

IME I PREZIME:

BROJ INDEKSA:

Radović Rikardo RR

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' + 3y' = 0$, uz $y(0) = 0$ i $y(3) = 0$.
Na kraju provjeri rješenje.

15

2. Nađi koliko iznosi $f(2.5)$ ako f zadovoljava $dy = (1 + x + \frac{y}{1-x^2}) dx$ i $y(0) = 0$.

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3. Za funkciju $f(x, y) = \frac{x}{y}$ odrediti domenu, kodomenu, razinske krivulje i limes u ishodištu (ako postoji).

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4. $\int_0^{\pi/2} e^x \cos x dx = ?$

20

5. $\int_{-\pi/2}^{\pi/2} \tan x dx = ?$

15

6. Integriranjem izračunati površinu trokuta zadanog točkama $A(0, 0)$, $B(-2, 2)$, $C(-1, -1)$.

15

Ukupno:

20

| f | $\frac{df}{dx}$ | Tablica nekih integrala | | |
|------------------------------|--------------------------|--|---|---|
| $x^\alpha (\alpha \neq 0)$ | $\alpha x^{\alpha-1}$ | $\int dx = x + C$ | $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$ | $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$ |
| $\ln x$ | $\frac{1}{x}$ | $\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$ | $\int \tan x dx = -\ln \cos x + C$ | $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$ |
| $\log_\alpha x (\alpha > 0)$ | $\frac{1}{x \ln \alpha}$ | $\int \frac{dx}{x} = \ln x + C$ | $\int \cot x dx = \ln \sin x + C$ | $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2} + C$ |
| e^x | e^x | $\int e^x dx = e^x + C$ | $\int \frac{dx}{\cos^2 x} = \tan x + C$ | $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$ |
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| $\sin x$ | $\cos x$ | $\int \sin x dx = -\cos x + C$ | $\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$ | |
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5. $\int_{-\pi/2}^{\pi/2} \tan x dx = \int_{-\pi/2}^{\pi/2} \frac{\sin x}{\cos x} dx = \int_{\cos(-\pi/2)}^{\cos(\pi/2)} \frac{\sin x}{u} \left(-\frac{1}{\sin x} \right) du = -\int_1^{-1} \frac{du}{u} = \int_1^{-1} \frac{du}{u}$

$\int f(g(x)) \cdot g'(x) dx = \int f(u) du, u = g(x)$
 $u = \cos x \quad du = -\sin x dx = -\frac{1}{\sin x} du$

$= (\ln |x|) \Big|_0^1 = \ln 1 = 0$

$$\textcircled{1} \int_0^{\frac{\pi}{2}} e^x \cos x dx = \begin{cases} u = e^x & du = e^x \\ dv = \cos x & v = \sin x \end{cases}$$

$$= (e^x \cdot \sin x) \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x \sin x dx = \begin{cases} u = e^x & du = e^x \\ dv = \sin x & v = -\cos x \end{cases}$$

$$= (e^x \cdot \sin x) \Big|_0^{\frac{\pi}{2}} - (e^x \cdot (-\cos x)) \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x - (-\cos x) dx$$

$$= (e^x \cdot \sin x) \Big|_0^{\frac{\pi}{2}} + (e^x \cdot \cos x) \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x \cdot \cos x dx \rightarrow \text{ISTI KAO NA POČETKU}$$

$$= \frac{e^x (\sin x + \cos x)}{2} \Big|_0^{\frac{\pi}{2}}$$

$$= \frac{e^{\frac{\pi}{2}} (\sin \frac{\pi}{2} + \cos \frac{\pi}{2})}{2} - \frac{e^0 (\sin 0 + \cos 0)}{2}$$

$$= \frac{e^{\frac{\pi}{2}} (1+0)}{2} - \frac{1(0+1)}{2} = \frac{e^{\frac{\pi}{2}}}{2} - \frac{1}{2} //$$

20 ✓ $\frac{e^{\frac{\pi}{2}}}{2} - \frac{1}{2} \approx 1.9$

② Koliko iznosi $f(2,5)$ ako f zadovoljava: $dy = (1+x + \frac{y}{1-x^2}) dx$ i

$y(0) = 0$

$dy = (1+x + \frac{y}{1-x^2}) dx, y(0) = 0$

$f(2,5) = f(x) = \frac{y'(1-2,5^2) + 2 \cdot 2,5 y}{(2,5)^4 - 2 \cdot (2,5)^2 + 1}$

$\frac{dy}{dx} = (1+x + \frac{y}{1-x^2})$ ✗

$y'' = 0 + 1 + (\frac{y}{1-x^2})'$ ✗

$y'' = 1 + \frac{y' \cdot (1-x^2) - y \cdot (1-x^2)'}{(1-x^2)^2}$

$y'' = 1 + \frac{y'(1-x^2) + y \cdot 2x}{1-2x^2 + x^4}$

$y'' = 1 + \frac{y'(1-x^2) + 2xy}{x^4 - 2x^2 + 1}$

~~0~~

③ Za funkciju $f(x,y) = \frac{x}{y}$ odrediti domenu, kodomenu, razine i krivulje i limes u ishodištu (ako postoji).

$$f(x,y) = \frac{x}{y}$$

$$y \neq 0 \rightarrow D(f) = \mathbb{R}^2 \setminus \{(x,0)\} \rightarrow \text{domena}$$

$$\text{ili } D(f) = \{(x,y) \in \mathbb{R}^2 : y \neq 0\} \quad \checkmark$$

$$(x,y) \in D \xrightarrow{f} u = f(x,y) \in \mathbb{R} \rightarrow \text{domena} \quad \underline{3}$$

$$\underline{f(O)} = \{u \mid u = f(x,y), (x,y) \in D\} = \mathbb{R} \rightarrow \text{kodomenu} \quad \checkmark \underline{2}$$

$$\lim_{(x,y) \rightarrow (0,0)} f(x,y) = \lim_{(x,y) \rightarrow (0,0)} \frac{x}{y} = \lim_{\substack{x,y \rightarrow 0 \\ y=kx}} \frac{x}{kx} = \frac{1}{k} \quad \rightarrow \text{ne postoji jer za}$$

različite k dobijemo različite vrijednosti \checkmark

\hookrightarrow ukoliko $(x,y) \rightarrow (0,0)$ ide po putovima koje određuju pravci $y=kx$ limesa $\underline{7}$

$$\forall z_0 \in f(O) \quad f(x,y) = z_0$$

$$x=0 \rightarrow z=0 \Rightarrow \text{braka} \quad \underline{1}$$

$$z=1 \rightarrow \frac{x}{y} = 1 \Rightarrow \text{razlinska krivulja}$$

} **MSLI STE**
SAMO DUJE
RAZINSKE KRIVULJE

MENE ZAMIMI SKICA RAZINSKIH KRIVULJA,

ODNOSNO VIZUALIZACIJA FUNKCIJE.

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⑥ Integriranjem izrač. površinu trokuta zadanog točkama
 $A(0,0)$, $B(-2,2)$, $C(-1,-1)$.

$$P = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$P = \frac{1}{2} |0 \cdot (2 + 1) + (-2)(-1 - 0) + (-1)(0 - 2)|$$

$$P = \frac{1}{2} |0 + 2 + 2| = \frac{1}{2} |4| = 2, \quad \text{~~0~~}$$

POVRŠINU JE TREBALO ODREDITI INTEGRIRANJE.

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: *Tibor Rak*

BROJ INDEKSA:

xxi

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

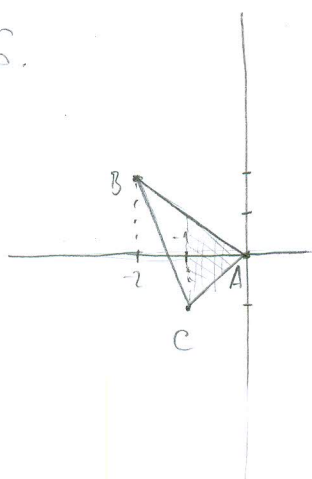
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6.



$A(0,0)$
 $B(-2,2)$
 $C(-1,-1)$

PRAVAC AB

$$x - x_1 = \frac{y_2 - y_1}{x_2 - x_1} = y - y_1$$

$$x - 0 = \frac{2 - 0}{-2 - 0} y + 2$$



$$4. \int_0^{\frac{\pi}{2}} e^x \cos x dx = \left(e^x (-\sin x) \right) \Big|_0^{\frac{\pi}{2}} = \left(e^{\frac{\pi}{2}} \left(-\sin \frac{\pi}{2} \right) \right) - \left(e^0 (-\sin 0) \right)$$

$$= e^{\frac{\pi}{2}} - \sin \frac{\pi}{2} - 1 - 0 = e^{\frac{\pi}{2}} - \sin \frac{\pi}{2} - 1$$

IME I PREZIME: *TONI PAŠTUDIĆ*

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③ $f(x, y) = \frac{x}{y} \quad \frac{x}{y} = C$

Df $y \in \mathbb{R} \setminus \{0\}$ ~~\emptyset~~

① $y'' + 3y' = 0 \quad y(0) = 0$
 $y(3) = 0$

② $dy = \left(1 + x + \frac{y}{1-x^2}\right) dx \quad y(0) = 0$

$\frac{dy}{y} = \left(1 + x + \frac{1}{1-x^2}\right) dx$ ~~$\frac{1}{1-x^2}$~~

$\ln |y| = x + \frac{x^2}{2} + \ln |1-x^2| + \ln |C|$

$y = x + \frac{x^2}{2} + (1-x^2) \cdot C \Rightarrow 0 = 0 + \frac{0^2}{2} + (1-0) \cdot C = 0$

$$y = x + \frac{x^2}{2} + (1 - x^2 \cdot c)$$

$$y = 2.5 + \frac{2.5^2}{2} + 1 = 6.625$$

$$f(2.5) = 6.625$$

$$(4) \int_0^{\frac{\pi}{2}} e^x \cos x \, dx = \left. \begin{array}{l} u = \cos x \\ du = -\sin x \, dx \\ v = e^x \\ dv = e^x \, dx \end{array} \right\} \begin{array}{l} = \cos x \cdot e^x - \int e^x \cdot \sin x \, dx \\ = \cos x \cdot e^x - e^x \end{array}$$

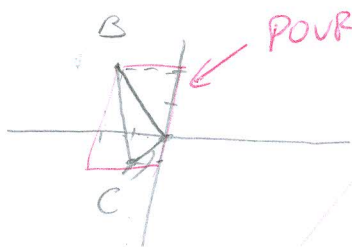
$u \cdot v - \int v \cdot du$

$$(5) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan x \, dx = -\ln|\cos x| + c \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \lim_{\epsilon \rightarrow 0} -\ln|\cos \epsilon| - \lim_{\delta \rightarrow 0} \ln|\cos \delta|$$

OVO JE NEPRAVI INTEGRAL KOJI DIVERGIRA

= $\ln|0| - \ln|0|$
= nema realne površine koju računamo tamo gdje su granice

- (6) A(0,0)
B(-2,2)
C(-1,1)



POVRŠINA PRAKUTNIKA JE P=6

AB $\Rightarrow y = -x$ $f(x) = -x$

BC $\Rightarrow f(x) = -4x - 6$ $y = -x$

CA $\Rightarrow y = x$ $f(x) = x$ $y = x$

$$P = \int (BA - AC) - \int (BC - AC)$$

$$P_1 = \int BA - AC$$

$$P_2 = \int BC - AC$$

BC CB

$$(y - y_1) \cdot (x_2 - x_1) = (x - x_1) \cdot (y_2 - y_1)$$

$$y = -4x - 6$$

TROKUT JE MANJI!!!

$$P = 12.576$$

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xxi

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME:

ANKITE STANISIĆ

BROJ INDEKSA:

17-1-0066-2011

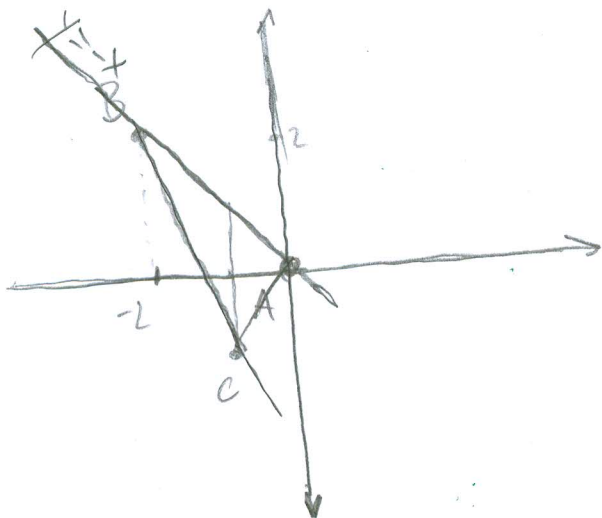
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$A(0,0)$ $B(-2,2)$ $C(-1,-1)$



$B(-2,2)$ $C(-1,-1)$
 $x_1 \ y_1 \quad x_2 \ y_2$

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 2 = \frac{-1 - 2}{-1 - (-2)} (x + 2)$$

$$y - 2 = -3x - 6$$

$$y = -3x + 2 - 6$$

$$\boxed{y = -3x - 4} \quad BC$$

$A(0,0)$ $C(-1,-1)$
 $x_1 \ y_1 \quad x_2 \ y_2$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 0 = \frac{-1 - 0}{-1 - 0} (x - 0)$$

$$\boxed{y = 1x} \quad AC$$

~~AB~~ $B(-2,2)$ $A(0,0)$
 $x_1 \ y_1 \quad x_2 \ y_2$

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 2 = \frac{0 - 2}{0 - (-2)} (x + 2)$$

~~$$y = 2 + (-1)(x) - 2$$~~

$$\boxed{y = -1x + 2} \quad BA$$

$$P_1 = \int_{-2}^{-1} ((-x) - (-3x - 4)) dx = \int_{-2}^{-1} (-x + 3x + 4) dx = \int_{-2}^{-1} (2x + 4) dx$$

$$P_1 = 2 \cdot \frac{x^2}{2} + 4x \Big|_{-2}^{-1} = (2 \cdot \frac{(-1)^2}{2} + 4 \cdot (-1)) - (2 \cdot \frac{(-2)^2}{2} + 4 \cdot (-2)) = -3 + 4 = 1$$

$$P_2 = \int_{-1}^0 ((-x) - (1x)) dx = \int_{-1}^0 (-x - x) dx = P_2 = 0$$

KAKO TA
 POKRŠINA MOŽE
 BITI NULA?

$$\boxed{P = P_1 + P_2 = 1}$$

$P = 1$



1) D: R X

5) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan x dx = 0$

| | | | |
|----------------|------------------|---|-----------------|
| K | 0 | 1 | 2 |
| x _K | $-\frac{\pi}{2}$ | 0 | $\frac{\pi}{2}$ |
| f _K | -0,0279 | 0 | 0,0279 |

$\frac{\pi}{2} \frac{0}{1} (-0,0279 + 0,279) = 0$ X

9)

$$\int_0^{\frac{\pi}{2}} e^x \cos x dx$$

$$u = e^x$$
$$du = e^x dx$$

$$dv = \cos x dx$$

$$v = \int \cos x dx = \sin x$$

$$I = \sin x e^x - \int \sin x e^x dx$$

$$= \sin x e^x + \cos x - e^x$$

$$= \sin \frac{\pi}{2} e^{\frac{\pi}{2}} + \cos \frac{\pi}{2} - e^{\frac{\pi}{2}}$$

$$= -3,67888$$

IME I PREZIME: *MARIE ČOSIĆ*

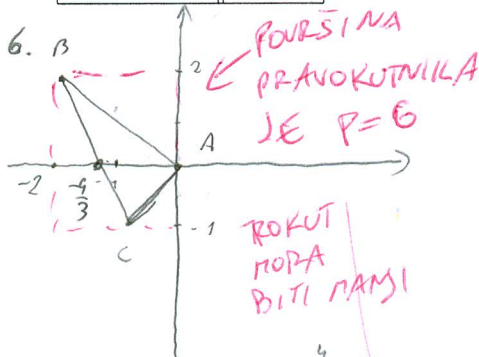
BROJ INDEKSA: *55924*

POPUNJAVA
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$$AB: y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$BC: y - 2 = \frac{-1 - 2}{-1 + 2} (x + 2)$$

$$y - 0 = \frac{2 - 0}{-2 - 0} (x - 0)$$

$$y - 2 = -3x - 6$$

$$y = -3x - 4$$

$$y = -x$$

$$AC: y - 0 = \frac{-1 - 0}{-1 - 0} (x - 0)$$

$$y = x$$

$$-3x - 4 = 0$$

$$-3x = 4$$

$$x = -\frac{4}{3}$$

$$P = \int_{-2}^0 -x dx - \int_{-2}^{-4/3} (-3x - 4) dx - \int_{-4/3}^{-1} x dx - \int_{-4/3}^{-1} (-3x - 4) dx = \left[-\frac{x^2}{2} \right]_{-2}^0 - \left[\frac{3x^2}{2} + 4x \right]_{-2}^{-4/3} - \left[\frac{x^2}{2} \right]_{-4/3}^{-1} - \left[-\frac{3x^2}{2} - 4x \right]_{-4/3}^{-1}$$

$$= -\left(-\frac{2^2}{2}\right) - \left(\frac{3 \cdot (-4/3)^2}{2} + 4 \cdot (-4/3)\right) - \left(-\left(\frac{3 \cdot (-2)^2}{2} - 4(-2)\right)\right) - \left(\frac{-1^2}{2} - \left(-\frac{4^2}{2}\right) - \left(-\frac{3(-1)^2}{2} - 4(-1)\right)\right) - \left(-\left(-\left(\frac{3 \cdot (-4/3)^2}{2} + 4 \cdot (-4/3)\right)\right)\right)$$

$$= 2 + 6 - \frac{1}{2} + \frac{11}{2} = 13$$

$$5) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan x \, dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin x}{\cos x} \, dx = \left. \begin{array}{l} \cos x = t \\ -\sin x \, dx = dt/(-1) \\ \sin x \, dx = -dt \end{array} \right| = - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{dt}{t}$$

$$= - \ln|t| \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = - \ln(\cos x) \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= - \left(\ln(\cos \frac{\pi}{2}) - (-\ln(\cos \frac{\pi}{2})) \right) =$$

$$= - \left(\ln(1) - (-\ln(1)) \right) = \underline{\underline{0}} \quad \times \quad \circ$$

$$4) \int_0^{\frac{\pi}{2}} e^x \cos x \, dx = \left. \begin{array}{l} e^x = u' \quad \cos x \, dx = dv \\ e^x = du \quad \sin x = v \end{array} \right|$$

$$e^x \sin x - \int \sin x e^x =$$

~~\times~~ ~~\circ~~

$$3) f(x, y) = \frac{x}{y}$$

Domain

$$xy \neq 0$$

$$D = \mathbb{R} \setminus \{0\} \quad \times$$

$$D = \mathbb{R} \times (\mathbb{R} \setminus \{0\})$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

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IME I PREZIME:

BROJ INDEKSA:

RINO KURTIN

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4. $\int_0^{\pi/2} e^x \cos x dx =$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

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IME I PREZIME: Ivan Klanac

BROJ INDEKSA: 0098-2011

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② $f(2.5)$

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IME I PREZIME: **TONI UGLEŠIĆ**

BROJ INDEKSA: **17-1-0065-2011**

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