

IME I PREZIME: MILHOVIĆ PEDIŠIĆ

BROJ INDEKSA: 17-2-0253-2012

1. Riješiti diferencijalnu jednačinu: $4y'' - y = 2x \sin x$ i na kraju provjeriti rješenje.

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2. Riješi diferencijalnu jednačinu $xyy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$.

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3. Obrazloži diferencijabilnost funkcije $f(x, y) = e^{xy} + 2y + x^2$.

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4. $\int_0^{\pi} \sin^5 x \cos^3 x dx = ?$

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5. $\int_0^{\pi} \frac{\sin x dx}{1 - \sin x} = ?$

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6. Integriranjem pronaći površinu trokuta koji je zadan točkama $A(3, 3)$, $B(0, 1)$, $C(3, -1)$.

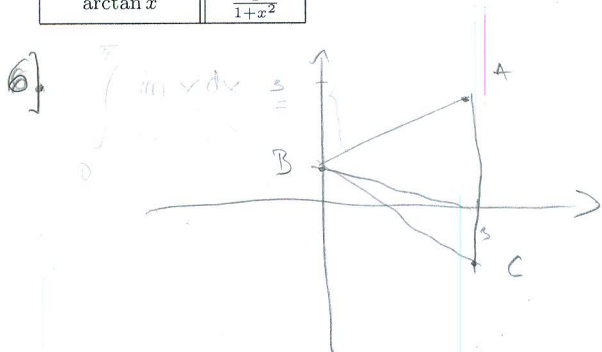
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Ukupno:

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f	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$a^x (a > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2} + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin(\frac{x}{a})] + C$	



$(y - y_1)(x_2 - x_1) = (y_2 - y_1)(x - x_1)$

BA $\rightarrow (y - 1)(3 - 0) = (3 - 1)(x - 0)$

$3y - 3 = 2x$

$3y = 2x + 3 \quad | :3$

$y = \frac{2}{3}x + 1$

BC $\rightarrow (y - 1)(3 - 0) = (-2)(x - 0)$

$3y - 3 = -2x$

$3y = -2x + 3$

$y = -\frac{2}{3}x + 1$

$\int_0^3 \left(\frac{2}{3}x + 1 \right) - \left(-\frac{2}{3}x + 1 \right) dx = \int_0^3 \left(\frac{2}{3}x + 1 + \frac{2}{3}x - 1 \right) dx = \int_0^3 \frac{4}{3}x dx = \frac{4}{3} \int_0^3 x dx =$

$\left(\frac{4}{3} \cdot \frac{x^2}{2} \right) \Big|_0^3 = \left(\frac{4}{3} \cdot \frac{9}{2} \right) - \left(\frac{4}{3} \cdot \frac{0}{2} \right) = 6$

$$2] \quad xy' = 1 - x^2 \quad y(1) = 1$$

$$yy' = \frac{1-x^2}{x}$$

$$y \frac{dy}{dx} = \frac{1-x^2}{x} \cdot dx$$

$$y dy = \frac{1-x^2}{x} \cdot dx \int$$

$$\int y dy = \int \frac{1-x^2}{x} dx$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + \ln|x| + C \quad | \cdot 2$$

$$\boxed{y^2 = -x^2 + 2\ln|x| + C}$$

$$y(1) = 1 \quad \dots \quad y^2 = -x^2 + 2\ln|x| + 2 \quad \checkmark$$

$$5] \quad \int_0^{\pi} \frac{\sin x dx}{1 - \cos x} = \int_0^{\pi} \frac{u = \sin x \quad du = \frac{dx}{1 - \cos x}}{u = \cos x \quad 1 - \cos x} = \begin{cases} u = \sin x \\ du = \cos x \end{cases} \quad \left\{ \begin{array}{l} \frac{du}{1 - \cos x} \\ \frac{du}{1 - \cos x} \end{array} \right.$$

$$* \int \frac{1-x^2}{x} \quad ; \quad \frac{1-x^2}{x} : x = -x + \frac{1}{x}$$

$$\int -x dx + \int \frac{1}{x} dx =$$

$$-\frac{x^2}{2} + \ln|x| + C$$

$$1 = -1 + 2\ln|1| + C$$

$$1 = -1 + C$$

$$C = 2$$

