

IME I PREZIME:

BROJ INDEKSA:

*Radović Rikardo RR*

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete:  $y'' + 3y' = 0$ , uz  $y(0) = 0$  i  $y(3) = 0$ .  
Na kraju provjeri rješenje.

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2. Nađi koliko iznosi  $f(2.5)$  ako  $f$  zadovoljava  $dy = (1 + x + \frac{y}{1-x^2}) dx$  i  $y(0) = 0$ .

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3. Za funkciju  $f(x, y) = \frac{x}{y}$  odrediti domenu, kodomenu, razinske krivulje i limes u ishodištu (ako postoji).

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4.  $\int_0^{\pi/2} e^x \cos x dx = ?$

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5.  $\int_{-\pi/2}^{\pi/2} \tan x dx = ?$

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6. Integriranjem izračunati površinu trokuta zadanog točkama  $A(0, 0)$ ,  $B(-2, 2)$ ,  $C(-1, -1)$ .

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Ukupno:

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$f$	$\frac{df}{dx}$	Tablica nekih integrala		
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\ln x$	$\frac{1}{x}$	$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln  \cos x  + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$	$\int \frac{dx}{x} = \ln  x  + C$	$\int \cot x dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln  x + \sqrt{x^2 \pm a^2}  + C$
$e^x$	$e^x$	$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\alpha^x (\alpha > 0)$	$\alpha^x \ln \alpha$	$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\sin x$	$\cos x$	$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$	
$\cos x$	$-\sin x$	$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$	
$\tan x$	$\frac{1}{\cos^2 x}$			
$\cot x$	$\frac{-1}{\sin^2 x}$			
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$			
$\arctan x$	$\frac{1}{1+x^2}$			

5.  $\int_{-\pi/2}^{\pi/2} \tan x dx = \int_{-\pi/2}^{\pi/2} \frac{\sin x}{\cos x} dx = \int_{\cos(-\pi/2)}^{\cos(\pi/2)} \frac{\sin x}{u} \left(-\frac{1}{\sin x}\right) du = -\int_1^{-1} \frac{du}{u} = \int_1^{-1} \frac{du}{u}$

$\int f(g(x)) \cdot g'(x) dx = \int f(u) du, u = g(x)$   
 $u = \cos x \quad du = -\sin x dx = -\frac{1}{\sin x} du$

$= (\ln |x|) \Big|_0^1 = \ln 1 = 0$

$$\textcircled{1} \int_0^{\frac{\pi}{2}} e^x \cos x dx = \begin{cases} u = e^x & du = e^x \\ dv = \cos x & v = \sin x \end{cases}$$

$$= (e^x \cdot \sin x) \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x \sin x dx = \begin{cases} u = e^x & du = e^x \\ dv = \sin x & v = -\cos x \end{cases}$$

$$= (e^x \cdot \sin x) \Big|_0^{\frac{\pi}{2}} - (e^x \cdot (-\cos x)) \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x - (-\cos x) dx$$

$$= (e^x \cdot \sin x) \Big|_0^{\frac{\pi}{2}} + (e^x \cdot \cos x) \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x \cdot \cos x dx \rightarrow \text{ISTI KAO NA POČETKU}$$

$$= \frac{e^x (\sin x + \cos x)}{2} \Big|_0^{\frac{\pi}{2}}$$

$$= \frac{e^{\frac{\pi}{2}} (\sin \frac{\pi}{2} + \cos \frac{\pi}{2})}{2} - \frac{e^0 (\sin 0 + \cos 0)}{2}$$

$$= \frac{e^{\frac{\pi}{2}} (1+0)}{2} - \frac{1(0+1)}{2} = \frac{e^{\frac{\pi}{2}}}{2} - \frac{1}{2} //$$

20 ✓  $\frac{e^{\frac{\pi}{2}}}{2} - \frac{1}{2} \approx 1.9$

② Koliko iznosi  $f(2,5)$  ako  $f$  zadovoljava:  $dy = (1+x + \frac{y}{1-x^2}) dx$  i

$y(0) = 0$

$dy = (1+x + \frac{y}{1-x^2}) dx, y(0) = 0$

$f(2,5) = f(x) = \frac{y'(1-2,5^2) + 2 \cdot 2,5 y}{(2,5)^4 - 2 \cdot (2,5)^2 + 1}$

$\frac{dy}{dx} = (1+x + \frac{y}{1-x^2})$  ~~1~~ ~~x~~

$y'' = 0 + 1 + (\frac{y}{1-x^2})'$  ~~x~~

$y'' = 1 + \frac{y' \cdot (1-x^2) - y \cdot (1-x^2)'}{(1-x^2)^2}$

$y'' = 1 + \frac{y'(1-x^2) + y \cdot 2x}{1-2x^2 + x^4}$

$y'' = 1 + \frac{y'(1-x^2) + 2xy}{x^4 - 2x^2 + 1}$

~~0~~

