

**MATEMATIKA 2:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

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POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

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1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete:  $y'' + 5y' = 0$ , uz  $y(0) = 1$  i  $y'(0) = 0$ .  
Na kraju provjeri rješenje.

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2. Pronađi funkciju koja zadovoljava  $ydx + (2\sqrt{xy} - x) dy = 0$  i  $y(1) = 1$ .

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3. Zadana je funkcija za funkcija  $f(x, y) = \sqrt{(x+2)^2 + (y-1)^2} - 3$ . Ispitati domenu, kodomenu i razinske krivulje.

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4.  $\int_0^{\pi} \sin(x) e^x dx = ?$

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5.  $\int_0^2 \frac{2x^2 + x + 2}{x^2 - 1} dx = ?$

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6. Izračunati površinu lika omeđenog pravcem  $y = x + 1$  i parabolom  $y = x^2 - x - 2$ .

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Ukupno:

$f$	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
$e^x$	$e^x$
$\alpha^x (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln  \cos x  + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int \frac{dx}{x} = \ln  x  + C$	$\int \cot x dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln  x + \sqrt{x^2 \pm a^2}  + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right)] + C$	

1)  $y'' + 5y' = 0$

$y + 5y = 0$

$k^2 + 5k = 0$

$k | k + 5 | = 0$

$k = 0 \quad k = -5$

$y = 1 \quad y'(0) = 0$

$y = C_1 \cdot e^{-x} + C_2 \cdot e^{-5x}$

$y = C_1 + C_2 e^{-5x}$

$1 = C_1 + C_2$

$y' = -5C_2 \cdot e^{-5x}$

$0 = -5C_2$

$-\frac{1}{5}C_2$

$C_1 = 1$

$y = 1 - \frac{1}{5} e^{-5x}$

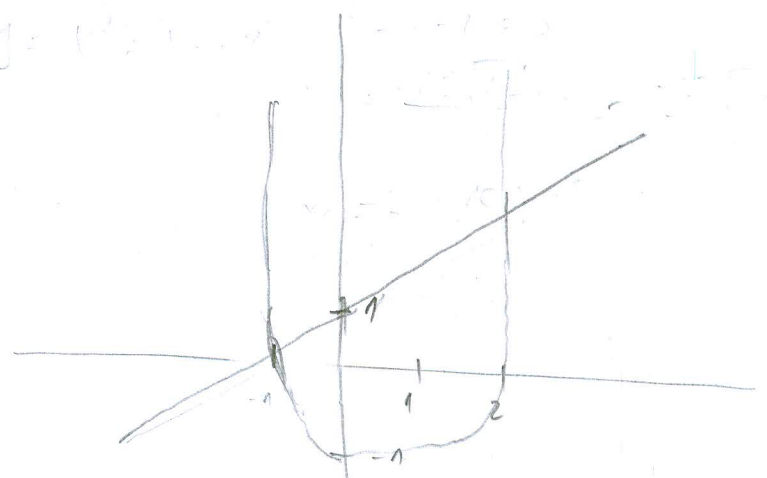
PROVERA?

6)  $y = x + 1$       $x =$

$y = x^2 - x - 2$

$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2}$

$x_{1,2} = \frac{-1 \pm 3}{2}$       $2 \times$



$P = \int_{-1}^2 [(x+1) - (x^2 - x - 2)] dx = ?$

4)  $\int_0^{\pi} \sin(x) \cdot e^x dx = \int e^x - u$

$\sin x = u$   
 $-\cos x = u'$

$= -e^x \cos x + \int \cos x \cdot e^x dx = \int e^x - u$       $\cos x = u$   
 $e^x dx = u'$       $\sin = u'$

$= -e^x \cos x + e^x \sin x - \int e^x \sin x dx$

$2I = -e^x \cos x + e^x \sin x \rightarrow ?$

$I = \frac{1}{2} (-e^x \cos x + e^x \sin x) \Big|_0^{\pi}$

$= \frac{1}{2} (e^{\pi} \sin \pi - e^{\pi} (-1) - (0 - e^0 \cdot 1))$

$= \frac{1}{2} (1 + e^{\pi})$

ŠTO JE OVO  
KOLEGA?  
JESTE LI VI  
PREPISIVALI  
S NEKOG IZVORA?

5)  $\int_0^2 \frac{2x^2 + x + 2}{(x-1)(x+1)} dx$

$\frac{2x^2 + x + 2}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$

$2x^2 + x + 2 = A(x+1) + B(x-1)$

VIDI RADOVIĆ

$5 = 2A \Rightarrow A = \frac{5}{2}$

$3 = B(-2) \Rightarrow B = -\frac{3}{2}$

$\frac{5}{2} \int_0^2 \frac{2x}{x-1} - \frac{3}{2} \int_0^2 \frac{dx}{x+1} = \frac{5}{2} \ln|x-1| \Big|_0^2 - \frac{3}{2} \ln|x+1| \Big|_0^2$

