \int_0^{\pi}\frac{\cancel{1-\sin x}}{\cancel{1-\sin x}}dx + \int_0^{\pi}\frac{dx}{1-\sin x} =$

$\int \Rightarrow \frac{1}{2}\ln\left|\frac{1+\sin x}{1-\sin x}\right|$

$\int_0^{\pi} dx + \int_0^{\pi}\frac{dx}{\sqrt{1}-\sqrt{\sin x}}$

$= \left(x + \frac{1}{2}\ln\left|\frac{1+\sin x}{1-\sin x}\right|\right)\Big|_0^{\pi} = \left(\pi + \frac{1}{2}\ln\left|\frac{1+\sin\pi}{1-\sin\pi}\right|\right) -$

$\left(\frac{1}{2}\ln\left|\frac{1+\sin 0}{1-\sin 0}\right|\right) = \underline{\pi}$

$\underset{0}{\shortparallel}$

$\emptyset$

4] $\int_0^{\pi} \sin x \sin x \sin x \sin x \sin x \cos x \cos x \cos x\, dx$ $\begin{cases}\sin^5 x = t\\ dt = 5\cos^4 x\, dx\end{cases}$

$dv = \frac{dt}{\cos t}$

$5\int_0^{\pi}\frac{t}{\cos t}dt = \left[\begin{matrix} u=t & dv=\cos t\,dt\\ du=dt & v=\sin t\end{matrix}\right] = t\sin t - \int \sin t\,dt =$

$= t\sin t + \cos t = \left(\sin^5 x\sin^6 x + \cos\sin^5 x\right)\Big|_0^{\pi} =$

$= \left(\sin^{11}x + \cos\sin^5 x\right)\Big|_0^{\pi} = \left(\sin^{11}\pi + \cos\sin^5\pi\right) -$

$\left(\sin^{11}0 + \cos\sin^5 0\right) = \underline{\underline{1}}$

$\emptyset$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xix

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: MATEO VUCIĆ **BROJ INDEKSA:** 17-2-0036-2010

1. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x \sin x$ i na kraju provjeriti rješenje. 20
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6. Integriranjem pronaći površinu trokuta koji je zadan točkama $A(3, 3)$, $B(0, 1)$, $C(3, -1)$. 15

Ukupno: 0

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
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6.) $A(3,3)$ $B(0,1)$ $C(3,-1)$

$x_1 \; y_1 \quad x_2 \; y_2$

$\overline{AB}$ $A(3,3)$ $B(3,-1)$

$(x_2 - x_1)(y - y_1) = (y_2 - y_1)(x - x_1)$

$(3-3)(y-3) = (-1-3)(x-3)$

$0(y-3) = (-4)(x-3)$

$-3y = (-4x) + 12$

$-3y + 4x = 12$ /:3

$\overline{AB}$ $y + \frac{4}{3}x = 4$

$y = 4 - \frac{4}{3}x$

BC

$x_1 \; y_1 \quad x_2 \; y_2$

$B(0,1)$ $C(3,-1)$

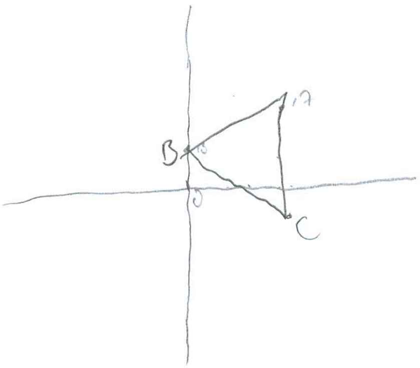
$(3-0)(y-1) = (-1-1)(x-0)$

$3(y-1) = -2x$

$3y - 3 = -2x$ /:3

$y - 1 = -\frac{2}{3}x$

$\overline{BC}$ $y = -\frac{2}{3}x + 1$



$\int_0^{0} \left[4 - \frac{4}{3}x - \left(-\frac{2}{3}x + 1\right)\right] = \int_0^{3} \left[4 - \frac{4}{3}x + \frac{2}{3}x - 1\right]$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xix

POPUNJAVA NASTAVNIK	Broj ↓ bodova

IME I PREZIME: ANTONIO BABARA **BROJ INDEKSA:** 17-2-0282-2013

1. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x \sin x$ i na kraju provjeriti rješenje. 20
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Ukupno: 0

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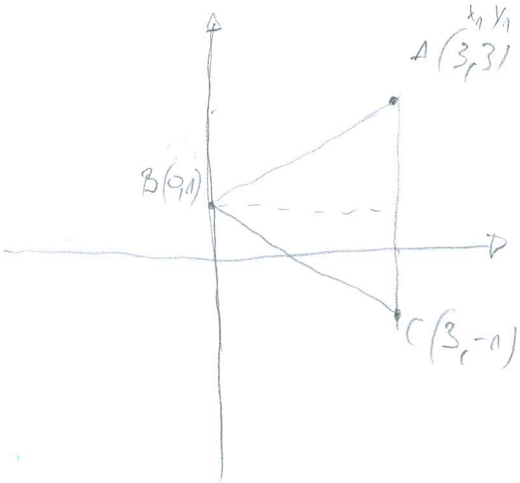
6. $A(3,3)$ $B(0,1)$ $C(3,-1)$



AB $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 3 = \frac{1-3}{0-3}(x-3)$

$y - 3 = \frac{2}{3}x - 2$

$y = \frac{2}{3}x + 1$

BC $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 1 = \frac{-1-1}{3-0}(x - 0)$

$y - 1 = \frac{-2}{3}x$

$y = -\frac{2}{3}x + 1$

CA $= y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y + 1 = \frac{3+1}{3-3}(x-3)$

$y + 1 = 0\,(x-3)$

$y = -1$

VIDI NAPOMENU KOD MAHADIĆ

$$\int_0^3 \frac{2}{3}x+1+1\,dx = \int_0^3 \frac{2}{3}x+2\,dx = \int \frac{2}{3}x\,dx + \int 2\,dx = \frac{2}{3}\int x\,dx + 2\int dx$$

$$= \frac{2}{3}\cdot\frac{x^2}{2} + 2x = \frac{x^2}{3} + 2x$$

$$P = \left(\frac{x^2}{3}+2x\right)\Big|_0^3 = \frac{9}{3} + 6 - (0) = 9$$

$$\int_0^3 -1+\frac{2}{3}x-1\,dx = \int_0^3 \frac{2}{3}x-2\,dx = \frac{2}{3}\int x\,dx - 2\int dx = \frac{2}{3}\cdot\frac{x^2}{2} - 2x$$

$$= \frac{x^2}{3} - 2x$$

$$P = \left(\frac{x^2}{3}-2x\right)\Big|_0^3 = \left(\frac{9}{3}-6\right) - (0) = 3$$

$$\underline{P = 12}$$

2. $xyy' = 1+x^2$

uz početni uvjet $y(1) = 1$

$xyy' = 1-x^2 \,/\, xy$

$y = 1 \qquad x = 1$

$y' = \frac{1-x^2}{xy} \,/\, y$

$1 = 2\ln 1 - 1$

$1 = -1$

$y'y = \frac{1-x^2}{x}$

KOJE JE RJEŠENJE?

$y'y = \frac{1}{x} - x$

$\frac{dy}{dx}\cdot y = \frac{1}{x} - x \,/\, dx$

$y\,dy = \left(\frac{1}{x} - x\right) dx \,/\, \int$

$\int y\,dy = \int \frac{1}{x}\,dx - \int x\,dx$

$\frac{y^2}{2} = \ln x - \frac{x^2}{2} + c \,/\, 2$

$y^2 = 2\ln x - x^2 + c \,/\, \sqrt{}$

$y = \pm\sqrt{2\ln x - x^2 + c}$

na krajū provjeri !!!

1. $4y'' - y = 2x \sin x$

$k^2 - k = 0$

$k(k-1) = 0$

$k_1 = 0 \quad k_2 = 1$

$yH = C_1 e^{k_1 x} + C_2 e^{k_2 x}$

$yH = C_1 e^{0x} + C_2 e^{x} \qquad \alpha = 0 \quad r = 1$

$y = 2x \sin x \, (Ax + B)$

$Y = yH + y$

$y = C_2 e^x + 2x \sin x \, (Ax + B)$ X

PROVJERA

$y' = C_2 e^x + C_2 e^x + (2x \sin x)'(Ax+B) + 2x \sin x (Ax+B)'$

$y' = C_2 e^x + C_2 e^x + (2 \sin x + 2x \cos x)(Ax+B) + (2x \sin x)(A)$

$y'' = C_2 e^x + C_2 e^x + C_2 e^x + C_2 e^x + (2 \sin x + 2x \cos x)'(Ax+B) + (2 \sin x + 2x \cos x)(Ax+B)' +$

$(2x \sin x)'(A) + (2x \sin x)(A)'$

$y'' = 4C_2 e^x + (2 \cos x + 2x \sin x)(Ax+B) + (2 \sin x + 2x \cos x)(A) + (2 \sin x + 2x \cos x)(A)$

$8C_2 e^x + 4(2\cos x - 2x \sin x)(Ax+B) + 4(\sin x + 2x \cos x)(A) + 4(2 \sin x + 2x \cos x)(A)$

$- C_2 e^x + 2x \sin x = 2x \sin x$

?

0

3. $f(x,y) = e^{xy} + 2y + x^2$

$f_x = e^y + 2x$ $\qquad e^y + 2x = 0$ $\qquad e^x + 2 = 0$

$f_y = e^x + 2$ $\qquad 2x = -e^y$ $\qquad e^x = -2 \,/\ln$

$x =$

?

4. $\int_0^{\pi} \sin^5 x \cos^3 x \, dx = \int \left(\frac{1}{1+t^2}\right)^5 \left(\frac{1}{1-t^2}\right)^3 dx = \int \frac{1}{(1+t^2)^5 (1-t)^3} dx$

?

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Popunjava nastavnik

Ime i prezime: Luka Radaš **Broj indeksa:** 57662

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$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha \, dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x \, dx = -\ln \lvert\cos x\rvert + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln \lvert x\rvert + C$	$\int \cot x \, dx = \ln \lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
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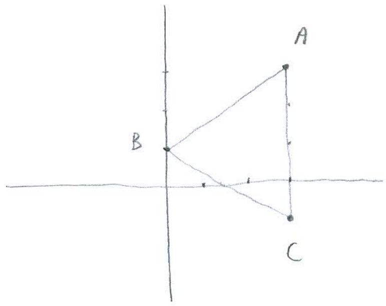
6.) $A(3,3)$

$B(0,1)$

$C(3,-1)$



$\overline{BA}$ $(y-1)(3-0) = (3-1)(x-0)$

$3y - 3 = 2x$

$3y - 2x - 3 = 0$

$x_{1,2} = \frac{2 \pm \sqrt{4 + 36}}{6}$?

$AC = (y-3)(3-3) = (-1-3)(x-3)$

$0 = -4x + 12$

$4x = 12$

$x = 3$

$\overline{BC}$: $(y-1)(3-0) = (-1-1)(x-0)$

$= 3y - 3 = -2x$

$3y + 2x - 3 = 0$

$x_{12} = \frac{-2 \pm \sqrt{4 + 36}}{6}$?

6.)

$$\int_0^3$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** xix

Popunjava nastavnik

Ime i prezime: Frane Mahadić

Broj indeksa: 17-1-0077-2011

	Broj ↓ bodova
1. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x \sin x$ i na kraju provjeriti rješenje.	20
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4. $\displaystyle\int_0^\pi \sin^5 x \cos^3 x \, \mathrm{d}x = ?$	20
5. $\displaystyle\int_0^\pi \frac{\sin x \, \mathrm{d}x}{1 - \sin x} = ?$	15
6. Integriranjem pronaći površinu trokuta koji je zadan točkama $A(3, 3)$, $B(0, 1)$, $C(3, -1)$.	~~15~~
Ukupno:	0

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
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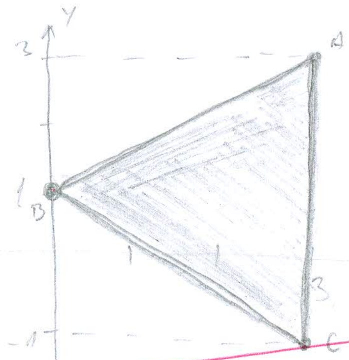
Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha \, \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, \mathrm{d}x = -\ln \lvert \cos x \rvert + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln \lvert x \rvert + C$	$\int \cot x \, \mathrm{d}x = \ln \lvert \sin x \rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int e^x \, \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, \mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, \mathrm{d}x = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x \, \mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2} \, \mathrm{d}x = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

5) $\displaystyle\int_0^\pi \frac{\sin x \, dx}{1 - \sin x} =$

$A(3,3),\ B(0,1)\quad C(3,-1)$



KORISTITE "BOLJU" FORMULU

$(y-y_1)(x_2-x_1)=(x-x_1)(y_2-y_1)$

AC $\quad A(3,3),\ C(3,-1)$ ($x_1\ y_1$, $x_2\ y_2$)

$$(y-y_1)=\frac{y_2-y_1}{x_2-x_1}\cdot(x-x_1)$$

$$(y-3)=\frac{-1-3}{3-3}\cdot(x-3)$$

$$(y-3)=\frac{-4}{0}\cdot(x-3)$$

$$\underline{y=3} \quad \times$$

AB $\quad A(3,3)\ B(0,1)$ ($x_1\ y_1$, $x_2\ y_2$)

$$(y-y_1)=\frac{y_2-y_1}{x_2-x_1}\cdot(x-x_1)$$

$$(y-3)=\frac{1-3}{0-3}\cdot(x-3)$$

$$y-3=\frac{-2}{-3}\cdot(x-3)$$

$$y-3=\frac{2}{3}\cdot(x-3)$$

$$y-3=\frac{2}{3}x-2+3$$

$$\underline{y=\frac{2}{3}x+1}$$

BC $\quad B(0,1),\ C(3,-1)$ ($x_1\ y_1$, $x_2\ y_2$)

$$(y-y_1)=\frac{y_2-y_1}{x_2-x_1}\cdot(x-x_1)$$

$$(y-1)=\frac{-1-1}{3-0}\cdot(x-0)$$

$$(y-1)=\frac{-2}{3}\cdot(x-0)$$

$$(y-1)=-\frac{2}{3}x$$

$$\underline{y=-\frac{2}{3}x+1}$$

$$\int_1^3\left(\frac{2}{3}x+1\right)-\left(-\frac{2}{3}x+1\right)=\int_1^3\left(\frac{2}{3}x+1\right)+\frac{2}{3}x-1=\frac{2}{3}\int_1^3 x+1\,dx+\frac{2}{3}\int_0^3 x-1\,dx$$

$\times$

$$=\frac{2}{3}\frac{x^2}{2}+x+\frac{2}{3}\cdot\frac{x^2}{2}-x\,\Big|_1^3$$

$$P_1=\frac{2}{3}\cdot\frac{3^2}{2}+3+\frac{2}{3}\cdot\frac{3^2}{2}-3=6$$

$$P_2=\frac{2}{3}\cdot\frac{1^2}{2}+1+\frac{2}{3}\cdot\frac{1^2}{2}-1=\frac{2}{3}$$

$$P=P_1+P_2=\frac{20}{3}=\underline{6.66666}$$

0

FRANE MAHADIĆ

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xix

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: Antonio Pribil **BROJ INDEKSA:** 57666

1. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x\sin x$ i na kraju provjeriti rješenje. 20
2. Riješi diferencijalnu jednadžbu $xyy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$. 15
3. Obrazloži diferencijabilnost funkcije $f(x, y) = e^{xy} + 2y + x^2$. 15
4. $\int_0^{\pi} \sin^5 x \cos^3 x \, dx = ?$ 20
5. $\int_0^{\pi} \frac{\sin x \, dx}{1 - \sin x} = ?$ 15
6. Integriranjem pronaći površinu trokuta koji je zadan točkama $A(3, 3)$, $B(0, 1)$, $C(3, -1)$. 15

Ukupno: ∅

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
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Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha \, dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
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$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** xix

Popunjava nastavnik

Ime i prezime: ROKO KADIJA **Broj indeksa:** 54957-2007

	Broj ↓ bodova
1. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x \sin x$ i na kraju provjeriti rješenje.	20
2. Riješi diferencijalnu jednadžbu $xyy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$.	15
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MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xix

POPUNJAVA NASTAVNIK

IME I PREZIME: Antonijo Knežević **BROJ INDEKSA:** 57672

	Broj ↓ bodova
1. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x \sin x$ i na kraju provjeriti rješenje.	20
2. Riješi diferencijalnu jednadžbu $xyy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$.	15
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Ukupno:	0

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$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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4. $\int_0^{\pi} \sin^5 x \cos^3 x \, dx =$